How much do dark matter initial conditions matter?

Numerical relativity simulations in DM environments

Katy Clough

Black hole merger simulations in wave dark matter environments
Jamie Bamber, Josu C. Aurrekoetxea, Katy Clough, Pedro G. Ferreira
Phys.Rev.D 107 (2023) 2, 024035 gr-qc 2210.09254
First ever black hole image released

By Pallab Ghosh
Science correspondent, BBC News

6 hours ago

Science & Environment

Einstein's gravitational waves 'seen' from black holes

By Pallab Ghosh
Science correspondent, BBC News

11 February 2016

What Brexit looks like from space
Interesting idea: Can we use this new data to learn about the environments of the black holes that we observe?
More concrete idea: Having additional matter around BHs will change the different parts of the waveform in a distinctive way.
Radiation of the environment

Dynamical friction and accretion

Change in curvature of space results in new trajectories
Plan:

- Generate GW templates with environments
- Match them to signals
- Detect environments
Potential problem: How important is it to have the “right” initial conditions for our simulations?
Concern:

We are not ready to generate template banks and do inference about the effects of matter environments on the merger part of the signal because we don’t have control over our initial conditions and understand how they affect the results.
What is numerical relativity? Why do initial conditions matter?
Curved spacetime

\[ ds^2 = (dt \ dx \ dy \ dz) \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} \]

“The spacetime metric”

\[ g_{ab}(t, \vec{x}) \]
What is the goal of NR?

The spacetime metric

\[ ds^2 = (dt \ dx \ dy \ dz) \]

\[ g_{ab}(t, \vec{x}) \]

The metric

\[
\begin{pmatrix}
g_{00} & g_{01} & g_{02} & g_{03} \\
g_{10} & g_{11} & g_{12} & g_{13} \\
g_{20} & g_{21} & g_{22} & g_{23} \\
g_{30} & g_{31} & g_{32} & g_{33}
\end{pmatrix}
\begin{pmatrix}
dt \\
dx \\
dy \\
dz
\end{pmatrix}
\]

“The spacetime metric”

The Einstein equation

\[ R_{ab} - \frac{R}{2} g_{ab} = 8\pi \ T_{ab} \]

f(\partial^2 g_{ab}, \partial g_{ab}, g_{ab})

“Curvature”

“Energy-Momentum”

“The metric

“Energy-Momentum”
The Einstein equation tells us how the metric should look, given some energy/matter distribution:

\[ \text{R}_{ab} - \frac{R}{2} \text{g}_{ab} = 8\pi \text{T}_{ab} \]

Four constraint equations for any time slice - non linear elliptic/Poisson equation:

\[ \frac{\partial^2 g}{\partial x^2} + \text{non linear terms} = f(\text{energy, momentum}) \]

An evolution equation for all time - non linear hyperbolic/wave equation:

\[ \frac{\partial^2 g}{\partial t^2} - \frac{\partial^2 g}{\partial x^2} + \text{non linear terms} = f(\text{energy, momentum}) \]

“Matter tells spacetime how to curve...”
The metric determines the motion of matter

\[ \nabla^a (R_{ab} - R/2 \ g_{ab}) = \nabla^a (8\pi \ T_{ab}) = 0 \]

Continuity equation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot j = \text{source} \]

“…spacetime tells matter how to move.”
In reality numerical relativity is done in a finite region of spacetime.

“local time”

Fill using Einstein equation

\[ g_{ab}(t+dt) = f(\partial_x g_{ab}, \partial_t g_{ab}, g_{ab}, T_{ab}) \]

boundary conditions \((g_{ab}, T_{ab})\)

Not a free choice!

initial data \((\partial_t g_{ab}, g_{ab}, T_{ab})\)

“space”
Numerical relativity - the initial condition problem

- describe BHs
  (Masses, momenta, using PN calculations)

- make 16 (arbitrary?) choices of free components

- solve 4 constraints on energy and momentum

Is this the system I was looking for?
(Are orbits actually circular, are masses what I wanted?)
Numerical relativity - the initial condition problem

- describe BHs (Masses, momenta, using PN calculations)

Iterate

- make 16 (arbitrary?) choices of free components

- solve 4 constraints on energy and momentum

**THese Aren't The Metric Components**

**You're looking for**

(Are orbits actually circular, are masses what I wanted?)
Numerical relativity - the initial condition problem

- Describe BHs
  (Masses, momenta, using PN calculations)

- This will just have to be good enough

- Is this the system I was looking for?

- Solve 4 constraints on energy and momentum

- Make 16 (arbitrary?) choices of free components
The “right” initial condition means:

1. Solves the energy and momentum constraints of GR

2. Is the correct physical scenario that we are looking for
Dark matter environments
Particle dark matter parameters

- Mass
- Self interaction
- Standard model interactions
- Fraction of total DM (locally/globally)
A vast range of potential masses

Wave DM
e.g. axions
$10^{-23}$ eV - 1 eV

Particle DM
e.g. WIMPS
1 eV - $10^{13}$ eV

Schive et al. 2014
Cosmic structure as the quantum interference of a coherent dark wave
A useful distinction is between wave-like and particle DM. 

Wave DM
- e.g. axions
- $10^{-23} \text{eV} - 1 \text{eV}$

Particle DM
- e.g. WIMPS
- $1 \text{eV} - 10^{13} \text{eV}$

Mass

Particle dark matter parameters

Schive et al. 2014
Cosmic structure as the quantum interference of a coherent dark wave
A useful distinction is between wave-like and particle DM

Particle dark matter parameters

Schive et al. 2014
Cosmic structure as the quantum interference of a coherent dark wave
Detection / constraints rely heavily on fraction of DM composed by the candidate, and its distribution (uniform/clumpy)
Bad news: average DM density is very low :-(

Barausse et al. 2014
Can environmental effects spoil precision gravitational-wave astrophysics?

