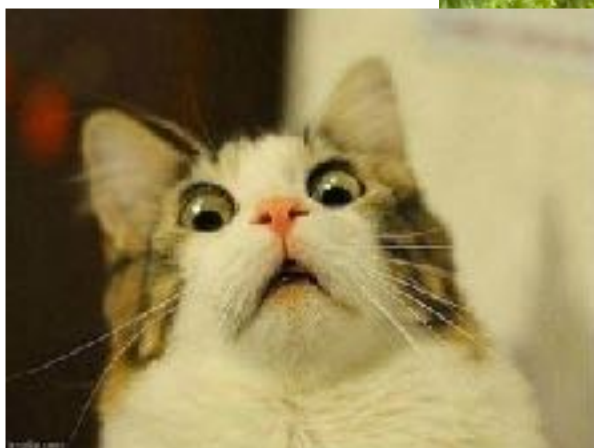
NBIA Gravity Seminar, May 2023

Classical Gravity from Quantum Amplitudes

Donal O'Connell
Edinburgh

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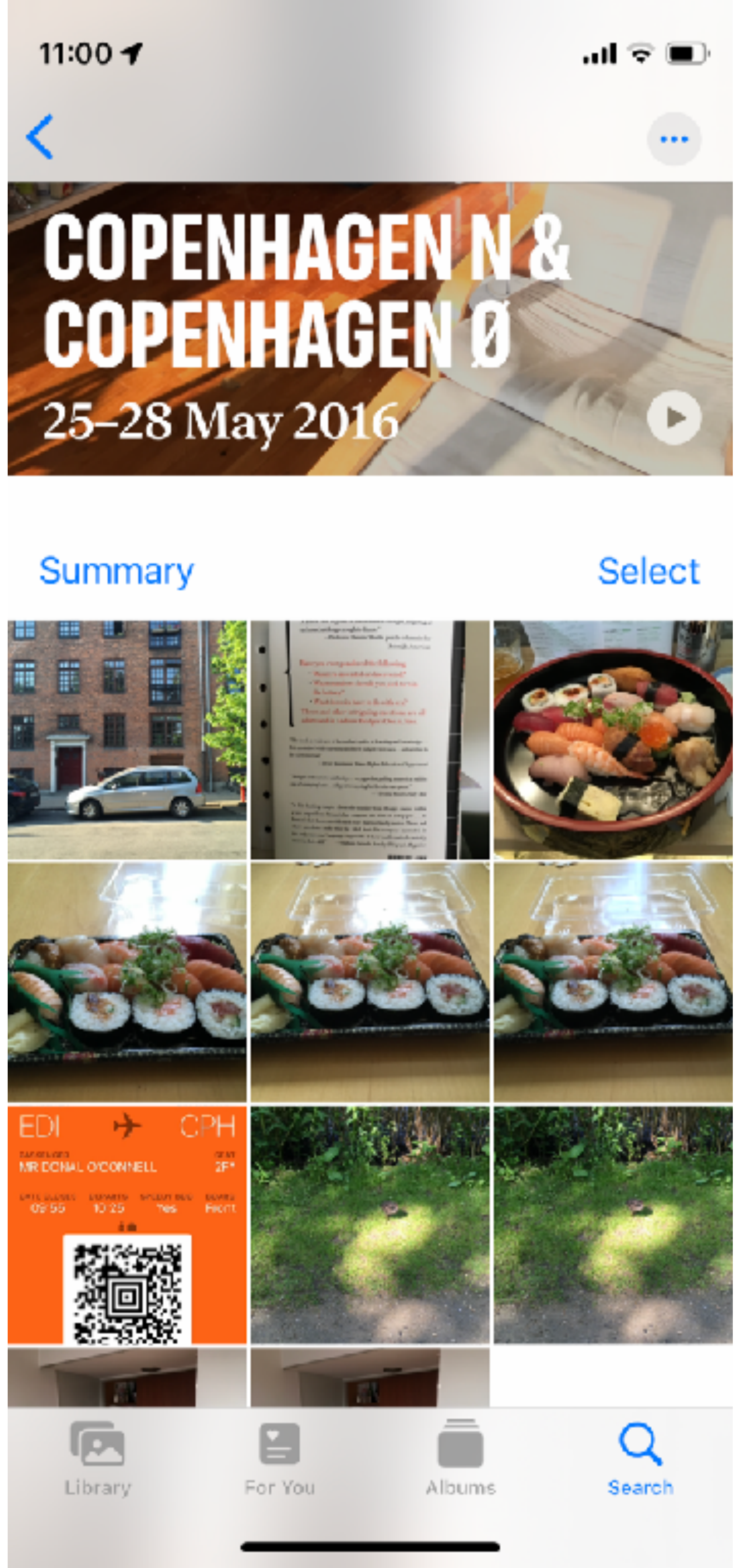
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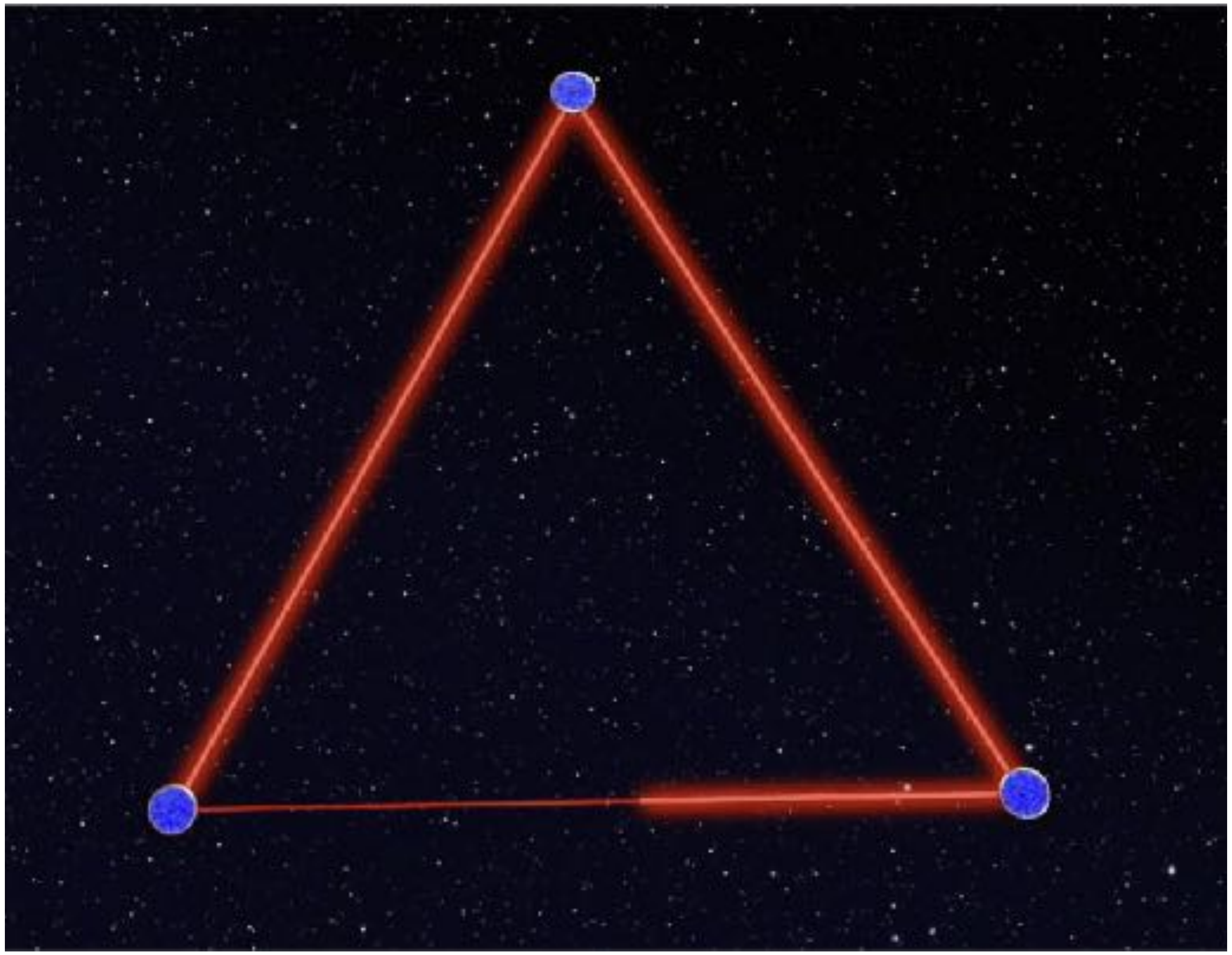
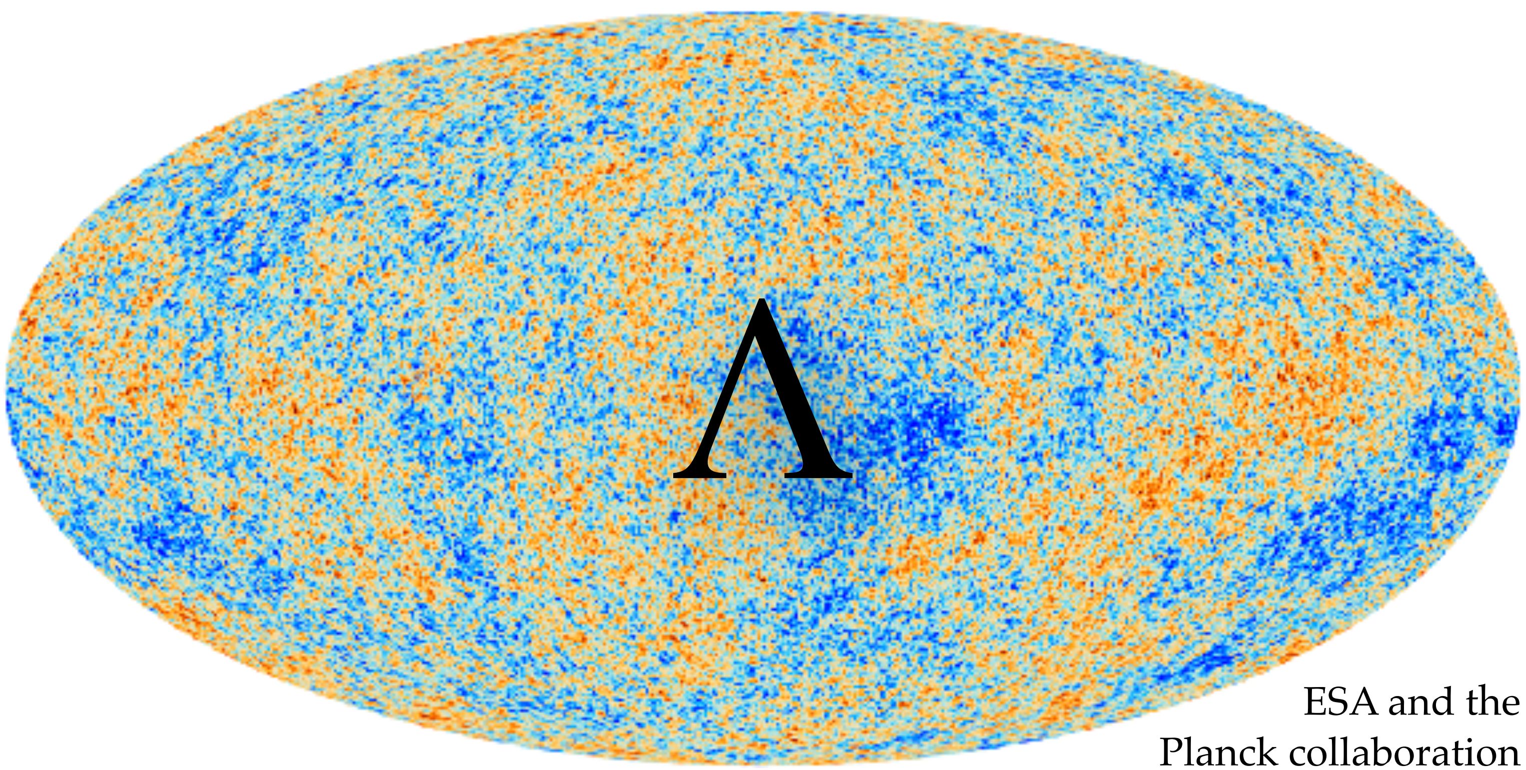


