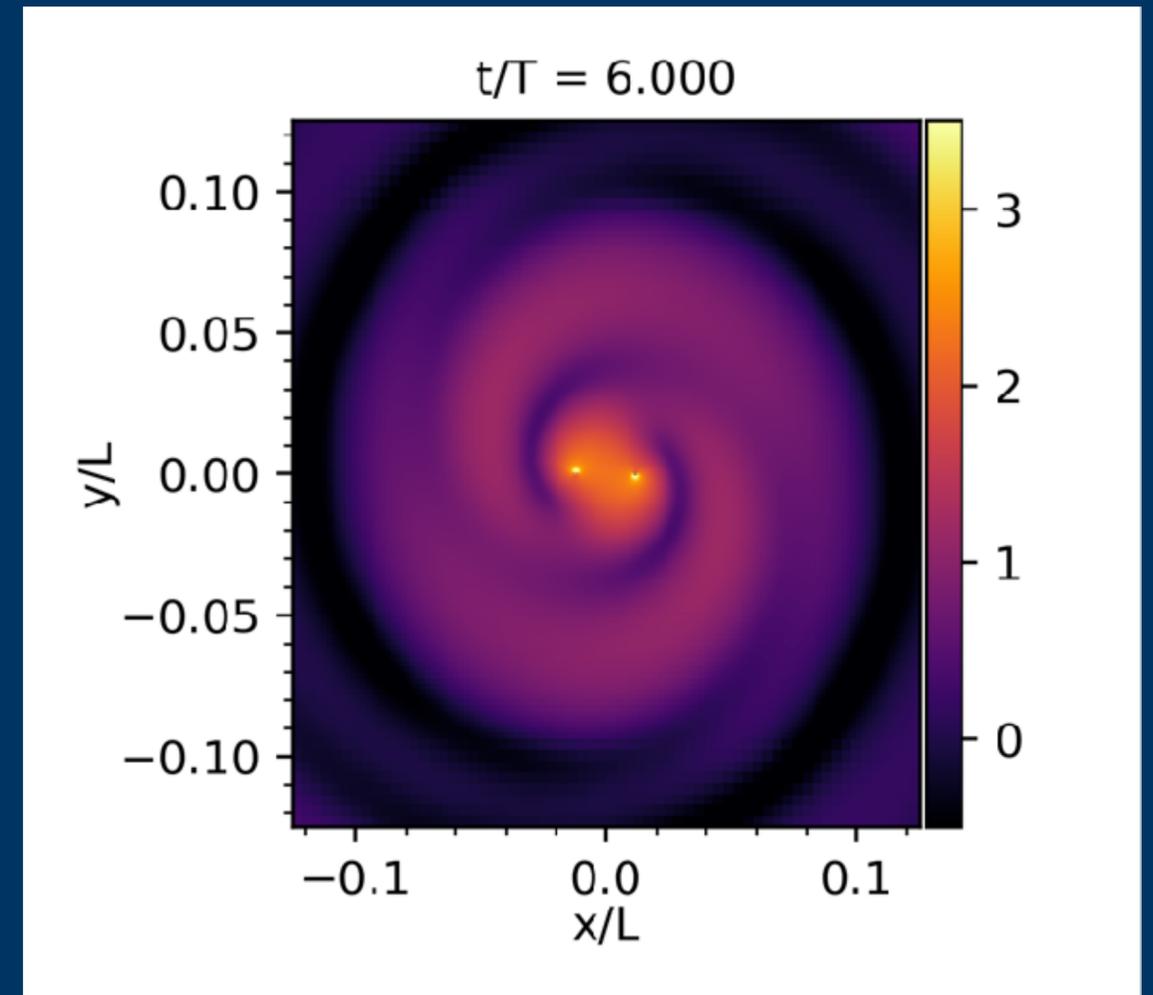


# How much do dark matter initial conditions matter?

Numerical relativity simulations in DM environments

**Katy Clough**



*Black hole merger simulations in wave dark matter environments*

*Jamie Bamber, Josu C. Aurrekoetxea, Katy Clough, Pedro G. Ferreira*

*Phys.Rev.D 107 (2023) 2, 024035 gr-qc 2210.09254*



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Science & Environment

# First ever black hole image released

By Pallab Ghosh  
Science correspondent, BBC News

6 hours ago

**BBC** Home News Sport More Search

**NEWS**

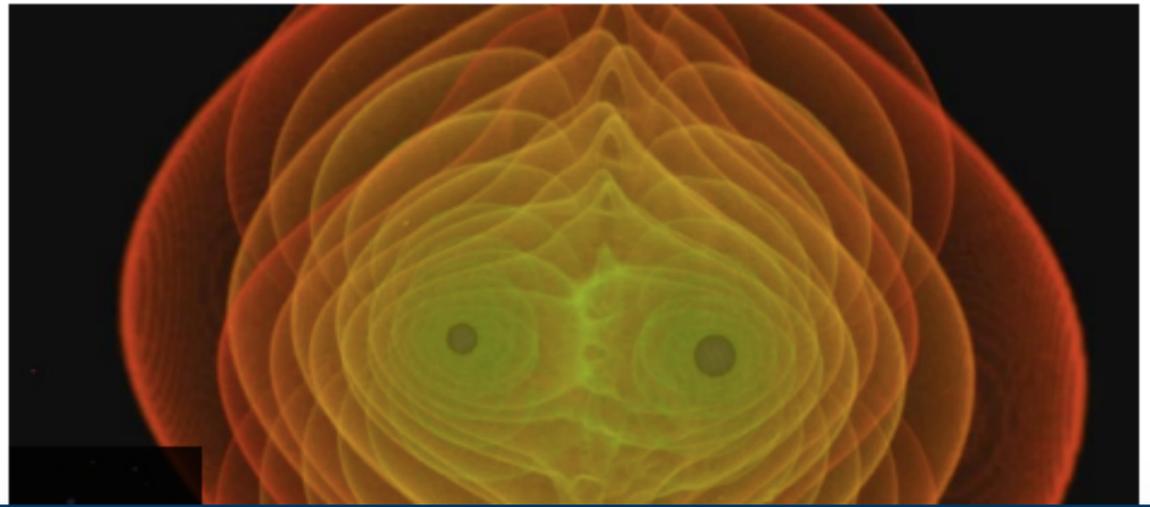
Science & Environment

# Einstein's gravitational waves 'seen' from black holes

By Pallab Ghosh  
Science correspondent, BBC News

11 February 2016

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**METRO** FREE THURSDAY, APRIL 11, 2019 THE WORLD'S MOST POPULAR FREE NEWSPAPER

**DARCEY: WHY I'M WALTZING OFF STRICTLY**  
GUILTY PLEASURES  
Pages 12-13

# AS EU DICTATES BRITAIN'S FUTURE

# WHAT BREXIT LOOKS LIKE FROM SPACE

DEAL APPEARS AS FAR, FAR AWAY AS THIS BLACK HOLE  
HELPLESS MAY SUCKED INTO THE BRUSSELS VORTEX

IT'S a gaping void that sucks in energy with an irresistible force. This first ever image of a black hole was yesterday revealed in Brussels - where the tortuous negotiations over Brexit were threatening to prove just as difficult to escape from.

EU leaders were deciding whether to agree to put off Britain's departure beyond tomorrow.

as the stalemate continues at Westminster.

But German Chancellor Angela Merkel earlier told her MPs they may well go for a longer delay than Theresa May (pictured) requested, to give time for a breakthrough.

The prime minister, who wants the UK to leave no later than June 30, gave her counterparts from the other 27 EU countries an hour-long briefing on progress.

But she then had to leave the room while they decided Britain's fate over dinner.

They had it in their power to refuse a delay - forcing her to choose between a no-deal Brexit tomorrow or invoking Article 50 to halt the process of leaving.

Irish PM Leo Varadkar said as he

Continued on Page 6

SPEEDBOAT KILLER IS FLOWN HOME TO FACE JUSTICE Page 2 HOTEL PAYS PRICE FOR OWNER'S HOMOPHOBIA Page 3

**7 DAY DEALS**

Warburton's 400g

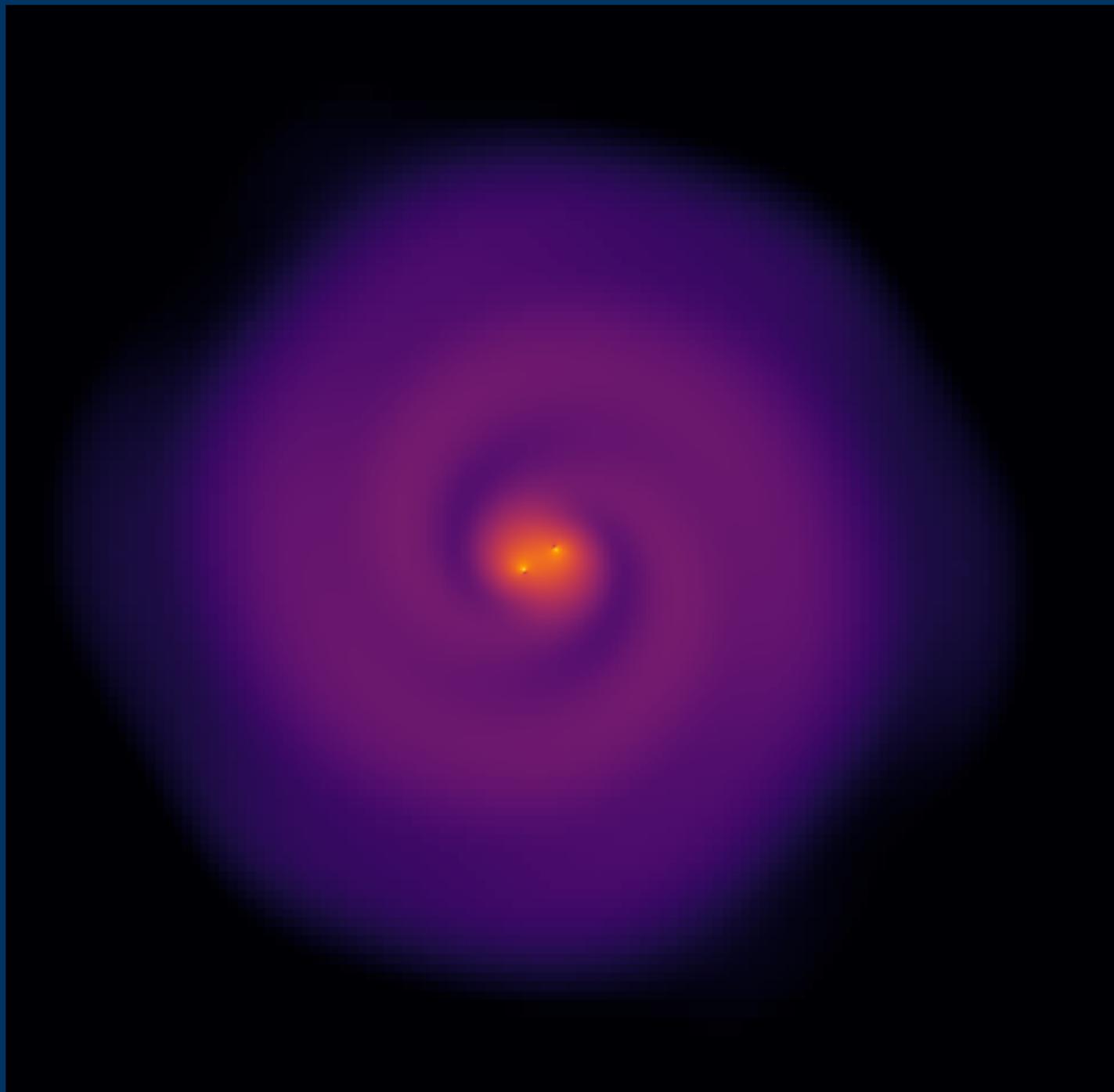
Half Price  
was £1 now 50p each

6 HOT CROSS BUNS 6 pack

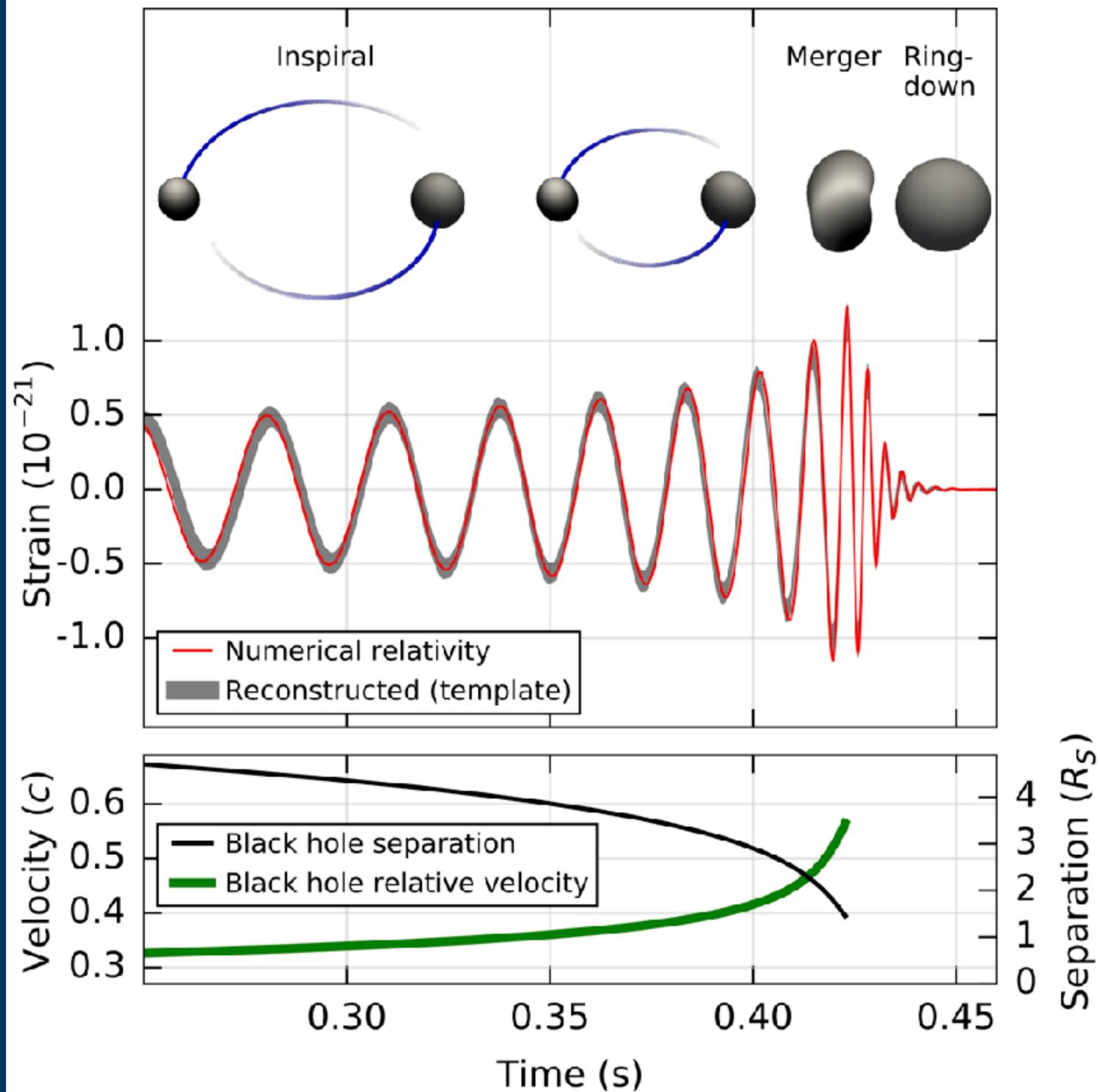
Available in store and at iceland.co.uk

Offer valid until 18.04.19 in store, online until 18.04.19, for online delivery until 18.04.19. Iceland Hot Cross Buns Recipe. Warburton's Hot Cross Buns 400g, 100g x 4. Subject to availability. UK only, excluding the Highlands. © 2019 Iceland Foods Limited.