(Answer: Broadly no - for inspiral and ringdown, assuming uniform density)
What do you mean “low”?

\[ \rho \sim 1 \text{ GeV/cm}^3 \text{ or } 1 \text{ M}_\odot/\text{pc}^3 \]

(Particle physicist) (Astrophysicist)
What do you mean “low”? 

\[
\frac{\rho}{1/R_s^2} \sim 10^{-30} \left( \frac{M_{BH}}{10^6 M_\odot} \right)^2
\]

(Numerical relativist)
What DM density enhancement is required to have an observable impact on GW signals? Do such enhancements arise naturally?
Superradiance
Review by Brito et. al. (updated 2020)
Superradiance: New Frontiers in Black Hole Physics

Interactions e.g. bremsstrahlung, or attractive self interactions

Dark matter overdensity scenarios

Exotic compact objects e.g. boson stars

Dark matter minispikes (adiabatic growth, accretion)

Dietrich et al. 2019
Cooling binary neutron star remnants via nucleon-nucleon-axion bremsstrahlung

Bamber et al. 2021
Growth of accretion driven scalar hair around Kerr black holes

Bustillo et al. 2021
GW190521 as a merger of Proca stars: a potential new vector boson of $8.7 \times 10^{-13}$ eV

Kavanagh et al. 2020, Coogan et al. 2022
Measuring the dark matter environments of black hole binaries with gravitational waves

Image credit: Helfer / Clough

FIG. 5. Snapshots of the time evolution of the energy density during the head-on collision of two PSs with $\omega/\mu \nu = 0.8936$. Time is given in code units.
Dark matter overlodensity scenarios

Interactions e.g. bremsstrahlung, or attractive self interactions

Exotic compact objects e.g. boson stars

Superradiance
Review by Brito et. al. (updated 2020)
Superradiance: New Frontiers in Black Hole Physics

Image credit: Helfer / Clough
Gondolo & Silk found that adiabatic growth led to a DM overdensity described by a power law

\[ \rho \sim \rho_0 \left( \frac{r}{r_0} \right)^{-\gamma} \]

- Becker et.al. 2021
Circularization vs. Eccentricity in Intermediate Mass Ratio Inspirals inside Dark Matter Spikes
Accretion in the low mass DM case - scalar accretion

DM overdensity described by power law plus oscillations on the scale of the Compton wavelength of the light particle

In the isolated BH case solutions are known exactly (they are the confluent Heun functions)

Black Hole Hair from Scalar Dark Matter
Lam Hui, Daniel Kabat, Xinyu Li, Luca Santoni, Sam S. C. Wong
JCAP 1906 (2019) no.06, 038

Energy density

Clough et. al 2019, Bamber et. al 2021
Growth of accretion driven scalar hair around Kerr black holes
Accretion in the wave DM case - scalar accretion

Field profile

Clough et al. 2019,
Bamber et al. 2021
Growth of accretion driven scalar hair around Kerr black holes
Ok, so maybe you have concentrations of dark matter around isolated black holes, but do you have them around binaries?
Binaries in the particle DM case

\[ M_{PBII} = 30 M_\odot; \ a_i = 0.01 \text{ pc}; \ e_i = 0.995 \]

\[ T = 0.00 \text{ kyr} \]
As a result there is a focus on high mass ratio merger events.

Kavanagh et. al. 2020, Coogan et. al. 2022
Detecting dark matter around black holes with gravitational waves: Effects of dark-matter dynamics on the gravitational waveform.
Binaries in the wave DM case

Bamber et. al., 2022
Black hole merger simulations in wave dark matter environments
Binaries in the wave DM case

Bamber et. al., 2022
Black hole merger simulations in wave dark matter environments
Using fixed orbit simulations to simulate stationary orbits, we find a stationary profile with a scaling symmetry.

Field profiles

- $t/T = 0.0$
- $t/T = 2.0$
- $t/T = 4.0$
- $t/T = 6.0$
So there is a “right” initial condition for the dark matter component. But what if we start with something else?
In full NR, different initial clouds converge to the same solution within a few orbits but there are significant transients.
How does this affect the solution of the constraint equations?
Numerical relativity is done in a finite region of spacetime.

Fill using Einstein equation:

\[ g_{ab}(t+dt) = f(\partial_x g_{ab}, \partial_t g_{ab}, g_{ab}, T_{ab}) \]

Boundary conditions (\( g_{ab}, T_{ab} \))

Not a free choice!

Initial data (\( \partial_t g_{ab}, g_{ab}, T_{ab} \))

“local time”

“space”
1. The initial matter density will push the BHs off circular orbits compared to the vacuum case.

-> Need to adjust the initial momenta so circular again.
Problem 2

2. The subsequent transient evolution of the field will potentially look like a signal.

-> Need to start with the closest initial condition to the “stationary” case.
Key points

- To see any signal we need a DM density enhancement mechanism - one possibility is accretion of wave like dark matter

- Numerical relativity simulations will not give the right answer unless you give them the right initial conditions

- We need to do better in modelling physical environments of interest before generating waveforms
Thank you, questions?
Engrenage (the code formerly known as BabyGRChombo)

Engrenage is a spherically symmetric BSSN code designed for teaching Numerical Relativity (NR), which is the solution of the Einstein Equations of General Relativity (GR) using numerical methods. The code includes a scalar field (obeying the Klein Gordon equation for a minimally coupled spin 0 field) as the matter source of the metric curvature. It currently includes two physical examples - a black hole and a real scalar boson star (or oscillator).