On the propagation of gravitational waves:
difraction, dispersion & birefringence

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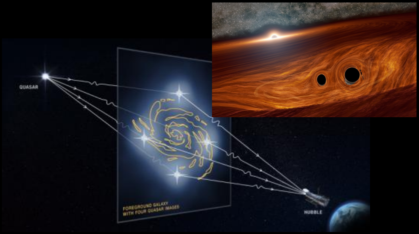
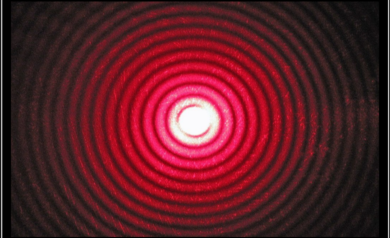
Collaborators: S. Savastano, G. Tambalo, H. Villarrubia-Rojo, R. Stiskalek, M. Oancea, L. Dai, HY Cheung,

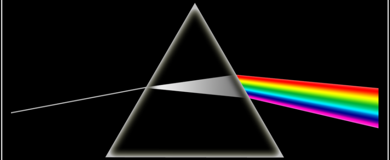
Wave propagation



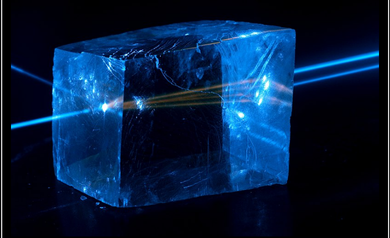
Diffraction/Interference



Dispersion



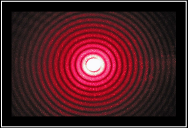
Birefringence

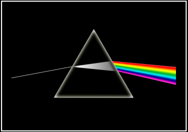


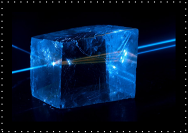
Outline

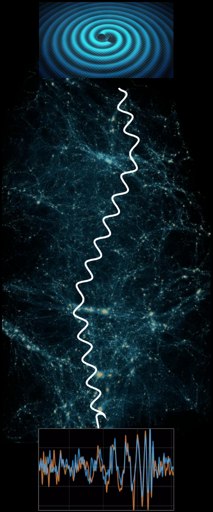
- Introduction
- Wave-optics lensing
  - Lens (sub)structure
  - Light halos & dark matter
- Strong-field lensing
  - Extreme environments
- ~~Lensing beyond Einstein~~

  $\rightarrow$ ask me latter (or JM anytime)
- Outlook & conclusions









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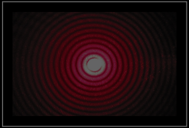
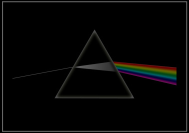
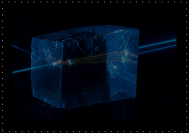
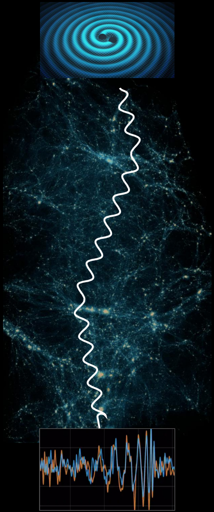
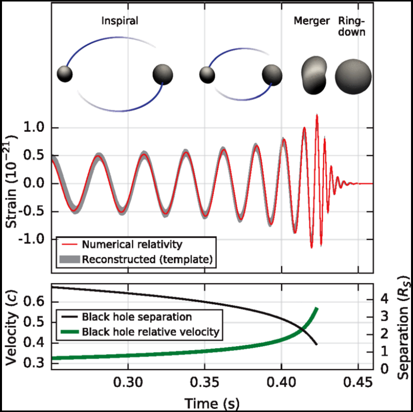


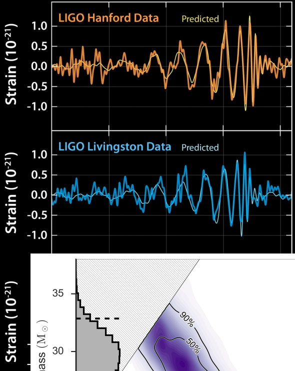


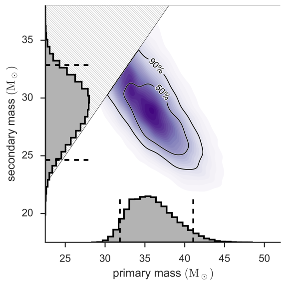




GW150914: first detection (LIGO '16)

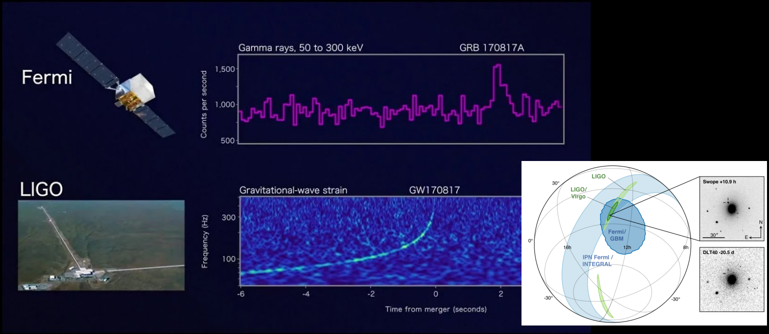






→ source <u>masses</u>, spins, distance, test gravity...

GW170817: only multi-messenger (LIGO '17, Fermi'17, $\cdots$)

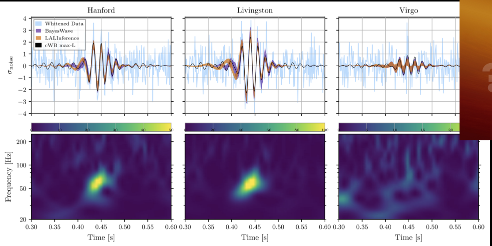


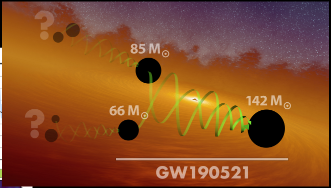
Neutron stars $\rightarrow$ Nuclear physics, nucleosynthesis,
Hubble's constant...

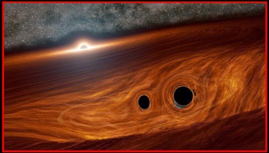
GW speed $|c_g/c - 1| < 10^{-15}$ $\rightarrow$ Stringent test of GR!

(Ezquiaga & MZ, Creminelli & Vernizzi, Baker, Bellini+... '17)

GW190521: a lot of questions! (2009.01075)





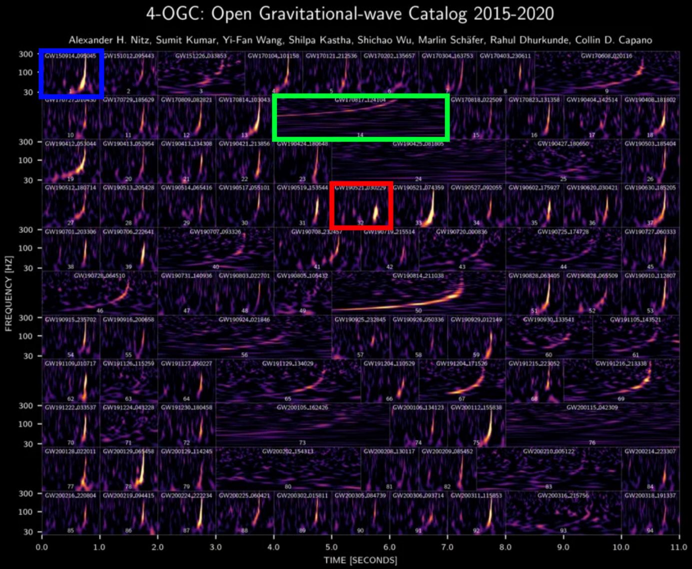


→ Mass ∈ pair-instability supernova gap♠
Eccentric?♦ → dynamical origin

→ Possible counterpart♣ → AGN environment

(♣Graham+19, ♠Esteles+21, ♦Romero-Shaw+22...)

$\mathcal{O}(100)$ GW events



→ astrophysics (BH origin), cosmology (distances, lensing), fundamental physics

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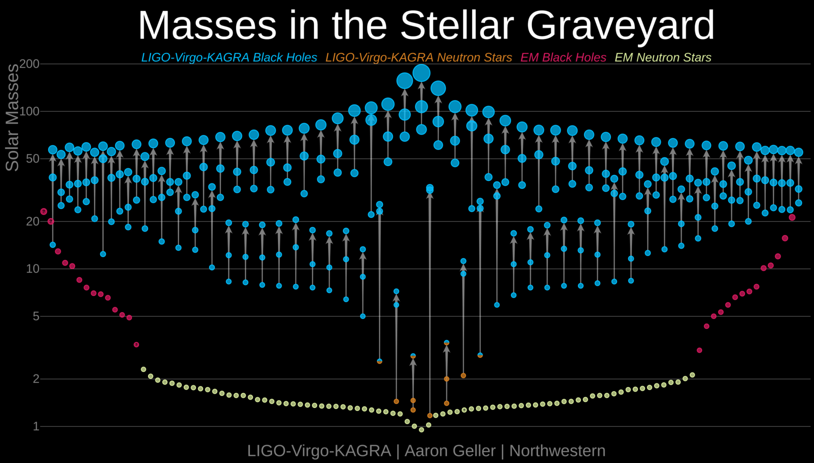

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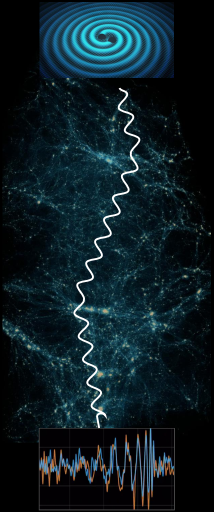
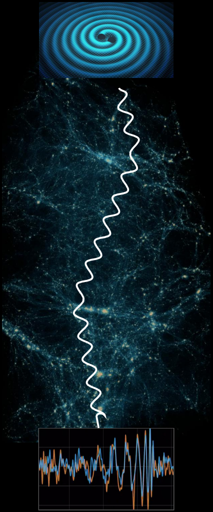
Why GW lensing?

- EM lensing → Large-scale structure, dark matter...



Why GW lensing?

