Classical Gravity from Quantum Amplitudes

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Data rich future for gravitational waves!
Motivation

Gravity: key challenge

ESA and the Planck collaboration

LIGO & Virgo collaborations
Motivation

Another data-rich, high-precision subject: LHC physics

Theoretical LHC tools now useful for gravity
Motivation

Higgs production amplitude $\mathcal{A} \sim \langle \text{out}|\text{in}\rangle$

Amplitude: sum of diagrams

Feynman diagram

Probability $= \int |\langle \text{out}|\text{in}\rangle|^2$
Motivation

Fast scattering waveform determined by similar amplitude!

$$\text{waveform} = \frac{1}{\text{distance}} \int \text{Graviton}$$

Distance from source to observer
Connection could be useful…

<table>
<thead>
<tr>
<th>Gauge / coordinate choice required (cancels in the end)</th>
<th>Gauge invariant throughout</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highly non-linear</td>
<td>Gravity = Yang-Mills$^2$</td>
</tr>
<tr>
<td>Complicated integrals</td>
<td>Complicated integrals very well studied</td>
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</table>
Gravity: exciting theory and data
Overview

1. Introduction to amplitudes
2. Scattering waveforms from amplitudes
3. Potentials for bound binaries
Introduction to Amplitudes
Why amplitudes?

Why are amplitudes relevant to gravitational waves / scattering?

*Universal time evolution* operator in QM

\[
|\psi\rangle \xrightarrow{\text{time}\rightarrow\infty} U(\infty, -\infty)|\psi\rangle
\]

Initial state  
Far past

Final state  
Far future
Why amplitudes?

Why are amplitudes relevant to gravitational waves / scattering?

*Universal time evolution* operator in QM

\[ |\psi\rangle \xrightarrow{\text{time} \to \infty} S|\psi\rangle \quad U(-\infty, \infty) = S = 1 + iT \]

Matrix elements of \( T \) are the amplitudes

\[ \langle q_1 \cdots q_m | T | p_1 \cdots p_n \rangle = A(p_1 \cdots p_n \rightarrow q_1 \cdots q_m) \delta^4(\text{total momentum}) \]

Momentum conservation
Why amplitudes?

Why are amplitudes relevant to gravitational waves / scattering?

Amplitudes tell us about long-time evolution
Amplitudes

Traditional construction: Feynman diagrams

Efficient unitarity method

Exploit known analytic structure to “cut” into simpler amplitudes
Recursive construction
Gauge invariant at all stages

Bern, Dixon, Dunbar, Kosower
Britto, Cachazo, Feng, Witten
Amplitudes

Traditional construction: Feynman diagrams

Efficient unitarity method

Build amplitudes of arbitrary complexity (loops/external legs)

Basic amplitudes: simple, universal

Bern, Dixon, Dunbar, Kosower
Britto, Cachazo, Feng, Witten
The double copy

Gravity = Yang-Mills$^2$:

Kawai, Lewellen, Tye
Bern, Carrasco, Johansson

Very bizarre from geometric point of view
The double copy

Gravity = Yang-Mills$^2$:

\[ A = gT^a \epsilon \cdot (p_2 - p_1) \]

Color structure

\[ M = \kappa (\epsilon \cdot (p_2 - p_1))^2 \]

Straight squares

\[ e^{\mu \nu} = \epsilon^\mu \epsilon^\nu \]

Kawai, Lewellen, Tye
Bern, Carrasco, Johansson
Loop integrals

Non-linearities lead to loop integrals

Well developed theory

Application to gravity very successful

Bini, Damour, Geralico, Laporta, Mastrolia

Local experts include: Frellesvig, Liu, Vergu, von Hippel, Zhang
Amplitudes

Advantages:

1. Universal
2. Construct efficiently (unitarity, double copy, gauge invariant)
3. Integration well studied

Price:

❖ Quantum mechanical formulation
Waveforms from Amplitudes
Waves from Amplitudes

Classical point particle approximation: finite size under control

Asymptotically Minkowski

Kosower, Maybee & DOC
Cristofoli, Gonzo, Kosower & DOC
Bautista & Siemonsen
Measure *expectation* of curvature component in classical limit

\[
\text{waveform} \equiv \langle \psi | S^\dagger R_{\ldots}(x) S | \psi \rangle
\]

**Newman-Penrose scalar** $\Psi_4$

Dominant curvature component at large distances

**Riemann curvature operator**

Final state Amplitudes!

\[
|\psi\rangle \sim \int \bigg| \bigwedge \bigwedge e^{ip_1 \cdot b} |p_1 p_2\rangle
\]

Classical Cauchy data

Correspondence principal

Ehrenfest theorem
Waves from Amplitudes

Measure *expectation* of curvature component in classical limit

\[ R_{\ldots}(x) = \partial_\mu \partial_\nu h_{\ldots}(x) \]

Graviton polarisation

\[ S = 1 + iT \]

Waveform

\[ \sim \int [kk \varepsilon \varepsilon e^{-ik \cdot x} \langle p_1' p_2' | S^\dagger a(k) S | p_1 p_2 \rangle + \text{c.c.}] \]

\[ \sim i \int kk \varepsilon \varepsilon e^{-ik \cdot x} \left[ \langle p_1' p_2' | a(k) T | p_1 p_2 \rangle - i \langle p_1 p_2 | T^\dagger a(k) T | p_1 p_2 \rangle \right] + \text{c.c.} \]

5 point amplitude

Product of amplitudes
Waves from Amplitudes

Measure *expectation* of curvature component in classical limit

\[ \Psi_4(\omega) = \frac{1}{\text{distance}} \int \delta^4(q_1) \delta^4(q_2) \delta^4(k - q_1 - q_2) e^{ib \cdot (q_1 - q_2)} \]

Integral: FT type

Frequency space

Five-point amplitude: double copy!