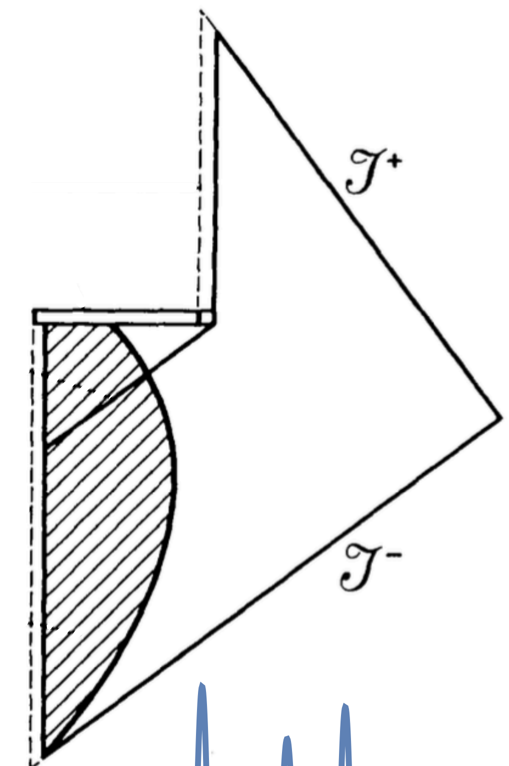
Data rich future for gravitational waves!