**Iceland** the food WAREHOUSE

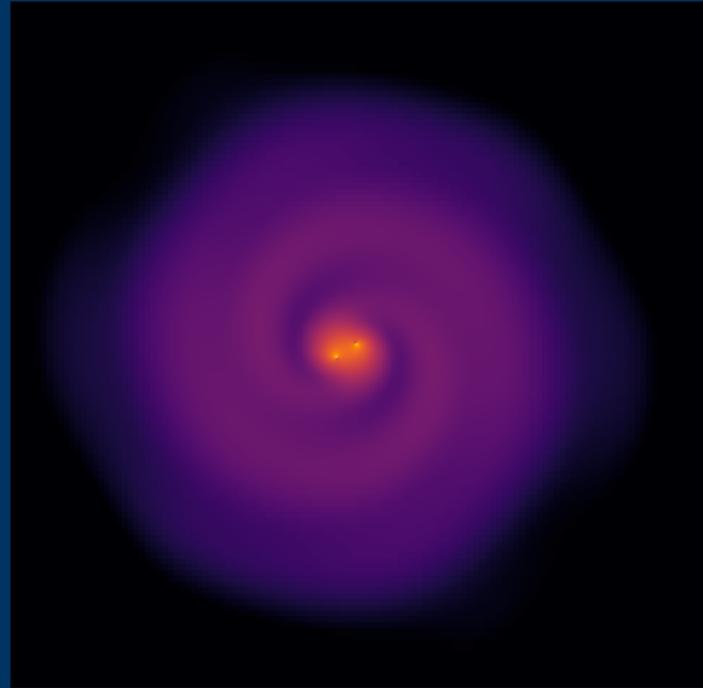


**Interesting idea:  
Can we use this new  
data to learn about the  
environments of the  
black holes that we  
observe?**

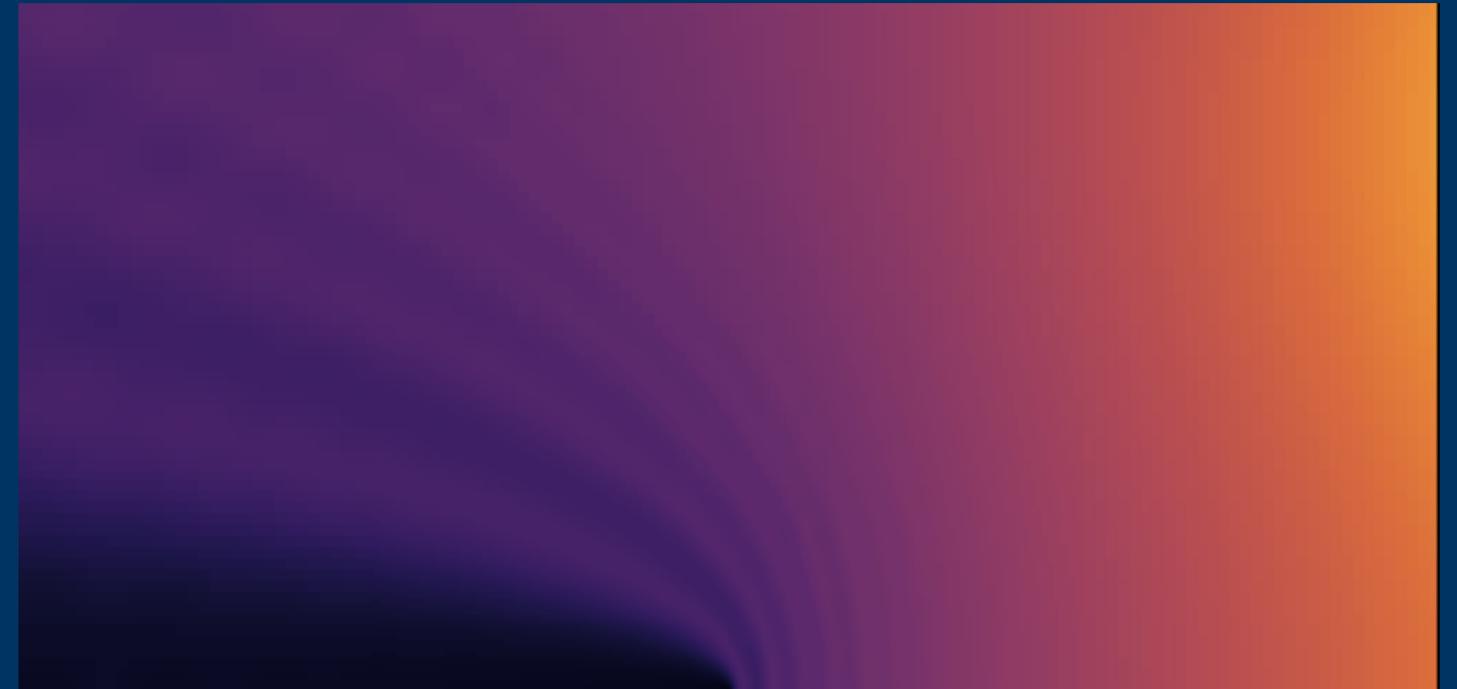


**More concrete idea:  
 Having additional  
 matter around BHs will  
 change the different  
 parts of the waveform  
 in a distinctive way**

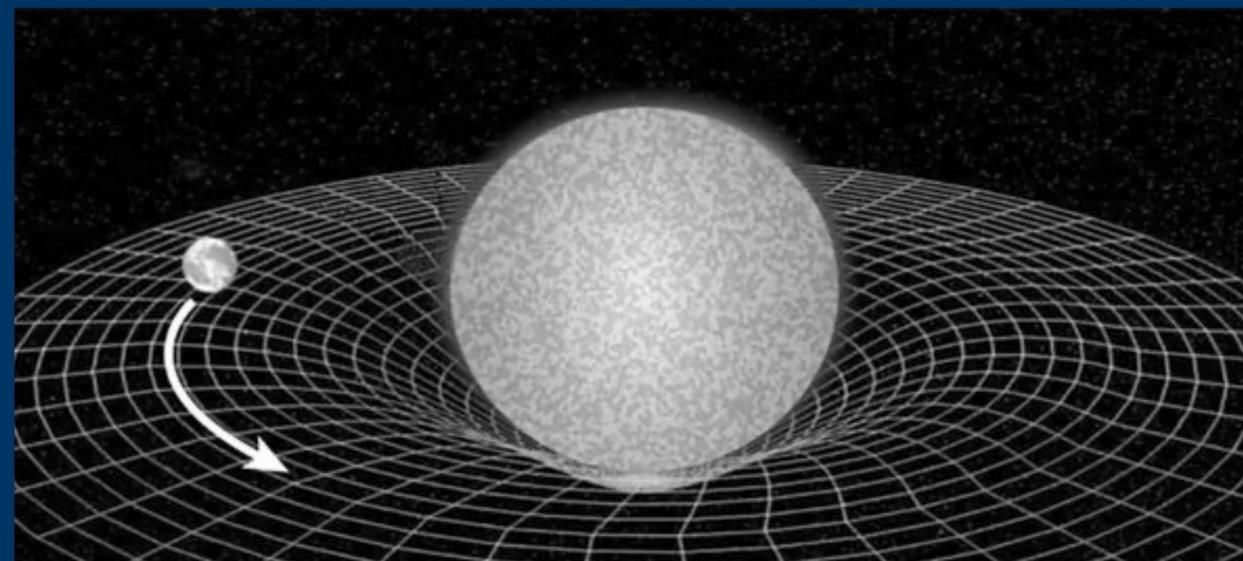
## Radiation of the environment



## Dynamical friction and accretion

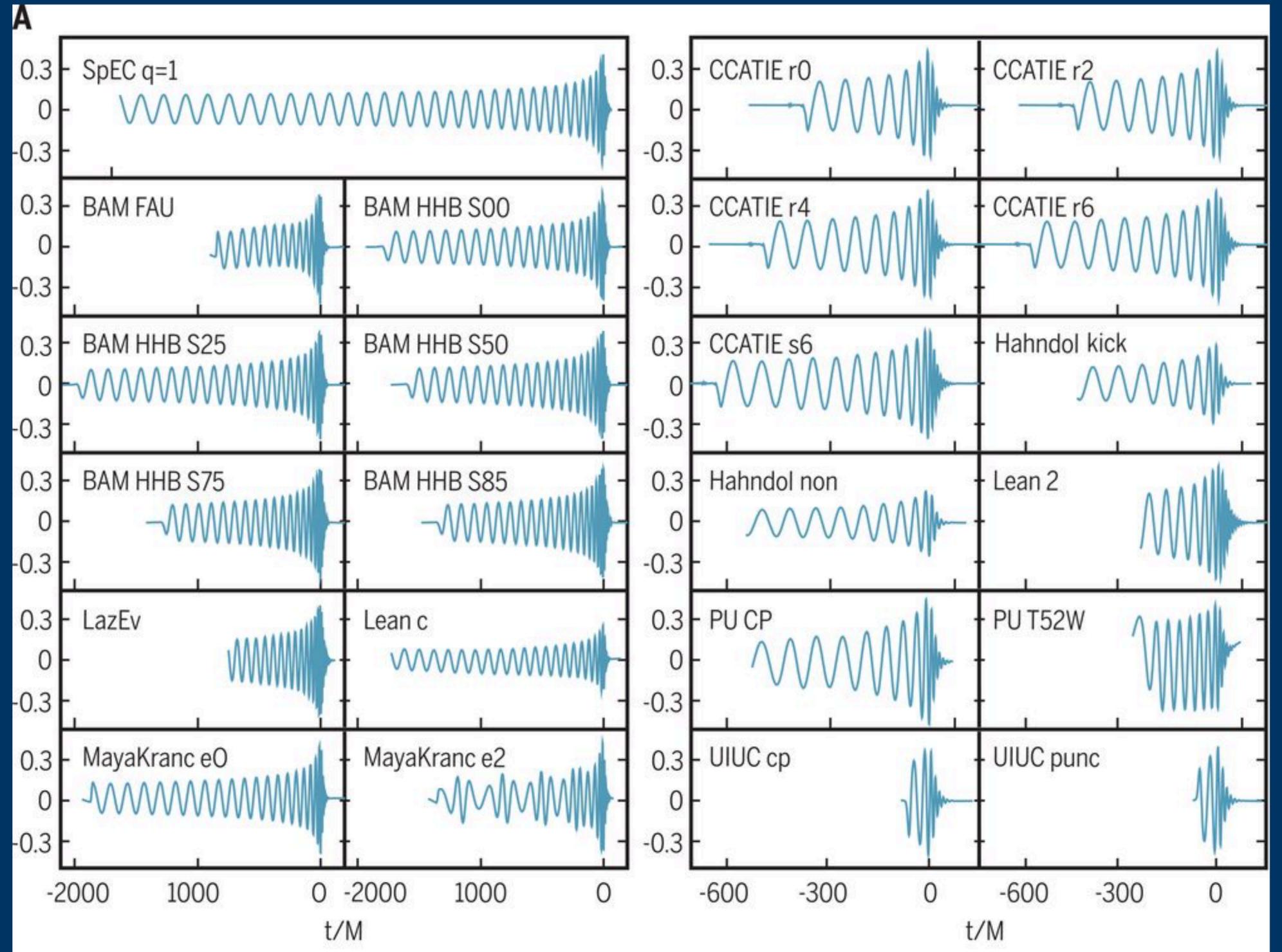


Change in curvature  
of space results in  
new trajectories

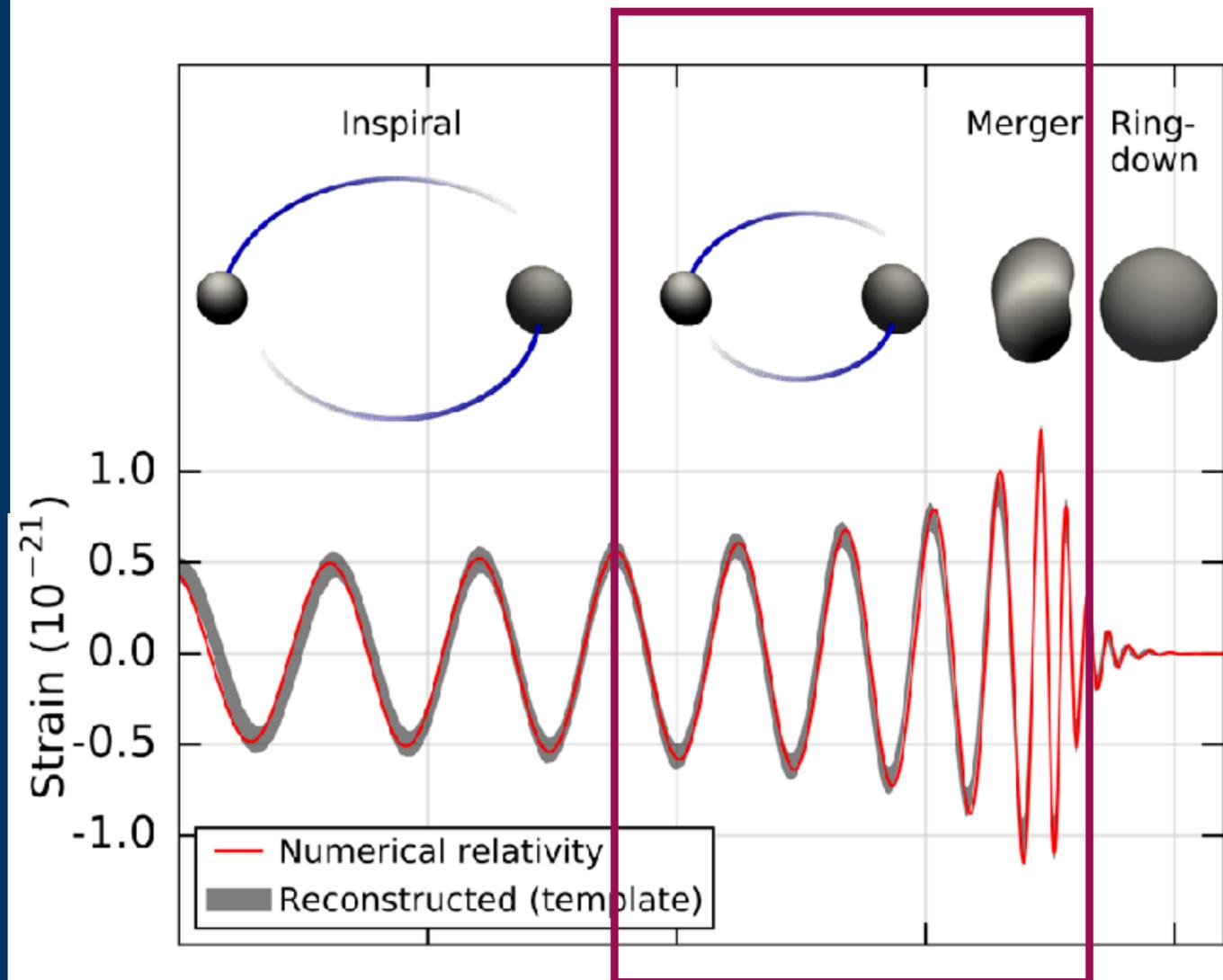


# Plan:

- Generate GW templates with environments
- Match them to signals
- Detect environments