- EM lensing $\rightarrow$ Large-scale structure, dark matter...
- GWs highly complementary:
  - Coherent, low frequency $\rightarrow$ wave effects
  - Weakly coupled $\rightarrow$ universe transparent to GWs
  - Well modeled $\rightarrow$ less uncertainty



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- GWs highly complementary:
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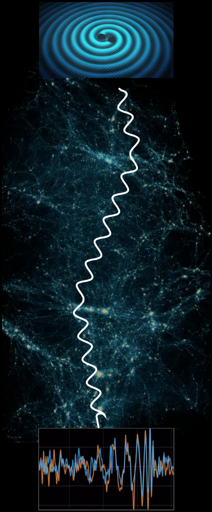
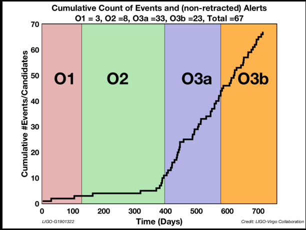
  - Weakly coupled → universe transparent to GWs
  - Well modeled → less uncertainty
- Many GW events → lensing increasingly relevant (e.g. LIGO/Virgo/Kagra searches)





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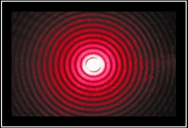
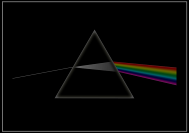
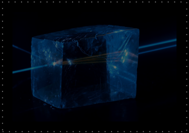
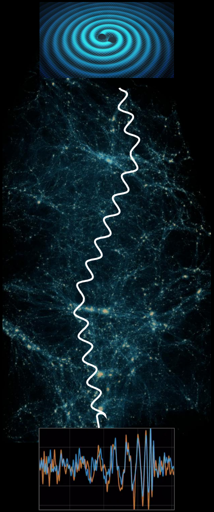


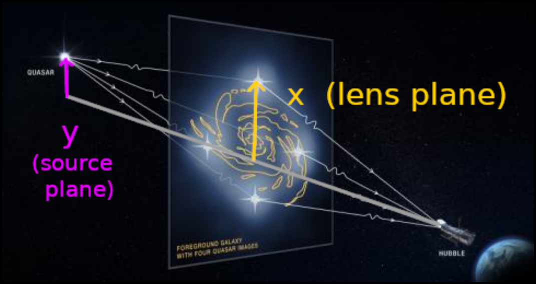






Geometric optics

(Schneider+ '92)



Lens equation:

$$\text{Images } \vec{x}_I(\vec{y}) \rightarrow \begin{cases} \text{magnification} & \mu_I \\ \text{time-delay} & T_I \\ \text{morse phase} & n_I \in (0, \pi/2, \pi) \end{cases}$$

Wave optics

(Takahashi & Nakamura 03)

Einstein radius

$$R_E = \sqrt{\frac{4GM_L D_L D_{LS}}{D_S}}$$

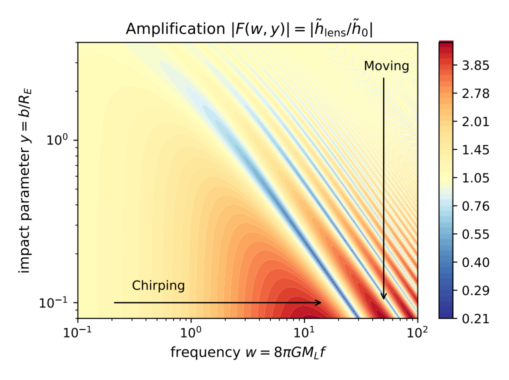
Dimensionless frequency

$$\boxed{w \equiv 8\pi G M_{Lz} f}$$

Amplification factor

$$\boxed{F(w) = \frac{\tilde{h}_{\text{lens}}}{\tilde{h}_{\text{flat}}}}$$

Point lens:



(Savastano, Vernizzi & MZ 2212.14697,
LISA cosmo white paper 2204.05434)

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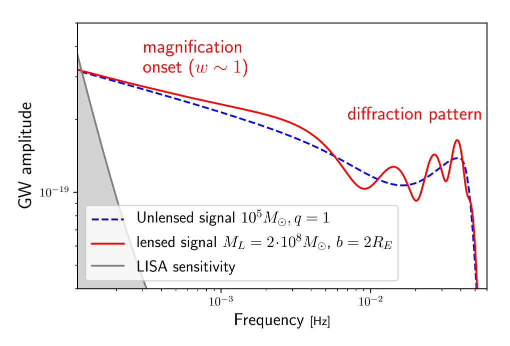

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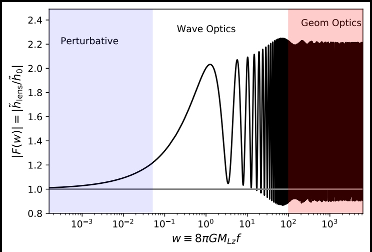
GW lensing (Schneider+ 93, Takahasi & Nakamura 03)

$$w \sim \left(\frac{M_L}{10^4 M_\odot}\right)\left(\frac{f}{\text{Hz}}\right)$$



(E.g. $30 + 30 M_\odot$ starting at 40Hz)

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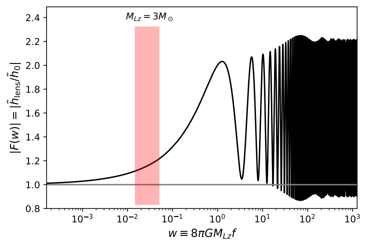

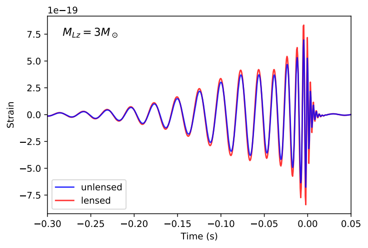

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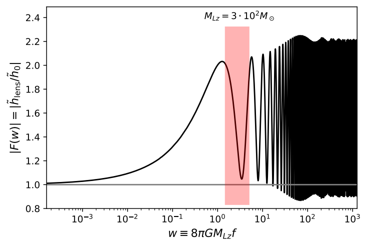
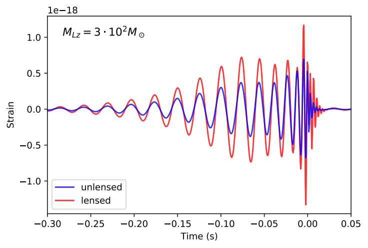
$$F = \frac{w}{2\pi i}\int d\vec{x} e^{iwT(\vec{x})}$$





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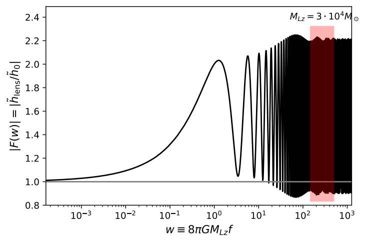
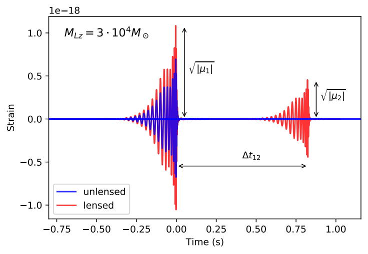
- Geometric optics $(w \to \infty)$

$$F \approx \underbrace{\sum_I}_{\text{images}} \underbrace{|\mu_I|^{1/2}}_{\text{magnification}} \underbrace{e^{i(wT_I + n_I)}}_{\text{time delay+phase}}$$





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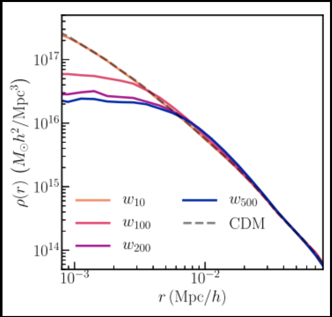
Probing a cored lens with GWs

(Tambalo, MZ+ *2212.11960*)

Cored profile

$$\rho(r) \propto \frac{1}{r^2 + r_c^2}$$

e.g. warm DM, ultra-light DM
self-interacting DM



(Nadler+20)

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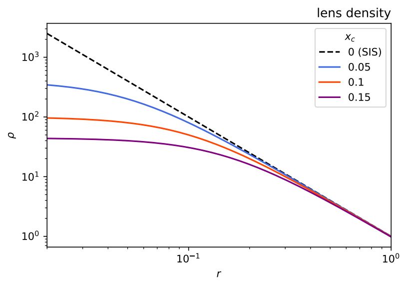

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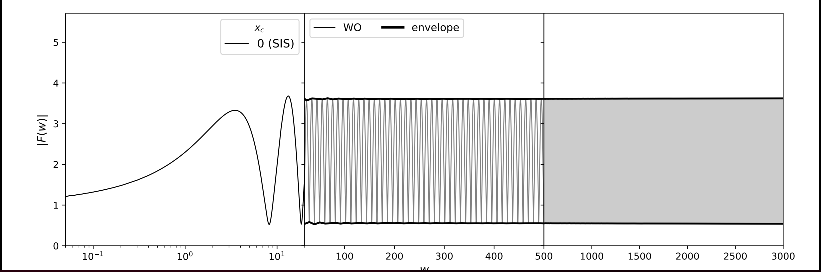

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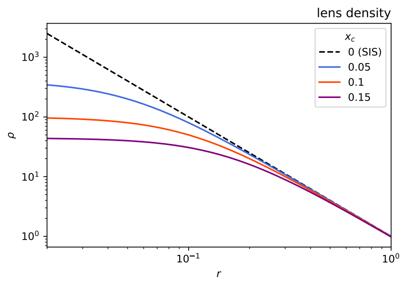

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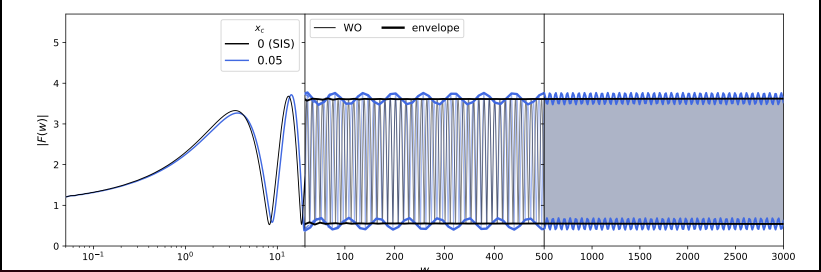