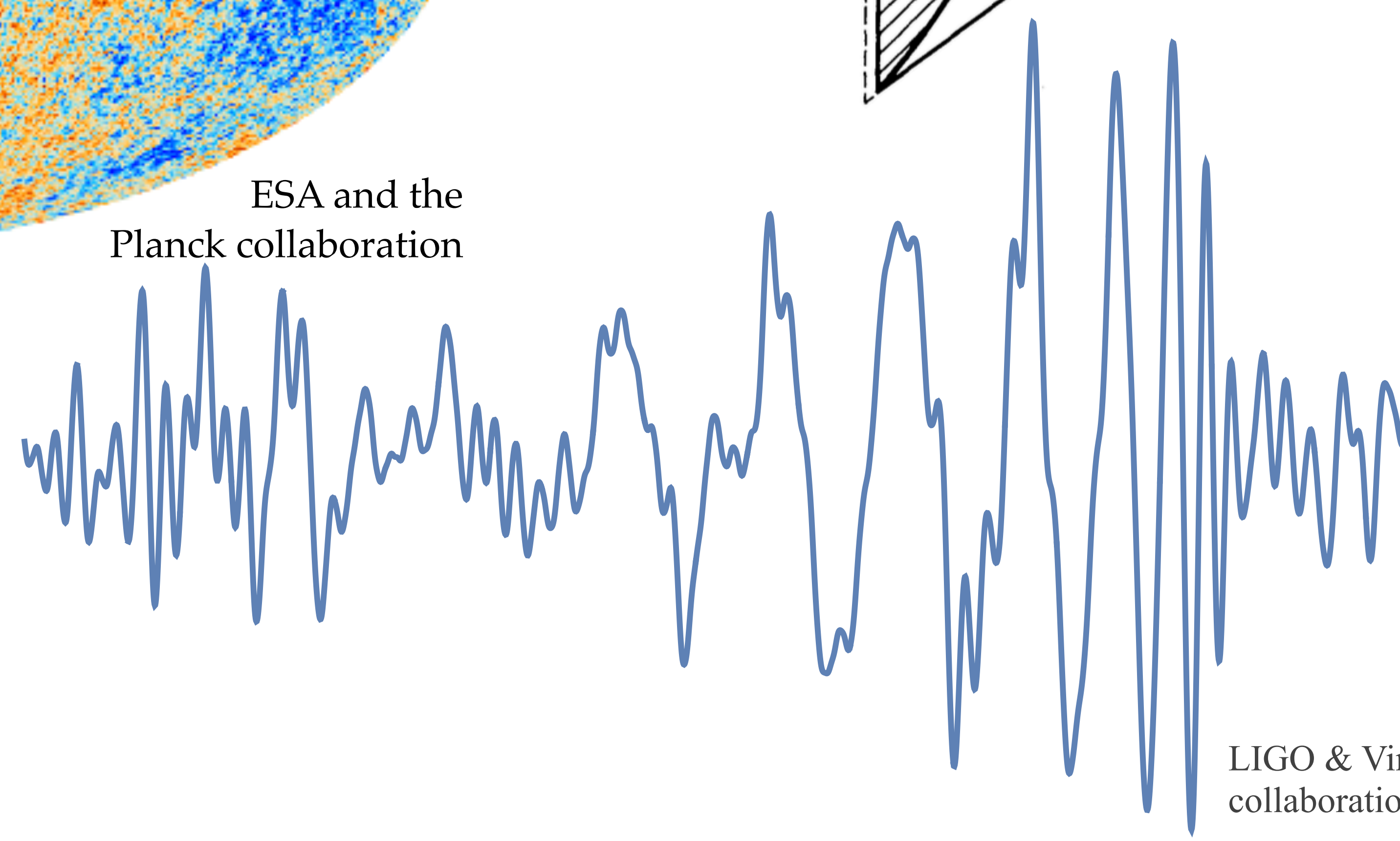
Motivation

Gravity: key challenge



ESA and the
Planck collaboration

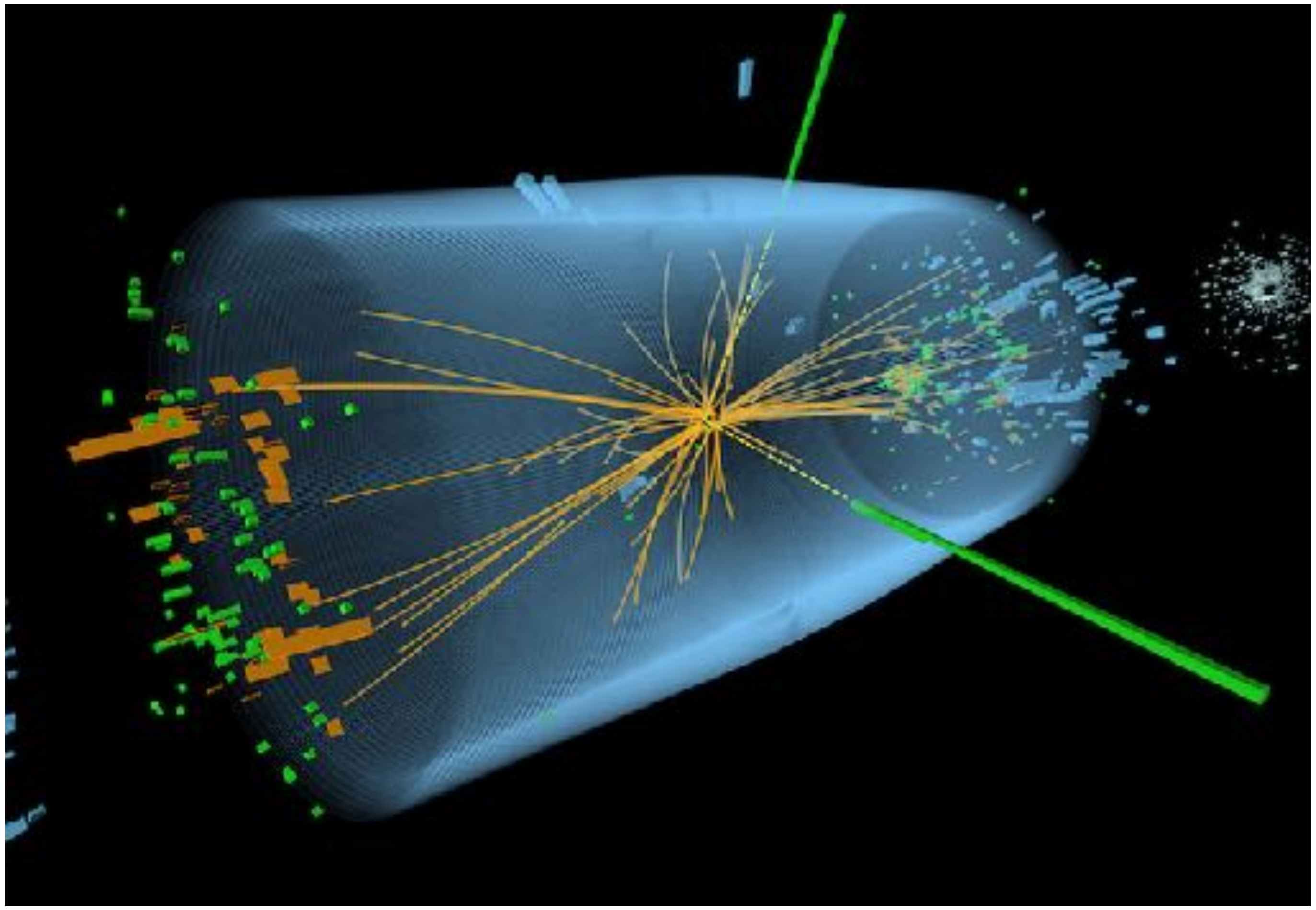




LIGO & Virgo
collaborations

Motivation

Another data-rich, high-precision subject: LHC physics



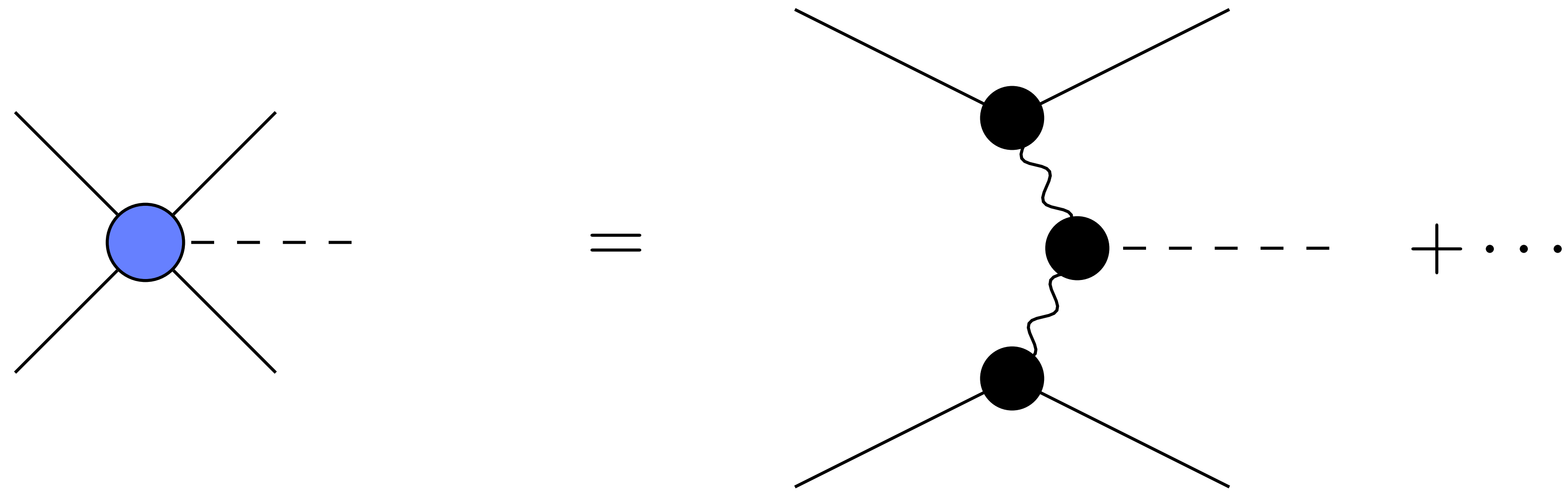
Higgs candidate event

CMS/CERN

Theoretical LHC tools now useful for gravity

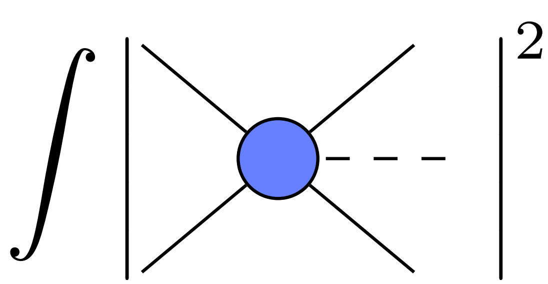
Motivation

Higgs production amplitude $\mathcal{A} \sim \langle \text{out} | \text{in} \rangle$