# Numerical relativity part



**Potential problem:  
How important is it to  
have the “right” initial  
conditions for our  
simulations?**

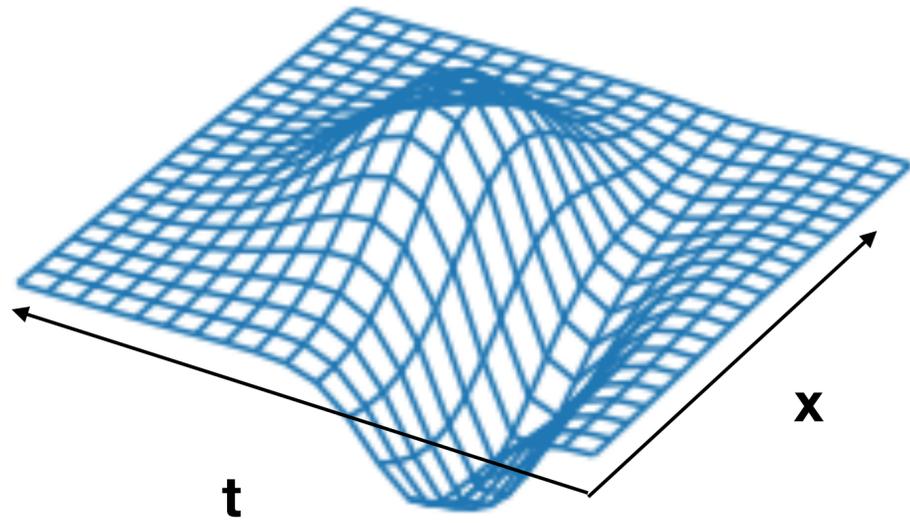
## Concern:

*We are not ready to generate template banks and do inference about the effects of matter environments on the merger part of the signal because we don't have control over our initial conditions and understand how they affect the results.*

**What is numerical relativity? Why do initial conditions matter?**

# Curved spacetime

$$ds^2 = (dt \quad dx \quad dy \quad dz) \underbrace{\begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}}_{\text{“The spacetime metric”}} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

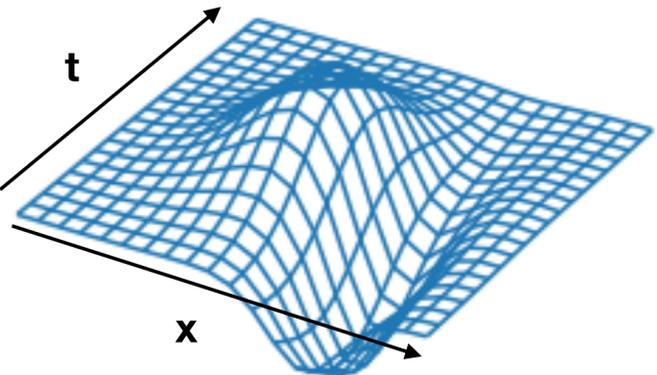


“The spacetime metric”

$$g_{ab}(t, \vec{x})$$

# What is the goal of NR?

## The metric

$$ds^2 = (dt \ dx \ dy \ dz) \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$


“The spacetime metric”

$$g_{ab}(t, \vec{x})$$

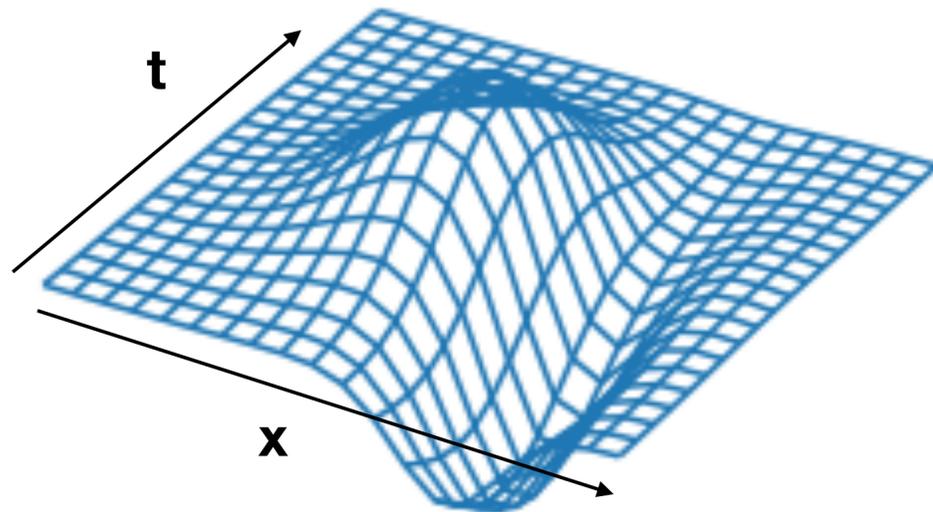
## The Einstein equation

$$\mathbf{R}_{ab} - \mathbf{R}/2 \mathbf{g}_{ab} = 8\pi \mathbf{T}_{ab}$$

$f(\partial^2 g_{ab}, \partial g_{ab}, g_{ab})$   
“Curvature”

“Energy-Momentum”

# The Einstein equation tells us how the metric should look, given some energy/matter distribution



$$R_{ab} - R/2 g_{ab} = 8\pi T_{ab}$$

Four constraint equations for any time slice - non linear elliptic/Poisson equation

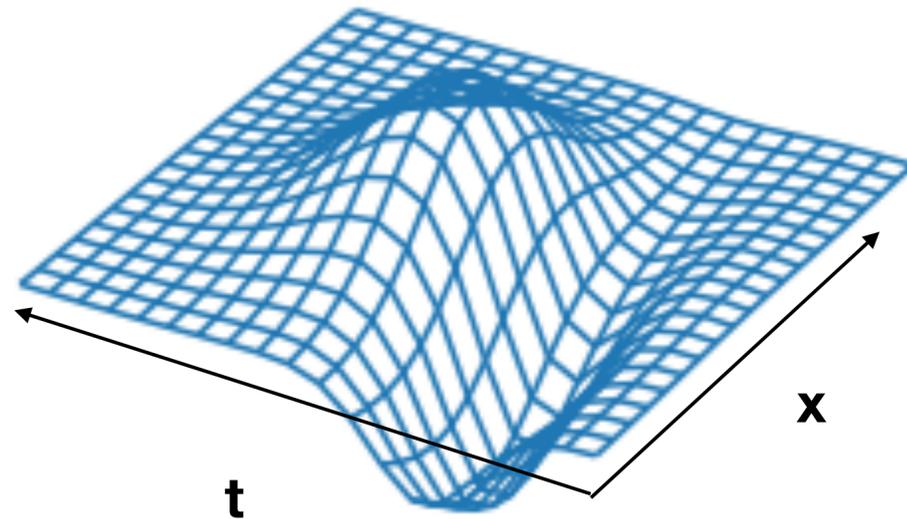
$$\frac{\partial^2 g}{\partial x^2} + \text{non linear terms} = f(\text{energy, momentum})$$

An evolution equation for all time - non linear hyperbolic/wave equation

$$\frac{\partial^2 g}{\partial t^2} - \frac{\partial^2 g}{\partial x^2} + \text{non linear terms} = f(\text{energy, momentum})$$

*“Matter tells spacetime how to curve...”*

# The metric determines the motion of matter



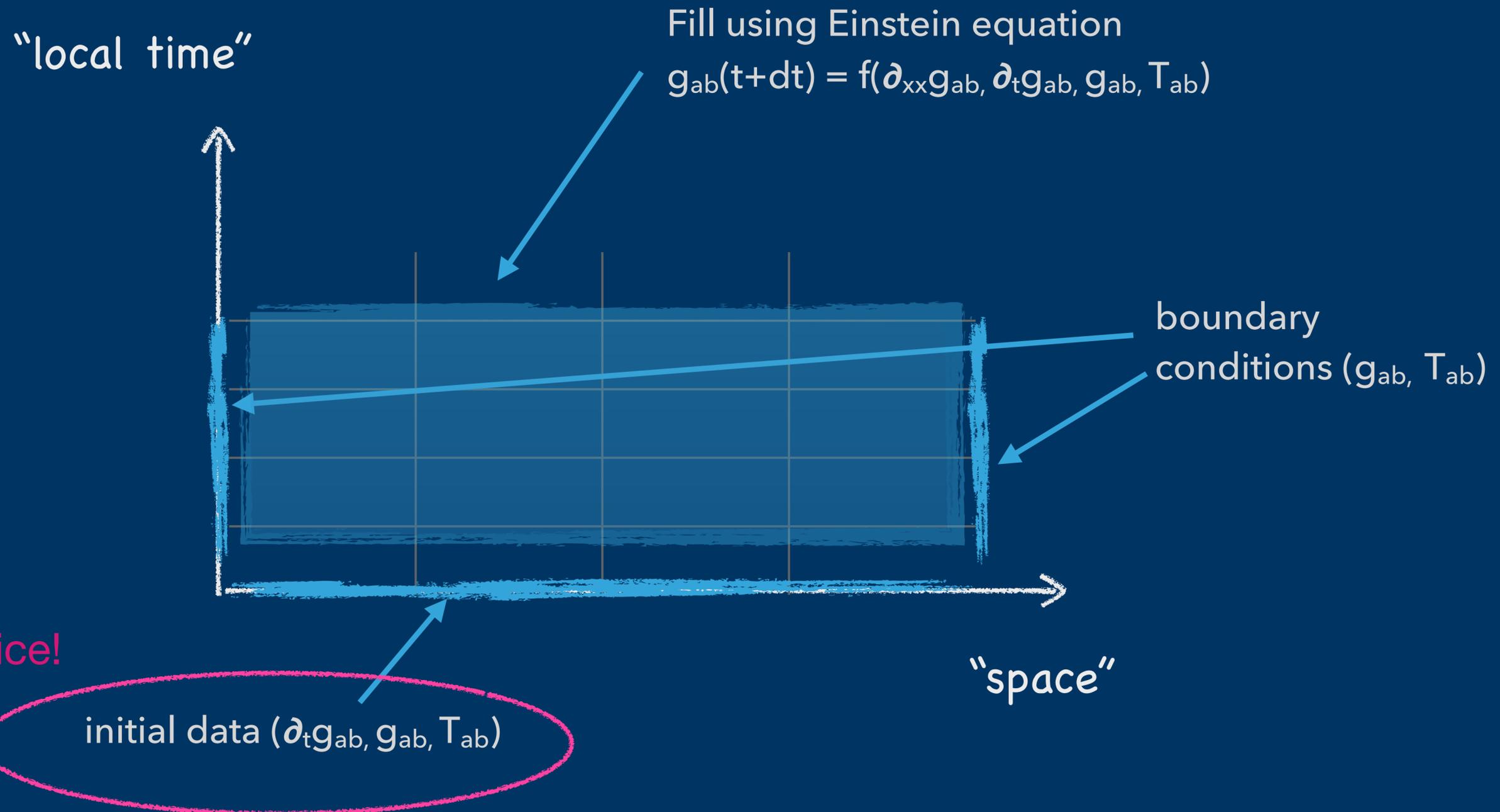
Continuity equation

$$\frac{\partial \rho}{\partial t} + \underbrace{\nabla}_{g_{ab}} \cdot \mathbf{j} = \underbrace{\text{source}}_{g_{ab}}$$

$$\nabla^a (R_{ab} - R/2 g_{ab}) = \nabla^a (8\pi T_{ab}) = 0$$

“...spacetime tells matter how to move.”

# In reality numerical relativity is done in a finite region of spacetime



Not a free choice!

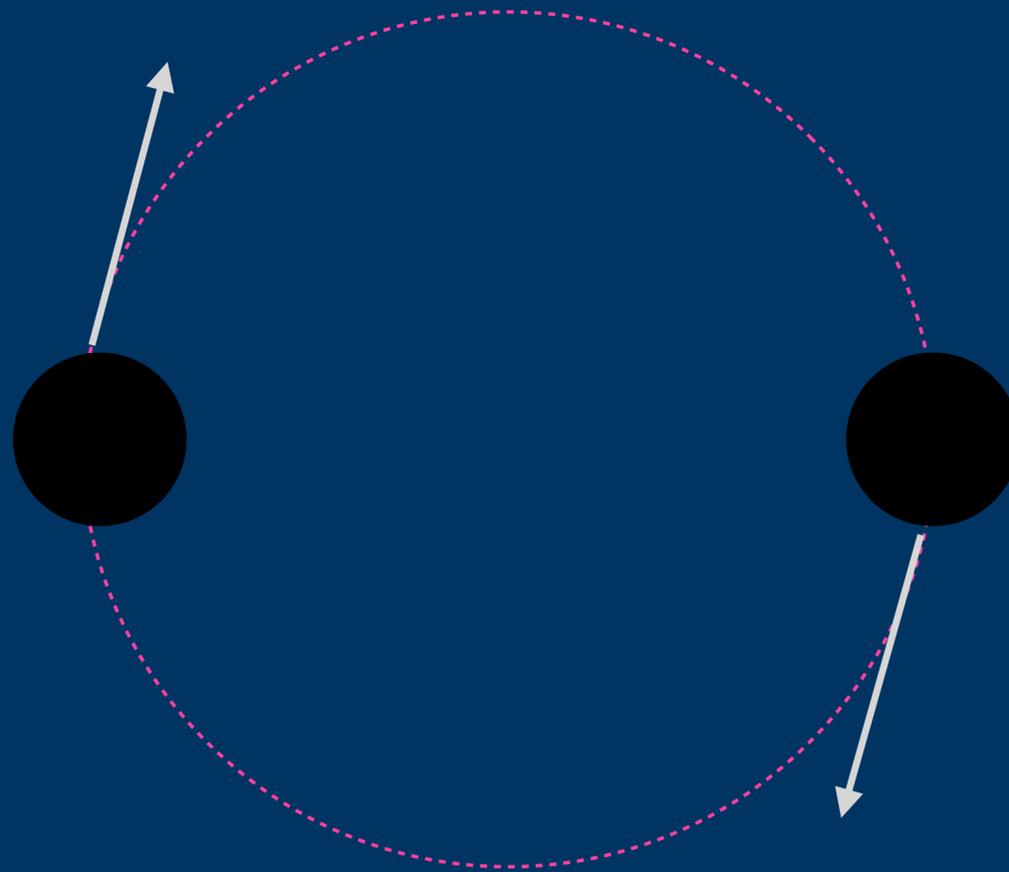
# Numerical relativity - the initial condition problem

describe BHs

(Masses, momenta, using PN calculations)



make 16 (arbitrary?)  
choices of free components



is this the system  
I was looking for?



solve 4 constraints  
on energy and momentum

(Are orbits actually circular, are masses what I wanted?)

# Numerical relativity - the initial condition problem

describe BHs

(Masses, momenta, using PN calculations)



make 16 (arbitrary?)  
choices of free components

Iterate



is this the system  
I was looking for?



solve 4 constraints  
on energy and momentum

(Are orbits actually circular, are masses what I wanted?)

