Seeing the edge of the universe

Subir Sarkar

Folkeuniversiteit I København, 5 & 7 Dec 2017
Is there an edge to the universe …
What happens when a spear is thrown across it?

Archytas of Tarentum (5th Century BC)
Medieval cosmology

“The Divine Comedy”, Dante Aligheri (1321)
The heliocentric universe

in ‘The Revolutions of the Celestial Spheres’, Nicholas Copernicus (1543)

... which had in fact been anticipated somewhat earlier!

“But Aristarchus of Samos brought out a book consisting of certain hypotheses in which the premises lead to the conclusion that the universe is many times greater than that now so called. His hypotheses are that the stars and the Sun remain motionless, that the Earth revolves about the Sun in the circumference of a circle, the Sun lying in the middle of the orbit ...”

‘The Sand Reckoner’, Archimedes of Syracuse (287-212 BC)
But it was still believed that space is finite with an edge

Immanuel Kant’s ‘Antimony of space’ (1783): Space has to be finite in extent and homogeneous in composition, and to obey the laws of Euclidean geometry... but all three assumptions cannot be true at once!
The infinite universe

Thomas Digges (1576)

“I could be bounded in a nutshell and count myself a king of infinite space”

“Hamlet”, William Shakespeare (1601)
“If this is true and if they are suns having the same nature as our Sun, why do these suns not collectively outshine our Sun in brilliance?

... you do not hesitate to declare that there are over 10,000 stars. The more there are and the more crowded they are, the stronger becomes my argument against the infinity of the universe.

This world of ours does not belong to an undifferentiated swarm of countless others ... Otherwise the whole celestial vault would be as luminous as the Sun!”

“Conversations with the Starry Messenger”

Johannes Kepler (1601)
... and the first (wrong!) solution

“... the more remote stars and those far short of the remotest vanish even in the nicest telescopes, by reason of their extreme minuteness; so that tho’ it were true, that some such stars are in such a place, yet their Beams, aided by any help yet known, are not sufficient to move our sense; after the same manner as a small Telescopical fixt star is by no means perceivable to the naked eye”

Edmund Halley (1720)

Clearly he had not appreciated his friend Isaac Newton’s newly invented calculus - distant stars do shrink in size ($\propto 1/r^2$) but their number grows ($\propto r^2$) to compensate!
However the distant stars are obscured by foreground ones, so the expectation would naturally be a sky totally covered by stars

i.e. $10^{14}$ times brighter than reality!

Why is the sky dark at night?
(later called “Olbers’ Paradox”)
“The enormous difference which we find between this conclusion and actual experience shows either that the sphere of the fixed stars is not infinite but that it is actually much smaller than the finite extent I have supposed for it, or that the power of light diminishes in greater proportion than the inverse square of the distances ... This latter supposition is plausible enough, it requires only that the heavens are filled with some fluid capable of intercepting light, however slightly ...”

Jean-Phillipe Loys de Cheseax (1744)

But **Lord Kelvin** showed later that this would not solve the problem ... the fluid would ultimately heat up and reradiate the light it absorbed
We cannot see through an infinite forest of trees

Otto von Guericke (1672)
... so how do we see through an infinite universe of stars?!

‘Why is the sky dark at night?’

Heinrich Wilhelm Mathäus Olbers (1826)
How far can we look in a forest before we see a ‘wall of trees’?

Lookout limit = $\frac{1}{2rn}$

$r$ : radius of tree trunk
$n$ : # trees/unit area

e.g. if trees are about 1 m in diameter and spaced on average 50 m apart, then we would be able to see for about 1 km …

How far can we look into the universe before we see a ‘wall of stars’?

Lookout limit = $\frac{1}{\pi R^2 N}$

$R$ : radius of star
$N$ : # stars/unit volume

… to calculate this we need to know how big stars are and how far apart they are on average
Measuring the universe: **Step 1 - the size of the Earth**

**Eratosthenes** (235 BC)

At noon on mid-summers day (22 June), the Sun is vertically overhead at Aswan ... but at Alexandria (~800 km due North) it casts a shadow, being 7.5° to the vertical

→ if 7.5° corresponds to 800 km (5000 “stadia”), then 360° corresponds to ~40000 km

**i.e. Earth’s radius is ~6000 km** (knowing the value of π ≈ 3.1 from Archimedes 250 BC)
Measuring the universe: **Step 2 - the distance to the Moon**

*Aristarchus (230 BC)*

The Moon subtends ~0.5° on the sky so its distance must be ~110 times its diameter (hold a coin at arm’s length to cover the Moon – measure its distance & diameter)

The absolute value can be obtained by careful observation of lunar eclipses ...