Amplitude: sum of diagrams

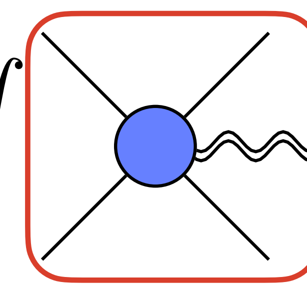
Feynman diagram

Probability $=$ 

Motivation

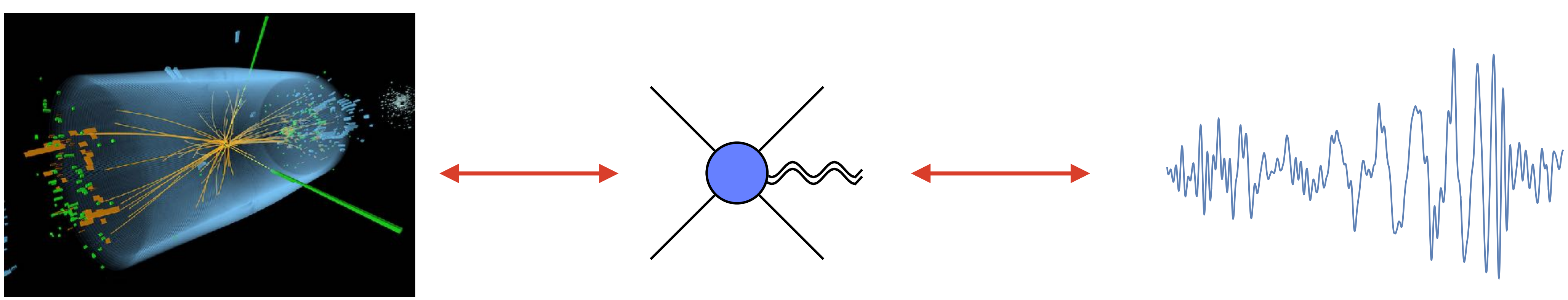
Fast scattering waveform determined by similar amplitude!

$$\text{waveform} = \frac{1}{\text{distance}} \int$$



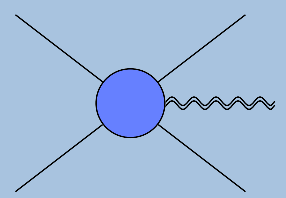
Graviton

Distance from source to observer



Motivation

Connection could be useful…

$G_{\mu\nu} = 8\pi\, T_{\mu\nu}$	
Gauge / coordinate choice required (cancels in the end)	Gauge invariant throughout
Highly non-linear	Gravity = Yang-Mills^2
Complicated integrals	Complicated integrals very well studied

Motivation

Gravity: exciting theory and data

Overview

1. Introduction to amplitudes
2. Scattering waveforms from amplitudes
3. Potentials for bound binaries

Introduction to Amplitudes

Why amplitudes?

Why are amplitudes relevant to gravitational waves / scattering?

Universal time evolution operator in QM

$$|\psi\rangle \xrightarrow{\text{time}\to\infty} U(\infty, -\infty)|\psi\rangle$$

Initial state
Far past

Final state
Far future

Why amplitudes?

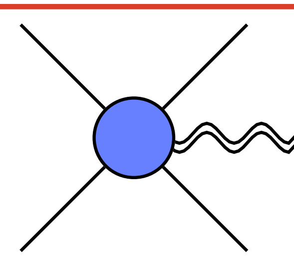
Why are amplitudes relevant to gravitational waves / scattering?

Universal time evolution operator in QM

$$|\psi\rangle \xrightarrow{\text{time}\to\infty} S|\psi\rangle \qquad U(-\infty,\infty) = S = 1 + iT$$

Matrix elements of T are the amplitudes

$$\langle q_1 \cdots q_m|T|p_1 \cdots p_n\rangle = \boxed{\mathcal{A}(p_1 \ldots p_n \to q_1 \ldots q_m)}\,\delta^4(\text{total momentum})$$



Momentum conservation

Why amplitudes?

Why are amplitudes relevant to gravitational waves / scattering?

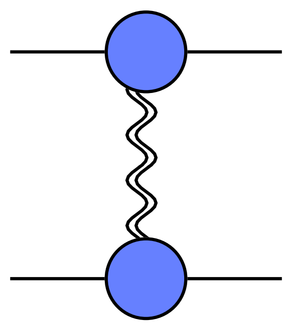
Amplitudes tell us about long-time evolution

Amplitudes

Traditional construction: Feynman diagrams

Bern, Dixon, Dunbar, Kosower
Britto, Cachazo, Feng, Witten

Efficient unitarity method



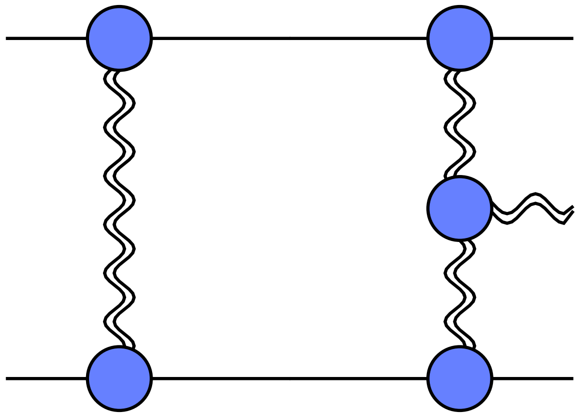
Exploit known analytic structure
to "cut" into simpler amplitudes
Recursive construction
Gauge invariant at all stages

Amplitudes

Traditional construction: Feynman diagrams

Bern, Dixon, Dunbar, Kosower
Britto, Cachazo, Feng, Witten

Efficient unitarity method



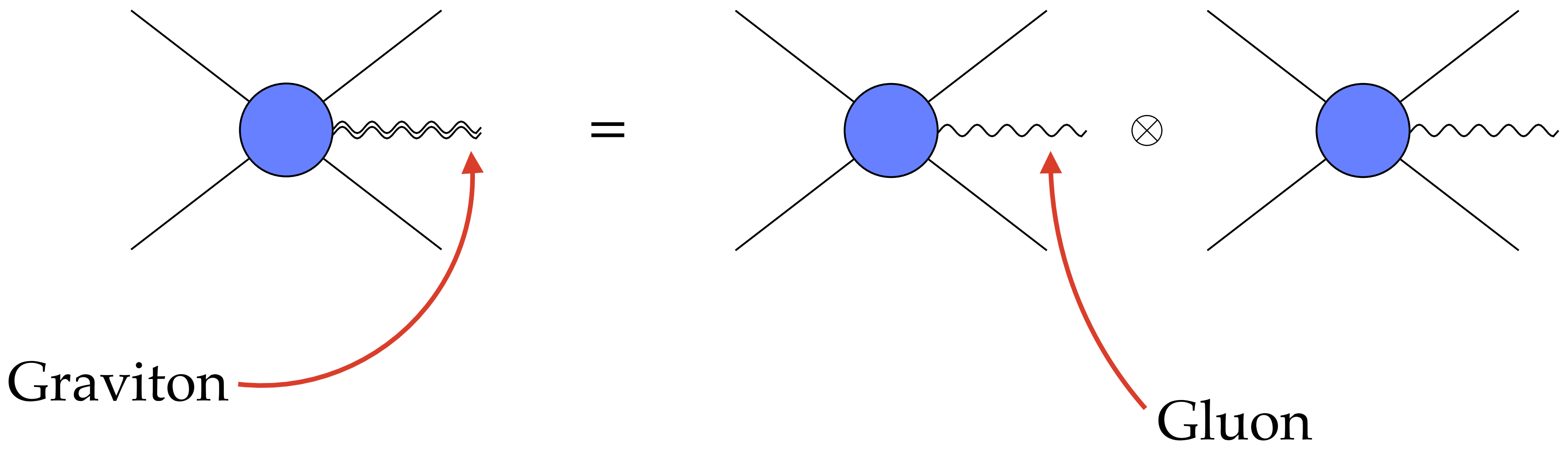
Build amplitudes of arbitrary complexity (loops/external legs)