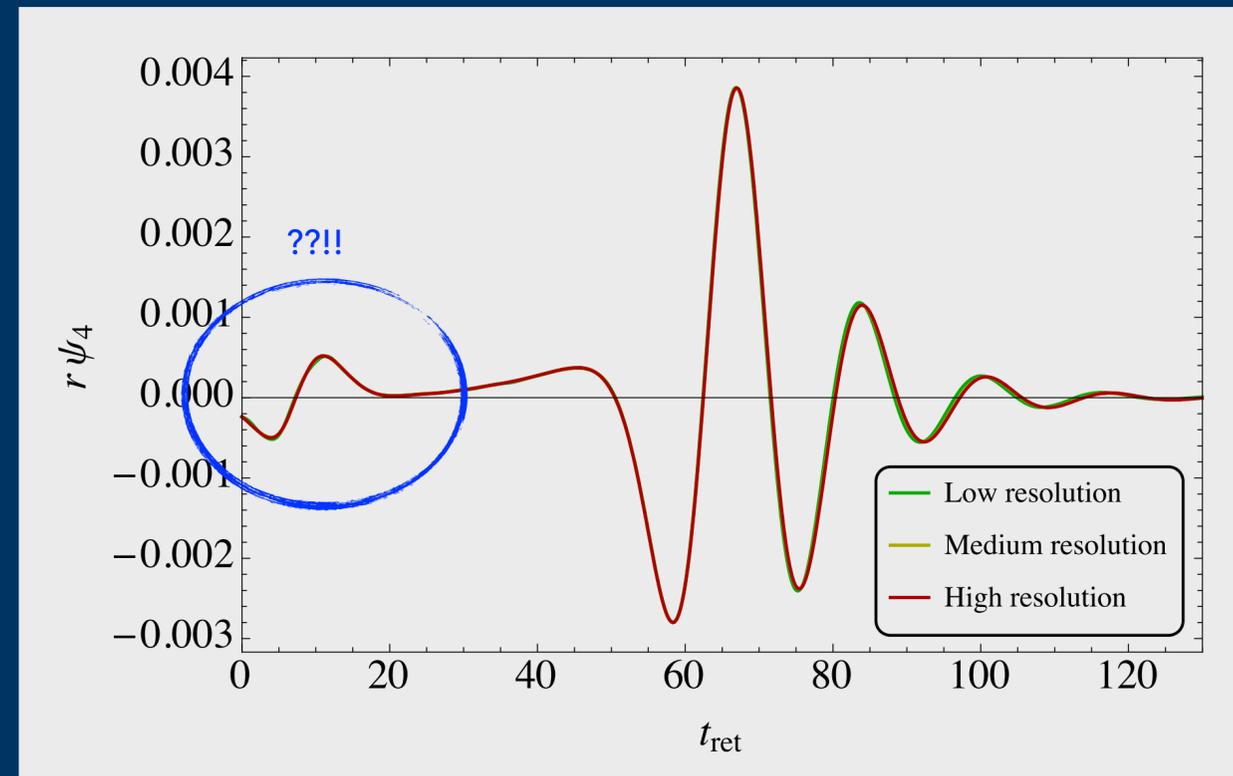
# Numerical relativity - the initial condition problem

describe BHs

(Masses, momenta, using PN calculations)

make 16 (arbitrary?) choices of free components

This will just have to be good enough



is this the system I was looking for?

solve 4 constraints on energy and momentum

**The “right” initial condition means:**

- 1. Solves the energy and momentum constraints  
of GR**
- 2. Is the correct physical scenario that we are  
looking for**

# Dark matter environments

Self  
interaction



**Particle dark matter  
parameters**



Fraction of  
total DM  
(locally/  
globally)

Mass



Standard  
model  
interactions

Self  
interaction

**(Small, ignore for this talk)**

Mass

**Particle dark matter  
parameters**

Fraction of  
total DM  
(locally/  
globally)

Standard  
model  
interactions

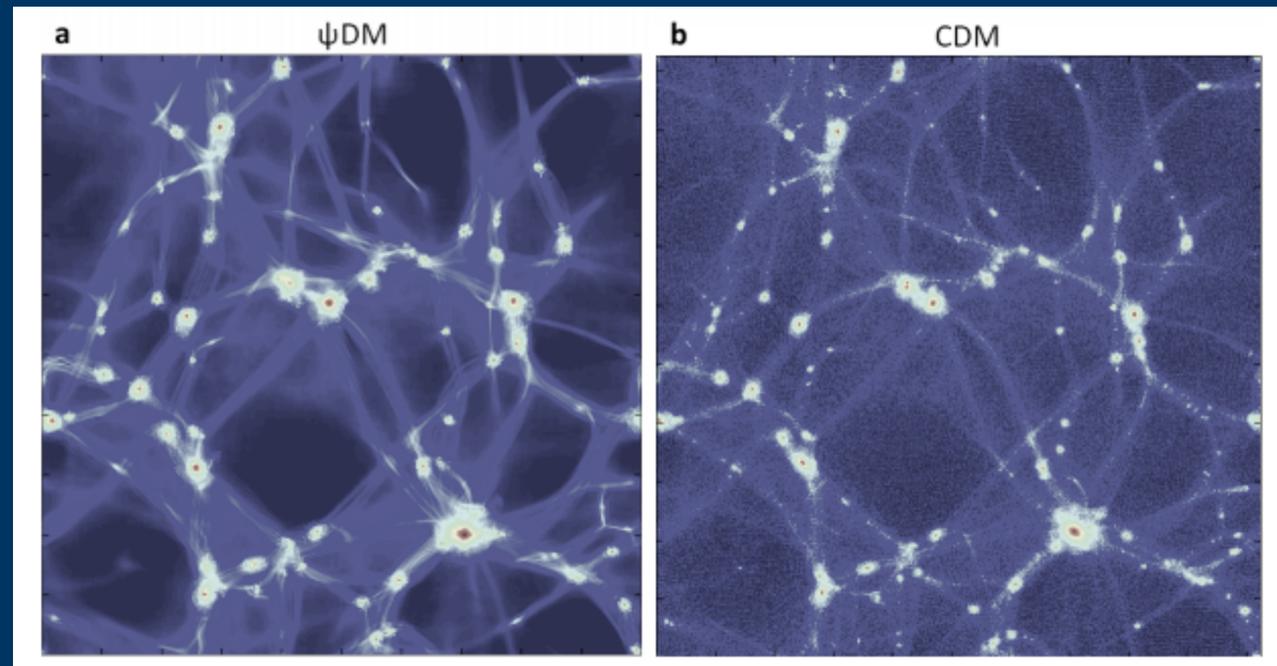
**(Small, ignore for this talk)**

# A vast range of potential masses

Mass

Particle dark matter  
parameters

Wave DM  
e.g. axions  
 $10^{-23}$  eV - 1 eV



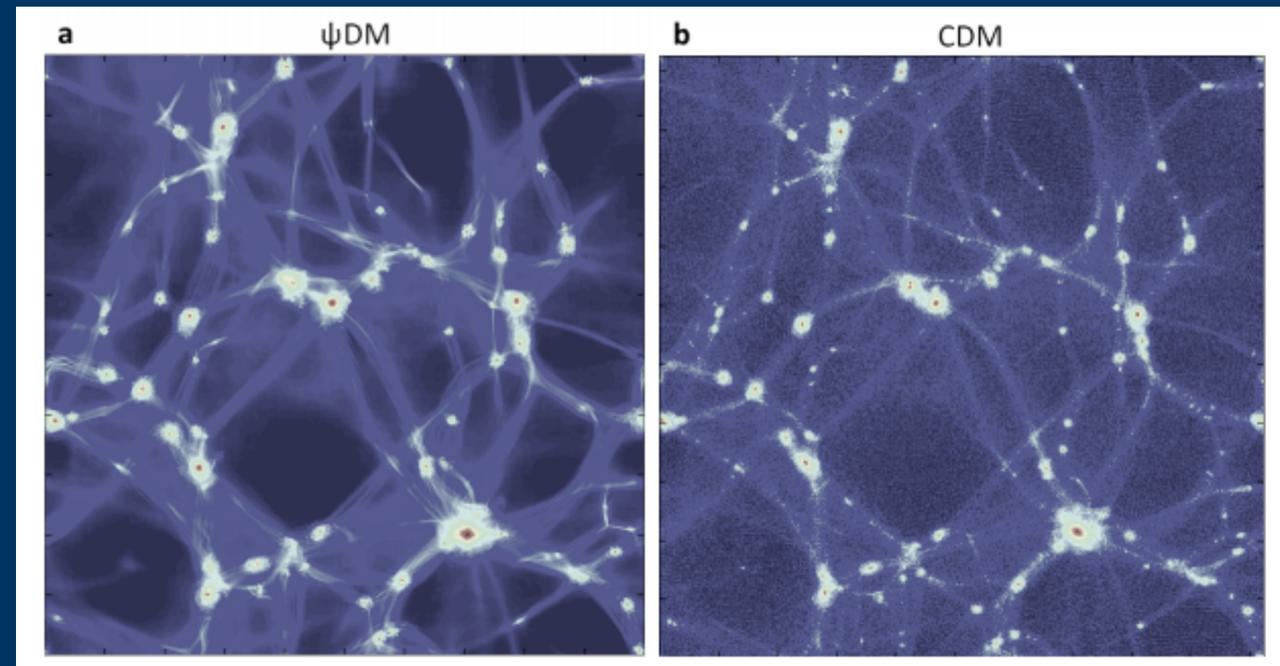
Particle DM  
e.g. WIMPS  
1 eV -  $10^{13}$ eV

# A useful distinction is between wave-like and particle DM

Mass

Particle dark matter parameters

Wave DM  
e.g. axions  
 $10^{-23}$  eV - 1 eV

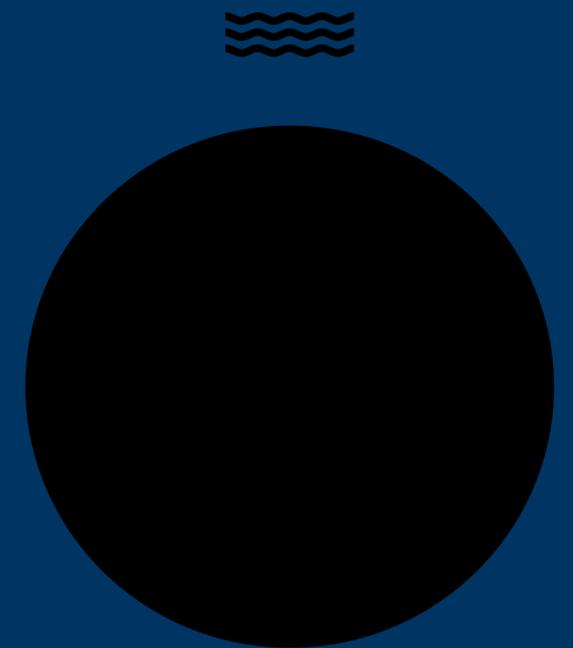
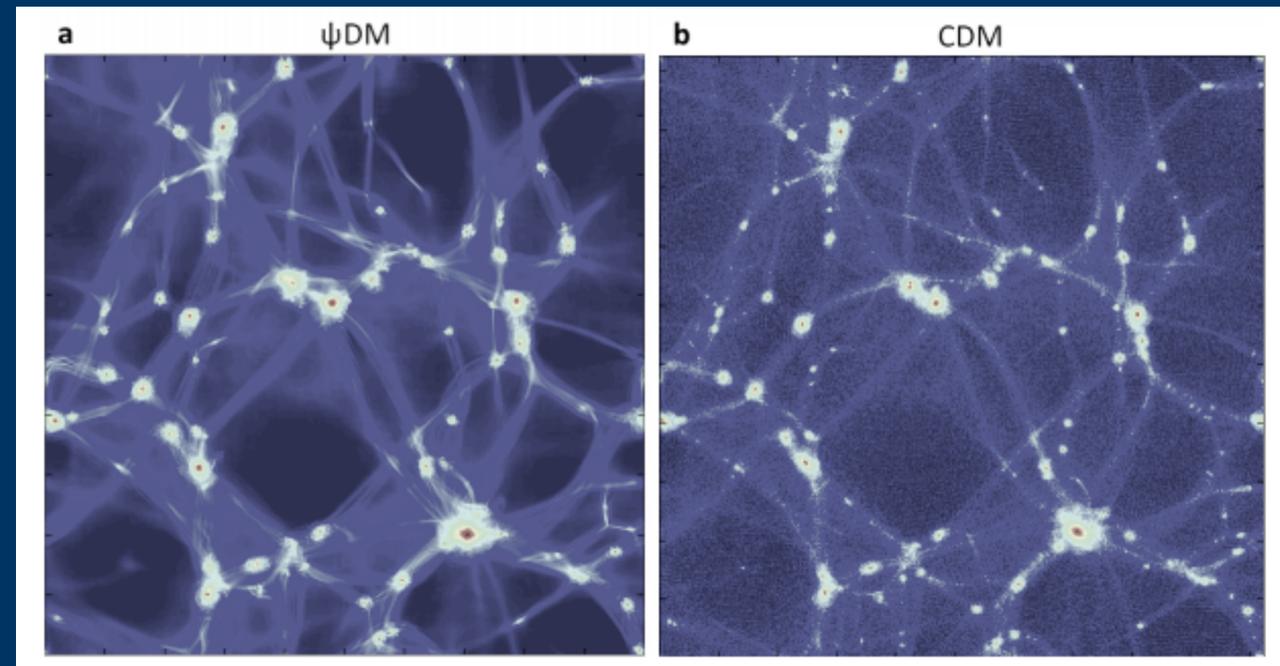
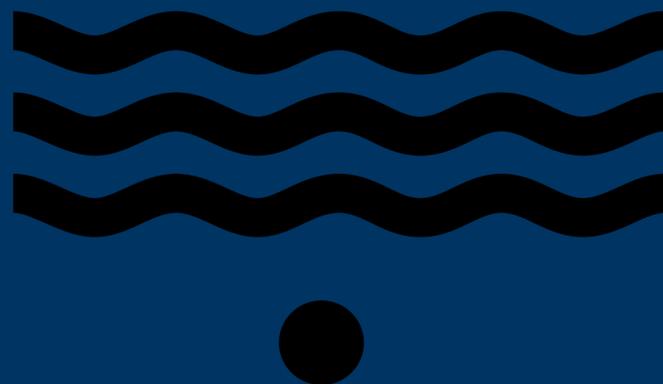


Particle DM  
e.g. WIMPS  
1 eV -  $10^{13}$ eV

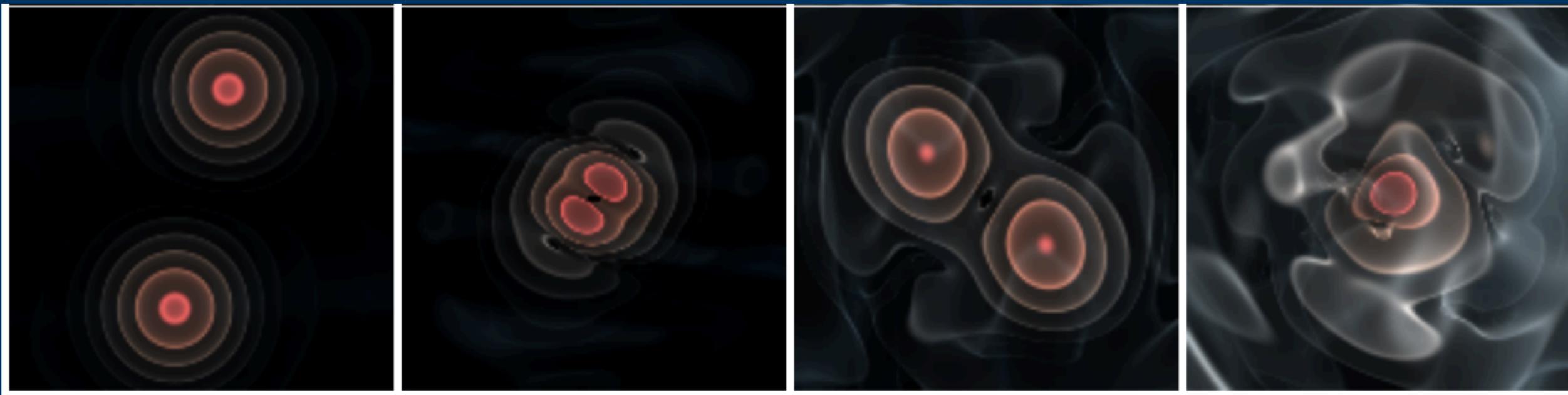
# A useful distinction is between wave-like and particle DM

Mass

Particle dark matter parameters



Schive et al. 2014  
Cosmic structure as the quantum interference of a coherent dark wave



Schwabe et al, 2016  
Simulations of solitonic core mergers in  
ultralight axion dark matter cosmologies

**Particle dark matter  
parameters**

**Fraction of  
total DM  
(locally/  
globally)**

**Detection / constraints rely heavily on fraction of  
DM composed by the candidate, and its  
distribution (uniform/clumpy)**

**Bad news: average DM density is very low :-)**

Barausse et al. 2014

Can environmental effects spoil precision gravitational-wave astrophysics?

