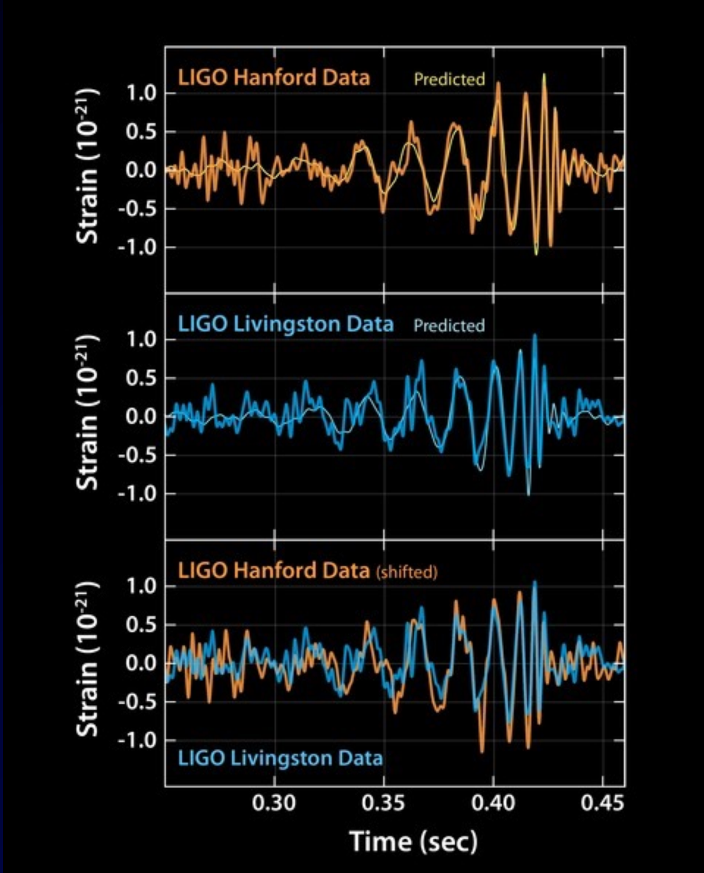
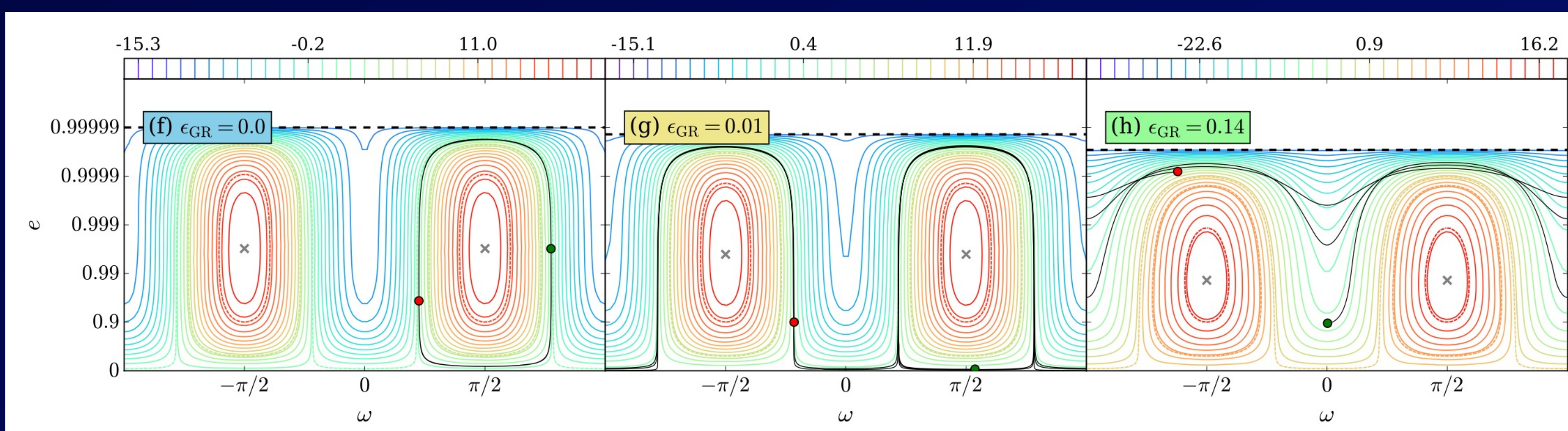


Cluster tides, stellar dynamics and LIGO/Virgo gravitational wave sources

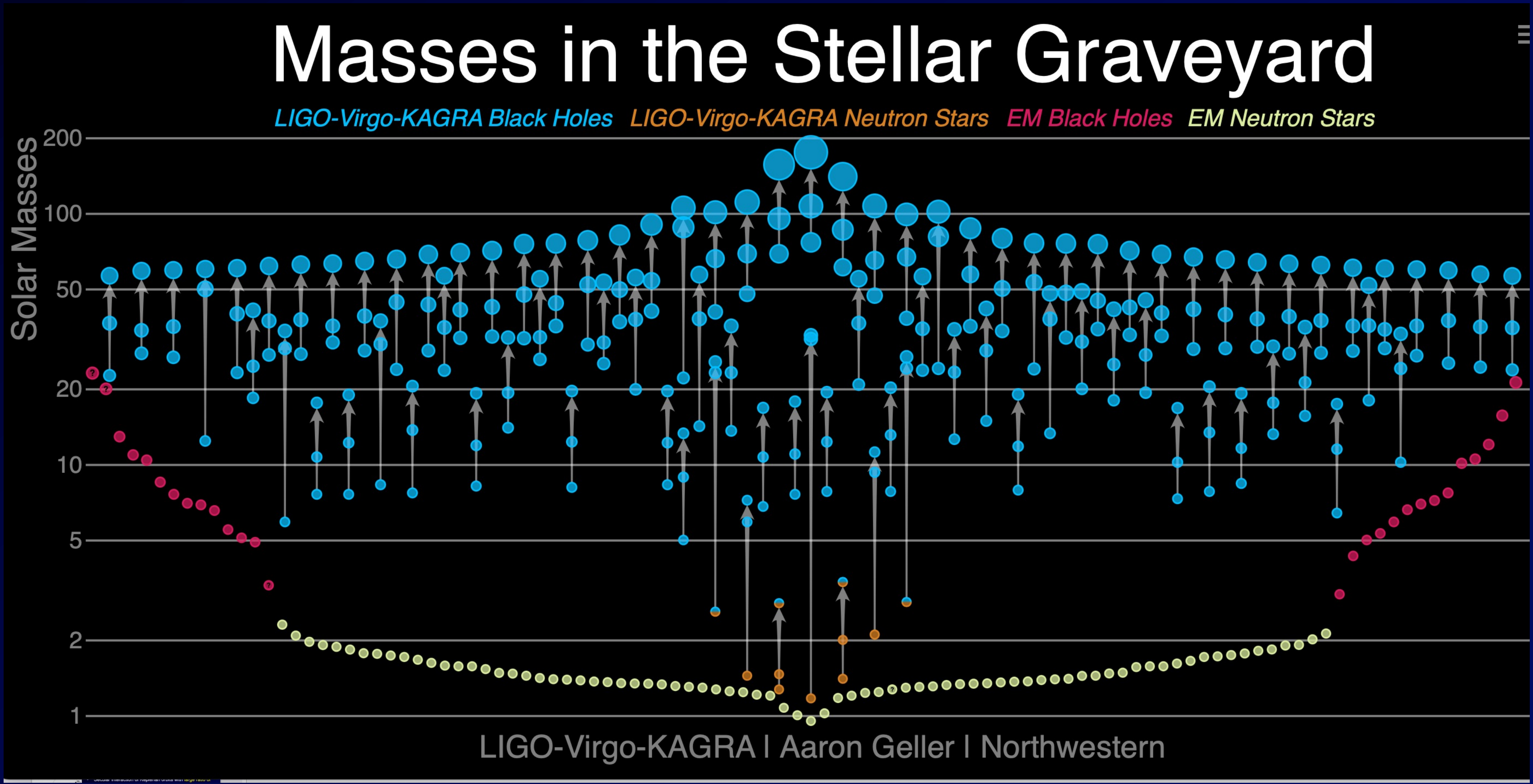
Roman Rafikov

UNIVERSITY OF CAMBRIDGE

Institute for Advanced Study



LIGO/Virgo discoveries of GW sources



LIGO et al 2018;
Venumadhav et al 2019

Origin of merging binaries

$(30+30)M_{Sun}$

Time to merge due to GW emission is long

$$T_{\rm m} \approx 10{\rm Gyr} \left(\frac{60 M_\odot}{m_1+m_2}\right)^2 \left(\frac{15 M_\odot}{\mu}\right) \left(\frac{a}{0.2{\rm AU}}\right)^4 (1-e^2)^{7/2}$$

Need the binary to be (1) compact (a<0.2 AU) or (2) very eccentric, $e \longrightarrow 1$, to merge in a Hubble time!

Main merger scenarios

Isolated stellar evolution

AGN disks

Dynamical evolution in stellar clusters

Dynamical evolution in isolated triples

Main merger scenarios

Isolated stellar evolution

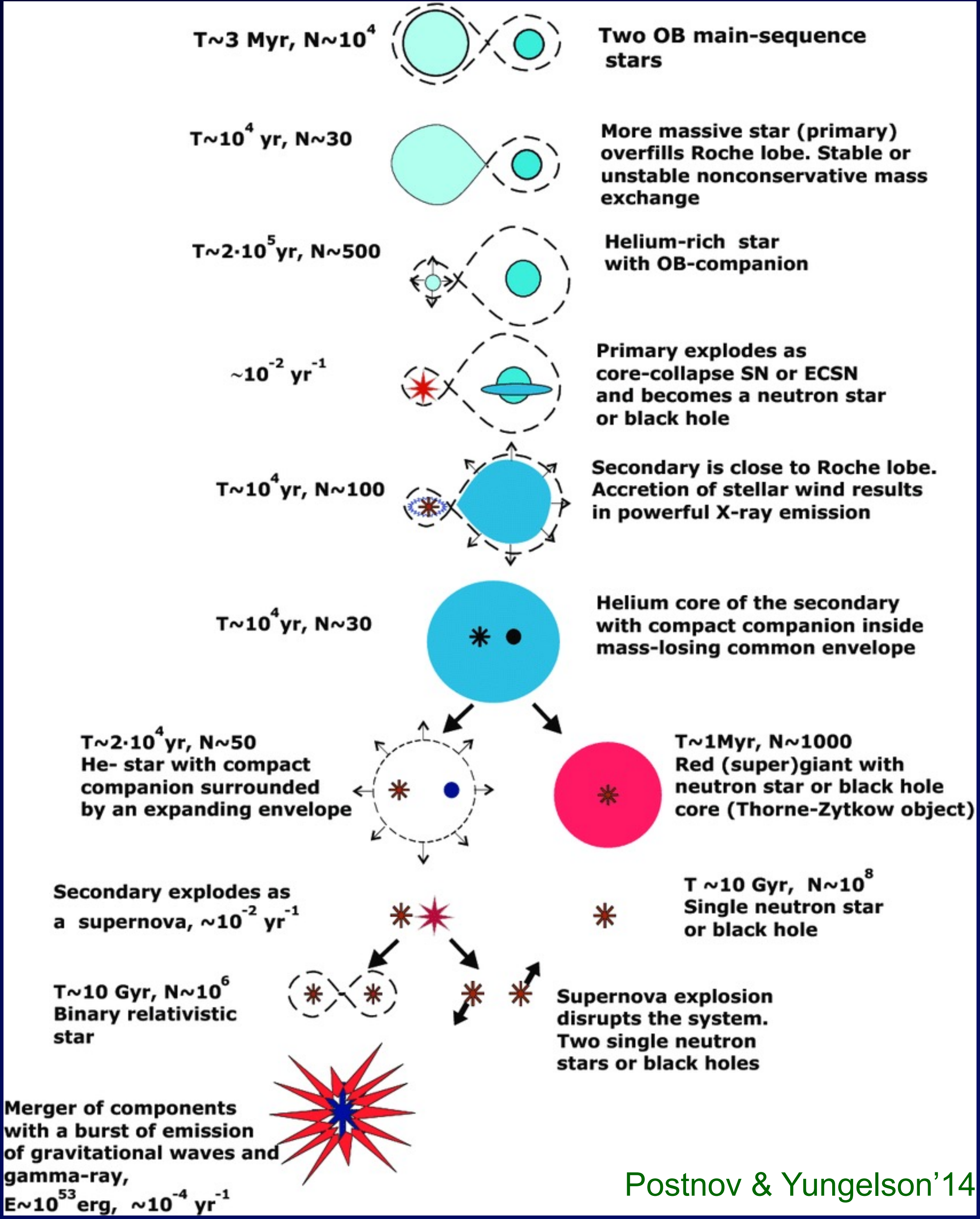
AGN disks

Dynamical evolution in stellar clusters

Dynamical evolution in stellar triples

Stellar evolution

- Massive stars often come in binaries
- Post-MS evolution produces compact objects (Tutukov & Yungelson 1973)
- Orbit can be shrunk through common envelope (Paczynsky 1971; Iben & Livio 1993)
- Can occur via a chemically homogeneous evolution in tight massive binaries (Mandel & de Mink 2016)