Basic amplitudes: simple, universal

The double copy

Gravity = Yang-Mills^2 :

Kawai, Lewellen, Tye
Bern, Carrasco, Johansson



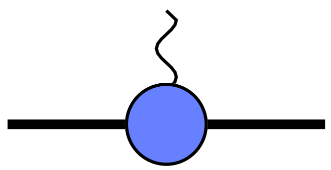
Very bizarre from geometric point of view

The double copy

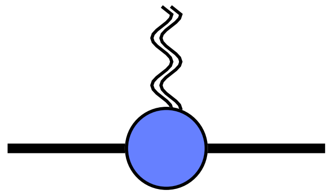
Gravity = Yang-Mills^2 :

Kawai, Lewellen, Tye
Bern, Carrasco, Johansson

Color structure

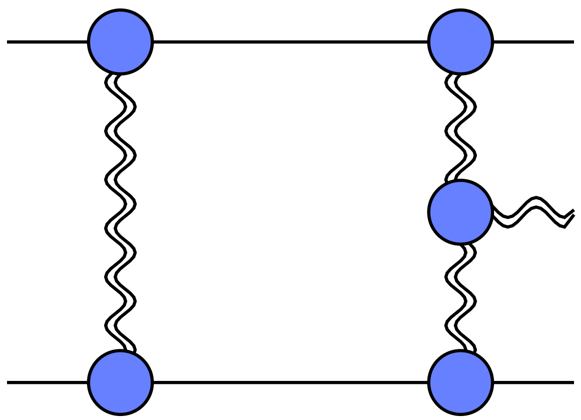
YM  $\mathcal{A} = gT^a\left(\epsilon \cdot (p_2 - p_1)\right)$

Straight squares

GR  $\mathcal{M} = \kappa\left(\epsilon \cdot (p_2 - p_1)\right)^2$ $\quad e^{\mu\nu} = \epsilon^\mu \epsilon^\nu$

Loop integrals

Non-linearities lead to loop integrals



Well developed theory

Application to gravity very successful

Bini, Damour, Geralico, Laporta, Mastrolia

Local experts include: ***Frellesvig*, *Liu*, *Vergu*, *von Hippel*, *Zhang***

Amplitudes

Advantages:

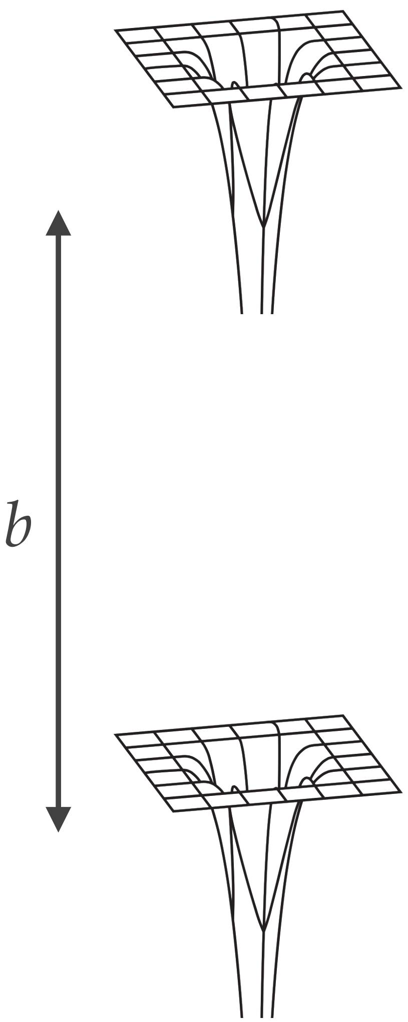
1. Universal
2. Construct efficiently (unitarity, double copy, gauge invariant)
3. Integration well studied

Price:

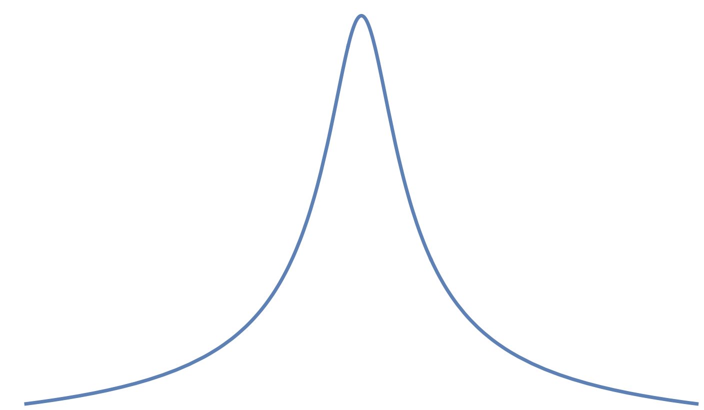
- Quantum mechanical formulation

Waveforms from Amplitudes

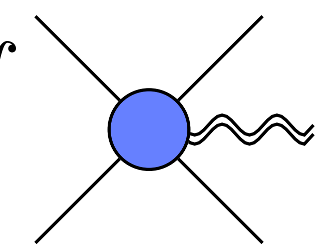
Waves from Amplitudes



waveform =



$$= \frac{1}{\text{distance}} \int$$



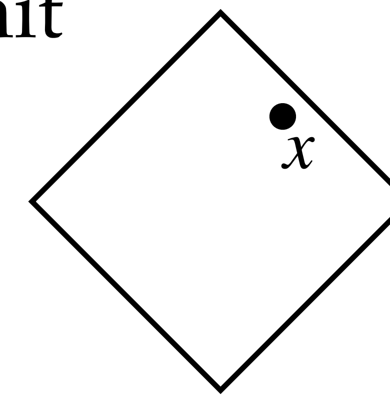
Classical point particle approximation: finite size under control

Asymptotically Minkowski

Kosower, Maybee & DOC
Cristofoli, Gonzo, Kosower & DOC
Bautista & Siemonsen

Waves from Amplitudes

Measure *expectation* of curvature component in classical limit



Riemann curvature

operator

$$\text{waveform} \equiv \langle\psi|S^\dagger \mathbb{R}\ldots.(x) S|\psi\rangle$$

Final state

Amplitudes!

Newman-Penrose scalar Ψ_4

Dominant curvature component

at large distances

$$|\psi\rangle \sim \int \bigwedge \; \bigwedge \; e^{ip_1\cdot b}|p_1\, p_2\rangle$$

Classical Cauchy data

Correspondence principal

Ehrenfest theorem

Waves from Amplitudes

Measure *expectation* of curvature component in classical limit

$$\mathbb{R}_{....}(x) = \partial.\partial.\mathbb{h}_{..}(x)$$

Graviton polarisation

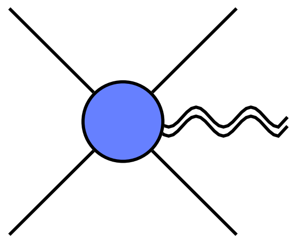
$$S = 1 + iT$$

$$\text{waveform} \sim \int \left[kk\, \varepsilon\varepsilon\, e^{-ik\cdot x} \langle p_1' p_2' | S^\dagger a(k) S | p_1 p_2 \rangle + \text{c.c.} \right]$$

$$\sim i \int kk\, \varepsilon\varepsilon\, e^{-ik\cdot x} \Big[\langle p_1' p_2' | a(k) T | p_1 p_2 \rangle - i \langle p_1 p_2 | T^\dagger a(k) T | p_1 p_2 \rangle \Big] + \text{c.c.}$$

5 point amplitude

Product of amplitudes



Waves from Amplitudes

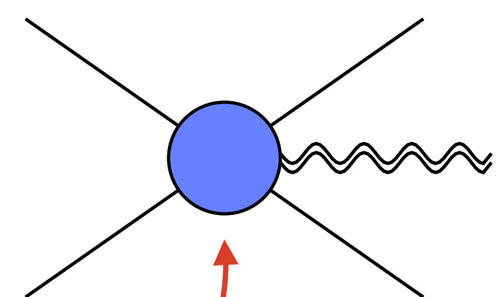
Measure *expectation* of curvature component in classical limit

Integral: FT type

$$\Psi_4(\omega) = \frac{1}{\text{distance}} \int \mathrm{d}^4 q_1 \mathrm{d}^4 q_2\, \delta(p_1 \cdot q_1)\delta(p_2 \cdot q_2)\delta^4(k - q_1 - q_2)\, e^{ib\cdot(q_1 - q_2)}$$

Frequency space

$\times$



$+ \cdots$

Five-point amplitude: double copy!