(Answer: Broadly no - for inspiral and ringdown, assuming uniform density)

# What do you mean “low”?

$$\rho \sim 1 \text{ GeV/cm}^3 \text{ or } 1 M_{\odot}/\text{pc}^3$$

(Particle physicist)

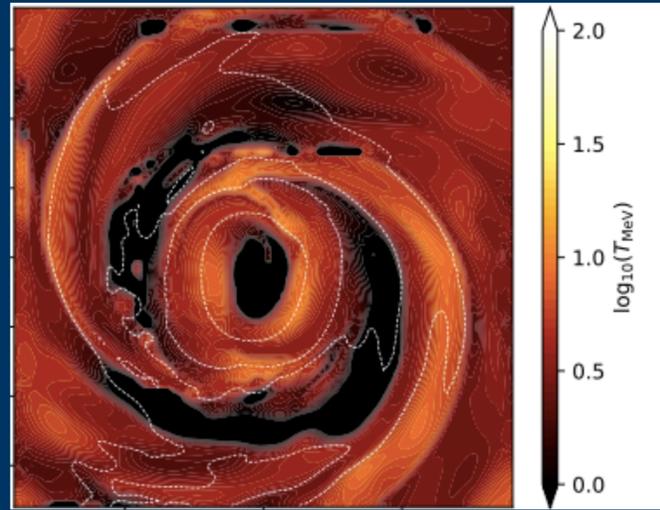
(Astrophysicist)

# What do you mean “low”?

$$\frac{\rho}{1/R_s^2} \sim 10^{-30} \left( \frac{M_{BH}}{10^6 M_\odot} \right)^2$$

(Numerical relativist)

**What DM density enhancement is required to have an observable impact on GW signals? Do such enhancements arise naturally?**



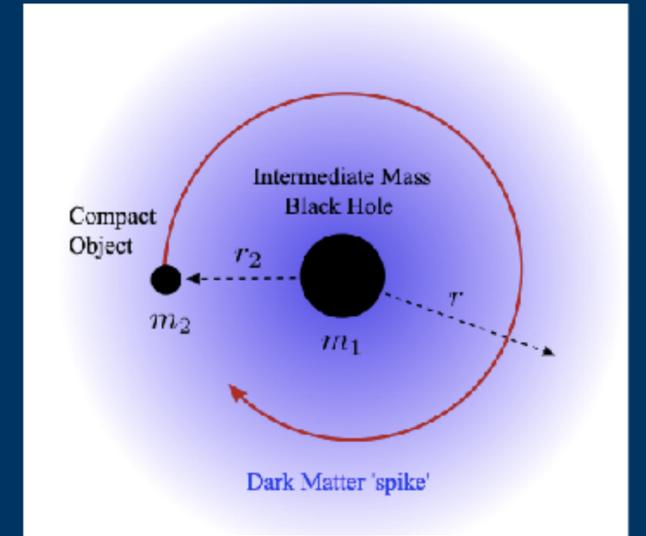
Dietrich et. al. 2019  
Cooling binary neutron star remnants via nucleon-nucleon-axion bremsstrahlung

Interactions e.g. bremsstrahlung, or attractive self interactions

Bamber et. al. 2021  
Growth of accretion driven scalar hair around Kerr black holes



Kavanagh et. al. 2020, Coogan et. al. 2022  
Measuring the dark matter environments of black hole binaries with gravitational waves



Dark matter minispikes (adiabatic growth, accretion)

## Superradiance

Review by Brito et. al. (updated 2020)  
Superradiance: New Frontiers in Black Hole Physics

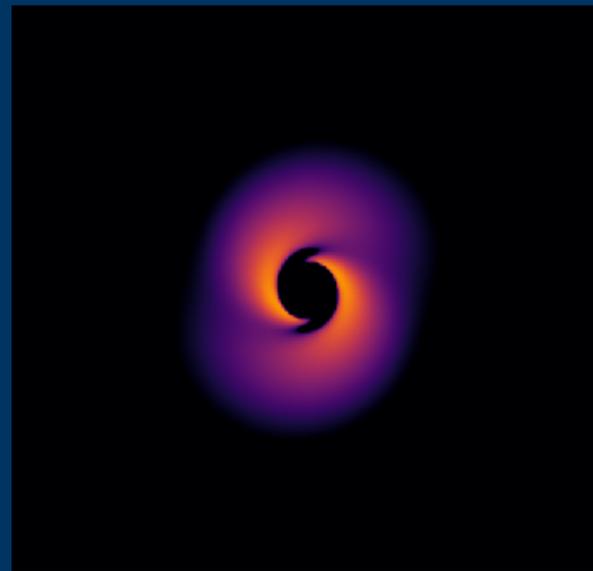


Image credit: Helfer / Clough

## Dark matter overdensity scenarios

Exotic compact objects e.g. boson stars

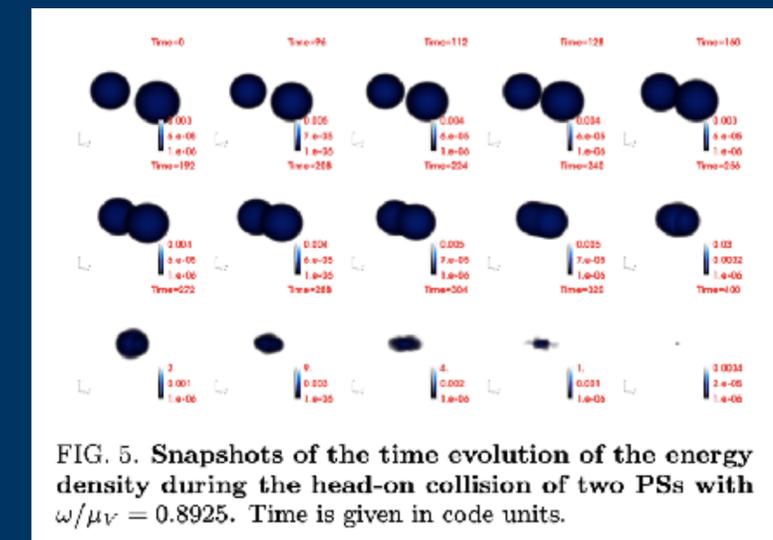
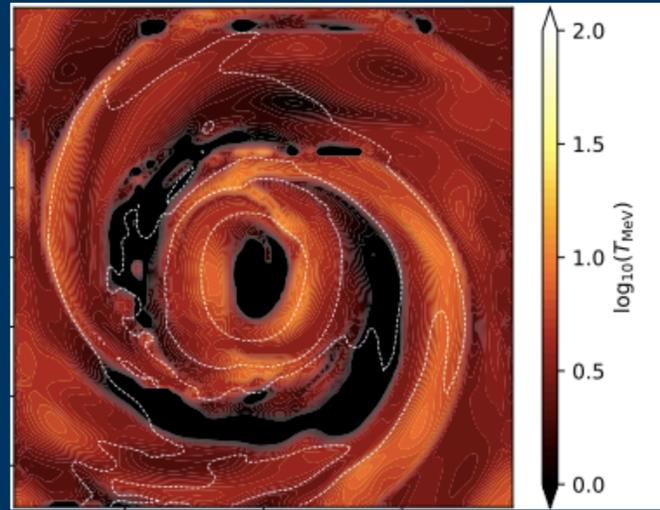


FIG. 5. Snapshots of the time evolution of the energy density during the head-on collision of two PSs with  $\omega/\mu_V = 0.8925$ . Time is given in code units.

Bustillo et. al. 2021  
GW190521 as a merger of Proca stars: a potential new vector boson of  $8.7 \times 10^{-13}$  eV



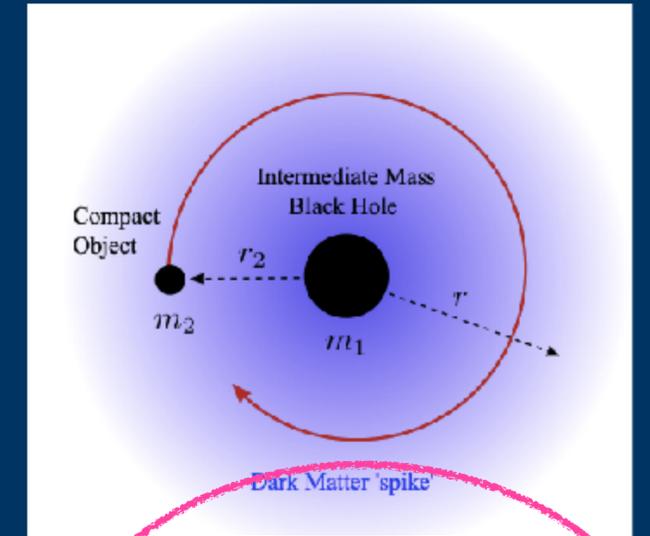
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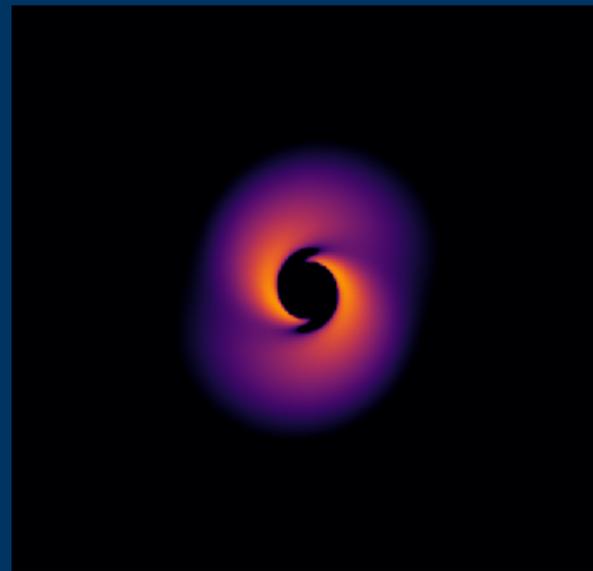


Image credit: Helfer / Clough

## Dark matter overdensity scenarios

Focus of this talk!

Exotic compact objects e.g. boson stars

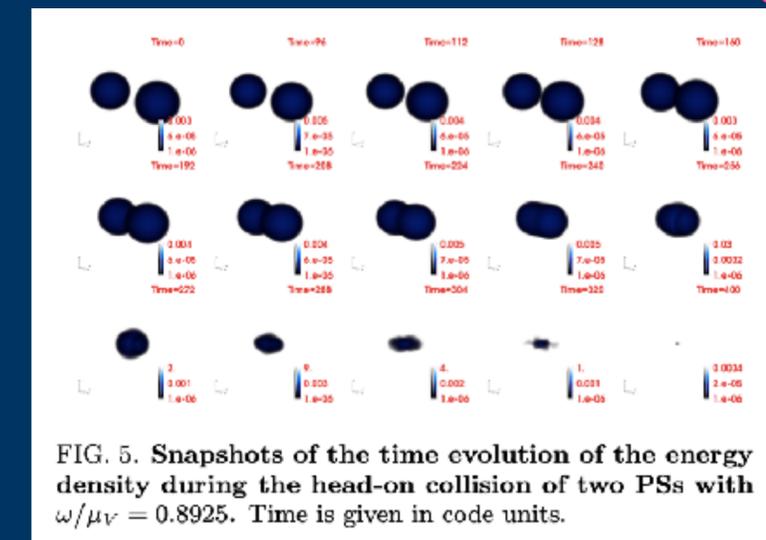
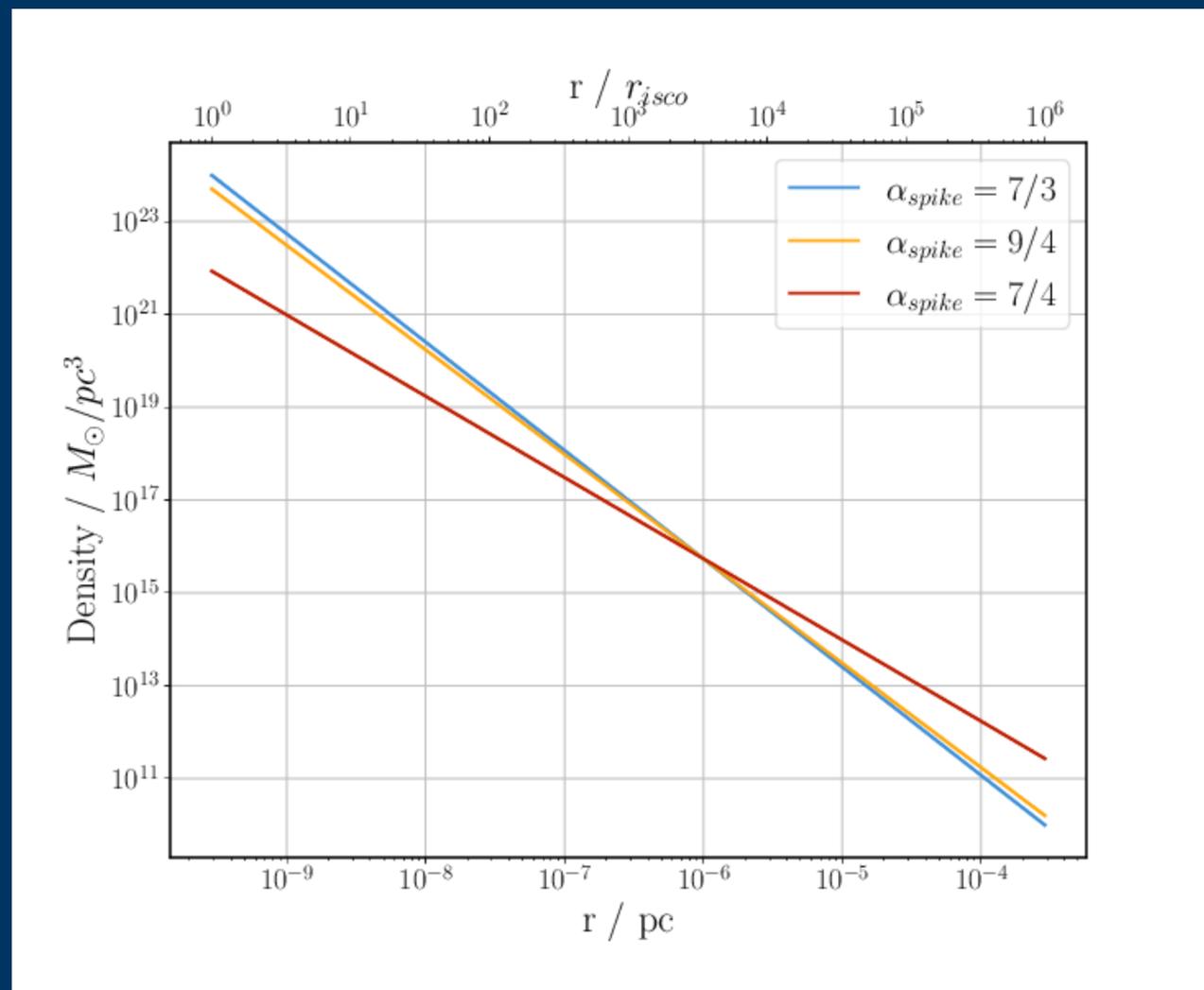


FIG. 5. Snapshots of the time evolution of the energy density during the head-on collision of two PSs with  $\omega/\mu\nu = 0.8925$ . Time is given in code units.