Postnov & Yungelson'14

Main merger scenarios

AGN disks

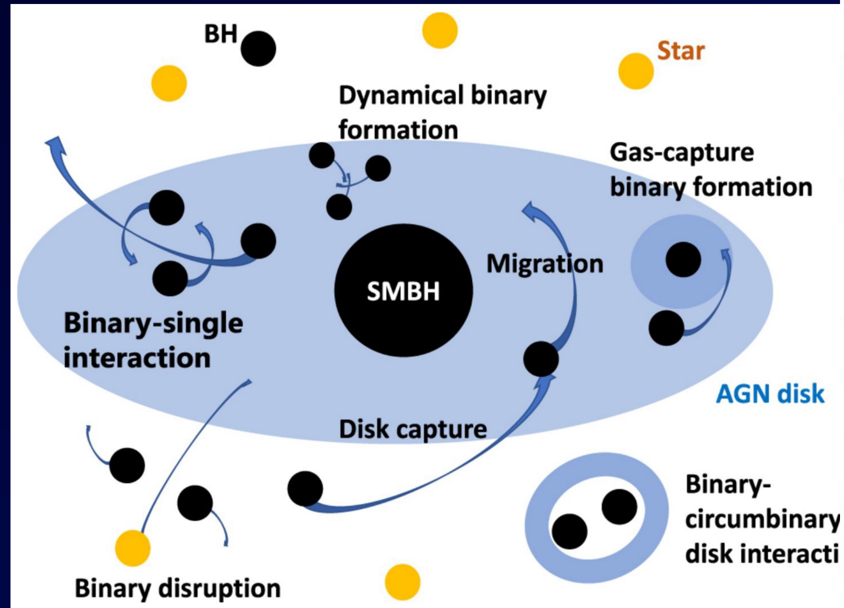
Isolated stellar evolution

Dynamical evolution in stellar clusters

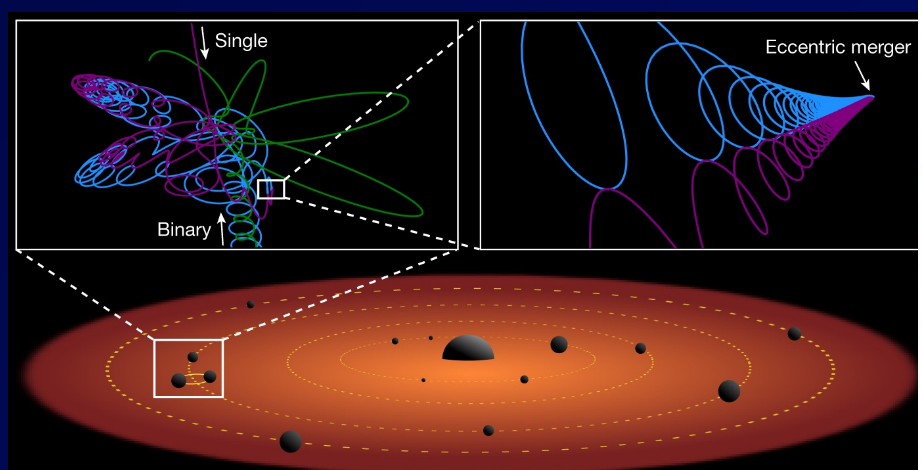
Dynamical evolution in stellar triples

AGN disks

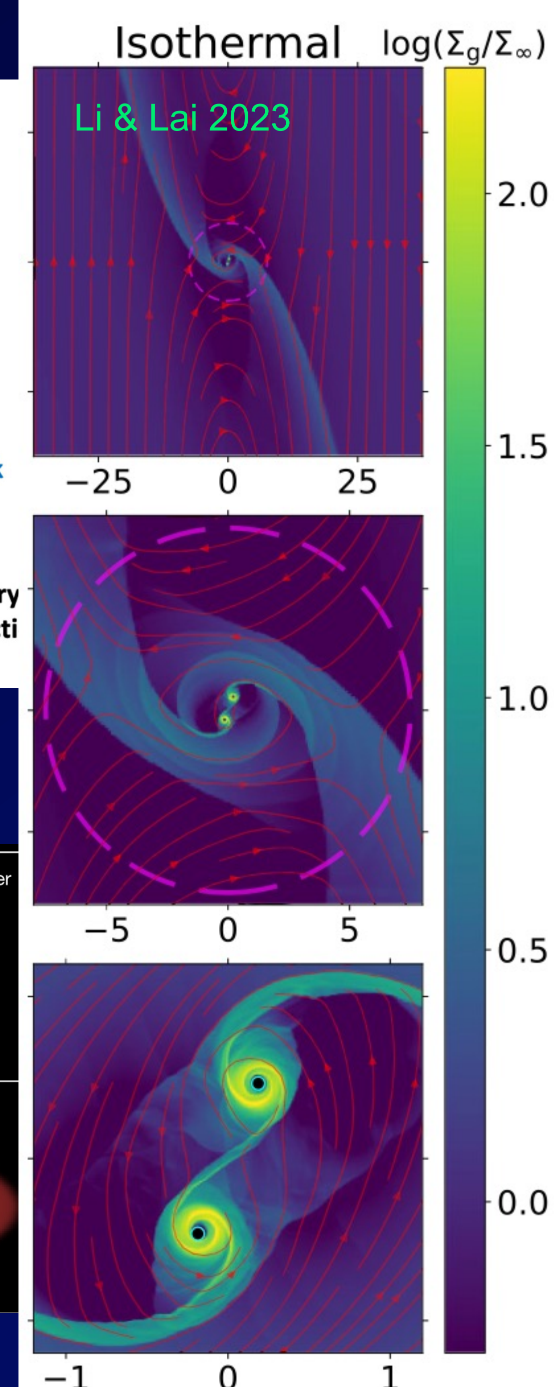
- BHs can be trapped by the disk or form in it as a result of evolution of massive stars
- They migrate, meet each other, form binaries
- Binaries shrink due to interaction with the gas and GW emission
- Eventually they merge



Tagawa et al 2020



Samsing et al 2022



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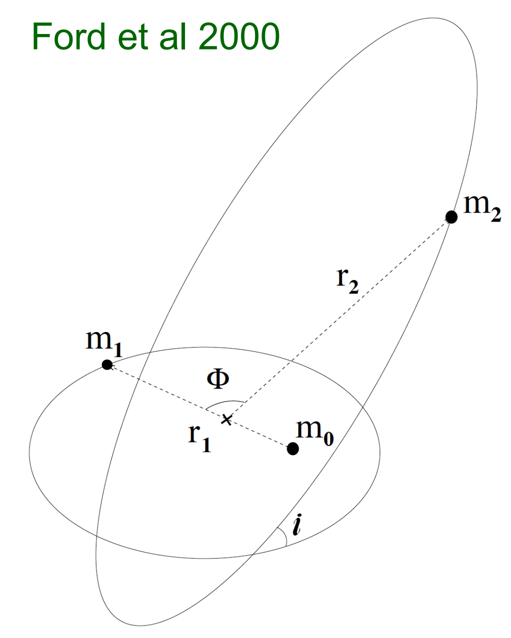


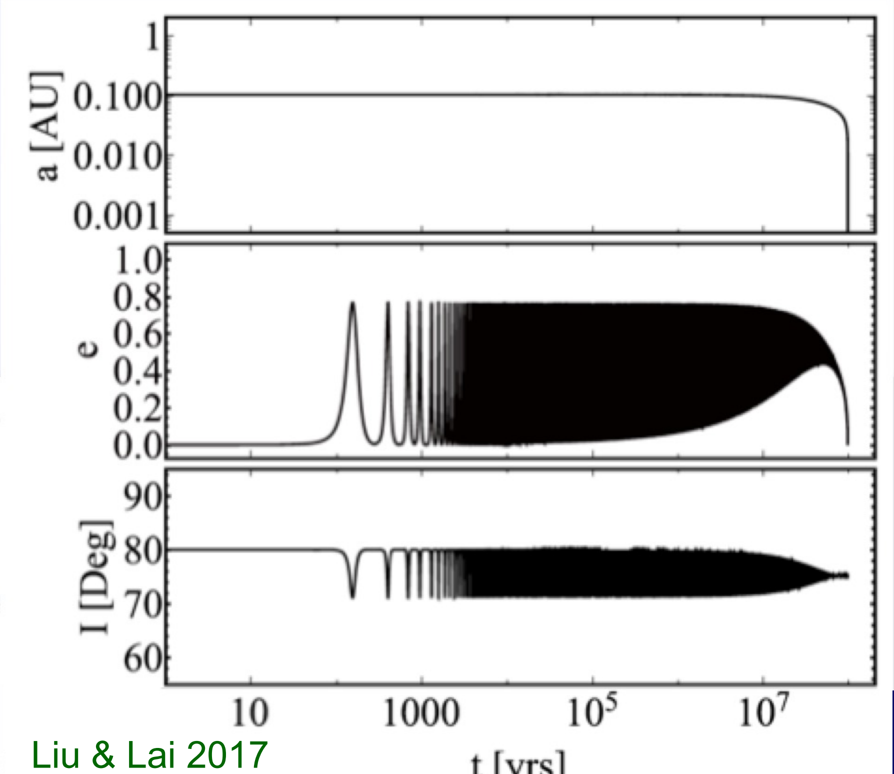
Ford et al 2000

Mergers via Lidov-Kozai in triples

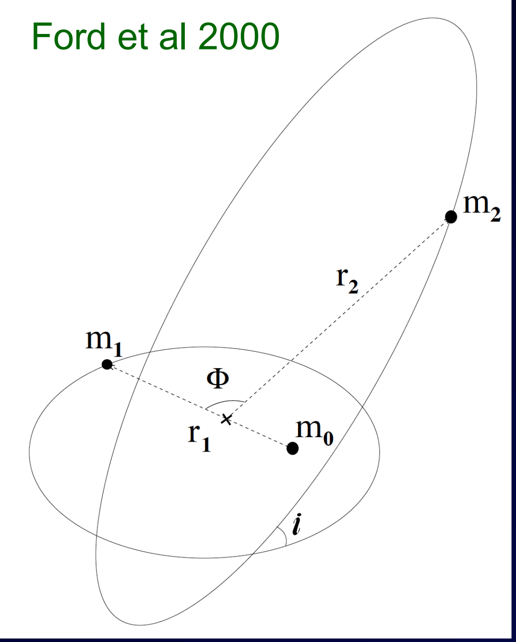
- Secular interaction of Keplerian orbits with large ratio of semi-major axes (hierarchical) (Lidov 1962; Kozai 1962)
- Can derive Hamiltonian (interaction potential) for arbitrary binary eccentricity e and inclination i

$$H = (2 + 3e^2)(1 - 3\cos^2 i) - 15e^2 \sin^2 i \cos 2\omega$$

- IoM – $L_z = \sqrt{1 - e^2}\cos i = const$
- For highly inclined orbits find large scale eccentricity excursions – LK cycles
- As e->1 gravitational wave emission gets boosted, shrinking binary semi-major axis
- Eventually results in a merger (Antonini et al 2014; Silsbee & Tremaine 2017; Liu & Lai 2017, etc.)