One additional term: bilinear in amplitudes

Cristofoli, Gonzo, Kosower, DOC

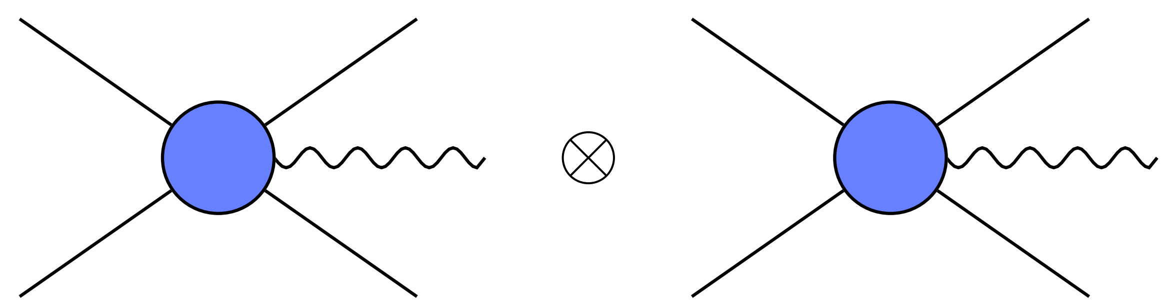
Waves from Amplitudes

Measure *expectation* of curvature component in classical limit

$$\Psi_4(\omega) = \frac{1}{\text{distance}} \int \mathrm{d}^4 q_1 \mathrm{d}^4 q_2\, \delta(p_1 \cdot q_1)\delta(p_2 \cdot q_2)\delta^4(k - q_1 - q_2)\, e^{ib\cdot(q_1 - q_2)}$$

Frequency space

$\times$



$+ \cdots$

Gravitational waves from YM!

Bypasses complexity of Einstein-Hilbert Lagrangian

Waves at NLO

Integrated NLO waveform

Herderschee, Roiban, Teng

*Brandhuber, Brown, **Chen**, De Angelis, Gowdy, Travaglini*

Georgoudis, Heissenberg, Vazquez-Holm

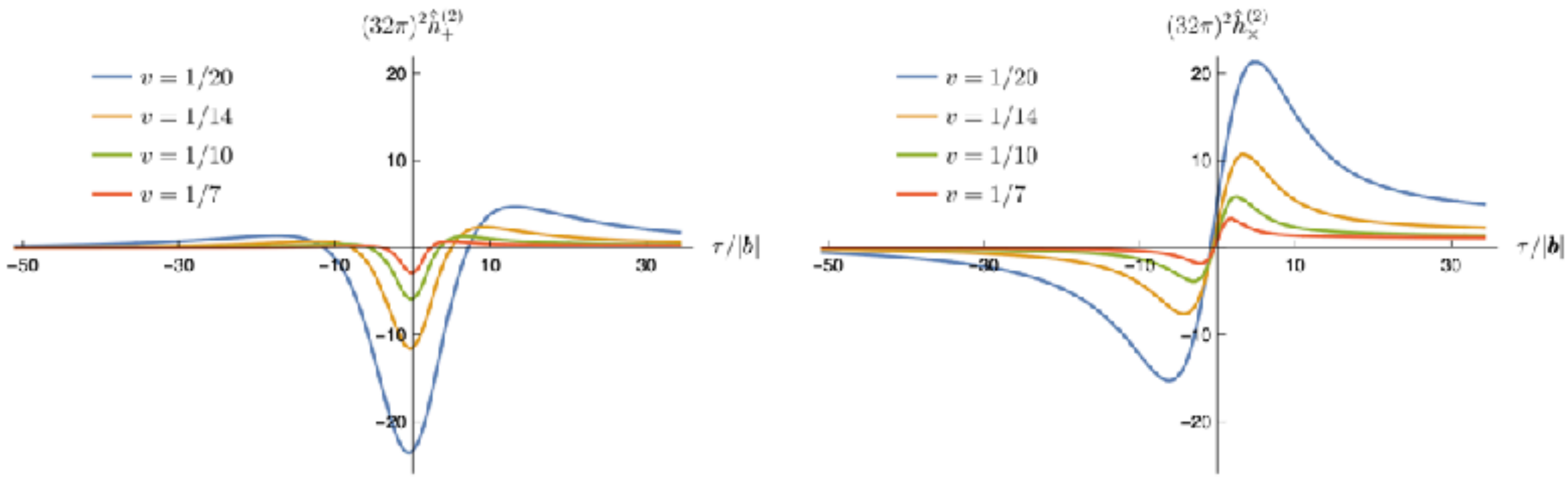


Figure from Herderschee et al

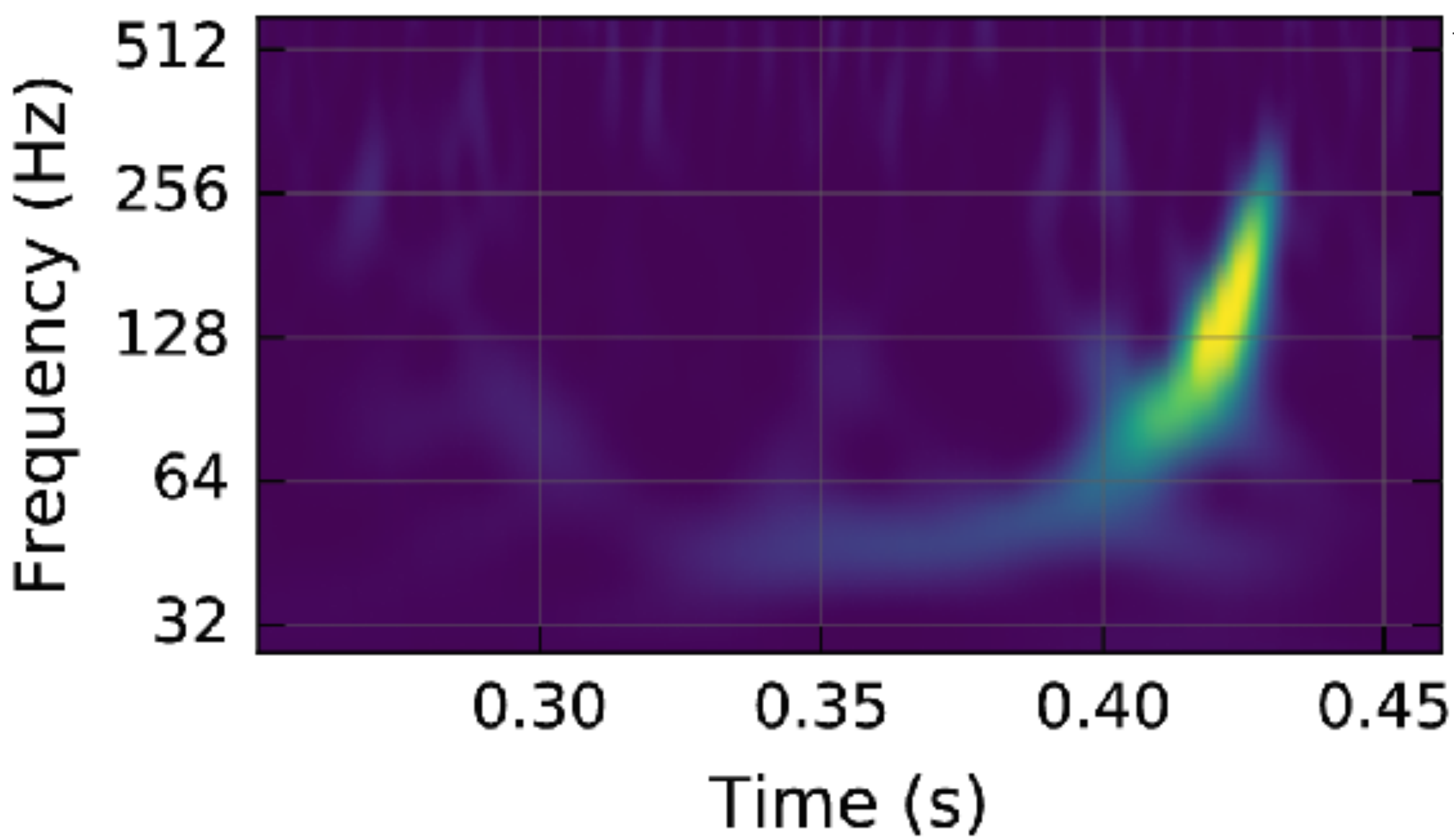
Collider methods: IBP, generalised unitarity, double copy, IR divergences, HQET