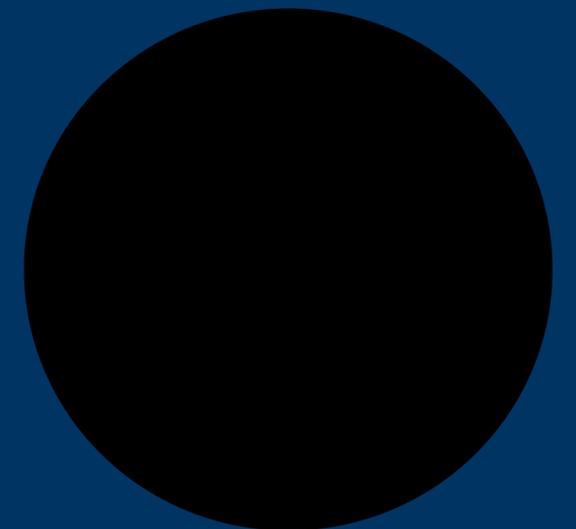
Bustillo et. al. 2021  
GW190521 as a merger of Proca stars: a potential new vector boson of  $8.7 \times 10^{-13}$  eV

# Accretion in the particle DM case - “DM spikes”



Gondolo & Silk found that adiabatic growth led to a DM overdensity described by a power law

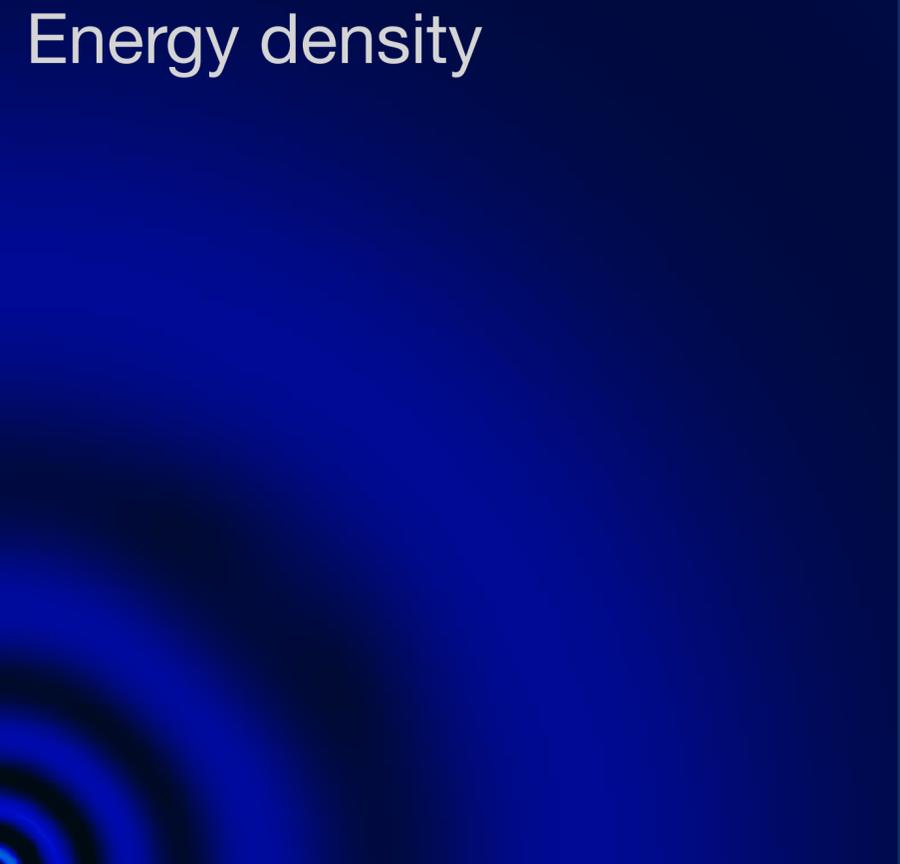
$$\rho \sim \rho_0 \left( \frac{r}{r_0} \right)^{-\gamma}$$



- Becker et.al. 2021  
Circularization vs. Eccentrification in Intermediate Mass Ratio  
Inspirals inside Dark Matter Spikes

# Accretion in the low mass DM case - scalar accretion

Energy density



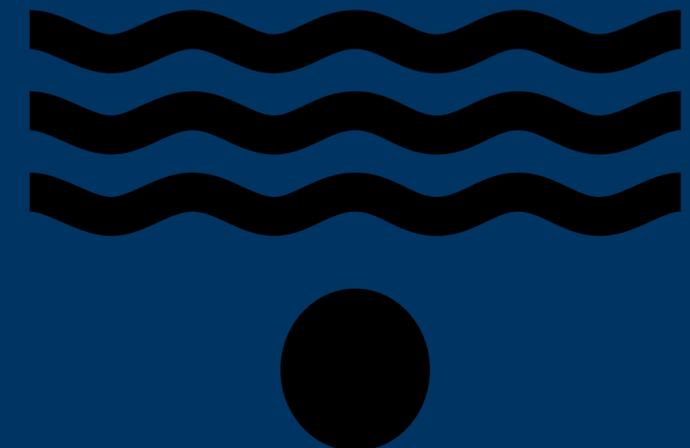
DM overdensity described by power law plus oscillations on the scale of the Compton wavelength of the light particle

In the isolated BH case solutions are known exactly (they are the confluent Heun functions)

Black Hole Hair from Scalar Dark Matter  
Lam Hui, Daniel Kabat, Xinyu Li, Luca Santoni, Sam S. C. Wong  
JCAP 1906 (2019) no.06, 038

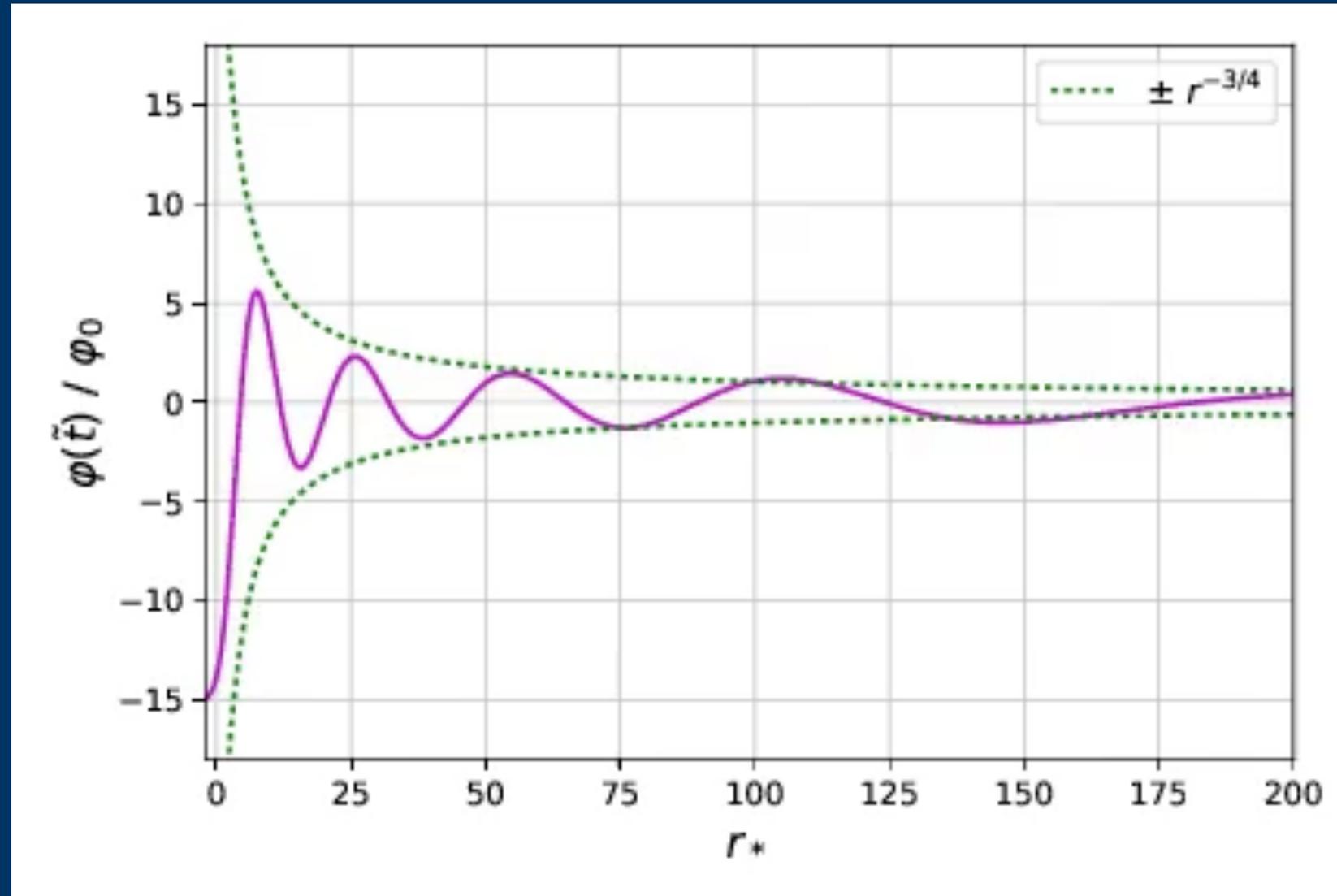
Clough et. al. 2019,  
Bamber et. al. 2021

Growth of accretion driven scalar hair around Kerr black holes



# Accretion in the wave DM case - scalar accretion

Field profile

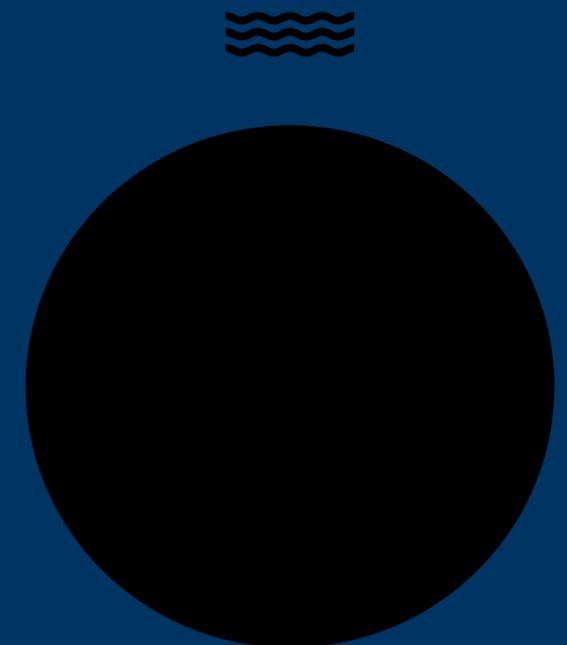
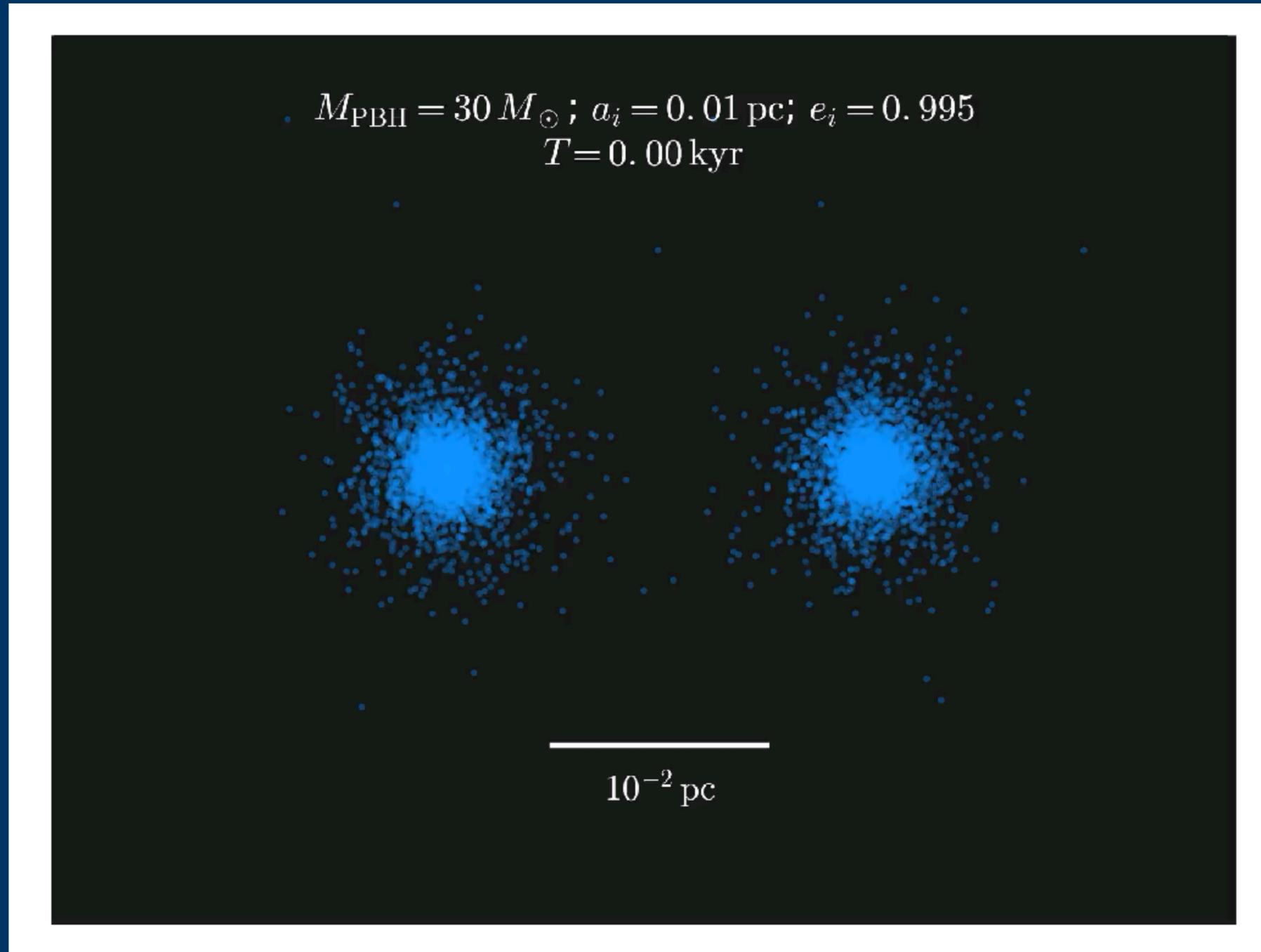


Clough et. al. 2019,  
Bamber et. al. 2021

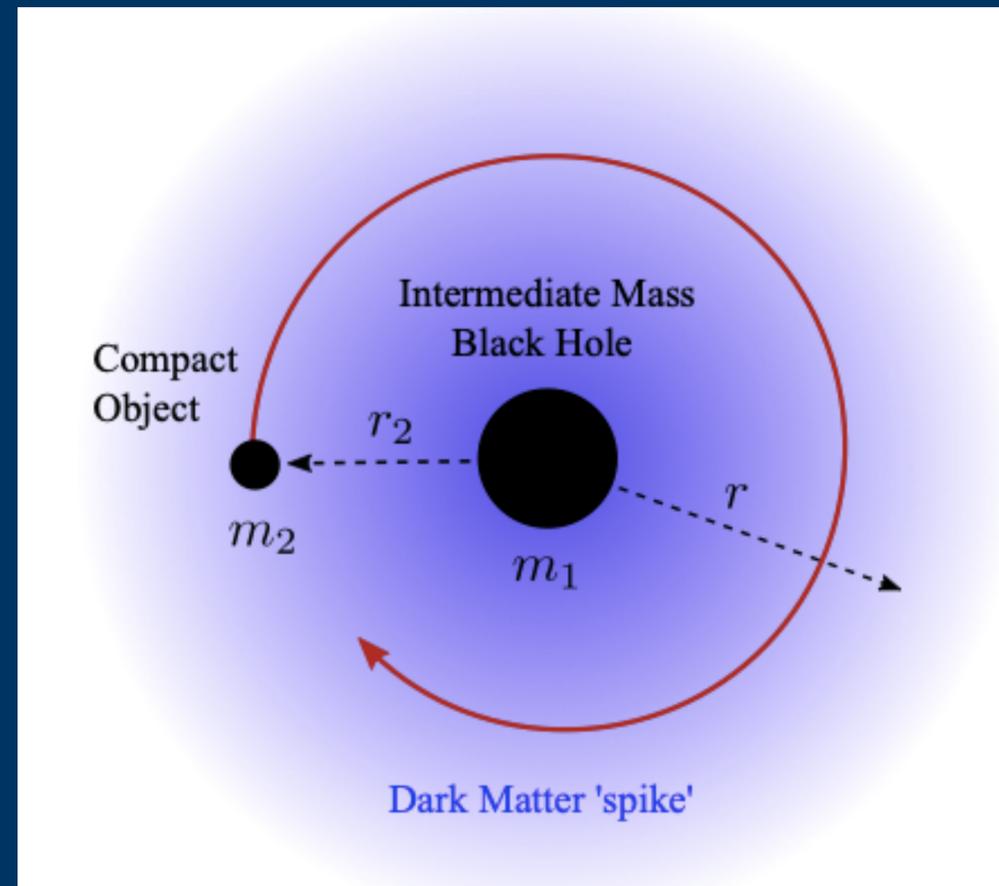
Growth of accretion driven scalar hair around Kerr black holes