By triangulation: Earth’s diameter – Moon’s diameter = 2.7 x Moon’s diameter

⇒ Moon’s distance = 110 x Moon’s diameter = 30 x Earth’s diameter ≈ 400,000 km
When the Moon is exactly half-full, light from the Sun must be falling on it exactly at right angles ... so have to just measure the angle S-E-M

Aristarchus’ guess was $87^0$ so he deduced that the Sun is 20 times further than the Moon ... in fact this angle is $89^050’$ so the Sun is actually 400 times further than the Moon

Clever logical arguments are not sufficient – science needs accurate measurements too!
The advent of precision astronomy ("big science")

Tycho Brahe (1582) and his great quadrant at Uraniborg... could measure angles as small as 30"
The distance to the Sun was first measured by Jeremiah Horrocks during the “Transit of Venus” 24th Nov 1639

The Sun's disc is ~33 times bigger than Venus but Venus is ~3.4 times closer than the Sun* hence the Sun is 112 times bigger than Venus

Assuming Venus is the same size as the Earth (fortuitously true!) the Sun’s distance \( \approx 110 \times 112 \times \text{Earth’s diameter} \sim 150 \text{ million km} \)

*The maximum angle between Venus and the Sun is 45° so the distance ratio \((\text{Sun-Venus})/(\text{Sun-Earth}) \approx 1/\sqrt{2}\) assuming the Earth and Venus move on circular orbits
The Sun also subtends $\sim 0.5^0$ on the sky, so its diameter is $1/110$ times the Sun-Earth distance, i.e. $\sim 1.5$ million km.

It is convenient to measure such huge scales in terms of the light travel time, e.g. the Sun is $\sim 5$ light-seconds across and $\sim 8.5$ light-minutes away.
Measuring the universe: **Step 4 - the distance to the stars**

Diameter of the Earth’s orbit is ~1000 light-seconds. So if the ‘parallax’ of a star is 1″, then its distance is 1 parsec ⇒ ~3.3 light-years

First measured in 1838 by **Wilhelm Bessel** for 61 Cygni (0.3″)

The *Hipparcus* satellite measured angles down to ~0.001″... *Gaia* (2013-) is now measuring down to ~0.00002″

To measure longer distances requires developing **The Cosmic Distance Ladder**
To do this we need “standard candles” – astronomical sources whose absolute luminosity is known from its correlation with some other property ... e.g. pulsation period in the case of Cepheid variable stars
Edwin Hubble (1923) used the 100” Mt. Wilson telescope to determine the distance to the Andromeda Nebula.

He was searching for "novae" (stars which suddenly increase in brightness) … instead he found a Cepheid variable, which had been shown by Henrietta Leavitt (1912) to be a star which can be used as a distance indicator.

Hubble discovered that Andromeda is not just a cloud of stars and gas in the Milky Way, but a large galaxy similar to our own at a very substantial distance … the known universe suddenly got a whole lot bigger!
In fact Andromeda (M31) is 2.2 million light-years away from us
The Hubble Space Telescope (1990-)
... could resolve Cepheids in galaxies much further away

Cepheid Variable in M100
HST • WFPC2
M100 is one of the galaxies in the Virgo cluster ... which is 54 million light-years away
Cepheids can be used to ‘calibrate’ other sources such as supernovae – exploding stars which are bright enough to be seen even further away.

... using supernovae we can now measure distances of billions of light-years.
So how far must we look to see the ‘wall of stars’?

\[
\text{Lookout limit} = \frac{1}{\pi R^2 n}
\]

\[R : \text{radius of star}\]
\[n : \# \text{ stars per unit volume}\]

→ for stars of typical size \(~5\) light-seconds, separated on average by \(~1000\) light-years,

we will have to look out for \(~10^{24}\) light-years

i.e. to get to us the light would have had to set out

1,000,000,000,000,000,000,000,000 years ago!

But have the stars been around for that long?

Earth is only \(4.6 \times 10^9\) yr old (from radioactive dating of uranium)

... can similarly date stars from uranium lines in their spectra
The synthetic spectrum was computed for the adopted abundances of the stable elements and for 4 different values of the abundance of uranium in the atmosphere of the star. The best fit is the middle (red) line, representing an uranium abundance of approximately 6% of the Solar value ...

→ this implies an age of “only” $12.5 \times 10^9$ years
there are not enough stars in the universe we can see to cover the sky
... this is why the sky is dark at night

“We're the succession of stars endless, then the background of the sky would present us an uniform luminosity, like that displayed by the Galaxy — since there would be absolutely no point in all that background, at which there would not exist a star. The only mode therefore in which under such a state of affairs we could comprehend the vistas which our telescopes find in innumerable directions, would be by supposing the distance of the invisible background to be so immense that no ray from it has yet been able to reach us at all”

‘Eureka’, Edgar Allan Poe (1848)
INTERMISSION
There are several thousand galaxies in this patch of sky ⇒ $\sim 10^{11}$ galaxies over the sky

We are seeing out of the ‘forest’ of galaxies – what lies beyond?
Looking far away is the same as looking back into our past …

We see the Sun as it was 8 minutes ago.