Bound Binaries

Potential

PN interaction potential important: time dependence of frequency

$$\text{Wave power} = \frac{\mathrm{d}}{\mathrm{d}t}(\text{potential energy}) \propto \frac{\mathrm{d}}{\mathrm{d}t}(\text{wave frequency})$$



LIGO Hanford

LIGO/Virgo collaboration

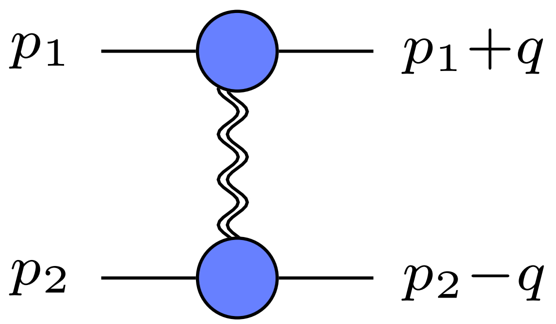
Potential

PN interaction potential important: time dependence of frequency

$$\text{Wave power} = \frac{\mathrm{d}}{\mathrm{d}t}(\text{potential energy}) \propto \frac{\mathrm{d}}{\mathrm{d}t}(\text{wave frequency})$$

Potential related to amplitudes: impulse

$$\Delta p^\mu = \int e^{iq\cdot b}\, iq^\mu \times$$



$$+ \cdots$$

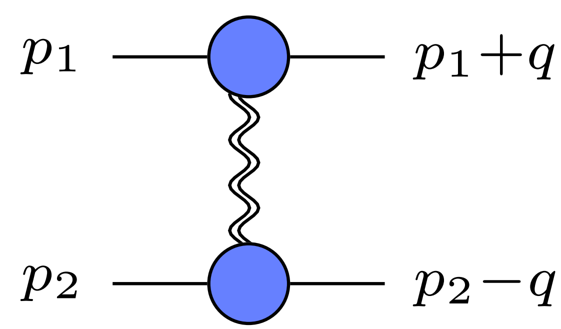
Potential

PN interaction potential important: time dependence of frequency

$$\text{Wave power} = \frac{\mathrm{d}}{\mathrm{d}t}(\text{potential energy}) \propto \frac{\mathrm{d}}{\mathrm{d}t}(\text{wave frequency})$$

Potential related to amplitudes: impulse

$$V(r) = \int e^{iq\cdot r} \times$$



$$+ \cdots$$

Systematic relationship

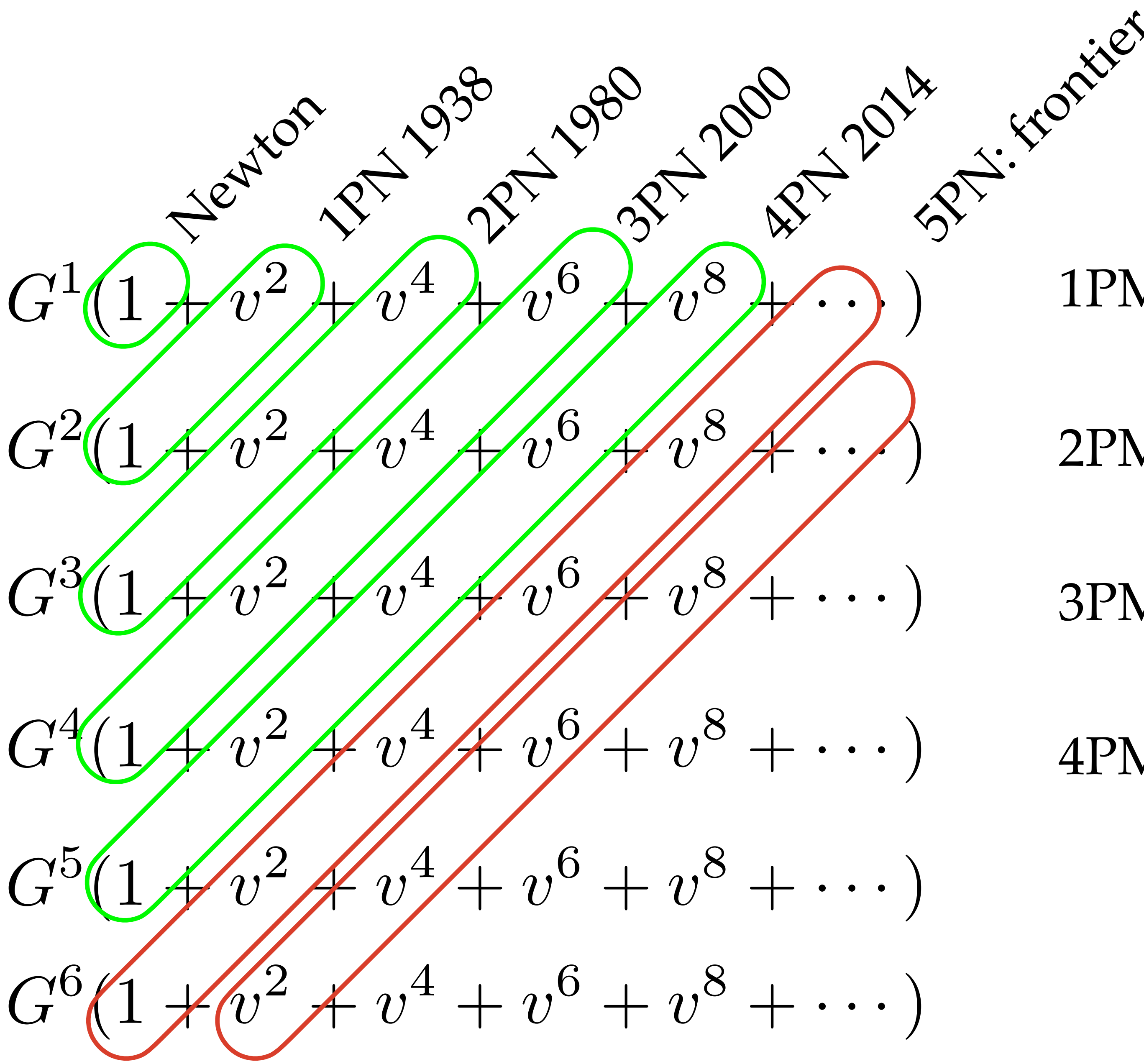
Donoghue, Holstein, ***Bjerrum-Bohr, Damgaard****, Vanhove, Neill, Rothstein, Cheung, Solon, Dlapa, Kälin,* ***Liu,*** *Porto, Bern, Herrmann, Parra Martinez, Roiban, Ruf, Shen, Solon, Zeng, Aoude, Haddad, Helset, Mogull, Plefka, Steinhoff, Brandhuber,* ***Chen****, Travaglini, Wen, …*