Liu & Lai 2017



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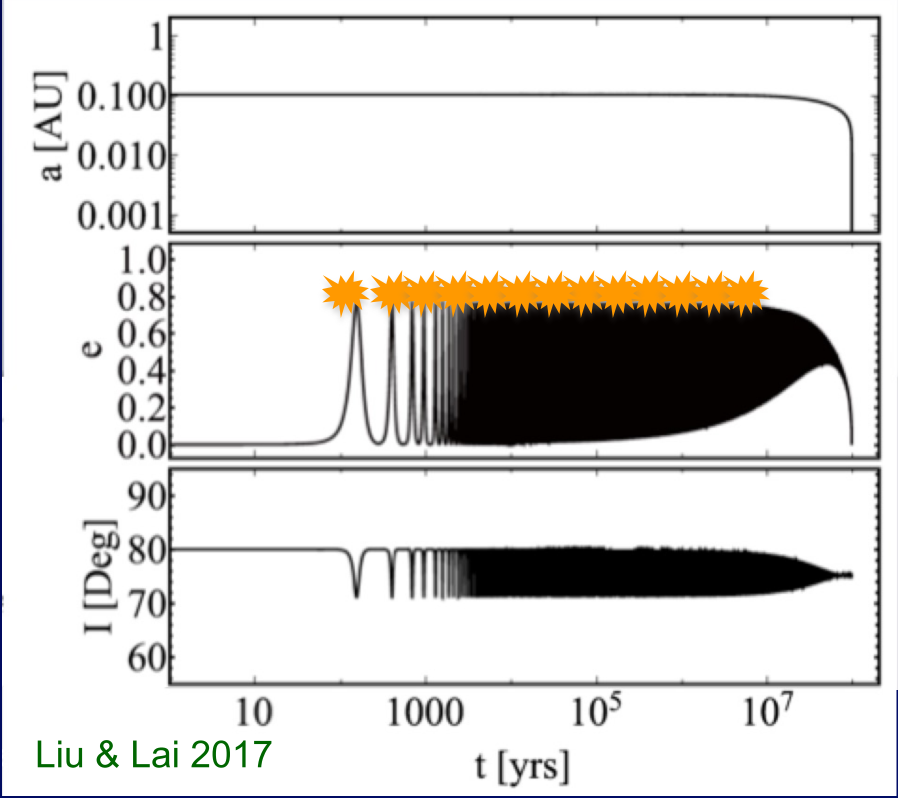

Liu & Lai 2017

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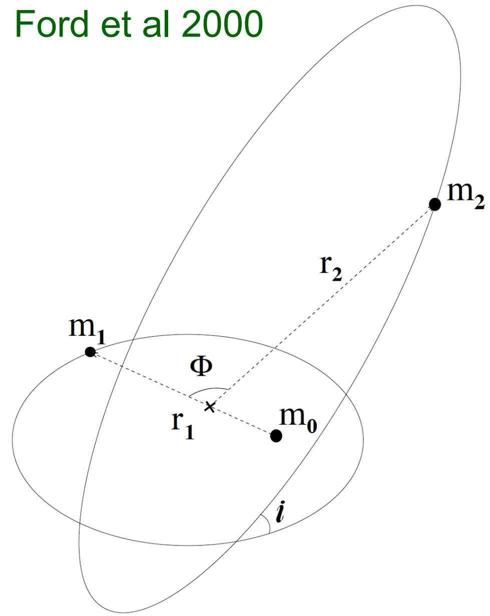

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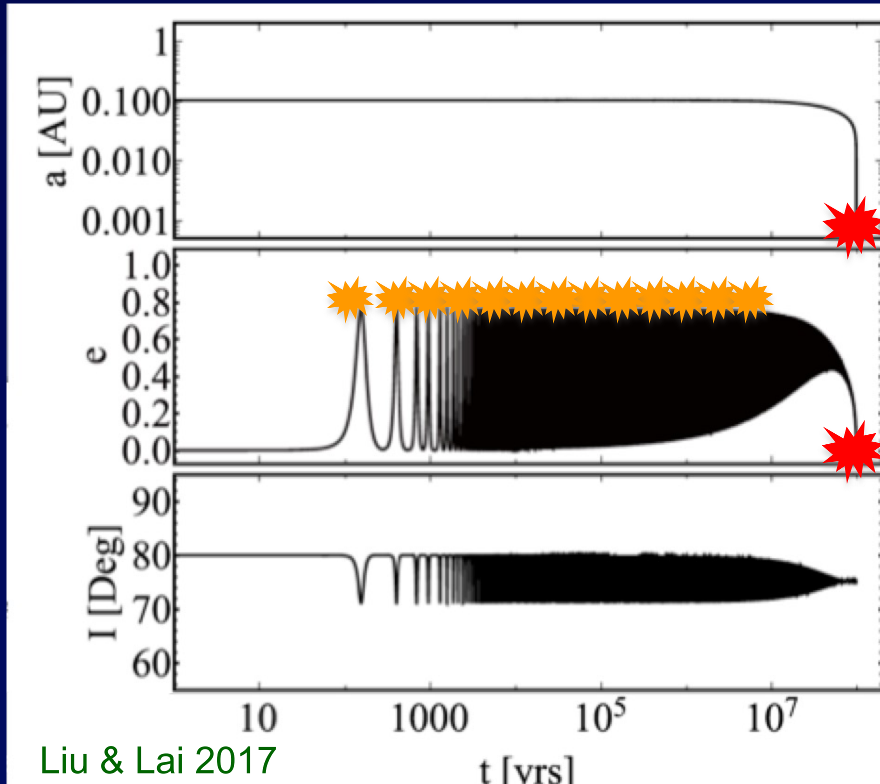
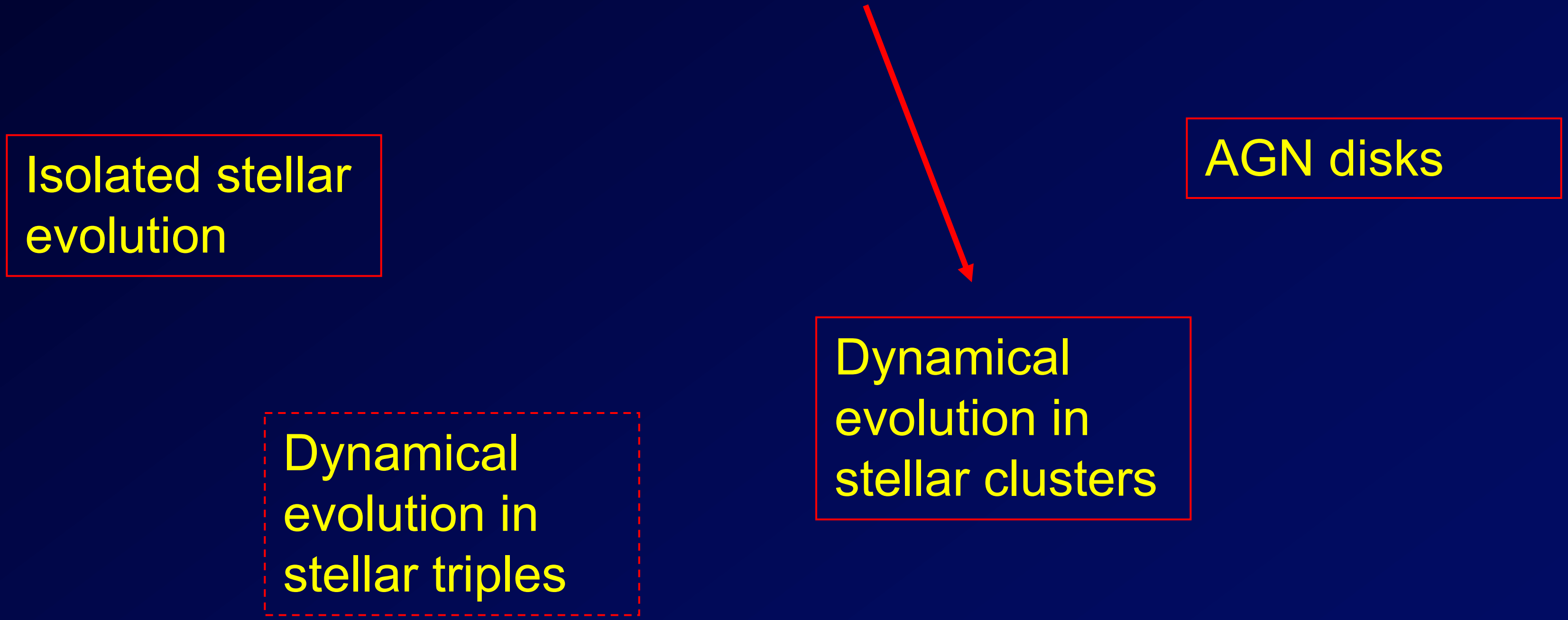


Liu & Lai 2017

Main merger scenarios



Stellar clusters

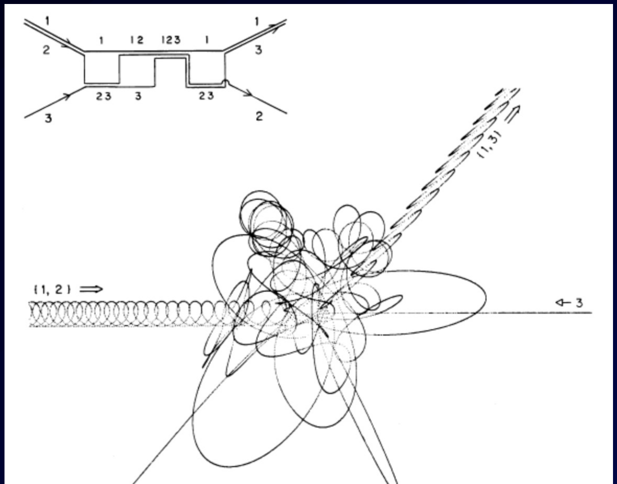
- Compact object binaries can also efficiently form in dense stellar systems – globular & nuclear clusters via many-body dynamical encounters

- Abundance of X-ray binaries (per unit stellar mass) is ~10^2 higher in globulars than in the field (Katz 1975; Clark 1975)

- Orbits can be shrunk by continuous hardening (stellar encounters) in cluster cores until the binary merges (Antonini & Rasio 2016; Leigh et al 2018)

- In clusters with central supermassive black hole (SMBH) Lidov-Kozai can work - SMBH is the outer (tertiary) companion (Antonini & Perets 2012; Hamers et al 2018, etc.)

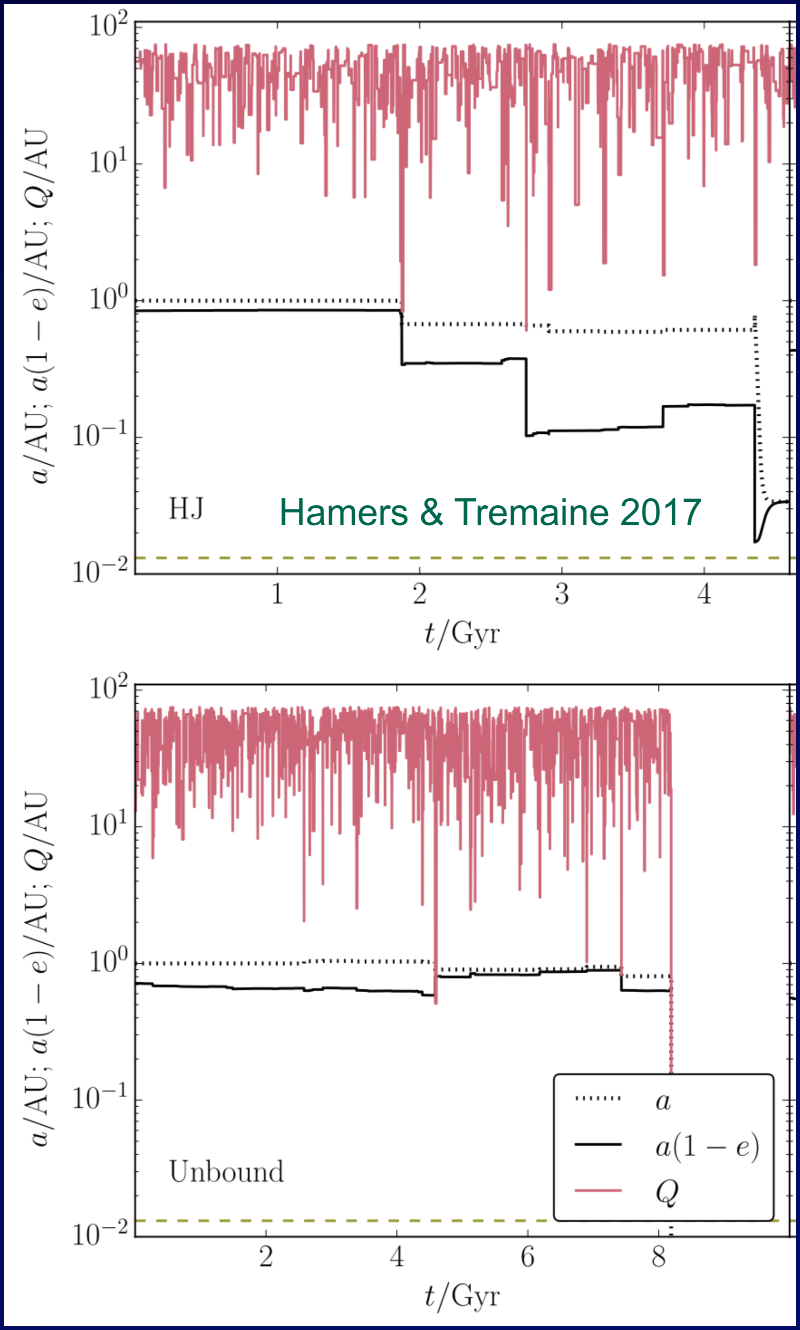
- Merger can be assisted by cluster oblateness (via nodal precession of the outer orbit, Petrovich & Antonini 2017) and GR spin-spin & spin-orbit coupling (Liu et al 2019)



Hut & Bahcall 1983

Stellar encounters hardening binaries

Heggie (1975), Hut + (1980s)



Hamers & Tremaine 2017

- Close stellar encounters strongly perturb binary orbit, directly change its semi-major axis
- Hard binaries harden - shrink, soft binaries soften - expand (Heggie's law, Heggie 1975)

$$a \lesssim \frac{GM_b}{\sigma^2}$$ - hard

- Eventually GW emission becomes important, causes orbital decay and merger

Rodrigues +, Kremer +, Samsing +, etc.