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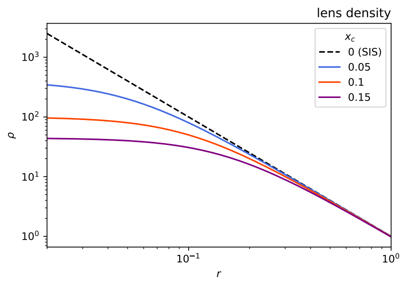

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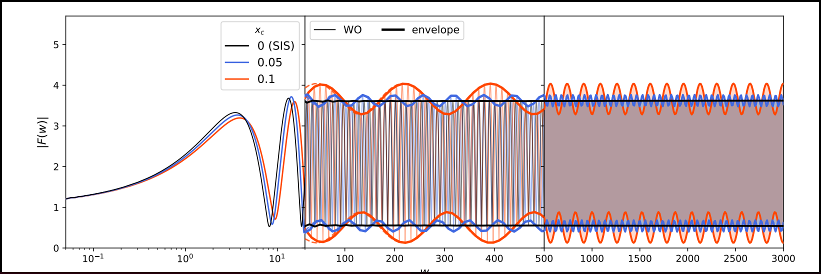

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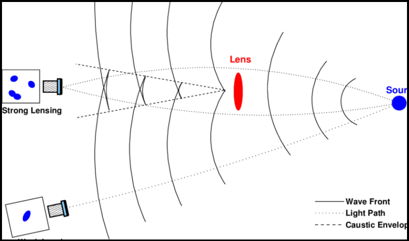

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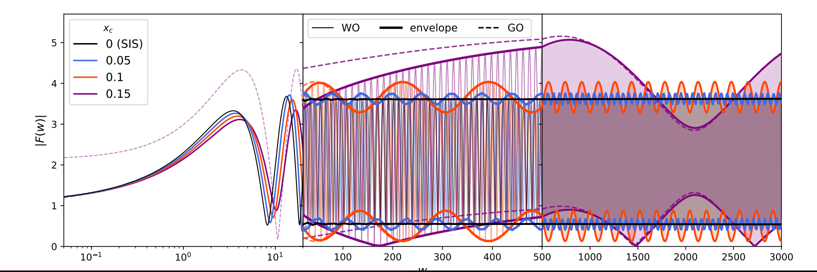
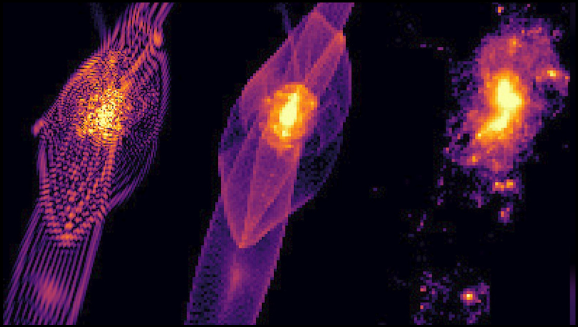


Tests of Dark Matter

(Mocz+20, Hui+16, $\cdots$)



fuzzy DM | warm DM | Cold DM

"Fuzzy" DM: ultra-light axion $r_c > 0.33\text{kpc}\frac{10^9 M_\odot}{M_c}\left(\frac{10^{-22}\text{eV}}{m_\phi}\right)^2$

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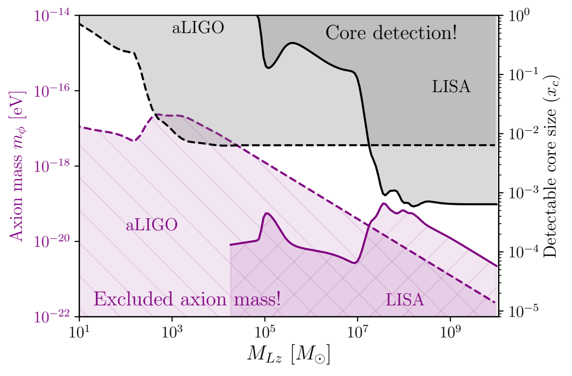

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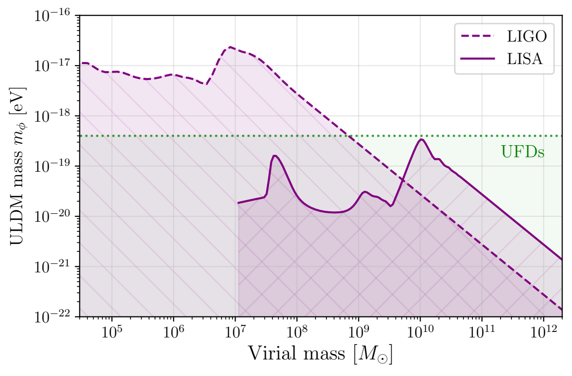

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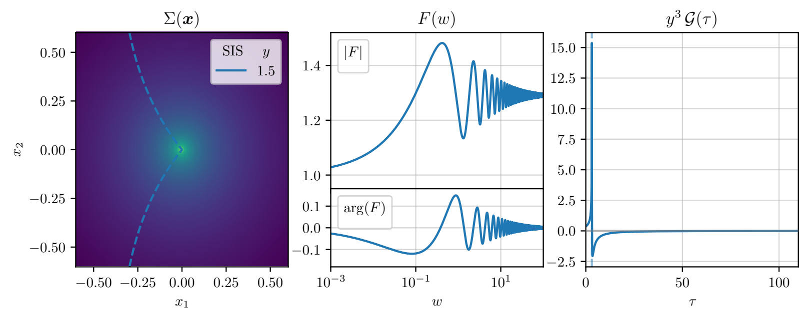
Assumes largest $R_E \to$ conservative (smallest x_c predicted)

Compare w/ (Dalal, Kravtsov 22)

Weak lensing limit

(Gao+21, Choi+21, Savastano+23)

$$\tilde{h}_L = F(w)\tilde{h}_0$$



Green's function $$h_L(t) = \int dt' \underbrace{G(t-t')}_{\mathcal{F}[F(f)]} h_0(t')$$

$$G(t) = \sqrt{\mu}\delta(t) + \mathcal{G}(t)$$

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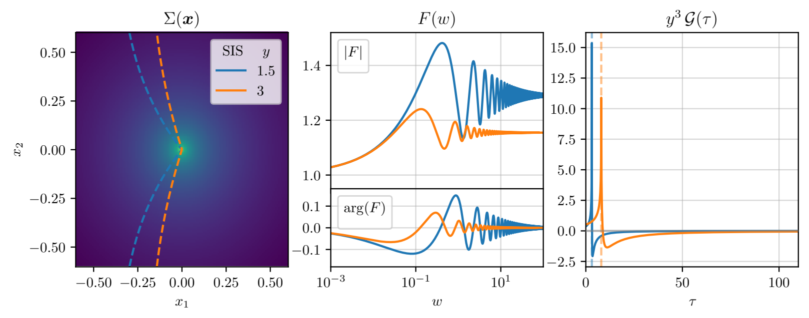

Green's function $$h_L(t) = \int dt' \underbrace{G(t-t')}_{\mathcal{F}[F(f)]} h_0(t')$$

$$G(t) = \sqrt{\mu}\delta(t) + \mathcal{G}(t)$$

Weak lensing limit

(Gao+21, Choi+21, Savastano+23)

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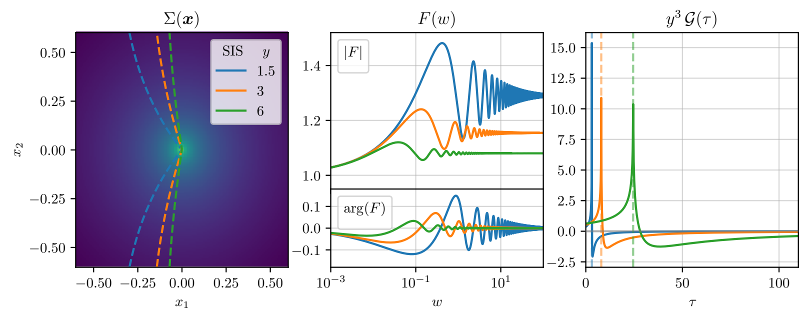

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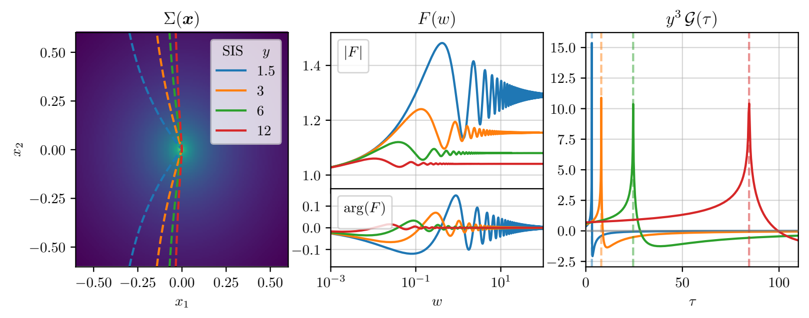

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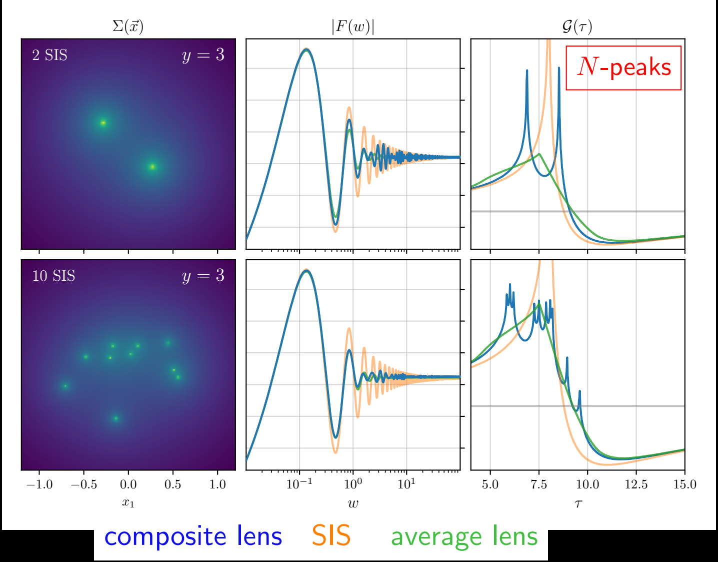
Probing Substructure

(Savastano+ *2306.05282*)