One additional term: bilinear in amplitudes

\textit{Cristofoli, Gonzo, Kosower, DOC}
Waves from Amplitudes

Measure *expectation* of curvature component in classical limit

\[ \Psi_4(\omega) = \frac{1}{\text{distance}} \int d^4q_1 d^4q_2 \delta(p_1 \cdot q_1) \delta(p_2 \cdot q_2) \delta^4(k - q_1 - q_2) e^{ib \cdot (q_1 - q_2)} \]

Gravitational waves from YM!

Bypasses complexity of Einstein-Hilbert Lagrangian
Waves at NLO

Integrated NLO waveform

Herderschee, Roiban, Teng
Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini
Georgoudis, Heissenberg, Vazquez-Holm

Collider methods: IBP, generalised unitarity, double copy, IR divergences, HQET

Figure from Herderschee et al
Bound Binaries
Potential

PN interaction potential important: time dependence of frequency

$$\text{Wave power} = \frac{d}{dt} (\text{potential energy}) \propto \frac{d}{dt} (\text{wave frequency})$$
Potential

PN interaction potential important: time dependence of frequency

\[
\text{Wave power} = \frac{d}{dt} (\text{potential energy}) \propto \frac{d}{dt} (\text{wave frequency})
\]

Potential related to amplitudes: impulse

\[
\Delta p^\mu = \int e^{iq \cdot b} iq^\mu \times p_1 \quad \quad \quad p_1 + q
\]

\[
\quad + \cdots \quad \quad \quad p_2 \quad \quad \quad p_2 - q
\]
Potential

PN interaction potential important: time dependence of frequency

\[
\text{Wave power} = \frac{d}{dt} (\text{potential energy}) \propto \frac{d}{dt} (\text{wave frequency})
\]

Potential related to amplitudes: impulse

\[
V(r) = \int e^{iq \cdot r} \times \begin{align*}
p_1 & \quad p_1+q \\
p_2 & \quad p_2-q
\end{align*} + \cdots
\]

Systematic relationship

Potential

Amplitudes naturally (specially) relativistic
Amplitudes naturally (specially) relativistic. Velocity expansion:

\[
G^1(1 + v^2 + v^4 + v^6 + v^8 + \cdots )
\]

1PM

\[
G^2(1 + v^2 + v^4 + v^6 + v^8 + \cdots )
\]

2PM, 1985

\[
G^3(1 + v^2 + v^4 + v^6 + v^8 + \cdots )
\]

3PM, 2019 (amplitudes)

\[
G^4(1 + v^2 + v^4 + v^6 + v^8 + \cdots )
\]

4PM, 2021-2023 (amplitudes + friends)

\[
G^5(1 + v^2 + v^4 + v^6 + v^8 + \cdots )
\]

\[
G^6(1 + v^2 + v^4 + v^6 + v^8 + \cdots )
\]

Bern et al, Liu, Porto, …
Spin

Classical spin — taught us a lot about spinning amplitudes

Guevara, Ochirov, Vines

\[ \mathcal{M}_3 \sim [\epsilon(k) \cdot p]^2 \]

Very active area

"Classical" double copy:

\begin{align*}
\Phi^{\text{Coul}}(x) &= \text{Re} \int \text{Integration measure} \, \left( d^4 k \, \delta(k^2) \delta(k \cdot p) \right) |k|^2 e^{-ik \cdot x} \\
\Psi^{\text{Schw}}(x) &= \text{Re} \int \text{Integration measure} \, \left( d^4 k \, \delta(k^2) \delta(k \cdot p) \right) |k|^4 e^{-ik \cdot x}
\end{align*}

Spinors (derivatives)

Amplitudes: double copy

Luna, Monteiro, White, DOC, Han, …
Spacetime curvature

“Classical” double copy:

\[ \Phi^{\text{Coul}}(x) = e \frac{(\text{spin structure})}{\sqrt{t_2^2 - x^2 - y^2}} + \delta(t_2^2 - x^2 - y^2)((2,2) \text{ signature}) \]

\[ \Psi^{\text{Schw}}(x) = m \frac{(\text{spin structure})^2}{\sqrt{t_2^2 - x^2 - y^3}} + (2,2) \text{ distributions} \]

\text{Luna, Monteiro, White, DOC, Han, ...}
Spacetime curvature

Coulomb

\[ x \to x + ia \]

\[ \sqrt{\text{Kerr}} \]

Schwarzschild (linearised)

\[ M_3 \to M_3 e^{k \cdot a} \]

“Newman-Janis shift”

\[ x \to x + ia \]

Kerr (linearised)
Conclusions

❖ Beautiful dialog between amplitudes and classical gravity

❖ Excitement about GW data leading to progress in unexpected areas
  ❖ Observables in quantum field theory
  ❖ Spinning amplitudes
  ❖ Post-Minkowski EFTs

❖ There’s much more to do!