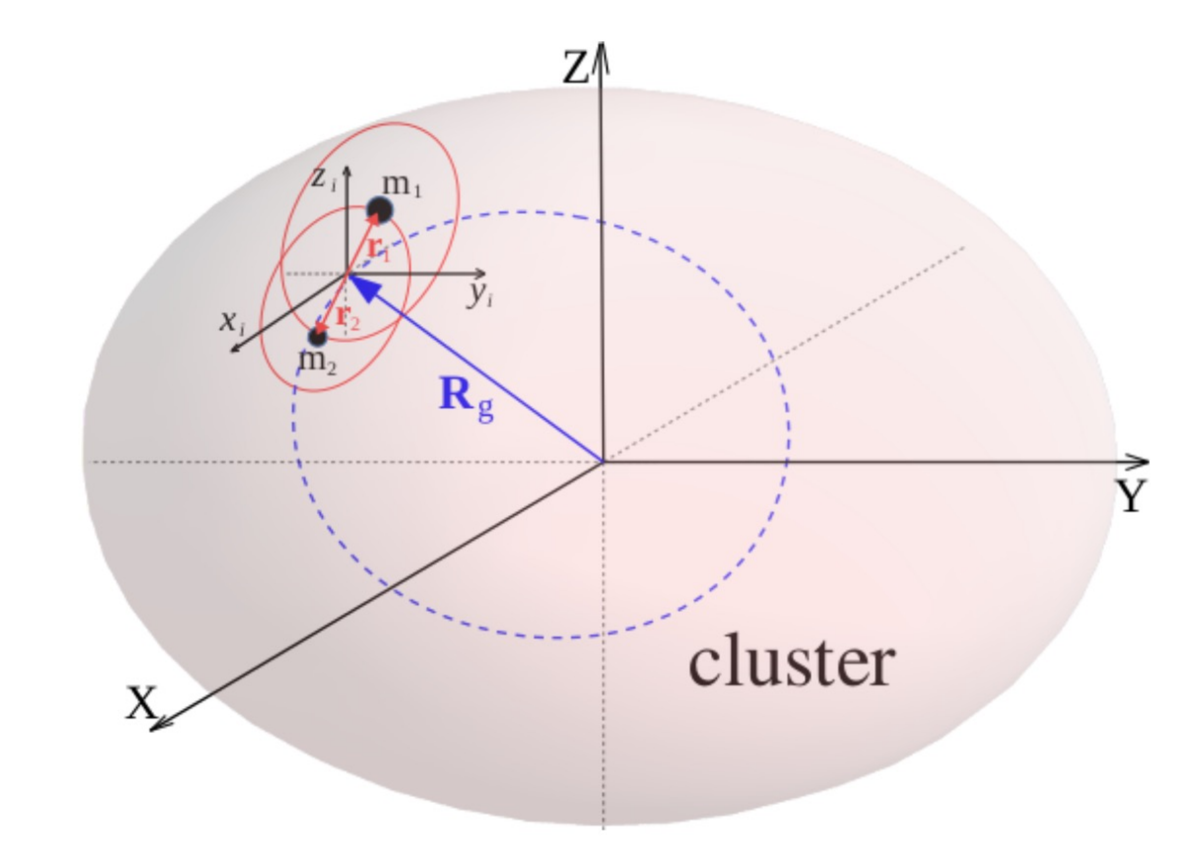
Cluster tides

With Chris Hamilton (IAS)

Cluster tides

- Cluster generates gravitational tide acting on the components of the binary

- Similar to a tide generated by a third body in the Lidov-Kozai case

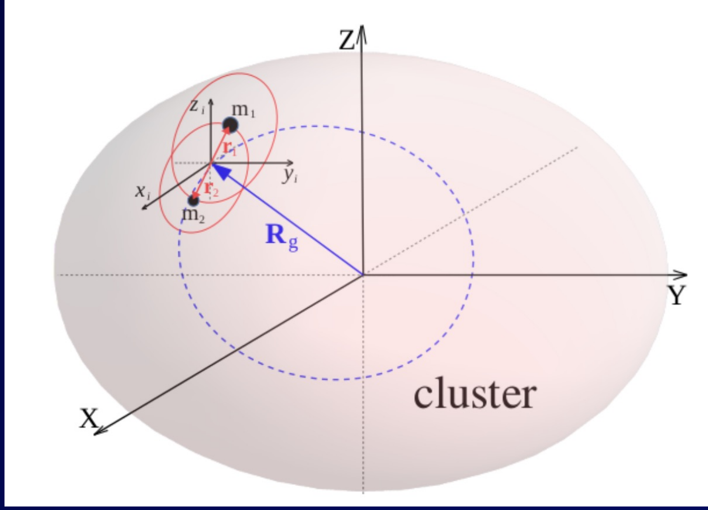


- Can be studied similarly, using secular perturbation theory

- In Hamilton & Rafikov (2019a,b) we explored tide-induced secular dynamics in axisymmetric clusters - subject of this talk

Tidal potential

Expand cluster potential around the binary barycenter, write down full interaction potential



$$H = H_0 + H_1$$

$$H_0 = \frac{1}{2}\mathbf{p}^2 - \frac{\mu}{r}$$

Newtonian 2-body interaction

$$H_1 = \frac{1}{2}\sum_{\alpha\beta} \Phi_{\alpha\beta}(\mathbf{R}_\mathrm{b})\, r_\alpha r_\beta$$

Tidal potential, quadrupole order

$$\Phi_{\alpha\beta} = \frac{\partial^2 \Phi}{\partial r_\alpha \partial r_\beta}$$

Averaging over inner orbit

$$\langle H \rangle_M = H_0 + \langle H_1 \rangle_M$$

- Upon averaging over the binary orbit

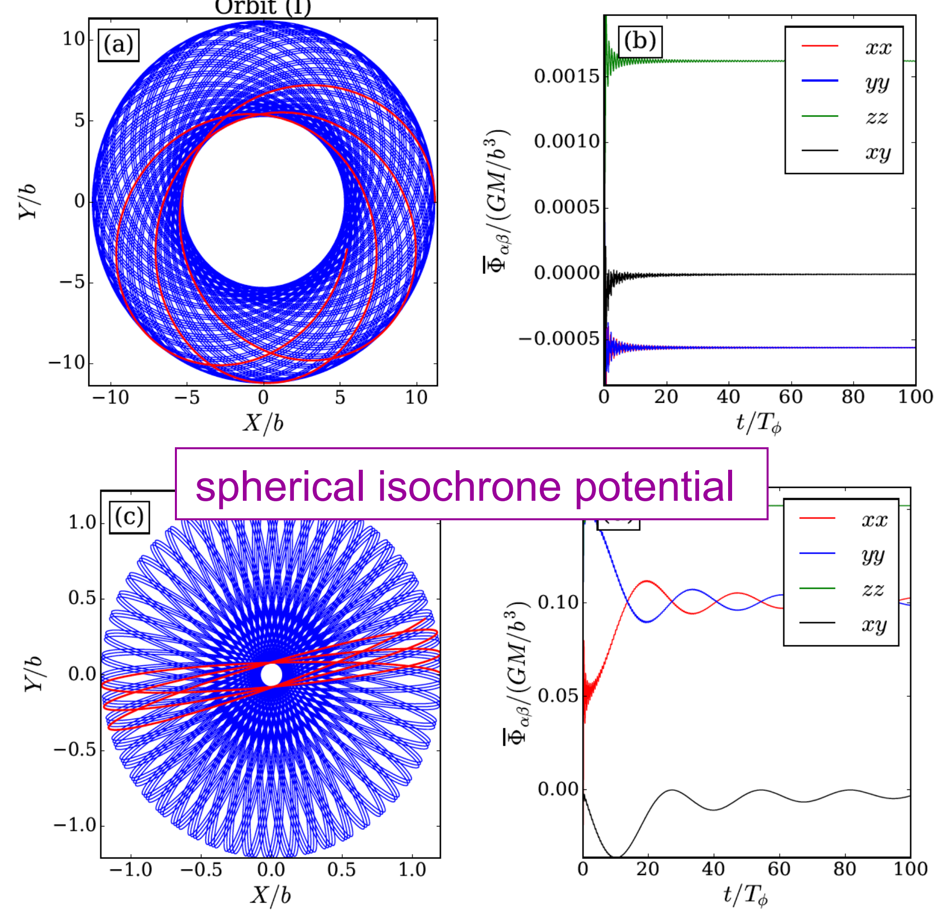
$$\langle H_1 \rangle_M = \frac{1}{2}\sum_{\alpha\beta} \Phi_{\alpha\beta} \langle r_\alpha r_\beta \rangle_M$$

- Singly-averaged (SA) tidal potential

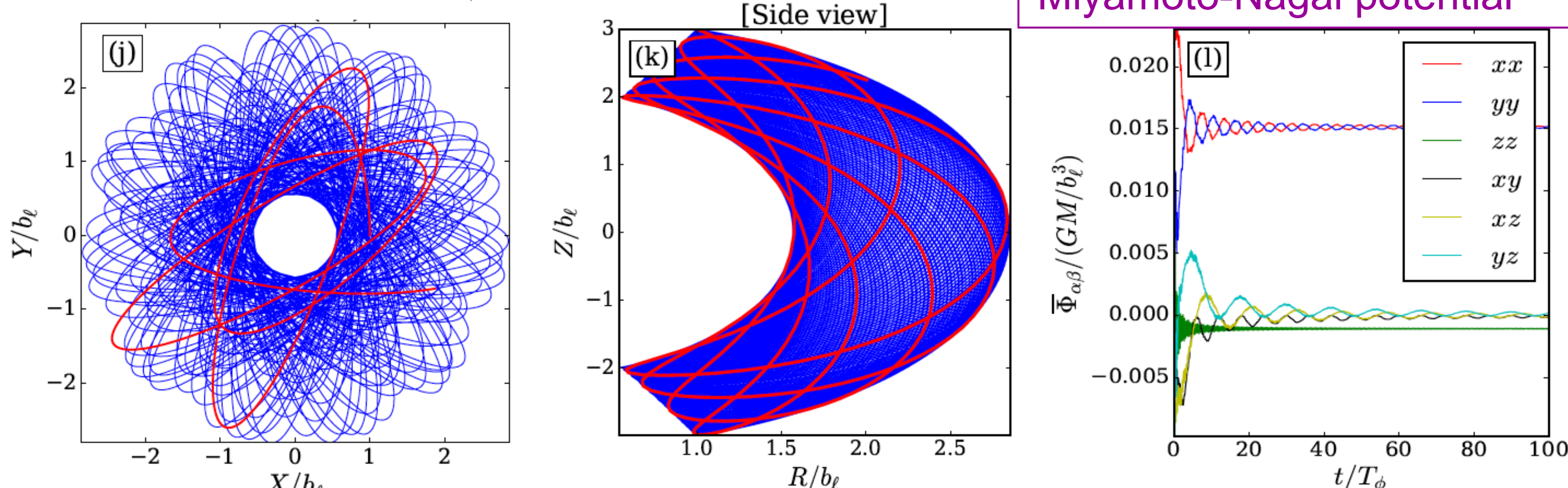
Averaging over the outer orbit

- Orbit fills 3D axisymmetric torus (planar annulus in a spherical cluster) over many outer periods
- Time-averaging of $\Phi_{\alpha\beta}$ results in axisymmetric tidal potential, L_z=const
- Convergence of $\langle\Phi_{\alpha\beta}\rangle$ is set by orbit-filling properties:
  faster filling = faster convergence
- Need convergence to occur faster than secular evolution (cf. Petrovich & Antonini 2017)

spherical isochrone potential



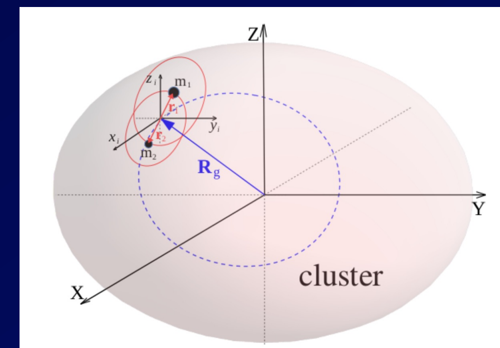
Miyamoto-Nagai potential



- In spherical clusters symmetry leaves only 2 independent components of $\langle\Phi_{\alpha\beta}\rangle$: $\langle\Phi_{xx}\rangle$ & $\langle\Phi_{zz}\rangle$