(toy model, not a realistic halo)

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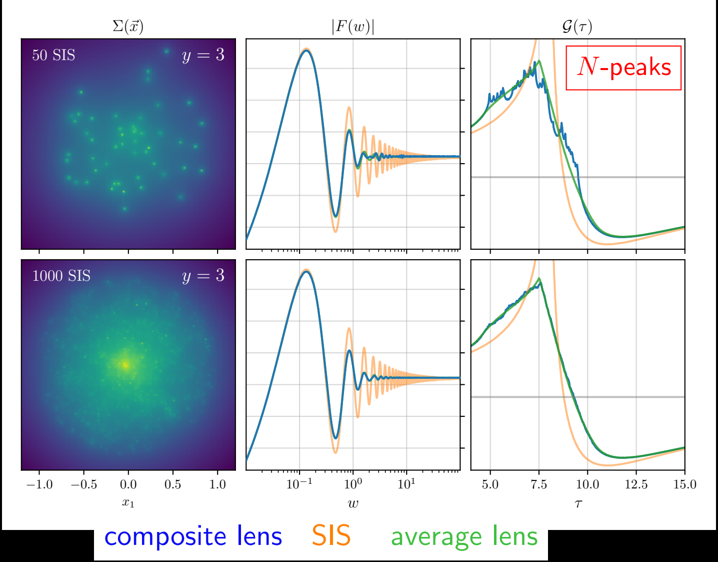
(Savastano+ *2306.05282*)



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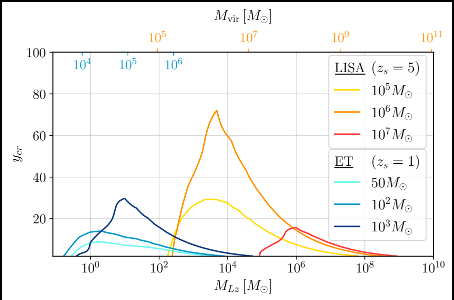
WO detection prospects

(Savastano+ *2306.05282*)

- Critical impact param $y_{\rm cr}$:

  $\mathcal{M} \cdot \mathrm{SNR}^2 > 1$ (optimistic)

  ($\sim 30\%$ larger than Caliskan+22)



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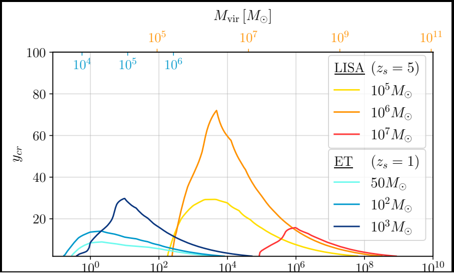
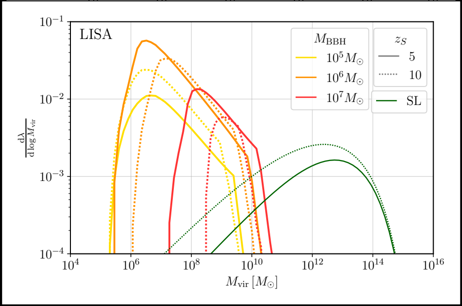
  No sub-halos (pesimistic)





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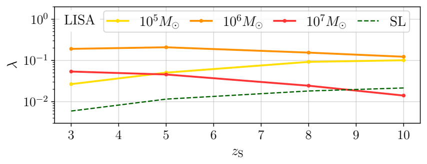
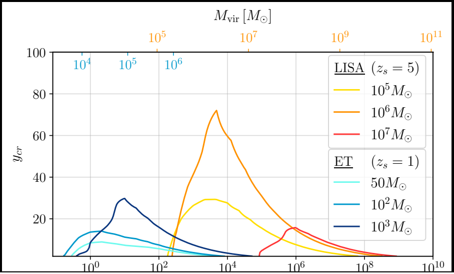
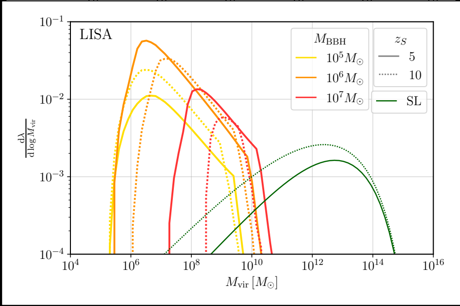
- MBH optical depth ~ 0.1



  (higher than Gao+22, Fairbairn+22)

- Probe $M_v \sim 10^7 M_\odot$ halos?





Outline

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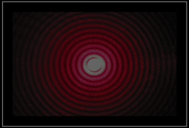
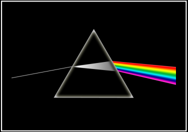
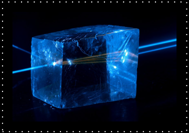
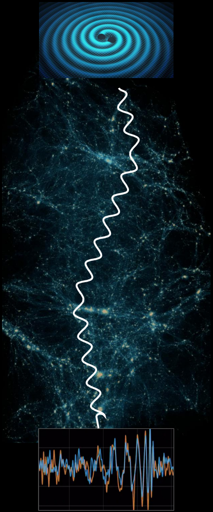






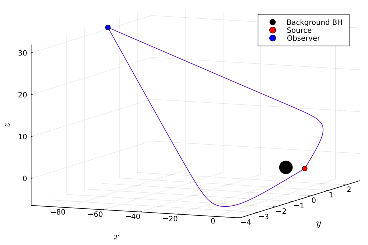


Strong field propagation

(Anderson+ '19, Oancea+ '22)

Gravitational spin-hall effect

$$\dot{x}^\nu \nabla_\nu p_\mu = -\frac{1}{2} \underbrace{\bar{R}_{\mu\nu\alpha\beta}}_{\text{Space-time}} p^\nu \underbrace{S^{\alpha\beta}}_{\text{GW spin}}$$



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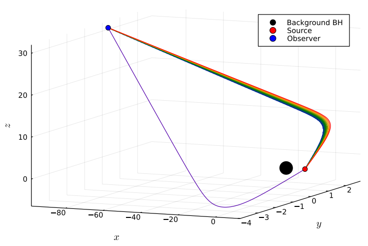
- Modified trajectory:

  $t_R - t_{\rm geo} \propto G\bar{M}/w^2$



dispersion, birefringence

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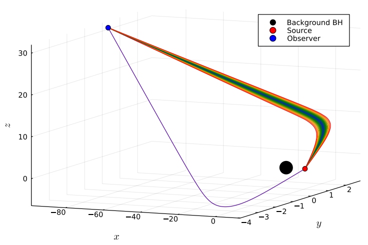

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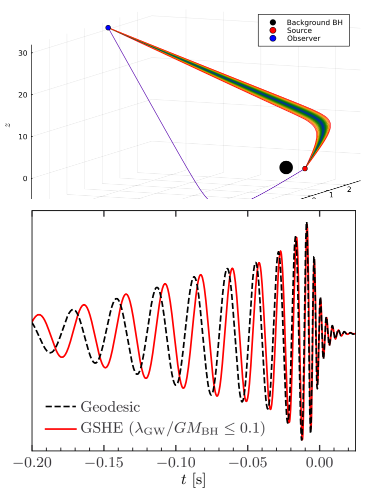
dispersion, birefringence

- Distorted waveform:

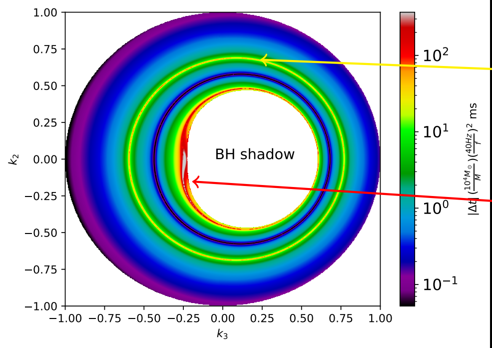
$$\Delta t = 15\text{ms}\,\beta \left(\frac{10^4 M_\odot}{\bar{M}}\right)\left(\frac{40\text{Hz}}{f}\right)^2$$



Configuration dependence

(Oancea+ 2209.06459)

$$\Delta t = 15\text{ms}\,\boxed{\beta(\hat{k})}\left(\frac{10^4 M_\odot}{\bar{M}}\right)\left(\frac{40\text{Hz}}{f}\right)^2$$



Einstein radius (magnified)

graze light-ring (demagnified)

source sphere, $r_{\rm src} = 10G\bar{M}$, $a = 0.99$

GSHE detection prospects

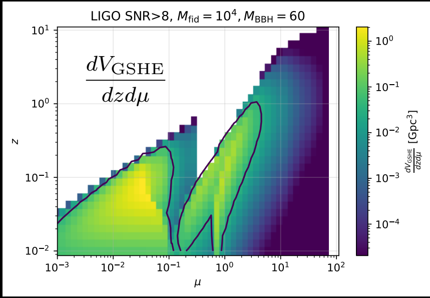
(Oancea+ *in prep.*)

- Source $\rightarrow$ observer:

$$dP \propto |\mu|^{-1}$$

- Detector sensitiviy, SNR distribution...