Potential

Amplitudes naturally (specially) relativistic

Potential

Amplitudes naturally (specially) relativistic. Velocity expansion:



Bini, Blanchet, Damour, Geralico, Jaranowski, Schäfer, …

1PM

2PM, 1985

3PM, 2019 (amplitudes)

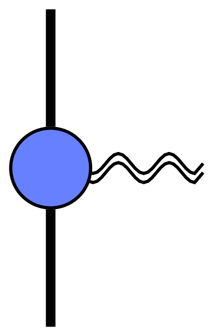
4PM, 2021-2023 (amplitudes + friends)

*Bern et al, **Liu**, Porto, …*

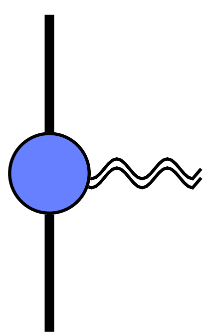
Spin

Classical spin — taught us a lot about spinning amplitudes

Guevara, Ochirov, Vines

Spin 0  $= \kappa\, \mathcal{M}_3$ $\qquad \mathcal{M}_3 \sim [\epsilon(k) \cdot p]^2$

Spin deformation

Spin large  $= \kappa\, e^{k \cdot a} \mathcal{M}_3$

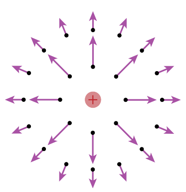
Very active area

Arkani-Hamed, Huang, Huang, Emond, Moynihan, Bern, Kosmopoulos, ***Luna, Liu****, Roiban, Teng,* ***Bjerrum-Bohr****,* ***Chen****,* ***Skowronek, Damgaard, Hoogeveen,*** *Levi, Mcleod,* ***Von Hippel****, Chung, Kim, Lee, Maybee, Febres Cordero, Krauss, Lin, Ruf, Zeng, Jakobsen, Mogull, Plefka, Steinhoff, Aoude, Haddad, Helset, Johansson, Cangemi, Chiodaroli, Pichini, Morales, Yin, …*

Spacetime curvature

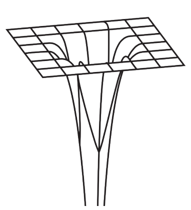
"Classical" double copy: *__Luna__, Monteiro, White, DOC, __Han__, …*

Integration measure



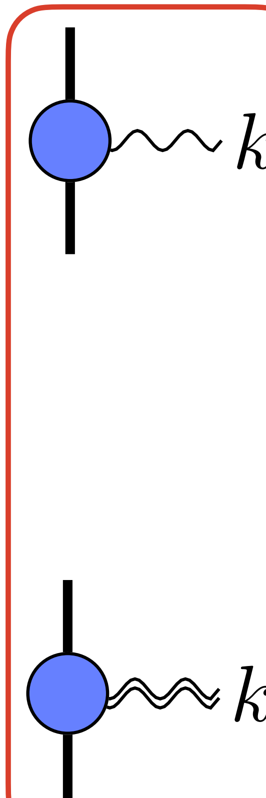
$$\Phi^{\mathrm{Coul}}(x) = \mathrm{Re} \int \mathrm{d}^4k\, \delta(k^2)\delta(k\cdot p)\, |k\rangle^2\, e^{-ik\cdot x}$$

Spinors (derivatives)



$$\Psi^{\mathrm{Schw}}(x) = \mathrm{Re} \int \mathrm{d}^4k\, \delta(k^2)\delta(k\cdot p)\, |k\rangle^4\, e^{-ik\cdot x}$$

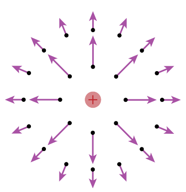
Integration measure



Amplitudes: double copy

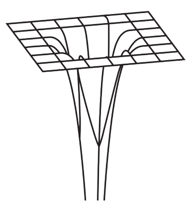
Spacetime curvature

"Classical" double copy: *__Luna__, Monteiro, White, DOC, __Han__, …*



$$\Phi^{\mathrm{Coul}}(x) = e\frac{(\text{spin structure})}{\sqrt{t_2^2 - x^2 - y^2}^{\,2}} + \delta(t_2^2 - x^2 - y^2)((2{,}2)\ \text{signature})$$

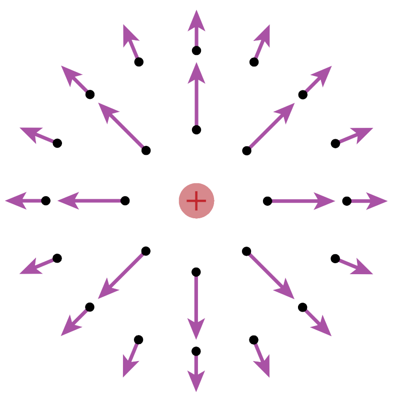
Minkowski Weyl spinors
inherit double copy



$$\Psi^{\mathrm{Schw}}(x) = m\frac{(\text{spin structure})^2}{\sqrt{t_2^2 - x^2 - y^2}^{\,3}} + (2{,}2)\ \text{distributions}$$

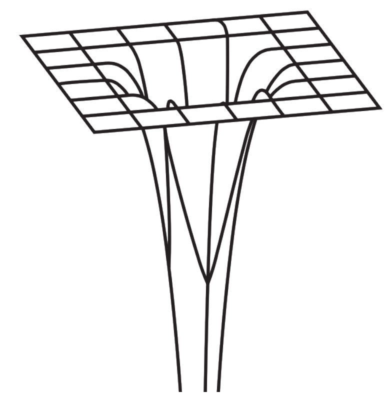
Spacetime curvature

Coulomb



Double copy

Schwarzschild (linearised)



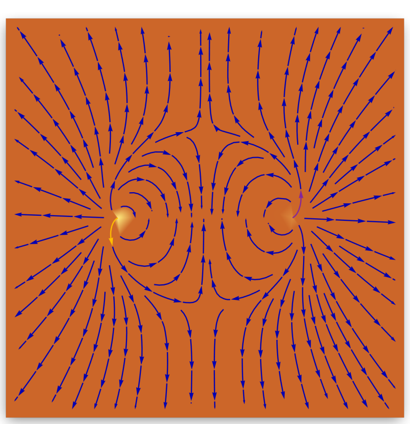
$x \to x + ia$

$\mathcal{M}_3 \to \mathcal{M}_3 e^{k \cdot a}$

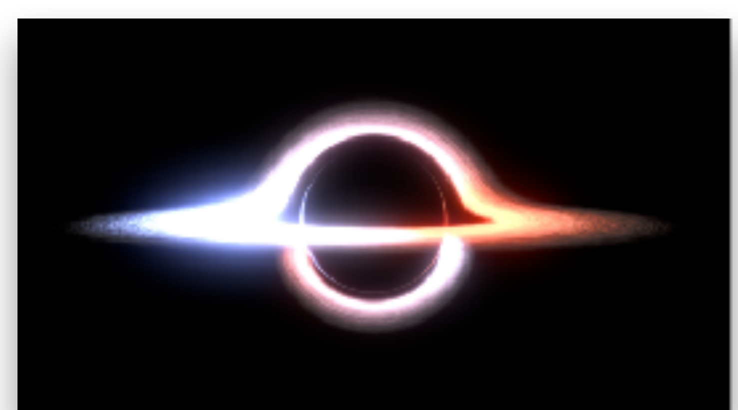
"Newman-Janis shift"

$x \to x + ia$

$\sqrt{\text{Kerr}}$



Kerr (linearised)



Conclusions

- Beautiful dialog between amplitudes and classical gravity
- Excitement about GW data leading to progress in unexpected areas
  - Observables in quantum field theory
  - Spinning amplitudes
  - Post-Minkowski EFTs
- There's much more to do!