- Define $A \equiv \overline{\Phi}_{zz} + \overline{\Phi}_{xx}, \quad B \equiv \overline{\Phi}_{zz} - \overline{\Phi}_{xx}, \quad \Gamma \equiv B/3A.$

- Tidal Doubly Averaged (DA) Hamiltonian becomes



$$\overline{\langle H_1 \rangle}_M = C H_1^* \quad \text{where} \quad C = Aa^2/8.$$

$$H_1^* = (2+3e^2)(1-3\Gamma\cos^2 i) - 15\Gamma e^2 \sin^2 i \cos 2\omega.$$

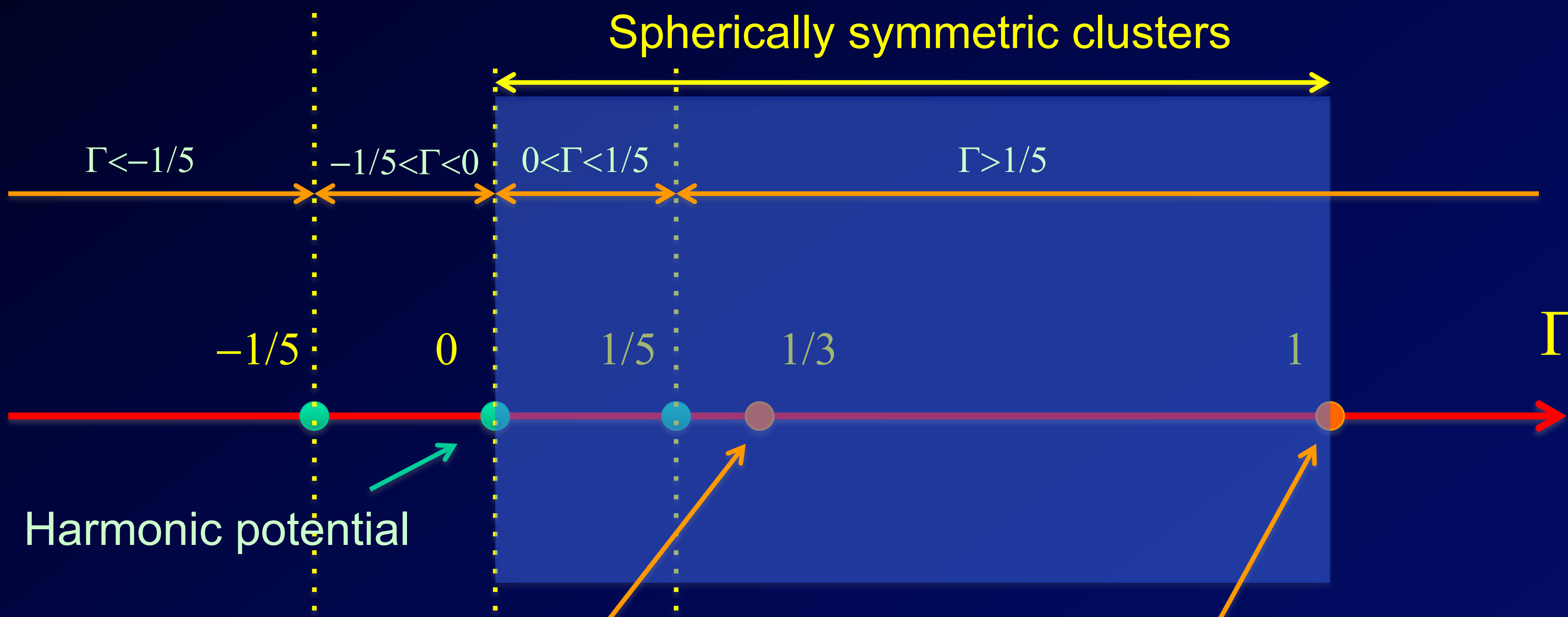
New interaction Hamiltonian due to cluster tide (Hamilton & Rafikov 2018a,b)

All cluster (and outer orbit) properties are absorbed into 2 parameters

- A – sets the timescale for the secular evolution $t_{sec} \sim n/A$, $A \sim GM_{cl}/b_{cl}^3$
- Γ - determines the phase space portrait

Γ regimes

$$H = (2+3e^2)(1-3\Gamma\cos^2 i) - 15\Gamma e^2 \sin^2 i \cos 2\omega$$



Galactic tide, Γ=1/3

$$H_{HT} = \sin^2 i(2+3e^2-5e^2\cos 2\omega)$$

Heisler & Tremaine 1986

Lidov-Kozai, Γ=1

$$H_{LK} = (2+3e^2)(1-3\cos^2 i) - 15e^2\sin^2 i\cos 2\omega$$

Lidov 1962; Kozai 1962

$\Gamma > 1/5$

For large Θ usual Laplace-Lagrange evolution

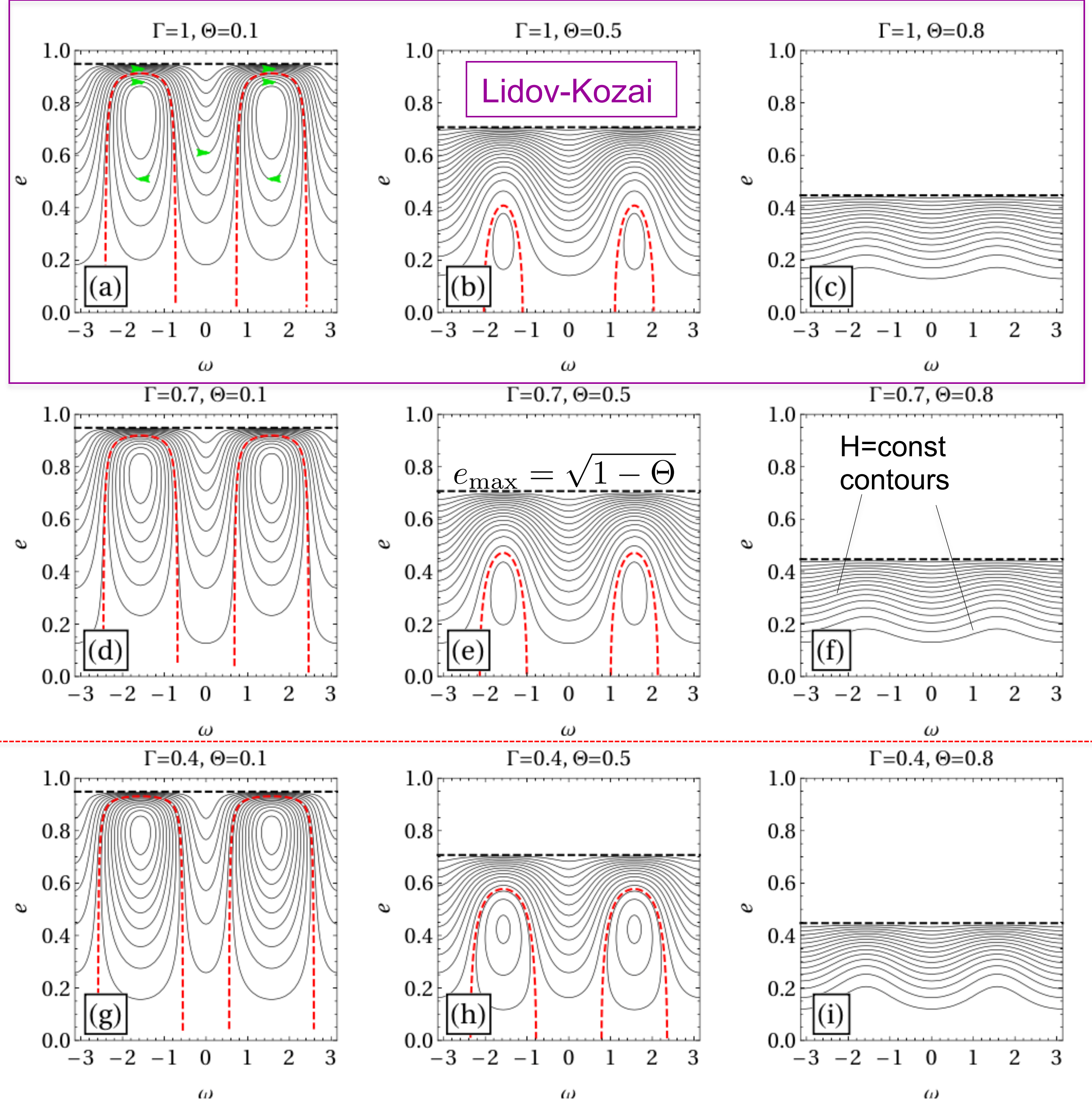
$$\Theta \equiv (1 - e^2)\cos^2 i$$

For low Θ fixed points and librating orbits appear.

Can take binary to high e

Circulating run above librating

Phase portraits are similar to the LK case.