- Define effective $V_{\rm GSHE}$



$r_{\rm src} = 10GM,\ M_{\rm BBH} = 60M_\odot$

GSHE detection prospects

(Oancea+ *in prep.*)

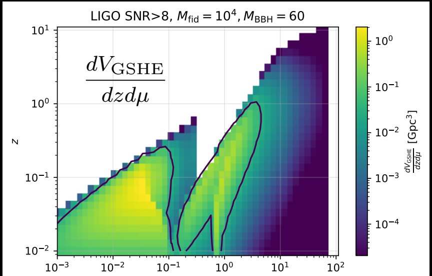
- Source $\rightarrow$ observer:

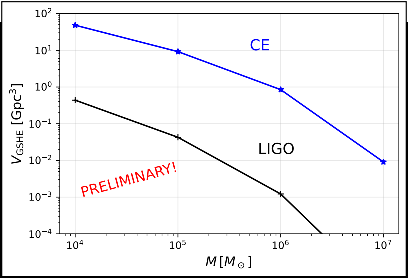
$$dP \propto |\mu|^{-1}$$

- Detector sensitiviy, SNR distribution...
- Define effective $V_{\rm GSHE}$
- Potential sources:
  - $\star$ Mergers *very* close to BHs?
  - $\star$ Last migration trap (Peng+21)

$$r_{\rm src} \sim 15GM \quad :)$$

$$M \gtrsim 10^7 M_\odot \quad :($$





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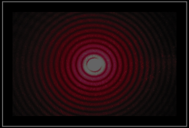
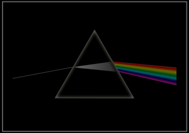
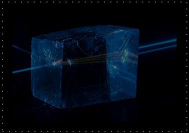
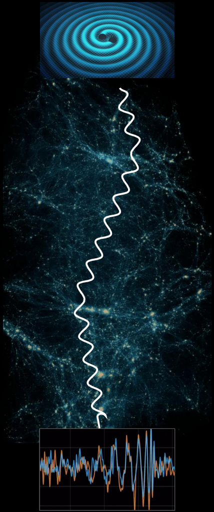








Outlook

$\sim 1/600$ strongly lensed events (aLIGO) (Ng+17)

Intriguing candidate? (Dai+ 2007.12709) or not? (LVC 2105.06384)

$\sim 1/\textit{week} \longrightarrow \sim 1/\textit{minute!}$

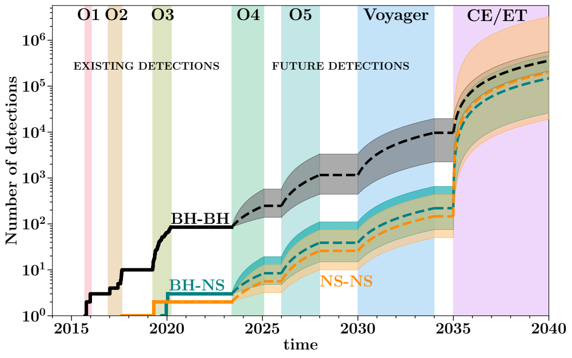


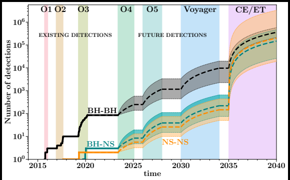
Figure: F. Broekgaarden

Searches for EM counterparts, strong lenses, small scale structure $\cdots$

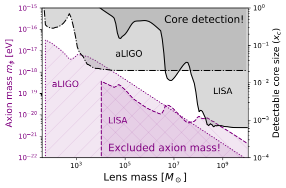
Conclusions

mange tak!

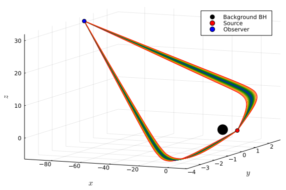
- More data is coming! ⟶



- GWs complement EM observations
  - ⋆ wave effects, low f
  - ⋆ probes gravtational d.o.f.s
- Diffraction from GW lensing
  - ⋆ dark matter ⟶
  - ⋆ lens (sub)-structure



- Dispersion in strong fields
  - ⋆ extreme environments ⟶



- Opportunities for fundamental physics

Backup Slides

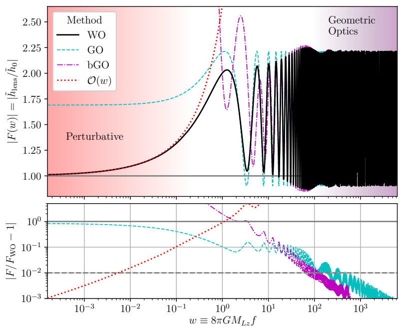
Computing amplification factor I

$$F = \frac{w}{2\pi i}\underbrace{\int d^2x\, e^{iwT(\vec{x},\vec{y})}}_{I(w)} \quad \xrightarrow{w\to 0} \quad 1 + Aw^{\alpha}$$

$$F = \frac{w}{2\pi i}\underbrace{\int d^2x\, e^{iwT(\vec{x},\vec{y})}}_{I(w)} \quad \xrightarrow{w\to \infty} \quad \sum_I \sqrt{|\mu_I|}\, e^{i(wT_I + \pi n_I)}$$

1) beyond Geometric Optics:

$$\sum_I \sqrt{|\mu_I|}\left(1 + i\frac{\Delta_I}{w}\right) e^{i(wT_I+\pi n_I)}$$



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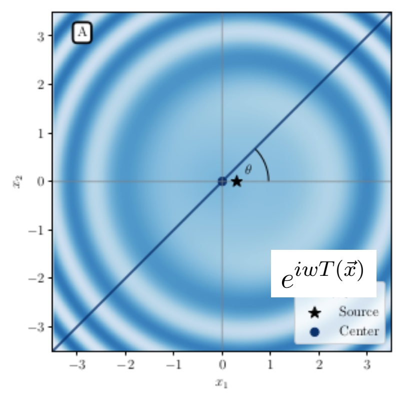
2) $\mathbb{C}$-deformation:

$$\vec{x} \to (r,\theta) \to (z(\lambda),\theta)$$

(Feldbrugge+, Tambalo, MZ+)



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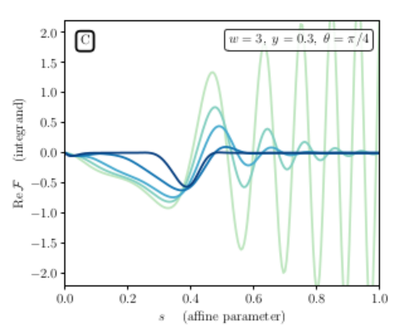

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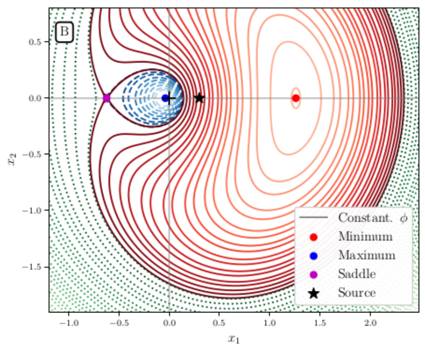
$$F = \frac{w}{2\pi i}\underbrace{\int d^2x e^{iwT(\vec{x},\vec{y})}}_{I(w)} \xrightarrow{w\to 0} 1 + Aw^{\alpha}$$

$$\xrightarrow{w\to\infty} \sum_I \sqrt{|\mu_I|}e^{i(wT_I+\pi n_I)}$$

Contour flow

$$\tilde{I}(\tau) = \int dw e^{-iw\tau} I(w)$$
$$= \int d^2x \delta(\tau - T(\vec{x}))$$

(Ulmer+ , Diego+, Tambalo, MZ+)



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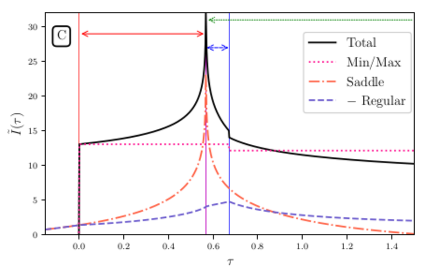

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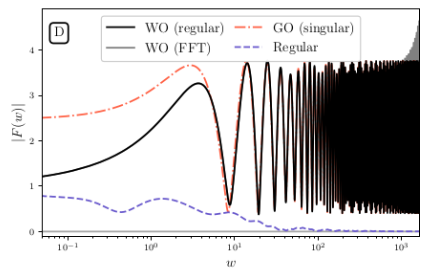
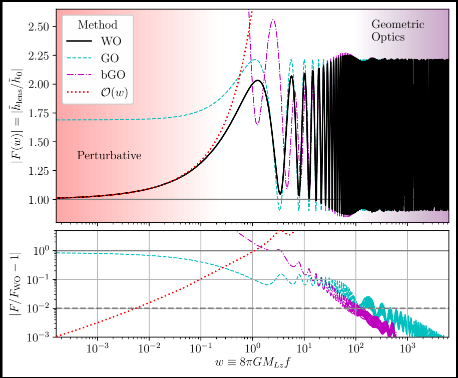


Wave Optics Methods & validation (Tambalo, MZ+ *2210.05658*)

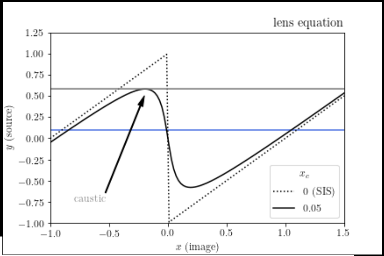
Accuracy $\lesssim \mathcal{O}(1\%)$ speed $\mathcal{O}(0.1)$s

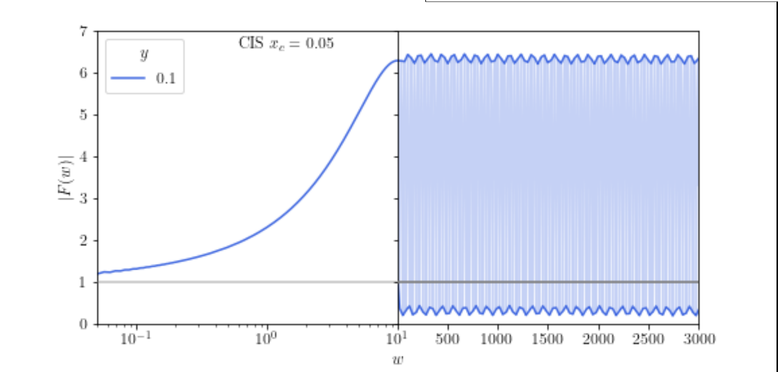


$2\times$ algorithms: ✓ point lens (analytic sol), ✓ extended lenses

Strong vs weak lensing

Fix core size $x_c = 0.05$





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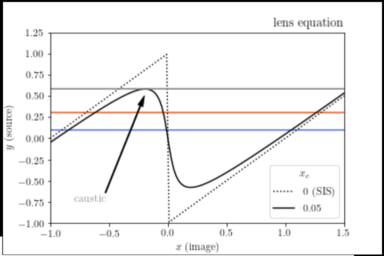