**Ok, so maybe you have concentrations of dark matter around isolated black holes, but do you have them around binaries?**

# Binaries in the particle DM case

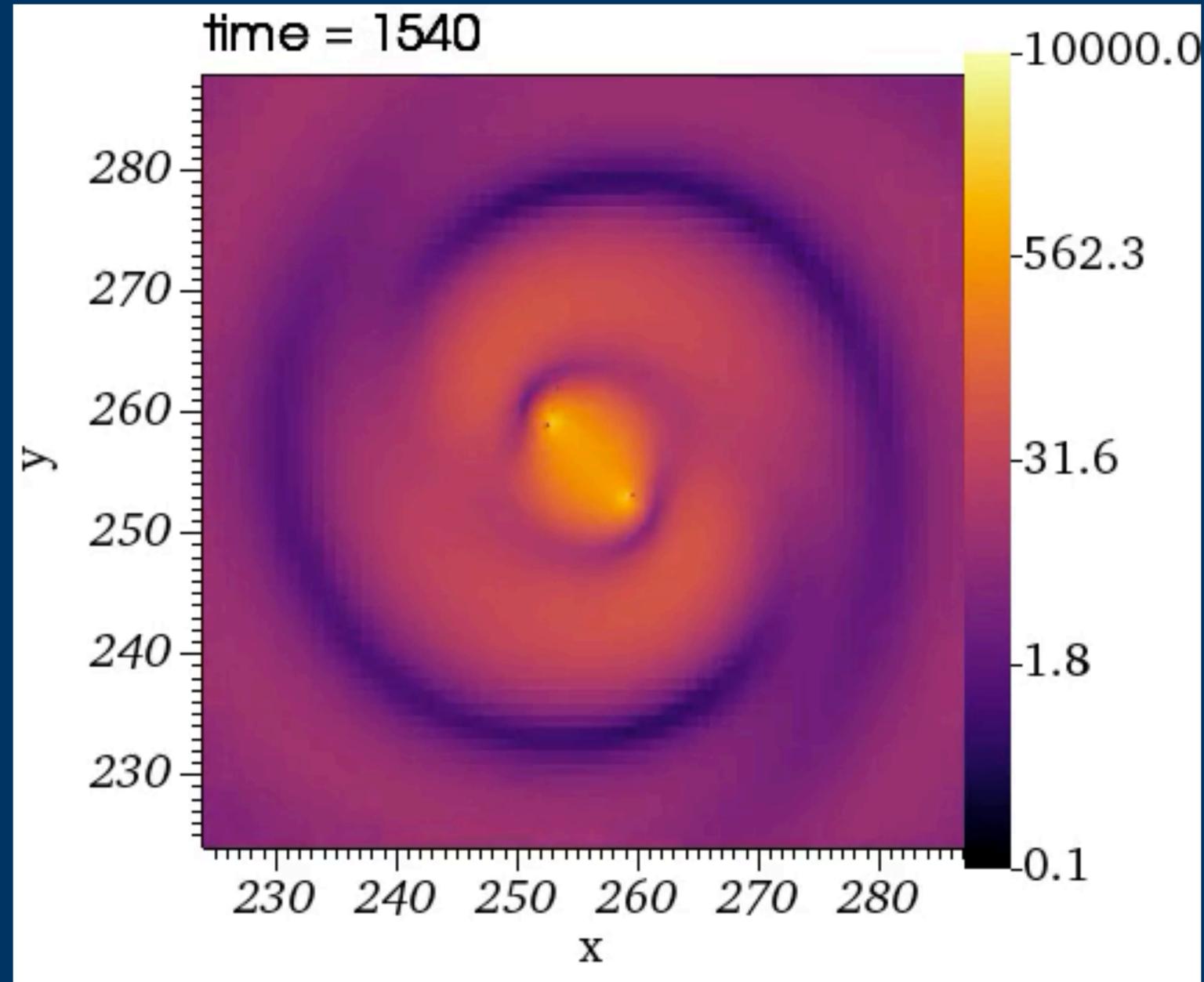


# As a result there is a focus on high mass ratio merger events



Kavanagh et. al. 2020, Coogan et. al. 2022  
Detecting dark matter around black holes with gravitational waves: Effects of dark-matter dynamics on the gravitational waveform

# Binaries in the wave DM case

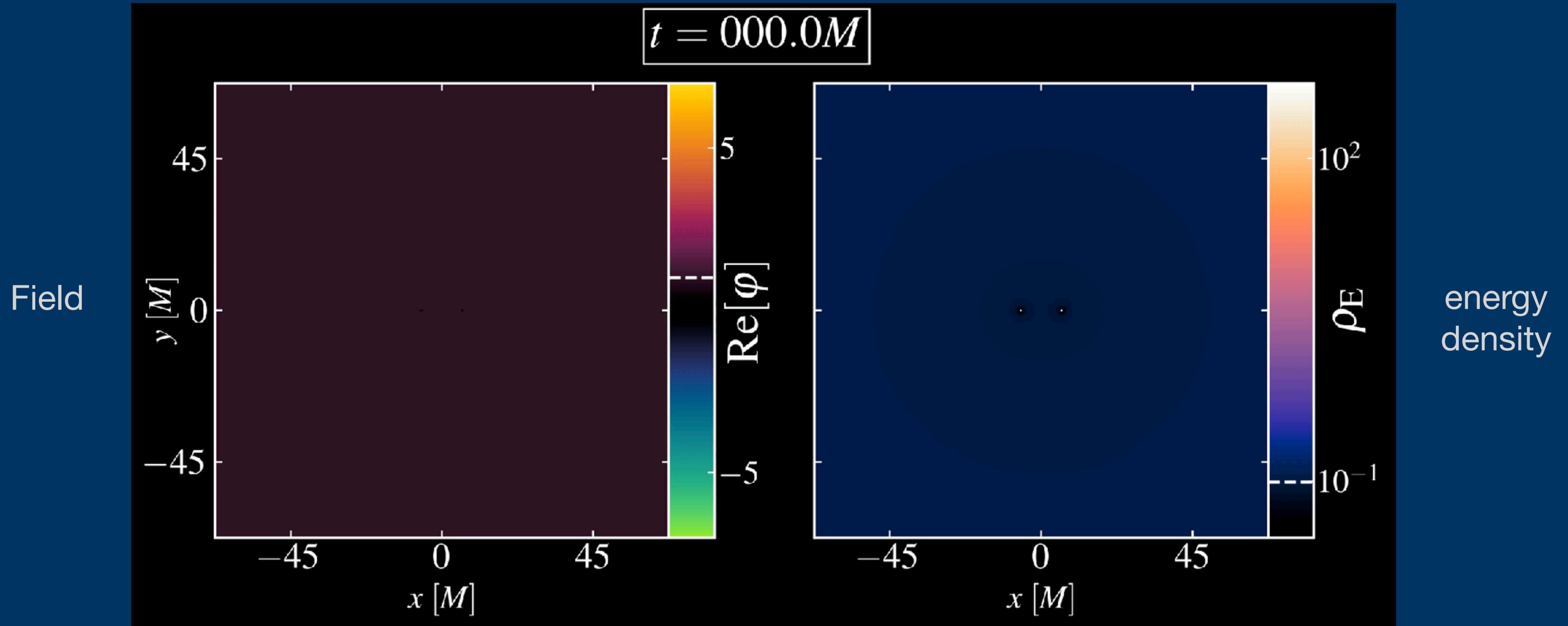


Bamber et. al., 2022

Black hole merger simulations in wave dark matter environments



# Binaries in the wave DM case

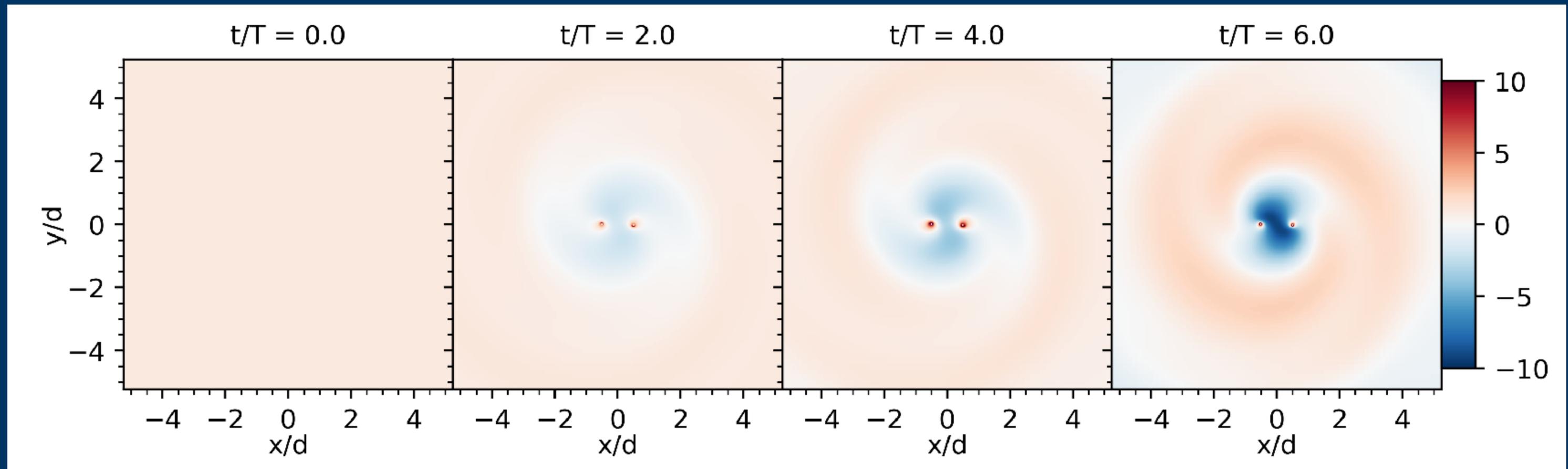
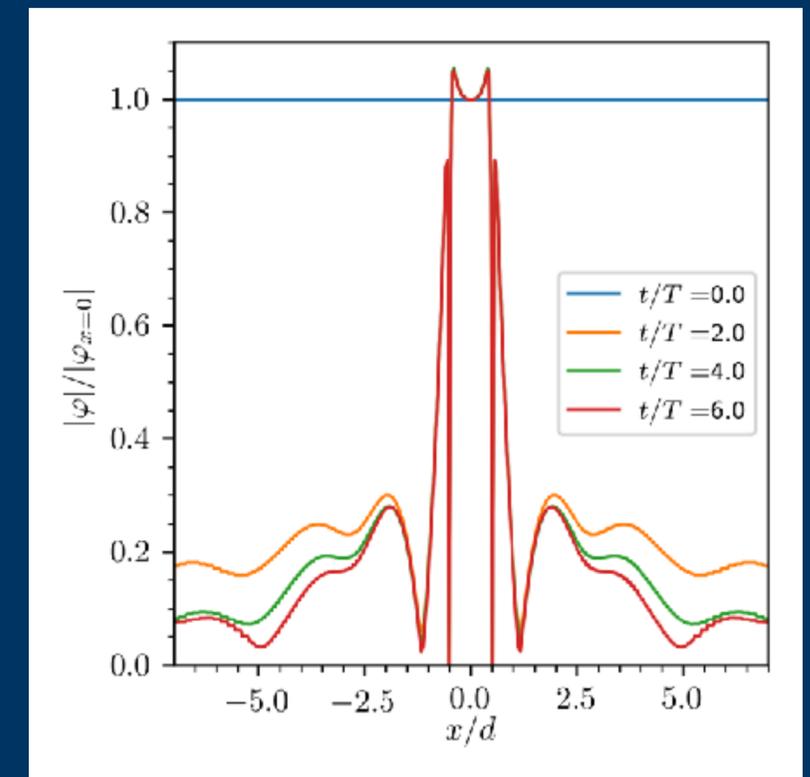


Bamber et. al., 2022

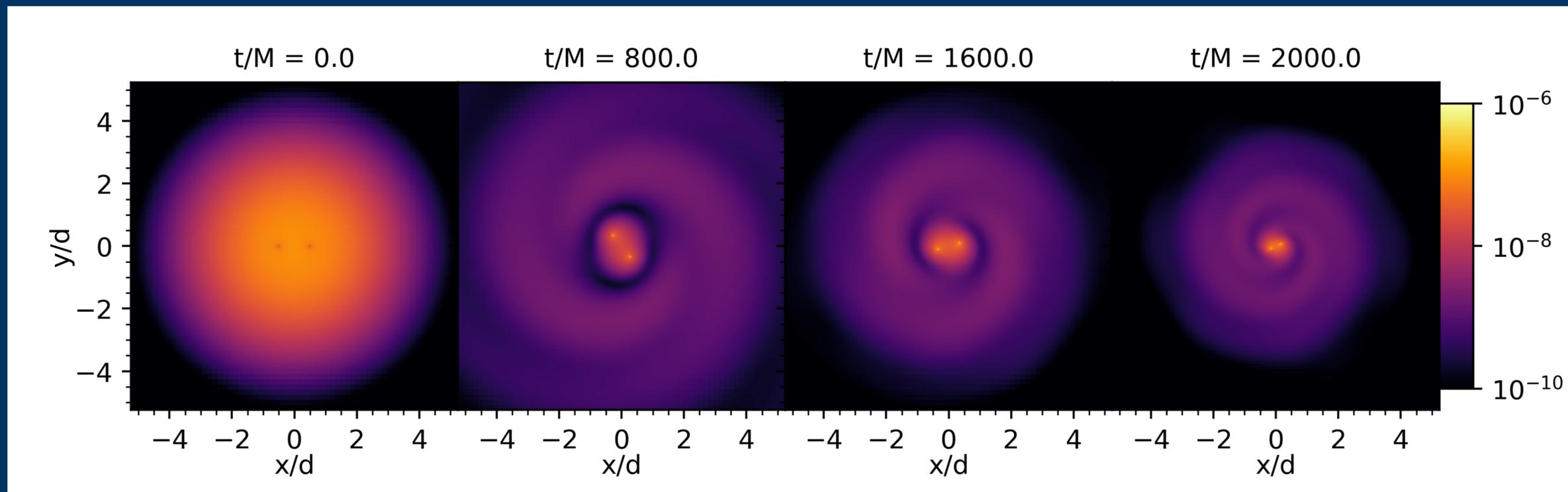
Black hole merger simulations in wave dark matter environments

# Using fixed orbit simulations to simulate stationary orbits, we find a stationary profile with a scaling symmetry

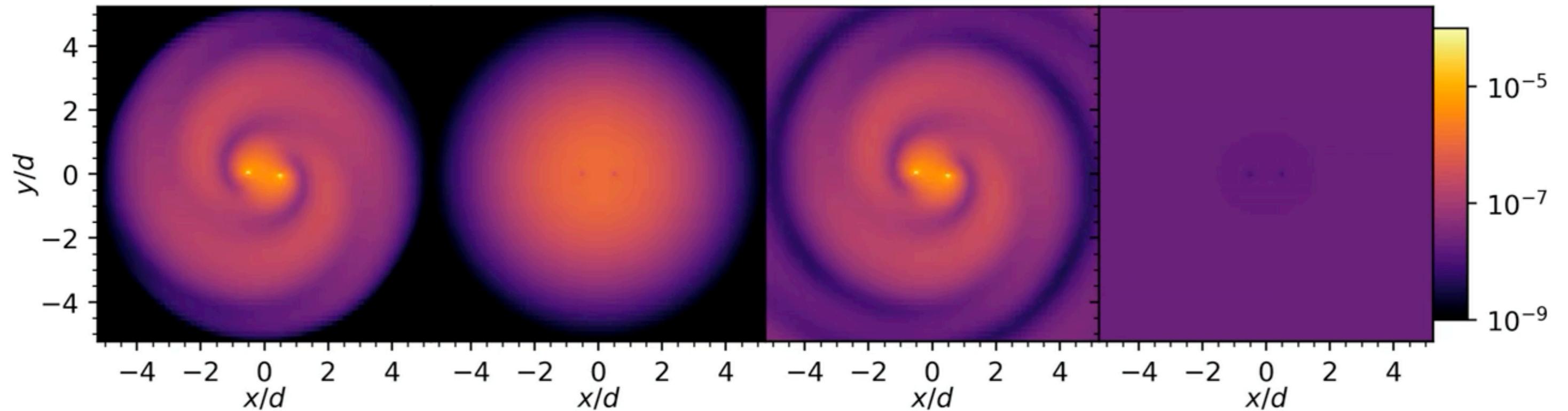
Field profiles



So there is a “right” initial condition for the dark matter component. But what if we start with something else?