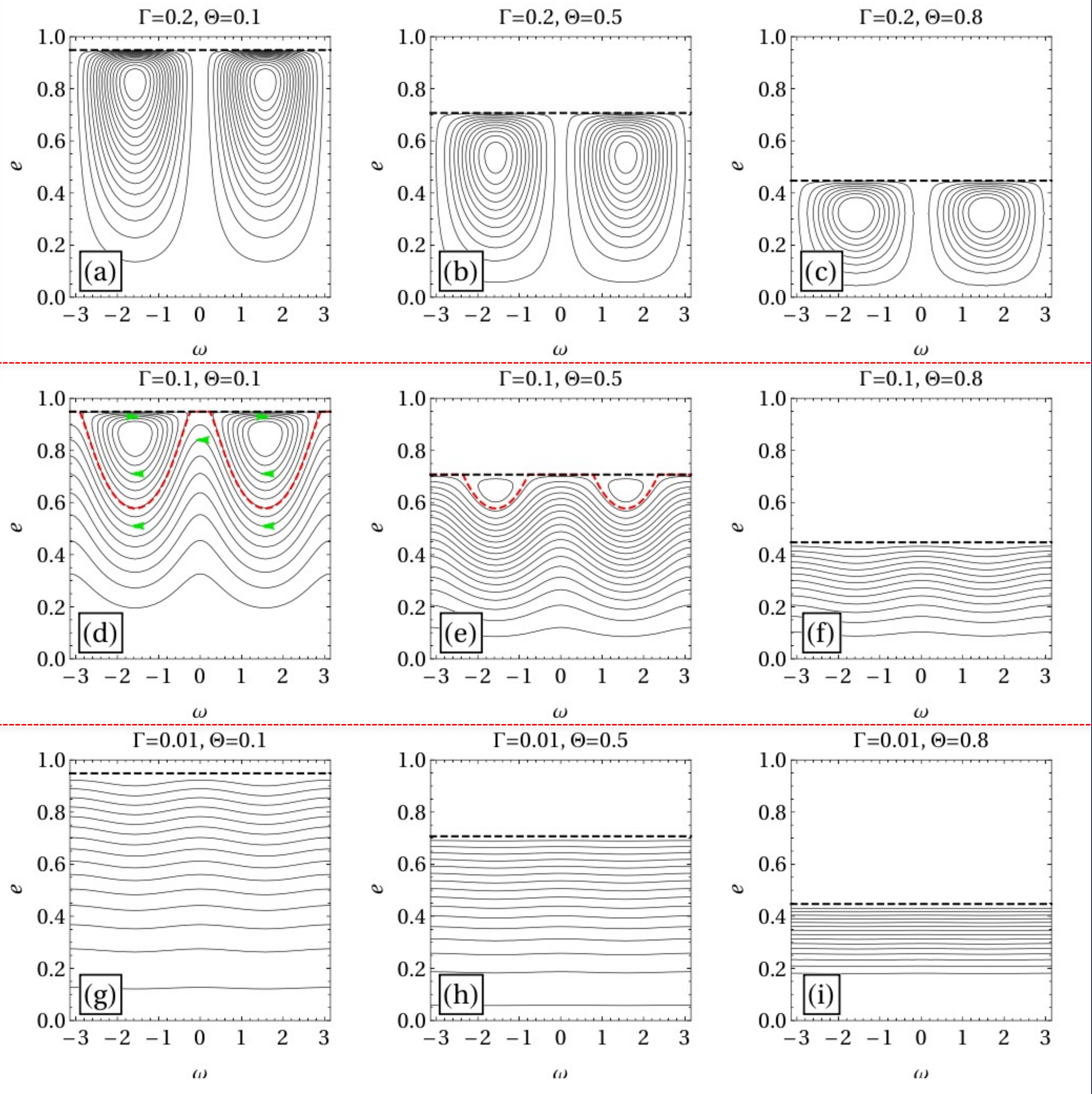
$0 < \Gamma < 1/5$

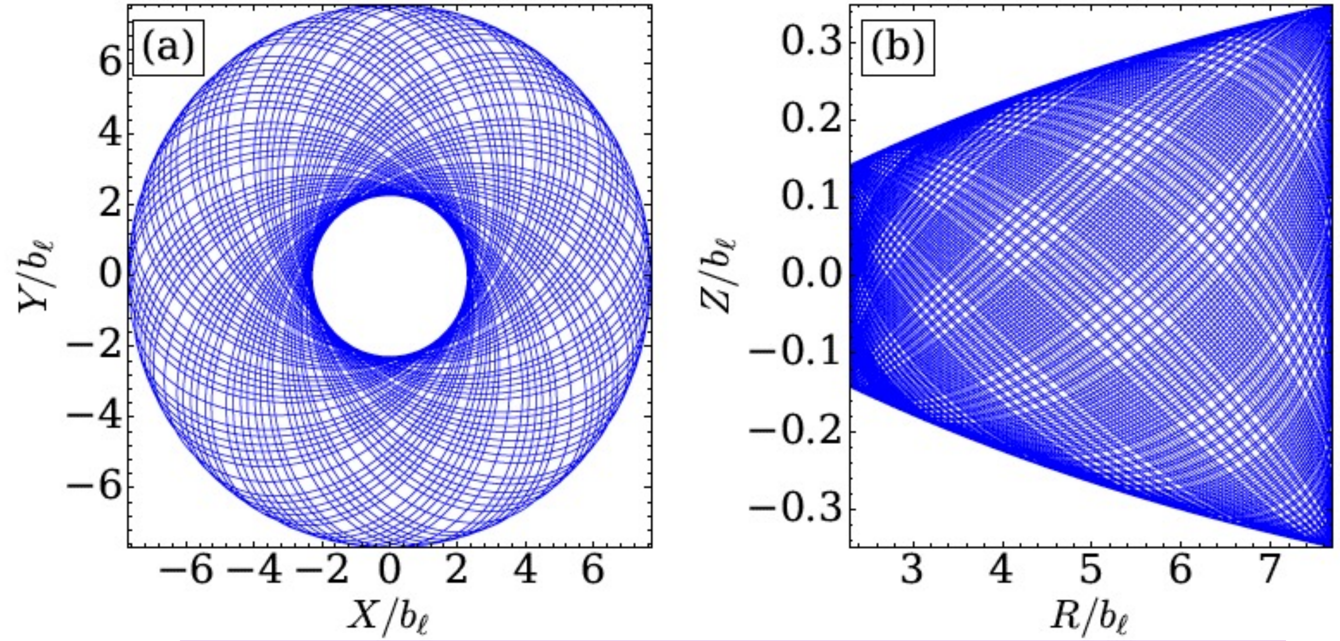
Phase portraits are different from the LK case.

Circulating orbits run below librating

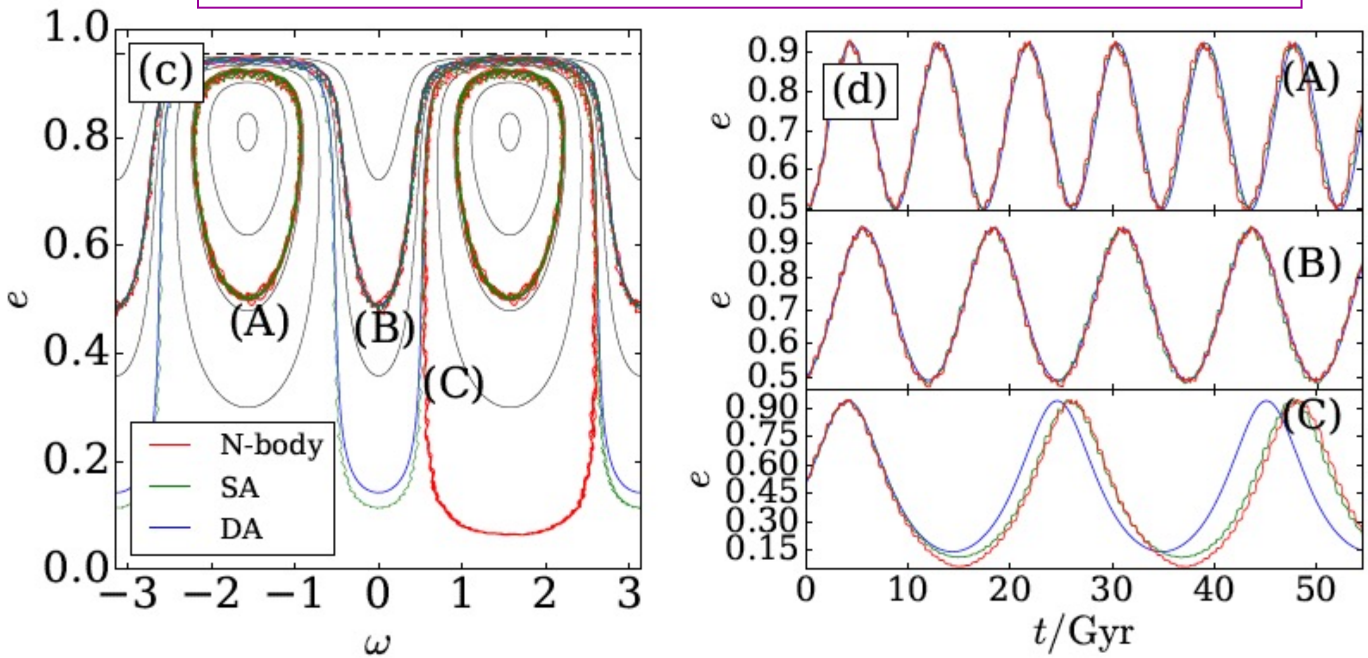
As Γ goes to zero (e.g. cores of clusters) fixed points disappear

Very difficult to reach high e starting with moderate eccentricity!



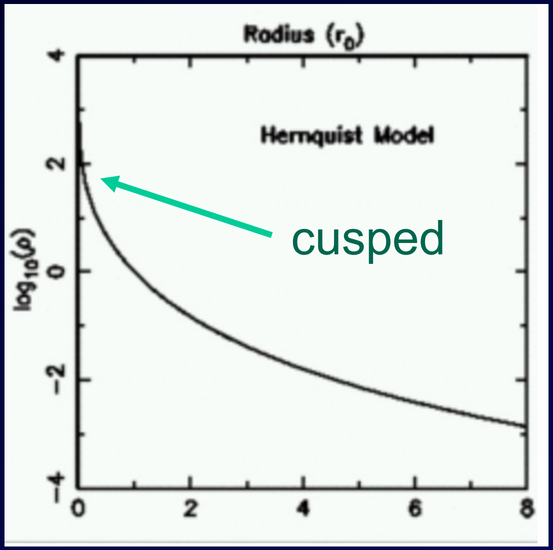


Test of the theory: Miyamoto-Nagai potential, Γ=0.37



Cluster potentials

Hernquist

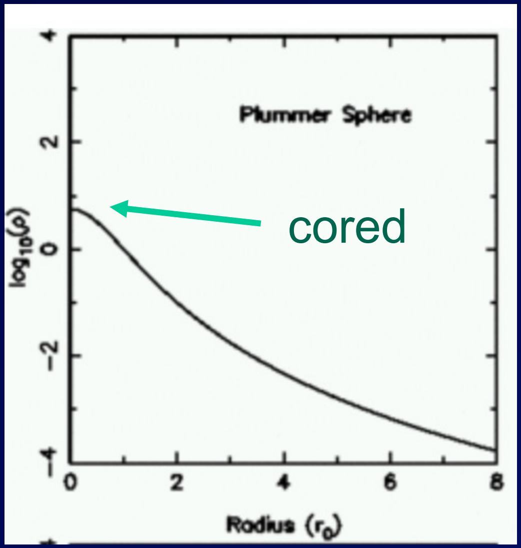


$$\Phi(r) = -\frac{GM_{cl}}{r+b}$$

$$\rho(r) = \frac{M_{cl}}{2\pi b^3}\frac{b^4}{r(r+b)^3}$$

May be suitable for
nuclear star clusters

Plummer



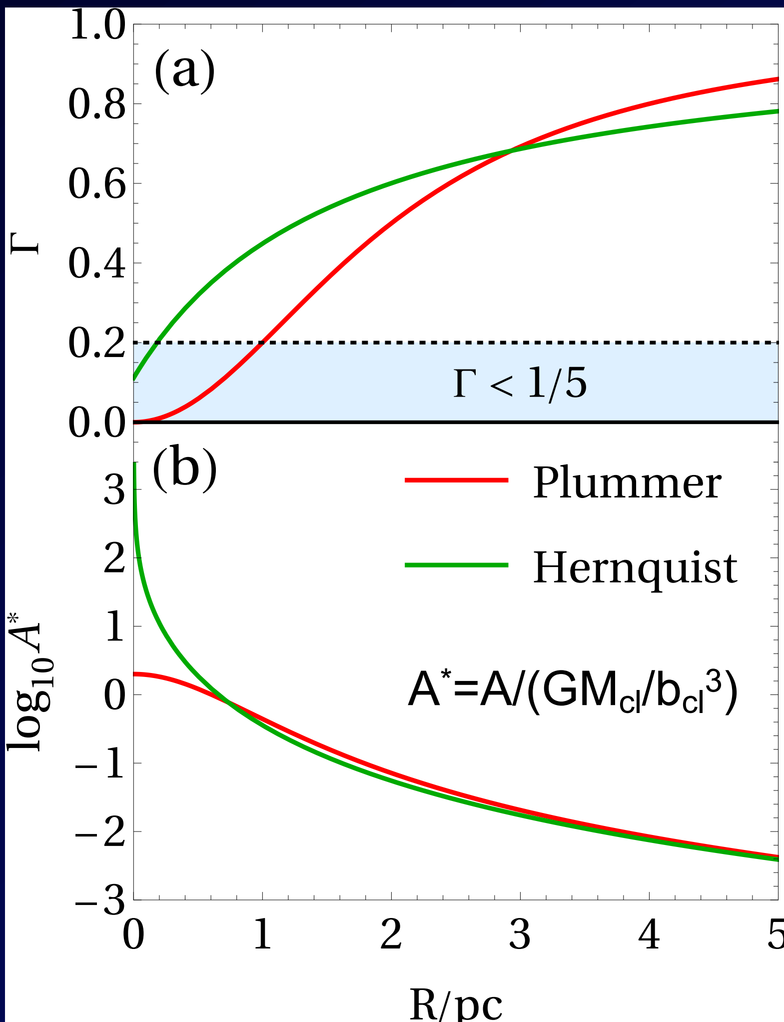
$$\Phi(r) = -\frac{GM_{cl}}{\sqrt{r^2+b^2}}$$