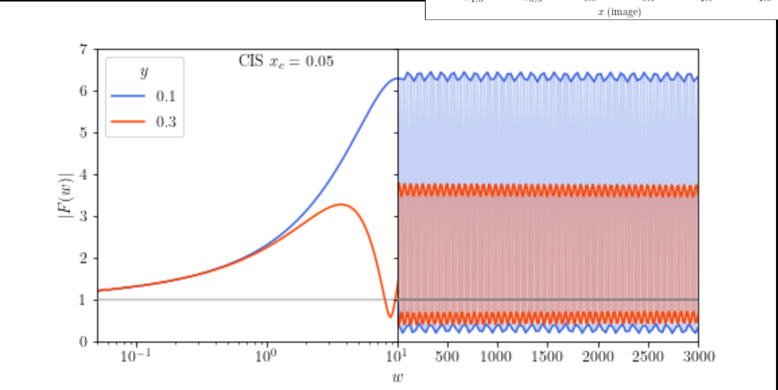

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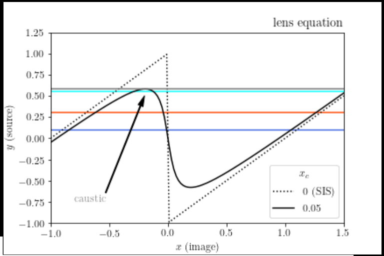

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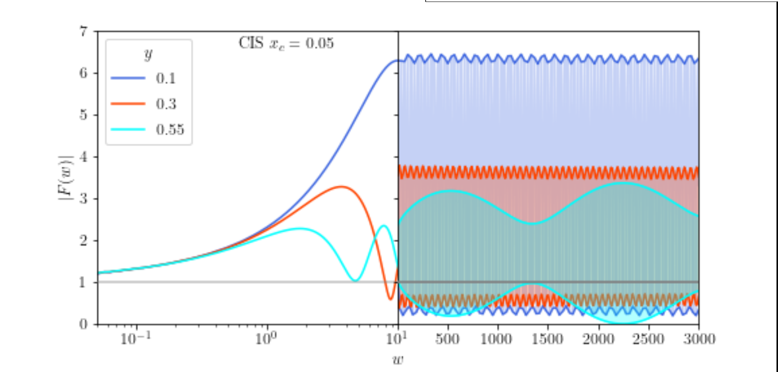

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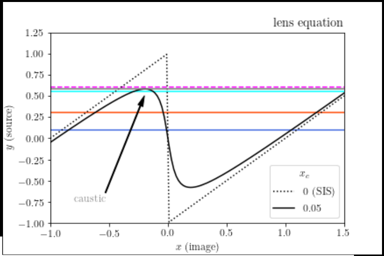

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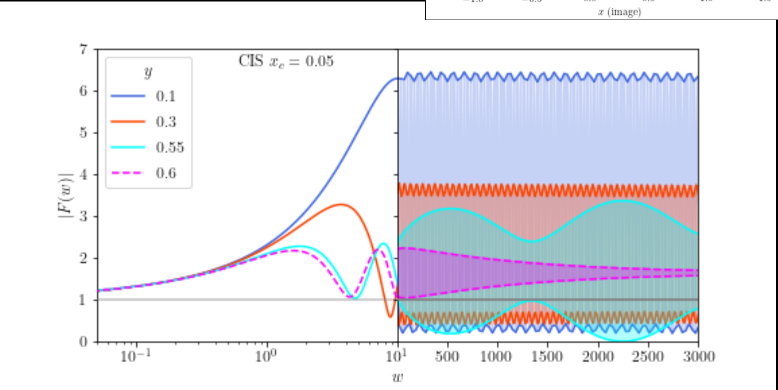
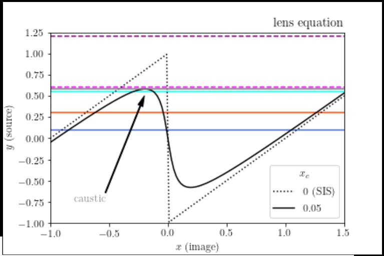
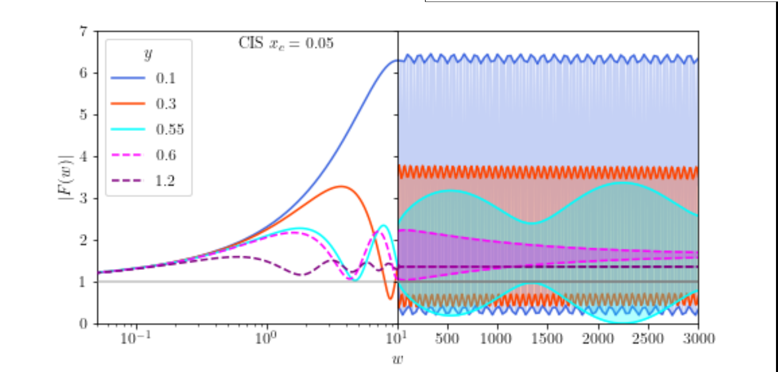


Strong vs weak lensing

Fix core size $x_c = 0.05$





Fisher matrix analysis

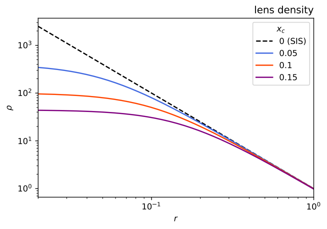
(Vallisneri '07)

$$F_{IJ} = \left(\frac{\partial h_L}{\partial \theta_I} \middle| \frac{\partial h_L}{\partial \theta_J} \right), \qquad (h|g) = 4\Re \left(\int \frac{df}{S_n(f)} \tilde{h}(f) \tilde{g}^*(f) \right)$$

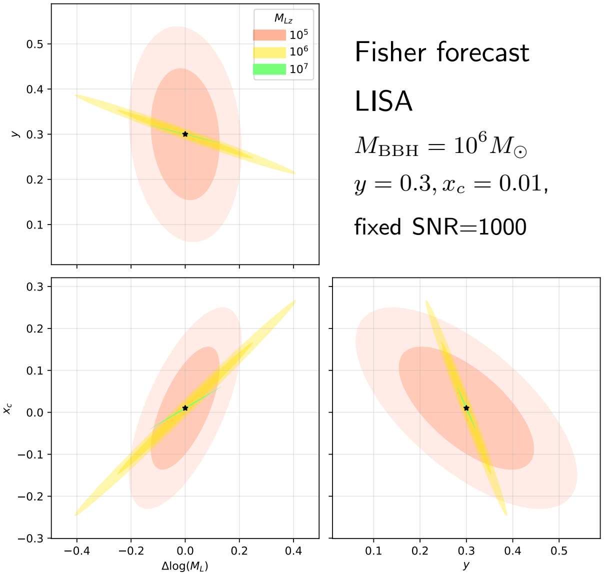
- $\tilde{h}_L(w, \vec{\theta}) = F(w)\tilde{h}(w)$
- $F \to \begin{cases} \text{WO} & (w < w_{\rm cut}) \\ \text{bGO} & (w > w_{\rm cut}) \end{cases}$
- $\theta_I \in (\underbrace{\log(D_L), \phi_0}_{\text{source}}, \underbrace{\log(M_{Lz}), y, x_c}_{\text{lens}})$
- Static single detector, optimal orientation...

(Caliskan+ '22 $\to$ detailed source modeling)

Lens with a core:



Reconstructing lens parameters (See also Takahashi+04, Caliskan+22)



Fisher forecast

LISA

$M_{\rm BBH} = 10^6 M_\odot$

$y = 0.3, x_c = 0.01,$

fixed SNR=1000

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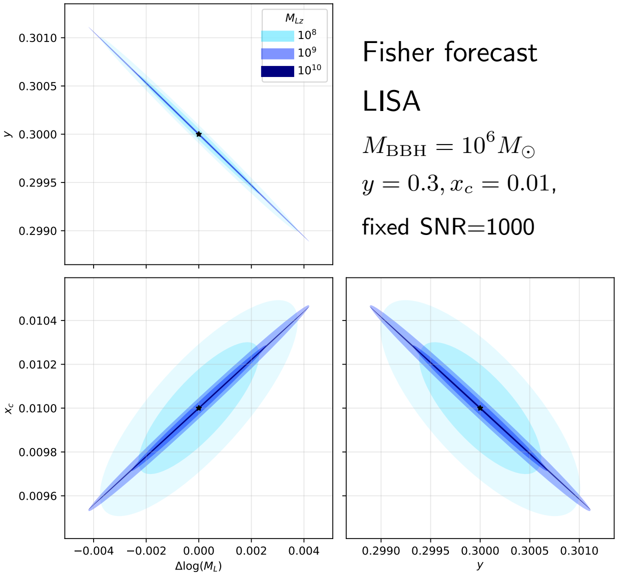

Fisher forecast

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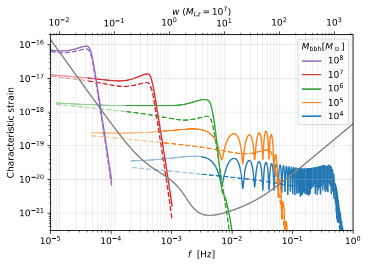
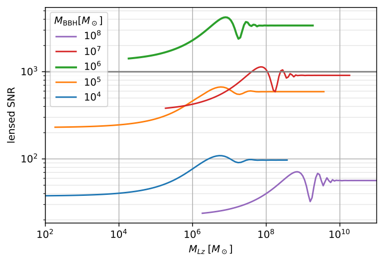
$y = 0.3, x_c = 0.01,$