We see the nearest star Proxima Centauri, as it was 4 years ago.

We see the Galactic centre as it was 30,000 years ago.

We see our nearest galaxy Andromeda as it was 2 million years ago.

We see the Virgo cluster as it was 50 million years ago.

We see galaxies in the Hubble Ultra Deep Field as they were up to 12 billion years ago.
We are looking right back to the time when the first galaxies were forming...
But there is something odd about the spectra of distant galaxies... they are all shifted towards the red end of the spectrum as if they are all moving away from us.

\[
\text{Red Shift: } z = \frac{\lambda_{\text{emitted}} - \lambda_{\text{observed}}}{\lambda_{\text{emitted}}} \approx \frac{v}{c}, \text{ for } z \ll 1
\]
Hubble discovered that the **further** a galaxy is, the **faster** it seems to be moving away from us.

“*Every time I see Edwin Hubble, he’s moving rapidly away from me!*”
The ‘expansion of the universe’

Hubble’s data (1929)

Modern data using supernovae (2002)
Galaxies *equidistant* from us, all moving away at the *same speed*

Galaxies *twice as far*, are moving away *twice as fast*
So going back in time, *all* galaxies will come together at the *same* instant at $\sim 1/H_0 \approx 14 \times 10^9$ yr (given the present expansion rate: $H_0 \approx 70 \text{ km/s/Mpc}$)

i.e. the entire universe originated in a ‘Big Bang’ about 14 billion years ago

... but this was the birth of space-time, *not an explosion* in space!
The redshift of distant galaxies should not be interpreted as a Doppler effect ... because it is not a concept appropriate to curved space-time.

"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect."
The redshift occurs because the wavelength of light is increased by the stretching of space-time.

\[
\frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} \equiv 1 + z = \frac{a_{\text{observed}}}{a_{\text{emitted}}}
\]

This analogy also illustrates that the expansion has \textit{no} ‘centre’
The Big Bang is the antipodal point of an expanding hypersphere.

... going back in time we are all subsumed in the initial singularity wherein all of space-time and matter seem to have been created.
Matter curves space-time (Einstein’s ‘General Theory of Relativity’) so when we look out at the universe we see …

‘Circle Limit III’, M.C. Escher (1959)
When the universe was younger, it was smaller therefore hotter ...

So if we can look back far enough in time, we should see a hot, dense ‘fireball’ covering the sky
This is the ‘bright sky’ we have been looking for ... but this light from the hot plasma of the early universe (~400,000 years old) has been redshifted to microwave frequencies.
... measured the spectrum of this relic radiation to be as perfect a blackbody as the calibrating source!

(John Mather, Nobel Prize 2007)

This requires the radiation to have originated from a hot and dense state ... and rules out the steady-state theory (wherein it is created by the thermalisation of starlight by dust)
But on closer inspection, the radiation is not quite uniform ...

Dipole anisotropy with $\delta T/T \sim 10^{-3}$

... due to our motion at $\sim 370$ km/s wrt the CMB rest frame
(there is however nothing special about this frame)
COBE also detected the expected tiny temperature fluctuations due to primordial density inhomogeneities (which have grown under gravity to form structure in the universe)

(George Smoot, Nobel prize 2007)

\[ \frac{\delta T}{T} \sim 2 \times 10^{-5} \]
But the COBE picture was rather fuzzy - like this one of the Earth.
By improving the resolution we start seeing detailed features …
these patches are believed to be due to **quantum fluctuations** generated during **inflation**, when the **entire universe was smaller than a nucleus**

... these excited sound waves in the plasma filling the early universe and provided the ‘seeds’ for the formation of galaxies
Planck (2009-13) … has measured these in exquisite quantitative detail and helped to establish the ‘standard model’ of cosmology

(The Planck reflectors were developed by a scientific Consortium (DK-Planck) led and funded by Denmark)
So this is the edge of the visible universe ...

“Our entire observable universe is inside this sphere of radius 13.3 billion light-years, with us at the center. Space continues outside the sphere, but this opaque glowing wall of hydrogen plasma hides it from our view. This censorship is frustrating, since if we could see merely 380000 light-years beyond it, we would behold the beginning of the universe”  Max Tegmark (2003)
One day we may indeed look back to the very beginning... perhaps with the Laser Interferometer Space Array (LISA) which aims to detect gravitational waves from the Big Bang itself.
"I'll tell you what's beyond the observable universe -- lots and lots of unobservable universe."
But this is a story we have to leave for another time …

Meanwhile, if cosmology interests you, then start with these books (which I have drawn on for this talk):

Edward Harrison, *Darkness at Night: A Riddle of the Universe* (1987)


... but to really understand curved space-time you will have to learn tensor calculus!
The End