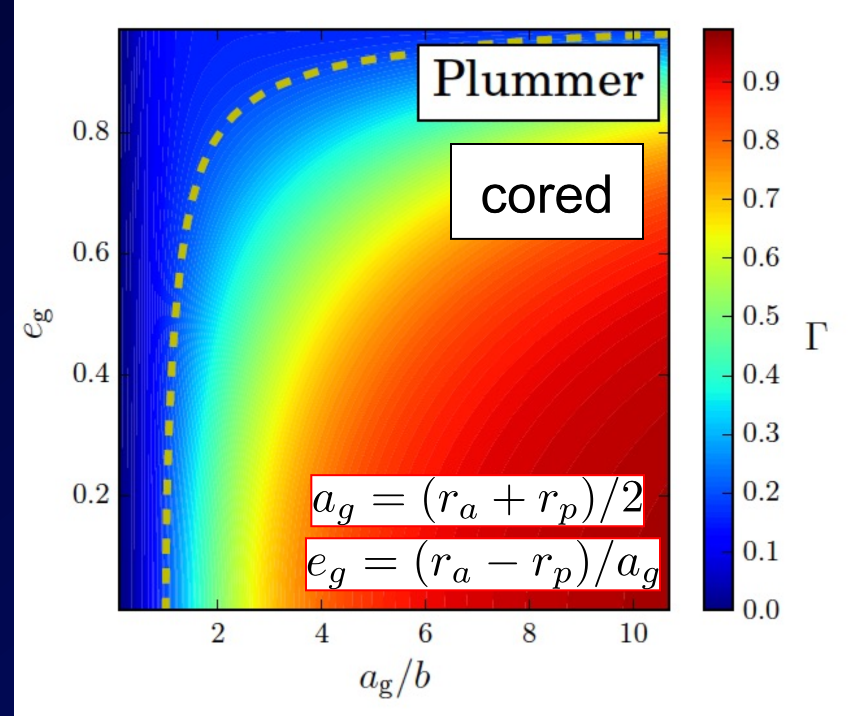
$$\rho(r) = \frac{3M_{cl}}{4\pi b^3}\frac{b^5}{(r^2+b^2)^{5/2}}$$

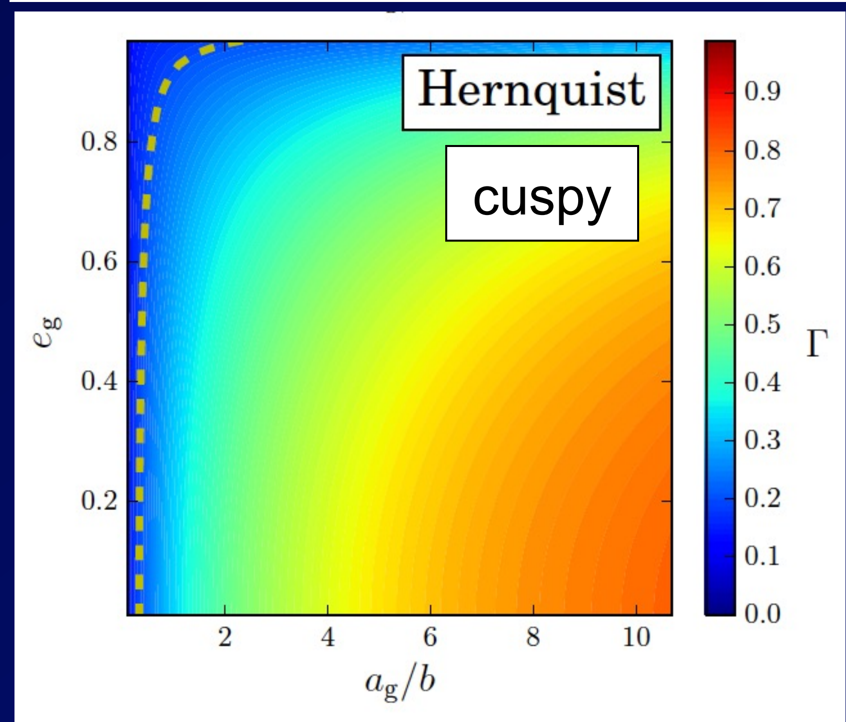
May be suitable for
globular clusters

Γ behavior: dependence on the potential and binary orbit properties

Circular orbits







Merger rate calculation (Hamilton & Rafikov 2019c)

For many binaries secular evolution timescale is shorter than t_{Hubble}

$$t_{\rm sec} \approx \frac{8}{3A}\sqrt{\frac{G(m_1+m_2)}{a^3}} \approx 100{\rm Myr}\left(\frac{0.5}{A^*}\right)\left(\frac{10^6 M_\odot}{M_{\rm cl}}\right)\left(\frac{b_{\rm cl}}{\rm pc}\right)^3\left(\frac{m_1+m_2}{M_\odot}\right)^{1/2}\left(\frac{10{\rm AU}}{a}\right)^{3/2}$$

May experience multiple secular cycles bringing e to high values, giving rise to GW emission and binary shrinking

Merger time is independent of t_{sec}: T_m

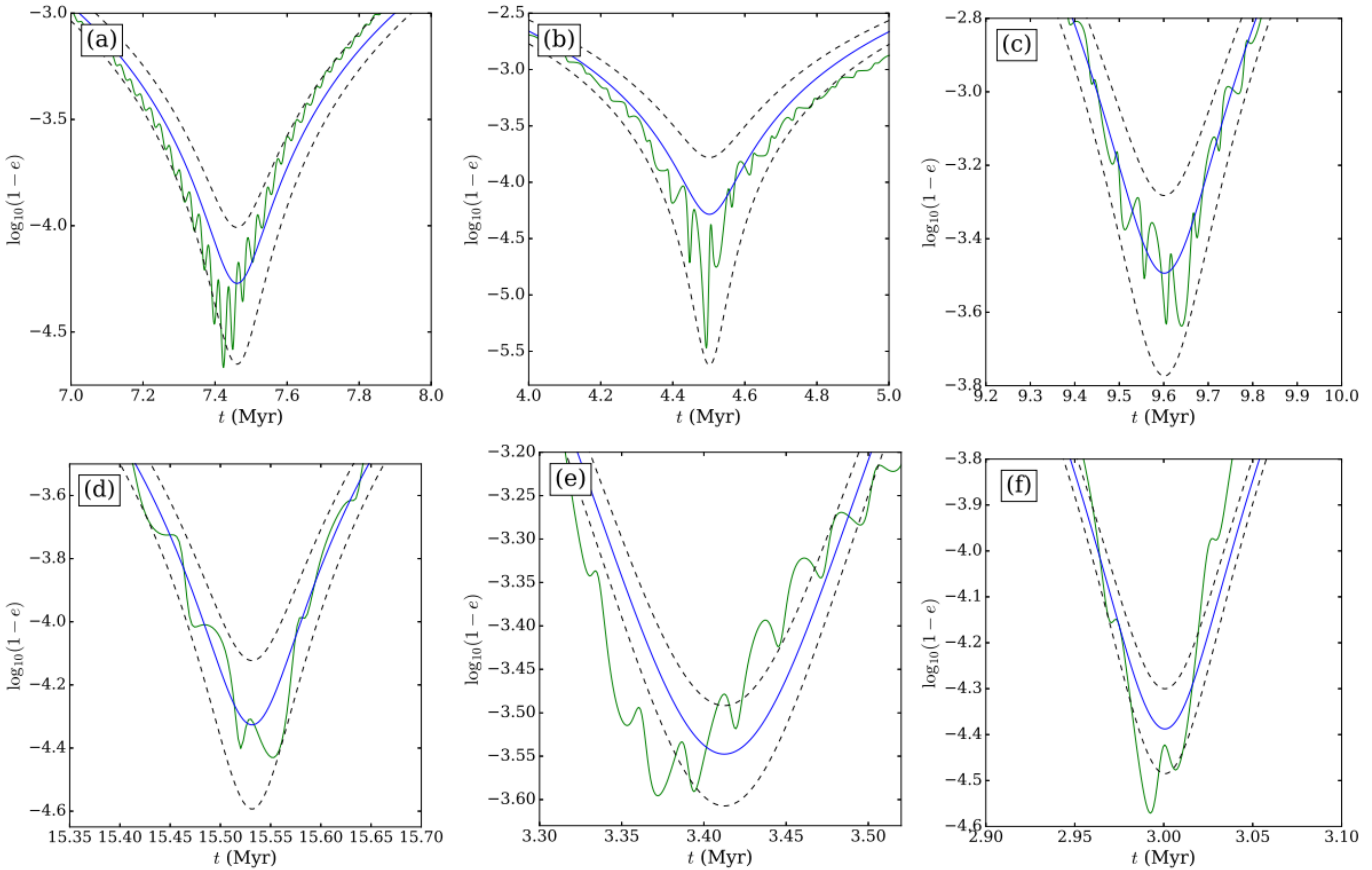
$$= 1.0\,{\rm Gyr}\left(\frac{m}{1.4M_\odot}\right)^{-3}\left(\frac{a_0}{10\,{\rm au}}\right)^4\frac{\psi(e_{\max},\tilde{e}_{\max})}{10^{-12}}$$

$$= 0.5\,{\rm Gyr}\left(\frac{m}{30M_\odot}\right)^{-3}\left(\frac{a_0}{30\,{\rm au}}\right)^4\frac{\psi(e_{\max},\tilde{e}_{\max})}{10^{-12}},$$

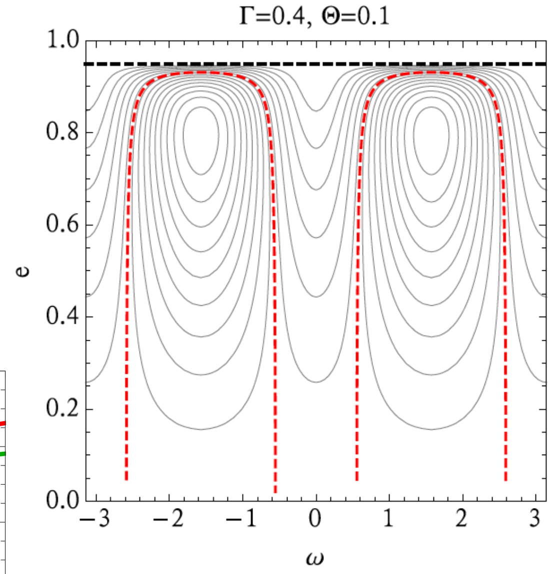
$$\psi(e_{\max},\tilde{e}_{\max}) = (1-\tilde{e}_{\max}^2)^{7/2}(1-e_{\max}^2)^{-1/2}$$

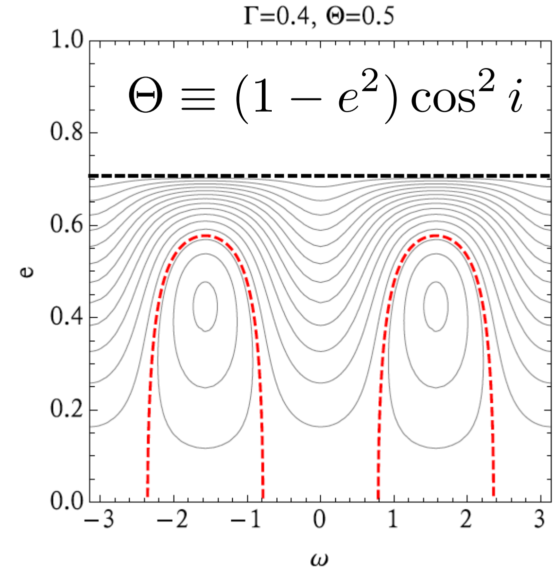
- Run MC-type calculation with $N=10^6$ binaries with randomly drawn initial parameters, compute e_{max} due to cluster tides for each binary
- Determine merger fraction $f_m(t)$ - fraction of the population that has $T_m < t$
- Account for the effect of GR precession (which dramatically reduces $f_m(t)$)