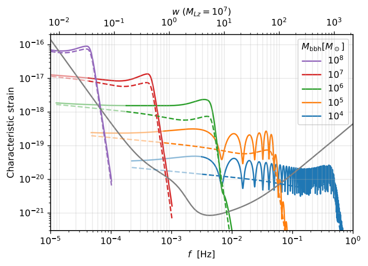
fixed SNR=1000

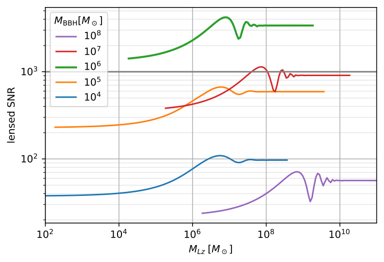
LISA: vary source mass



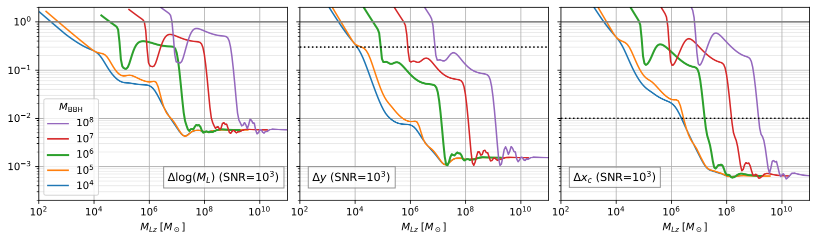


LISA: vary source mass





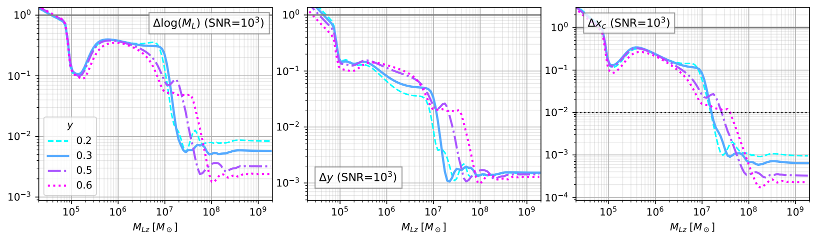
Fixed SNR=1000



Lens mass   Impact param. $(y = 0.3)$ Core size $(x_c = 0.01)$

Vary impact parameter

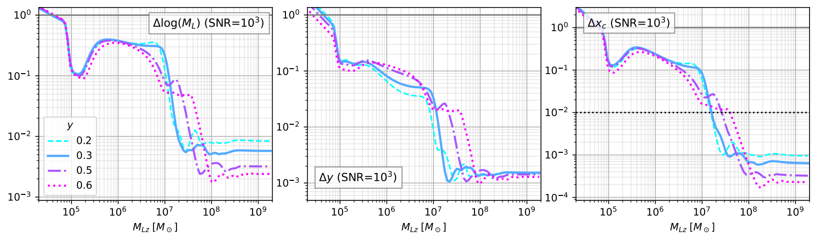
LISA ($M_{\rm BBH} = 10^6 M_\odot$, fixed SNR=1000)



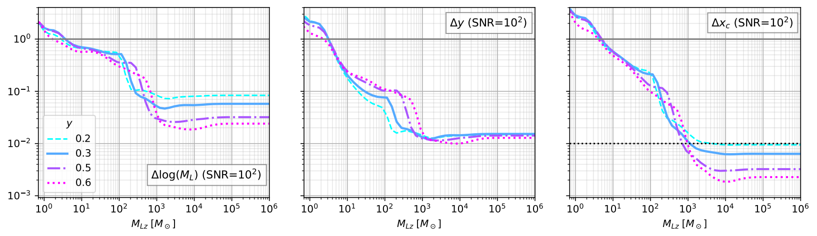
Lens mass Impact param. Core size

Vary impact parameter

LISA ($M_{\rm BBH} = 10^6 M_\odot$, fixed SNR=1000)



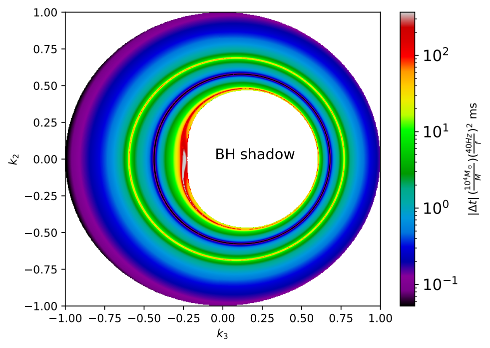
advanced LIGO ($M_{\rm BBH} = 30 M_\odot$, fixed SNR=100)



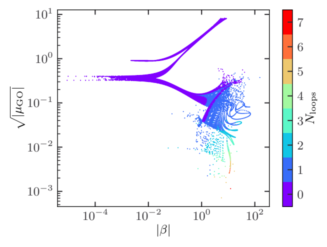
Lens mass Impact param. Core size

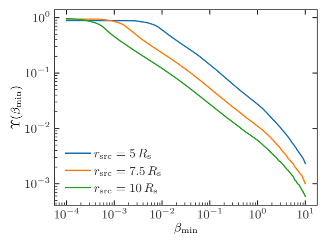
GSHE probabilities

$$\Delta t = 15\mathrm{ms}\,\beta \left(\frac{10^4 M_\odot}{M}\right)\left(\frac{40\mathrm{Hz}}{f}\right)^2$$



$r_{\mathrm{src}} = 10GM$

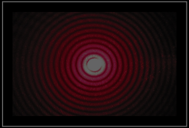
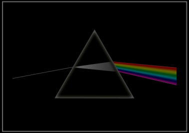
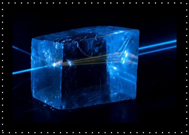
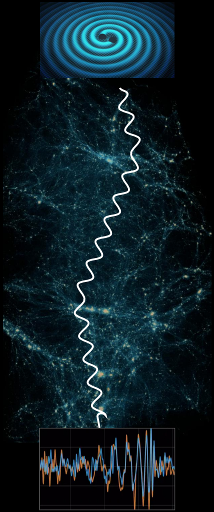




Outline

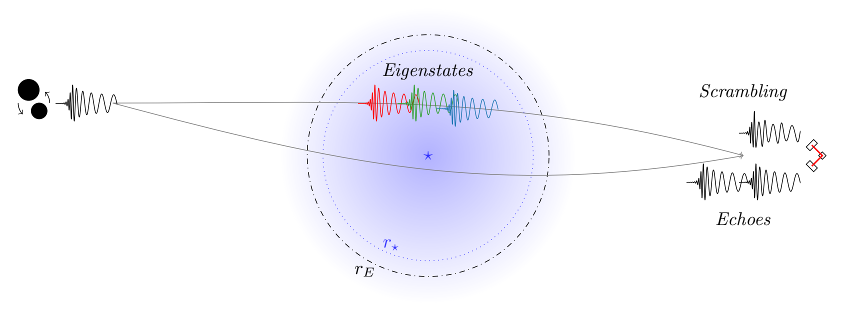
- Introduction
- Wave-optics lensing
  - Lens (sub)structure
  - Light halos & dark matter
- Strong-field lensing
  - Extreme environments
- ~~Lensing beyond Einstein~~

  $\rightarrow$ ask me latter (or JM anytime)
- Outlook & conclusions

GW lensing beyond GR

(Ezquiaga & MZ 20)



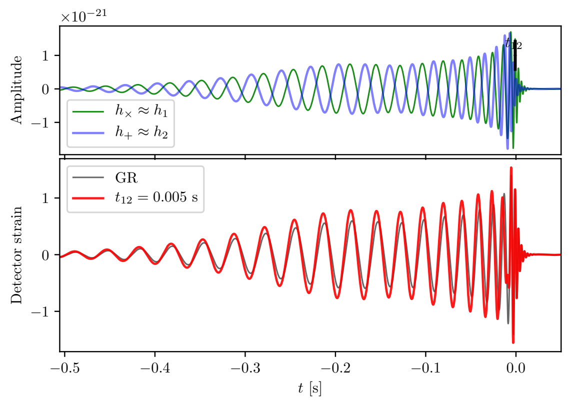
- Work in WKB ($f \to \infty$ expansion) at leading order
- GWs $(h_+, h_\times)$ + new d.o.f. $\leftrightarrow$ propagation eigenstates

(See also Dalang+ 20)

- Eigenstates split $\to$ interfere at detector (or "echoes")

Observing polarization Time Delays Δt_{12}

$$\Delta t_{12} = 5\text{ms}$$

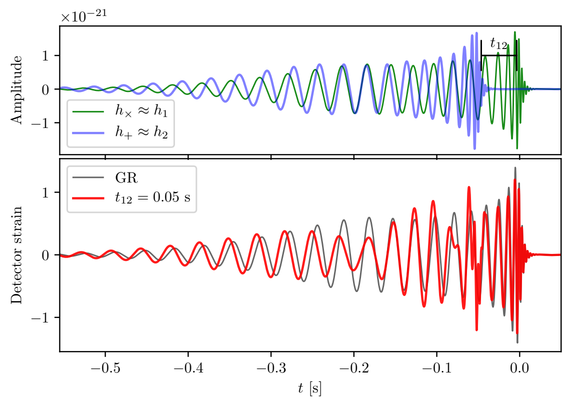


$\varphi \sim 0$, amplitude $H_i \sim h_i$, head-on $30-30M_\odot$ source,
$\mathcal{A}^+ = -0.38$, $\mathcal{A}^\times = 0.71$

No need for EM counterpart!

Observing polarization Time Delays Δt_{12}

$\Delta t_{12} = 50$ms



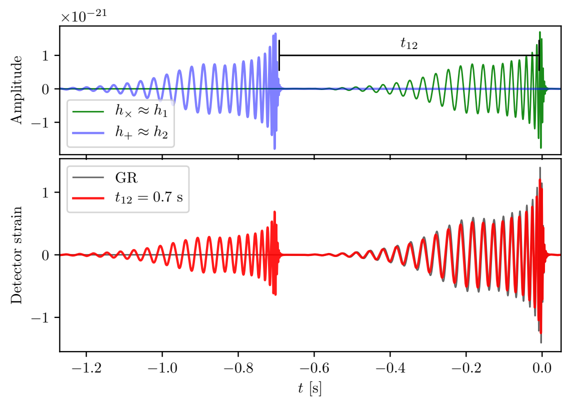
$\varphi \sim 0$, amplitude $H_i \sim h_i$, head-on $30 - 30 M_\odot$ source,

$\mathcal{A}^+ = -0.38$, $\mathcal{A}^\times = 0.71$

No need for EM counterpart!

Observing polarization Time Delays Δt_{12}

$\Delta t_{12} = 700\text{ms}$



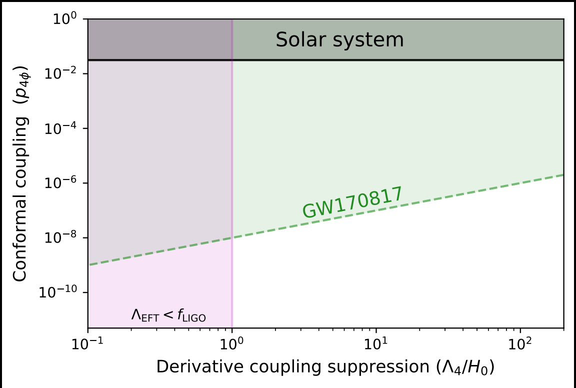
$\varphi \sim 0$, amplitude $H_i \sim h_i$, head-on $30 - 30 M_\odot$ source,

$\mathcal{A}^+ = -0.38$, $\mathcal{A}^\times = 0.71$

No need for EM counterpart!