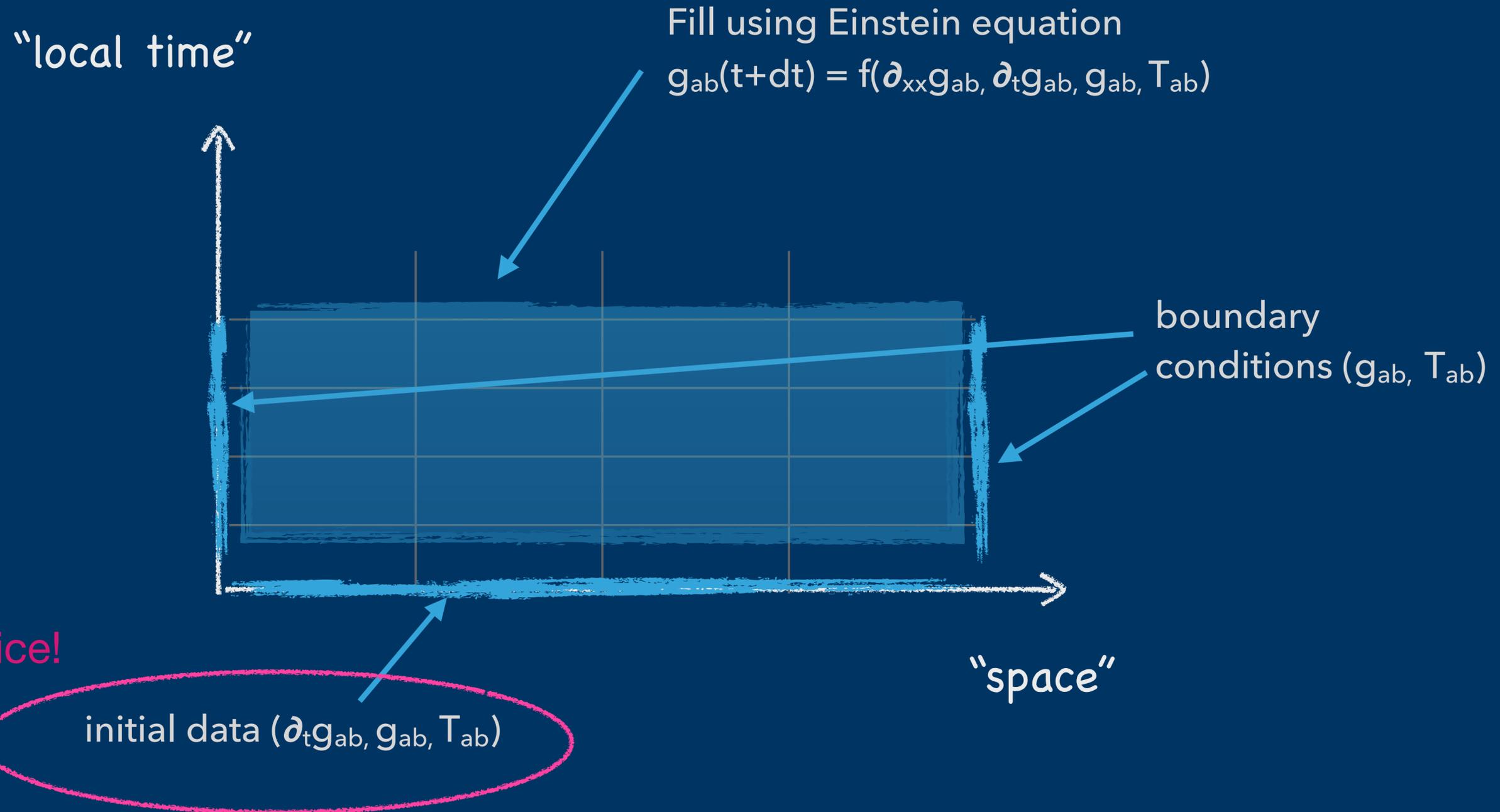
**In full NR, different initial clouds converge to the same solution within a few orbits but there are significant transients**



Energy density

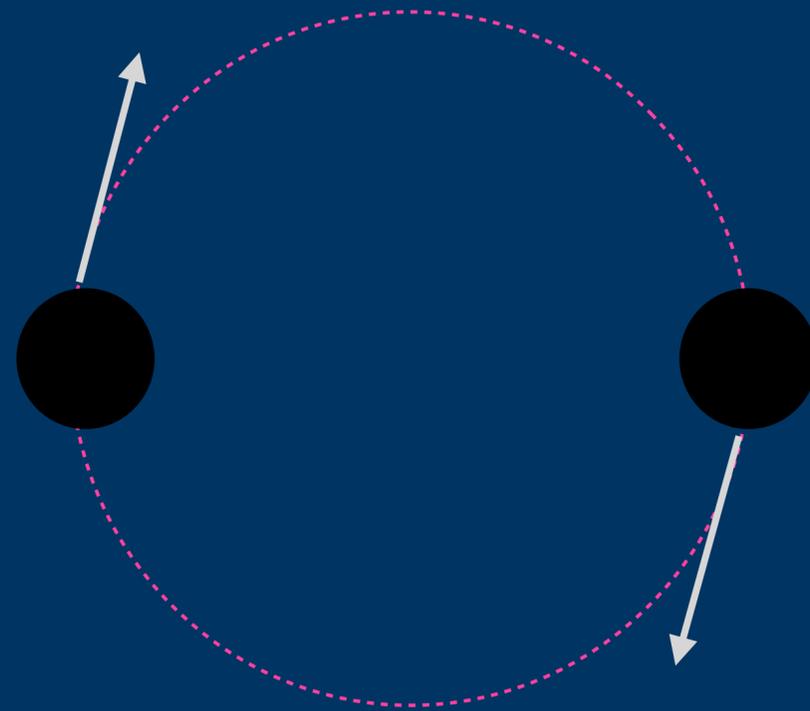
**How does this affect the solution of the constraint equations?**

# Numerical relativity is done in a finite region of spacetime



# Problem 1

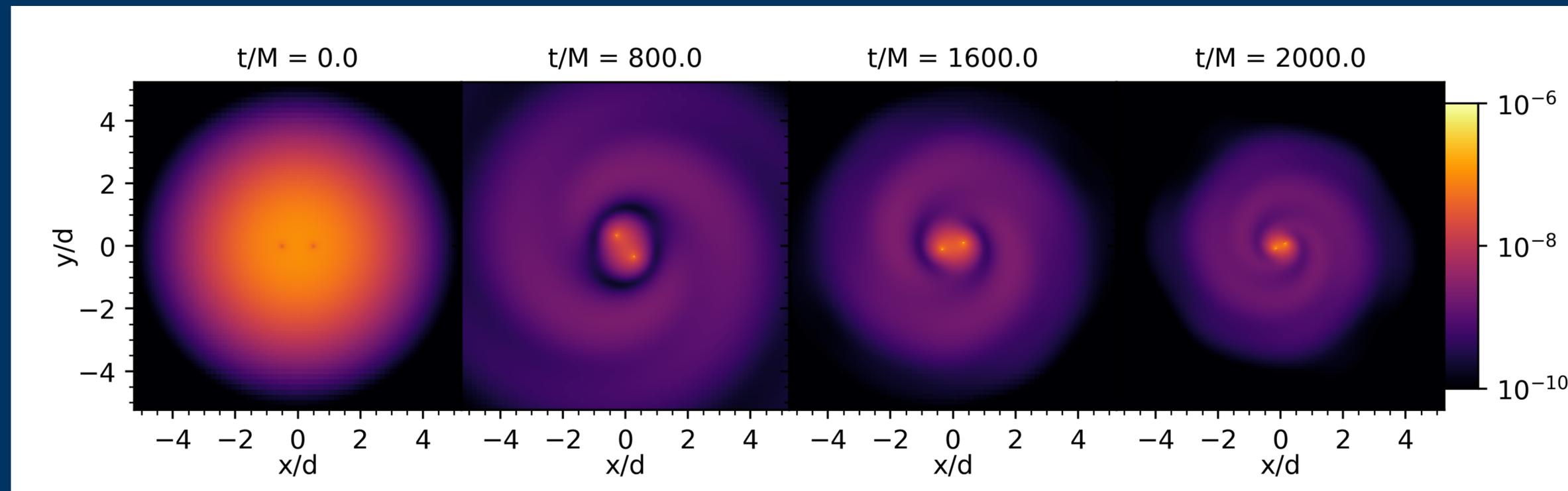
- 1. The initial matter density will push the BHs off circular orbits compared to the vacuum case**



**-> Need to adjust the initial momenta so circular again**

# Problem 2

2. The subsequent *transient* evolution of the field will potentially look like a signal



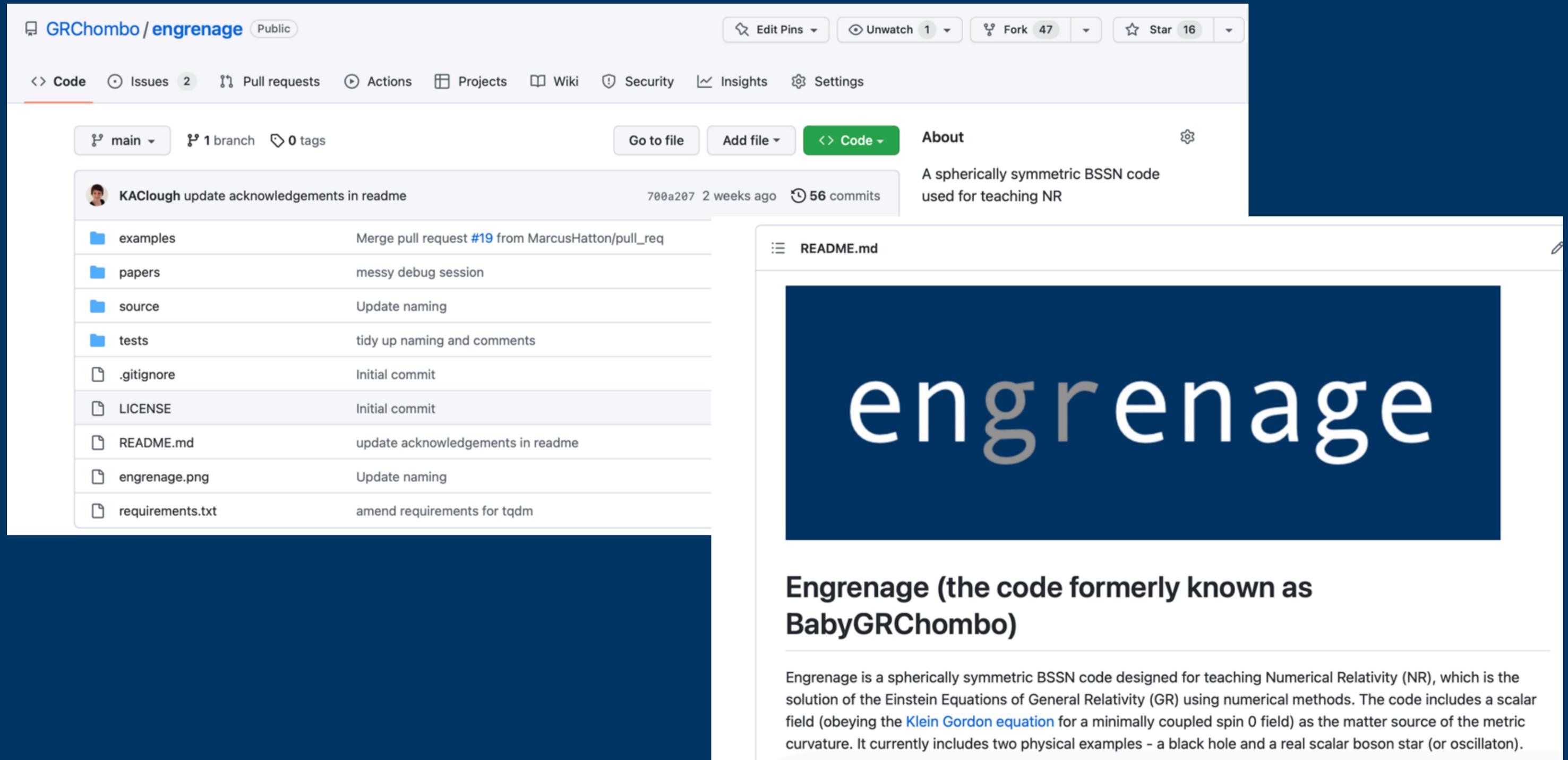
-> Need to start with the closest initial condition to the “stationary” case

## Key points

- To see any signal we need a DM density enhancement mechanism - one possibility is accretion of wave like dark matter
- Numerical relativity simulations will not give the right answer unless you give them the right initial conditions
- We need to do better in modelling physical environments of interest before generating waveforms

**Thank you, questions?**

# Commercial break:



GRChombo / engrenage Public

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main 1 branch 0 tags

Go to file Add file Code About

KAClough update acknowledgements in readme 700a207 2 weeks ago 56 commits

examples	Merge pull request #19 from MarcusHatton/pull_req
papers	messy debug session
source	Update naming
tests	tidy up naming and comments
.gitignore	Initial commit
LICENSE	Initial commit
README.md	update acknowledgements in readme
engrenage.png	Update naming
requirements.txt	amend requirements for tqdm

## README.md



### Engrenage (the code formerly known as BabyGRChombo)

Engrenage is a spherically symmetric BSSN code designed for teaching Numerical Relativity (NR), which is the solution of the Einstein Equations of General Relativity (GR) using numerical methods. The code includes a scalar field (obeying the [Klein Gordon equation](#) for a minimally coupled spin 0 field) as the matter source of the metric curvature. It currently includes two physical examples - a black hole and a real scalar boson star (or oscillaton).