An example theory (Horndeski) (Ezquiaga& MZ'20)

$$\mathcal{L} \sim \frac{M_P^2}{2}\left(1 + p_{4\phi}\frac{\phi}{M_P}\right)R + \frac{\phi}{\Lambda_4^2}\nabla_\mu\nabla_\nu\phi G^{\mu\nu}$$



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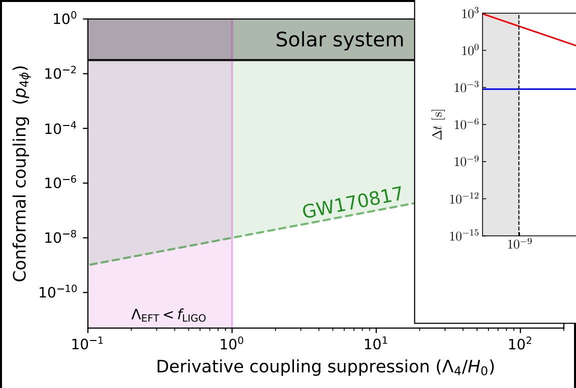
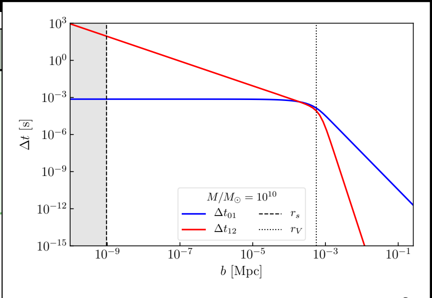
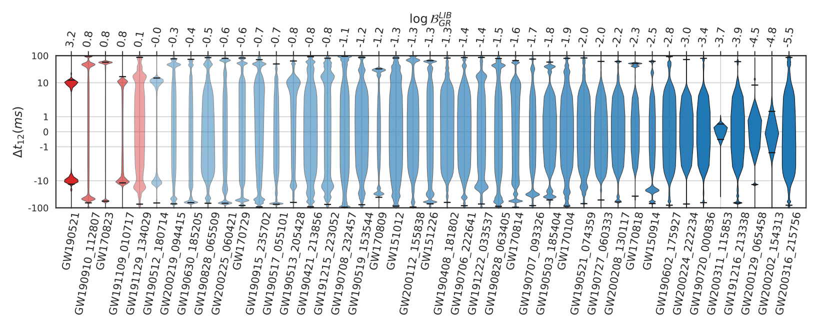




$(\Lambda_4 \sim H_0,\ p_{4\phi} \sim 10^{-8})$

GW Birefringence tests (Goyal+ *2301.04826*)



- Mismatch & Injection parameter estimation studies
- 43× GWTC-3 events analyzed (false alarm $\lesssim 10^{-3}/y$)
  - GW190521 $\rightarrow \Delta t_{12} \sim 9.5$ms $\rightarrow$ (likely noise fluctuation)
  - Constrain $P(\Delta t_{12})$ $\rightarrow$ (parameterization)

Constraining example theory (Goyal+ *2301.04826*)

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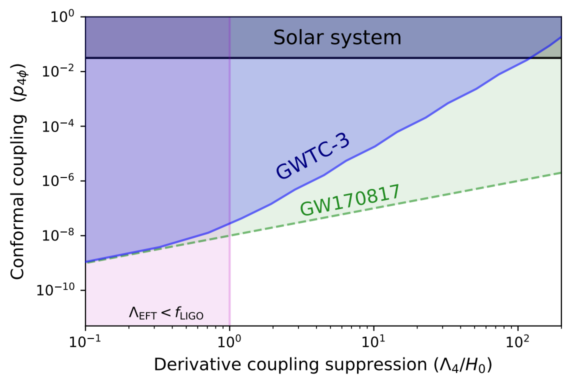

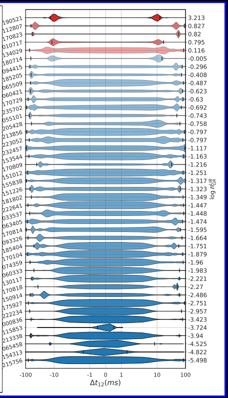

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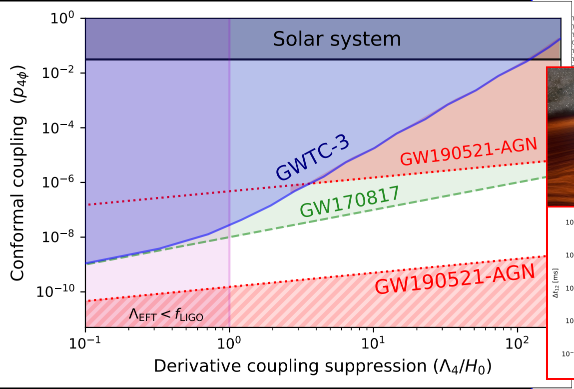

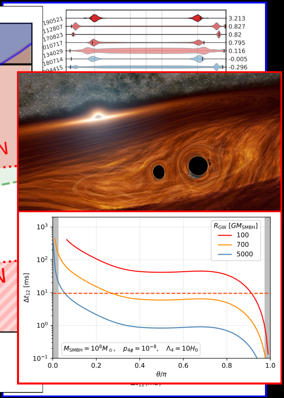

Propagation Eigenstates (e.g. neutrino flavor mixing)

- Leading order $\omega \to \infty$ in $k_\mu \equiv \theta_{,\mu}$ (phase)

$$\begin{pmatrix} \Box_h & 0 & M_\phi \Box_m \\ 0 & \Box_h & 0 \\ M_\phi \Box_m & 0 & \Box_s \end{pmatrix} \begin{pmatrix} h_+ \\ h_\times \\ \varphi \end{pmatrix} = 0$$

- Mixing M_{ij}, speeds $\underline{\Box_I \propto k_0^2 - c_I^2(\hat{k})|\vec{k}|^2}$

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  - Pure metric: $H_1 \propto h_\times$, $c_1^2 = c_h^2$
  - Mostly metric: $H_2 \propto h_+ + M_\phi \frac{\Delta c_{mh}^2}{\Delta c_{hs}^2} \varphi + \cdots$

$$\boxed{c_2^2 = c_h^2 + M_\phi^2 \frac{(\Delta c_{hm}^2)^2}{\Delta c_{hs}^2} + \cdots}$$

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  - Mostly metric: $H_2 \propto h_+ + M_\phi \frac{\Delta c_{mh}^2}{\Delta c_{hs}^2}\varphi + \cdots$

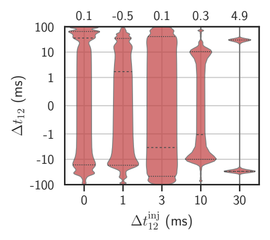
$$\boxed{c_2^2 = c_h^2 + M_\phi^2 \frac{(\Delta c_{hm}^2)^2}{\Delta c_{hs}^2} + \cdots}$$

  - Mostly scalar: $H_3 \sim \varphi + \mathcal{O}(M_\phi) h_+\,,\quad c_3 = c_s + \cdots$

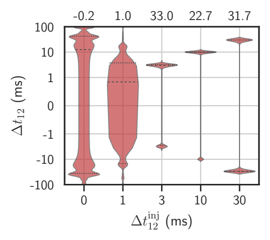
Lens-induced birefringence injections (Goyal+ *2301.04826*)

top: $\log(\mathcal{B}^{LIB}_{GR})$

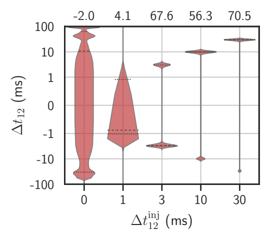
SNR=10



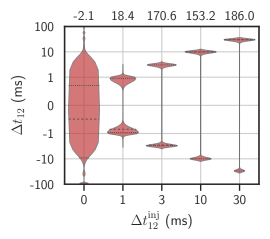
SNR=15



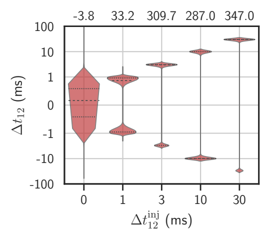
SNR=20



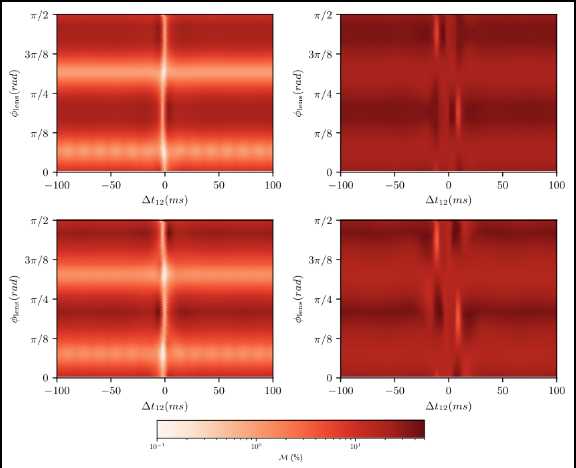
SNR=30



SNR=40



Lens-induced birefringence Mismatch (Goyal+ *2301.04826*)



Top: GW150914-like. Bottom: GW190814-like

Left: GR injection ($\Delta t^{inj} = 0$, $\phi^{inj}_{lens} = 0$)

Right: Non-GR injection $\Delta t^{inj} = 10$ms, $\phi^{inj}_{lens} = 0\pi/5$

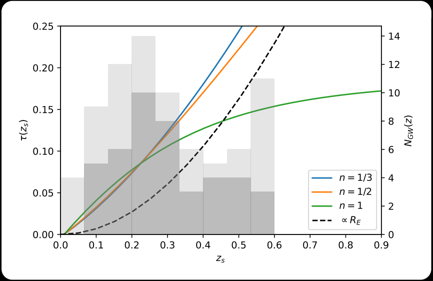
Lens-induced birefringence probability (Goyal+ *2301.04826*)

Probability (random lens):

$$P = \exp\left(-\sum_i^U \lambda_i\right)\prod_j^L \left(1 - e^{\lambda_j}\right)$$

Cross section:

$$\sigma = \pi R_{12}^2 \left(\frac{M}{10^{12} M_\odot}\right)^{2n}$$



Cosmology:

$$\lambda \propto \int_0^{z_s} dz \frac{(1+z)^2}{H(z)/H_0} \int d\log(M)\sigma(M,z) f(M,z)$$