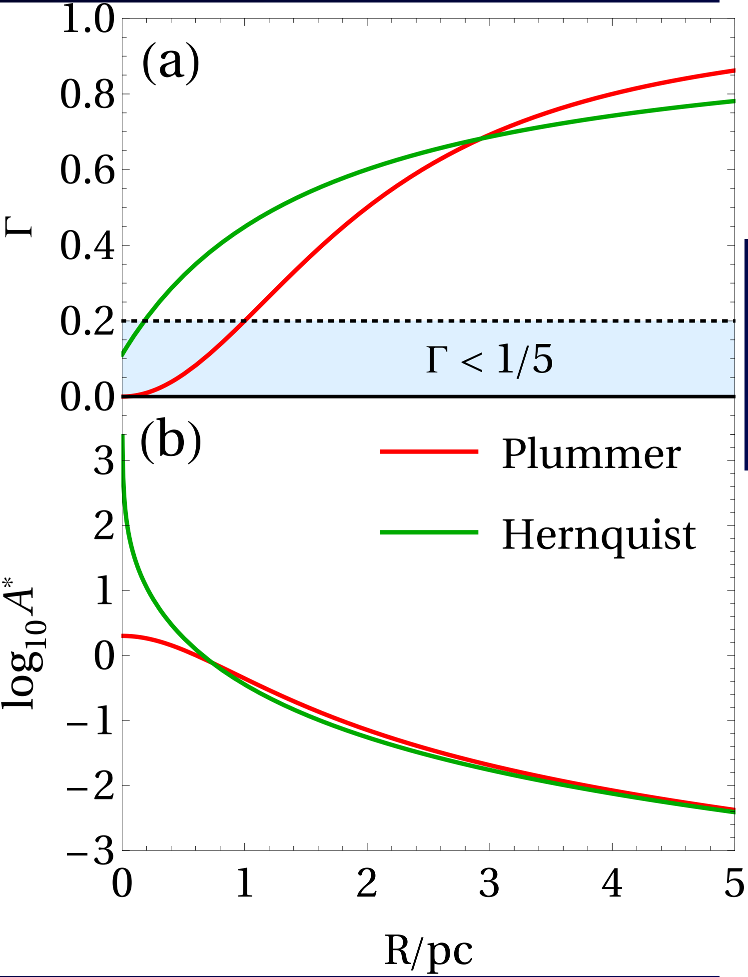
Singly-averaged (SA) eccentricity oscillations



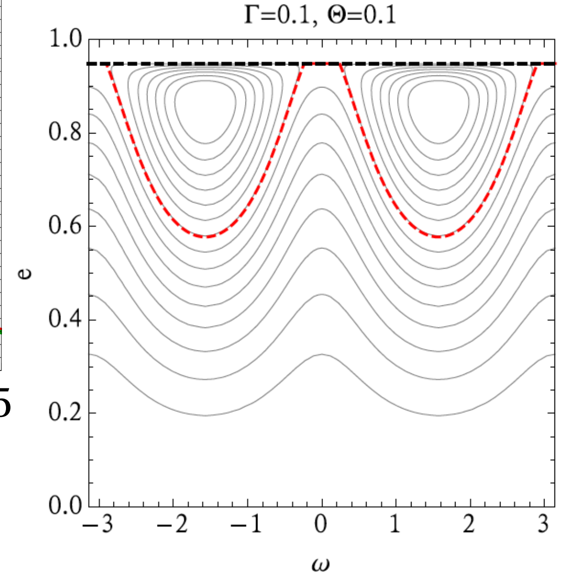
$\Gamma > 1/5$ regime dominates in cusped clusters - many binaries can reach high e

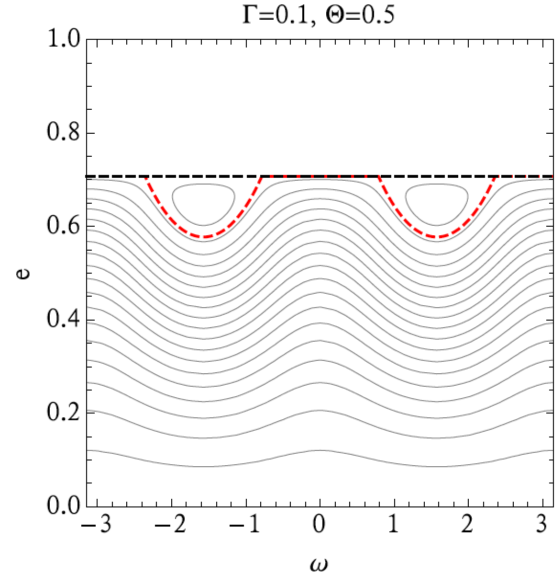






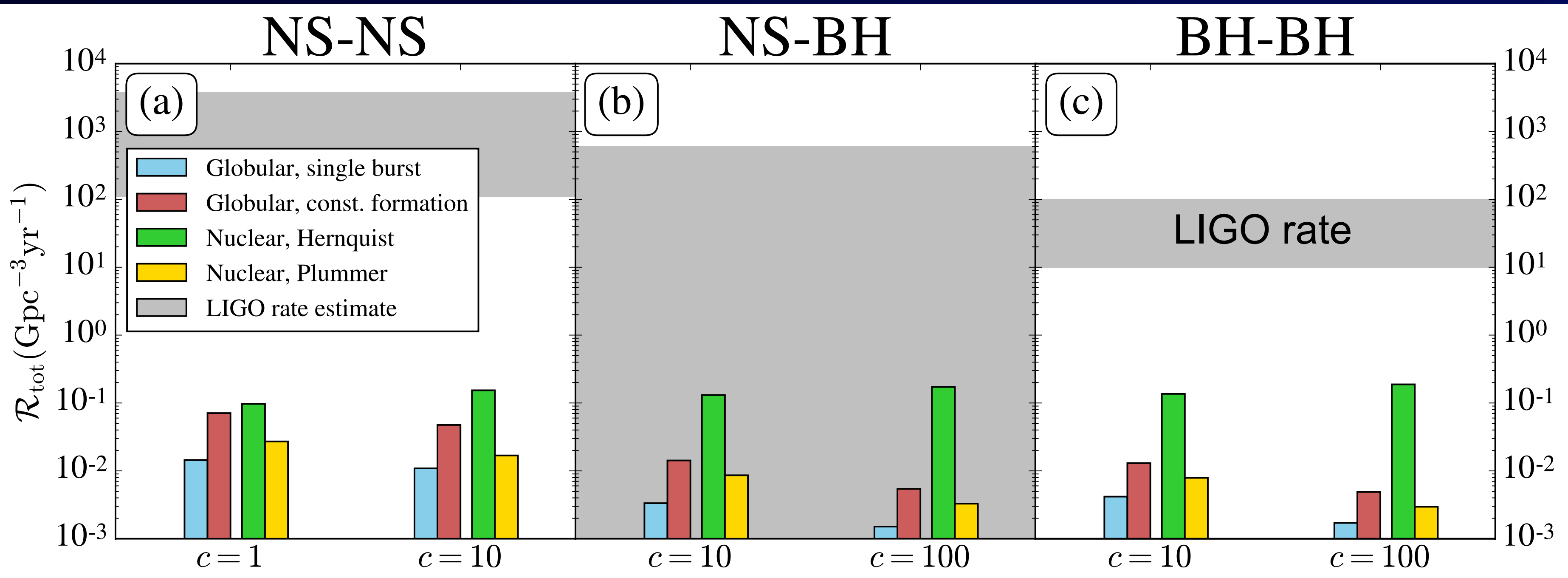
$0 < \Gamma < 1/5$ typical in cored clusters (e.g. globulars) - few binaries can reach high e





Merger rates

Knowing $f_m(t)$ compute merger rate for different compact binary birth histories:
(1) single burst or (2) continuous formation at a constant rate



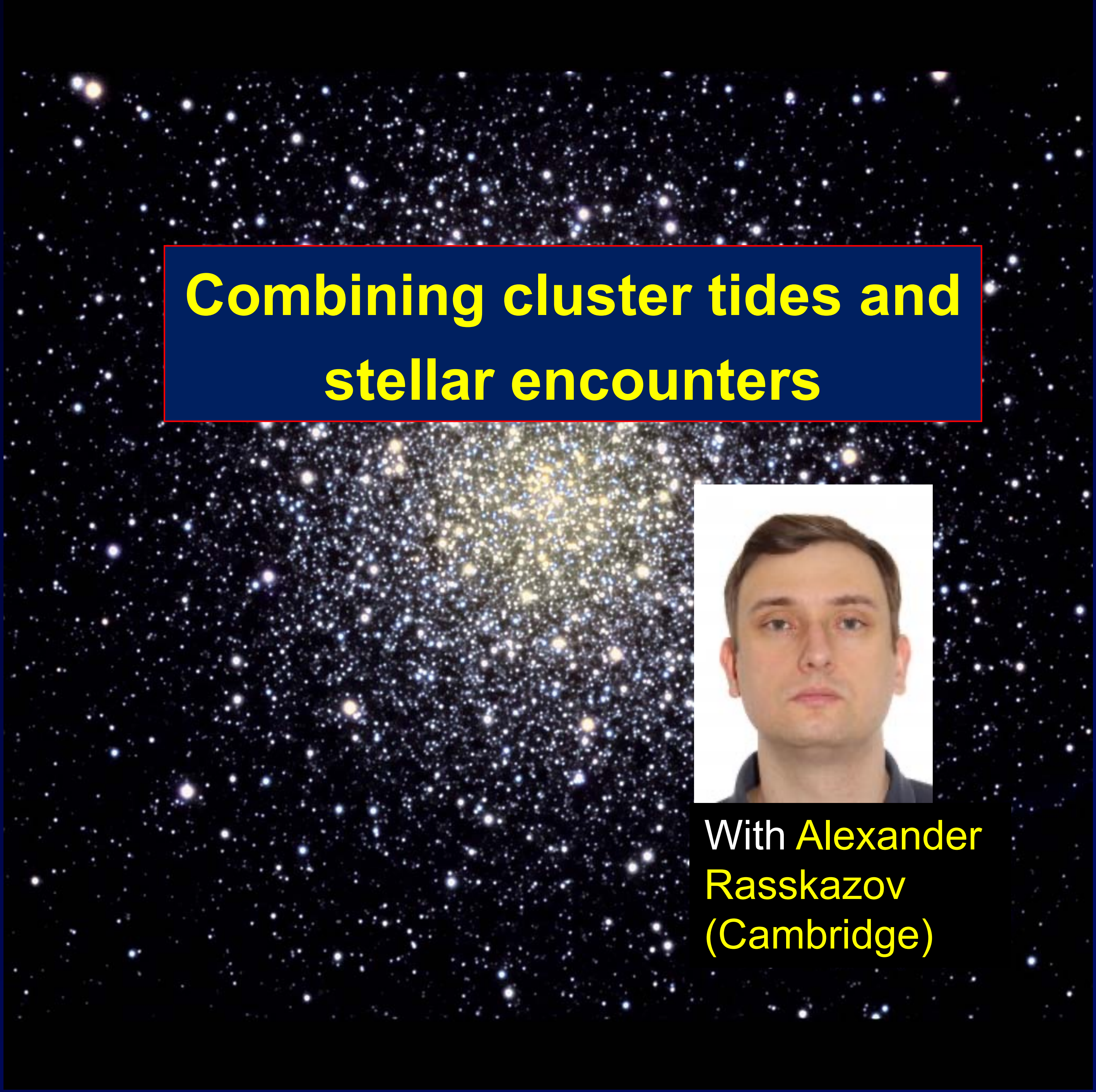
CONCLUSIONS: cluster tides acting alone (i.e. without central SMBH!)

- can account for several per cent of BH-BH merger rate
- contribute only weakly to NS-NS mergers
- rate is dominated by massive ($M_{cl} \sim 10^7 M_{Sun}$) cuspy nuclear clusters

Recent developments on cluster tides

- Bub & Petrovich (2020) extended calculation of cluster tides to triaxial potentials in the singly-averaged (SA) approximation – provided a code
- Hamilton & Rafikov (2021) explored the role of the 1pN apsidal precession due to the GR in the doubly-averaged (DA) approximation – suppresses eccentricity growth as *e* approaches unity
- Hamilton & Rafikov (2022) additionally included gravitational wave (GW) emission in the DA approximation - studied merger pathways of the binaries, following their evolution in the phase space
- Hamilton & Rafikov (2023) investigated binary evolution in the SA approximation with GR presession but no GW emission – found diffusive evolution of the DA integrals of motion, Relativistic Phase Space Diffusion (RPSD)
- Rasskazov & Rafikov (2023) looked at RPSD with GW emission numerically

Combining cluster tides and stellar encounters



With Alexander Rasskazov (Cambridge)

BESC – Binary Evolution in Stellar Clusters

Rasskazov & Rafikov (2023)

- Numerical framework for following evolution of the orbital elements of a binary in a cluster
- Also self-consistently follows the outer orbit of the binary in the cluster
- Considers a number of important physical processes

Cluster tides

•In the SA approximation (Bub & Petrovich 2017)

GR effects

•1pN apsidal precession

•GW emission

Stellar encounters

•Close encounters (ARCHAIN, Mikkola & Merritt 2008)

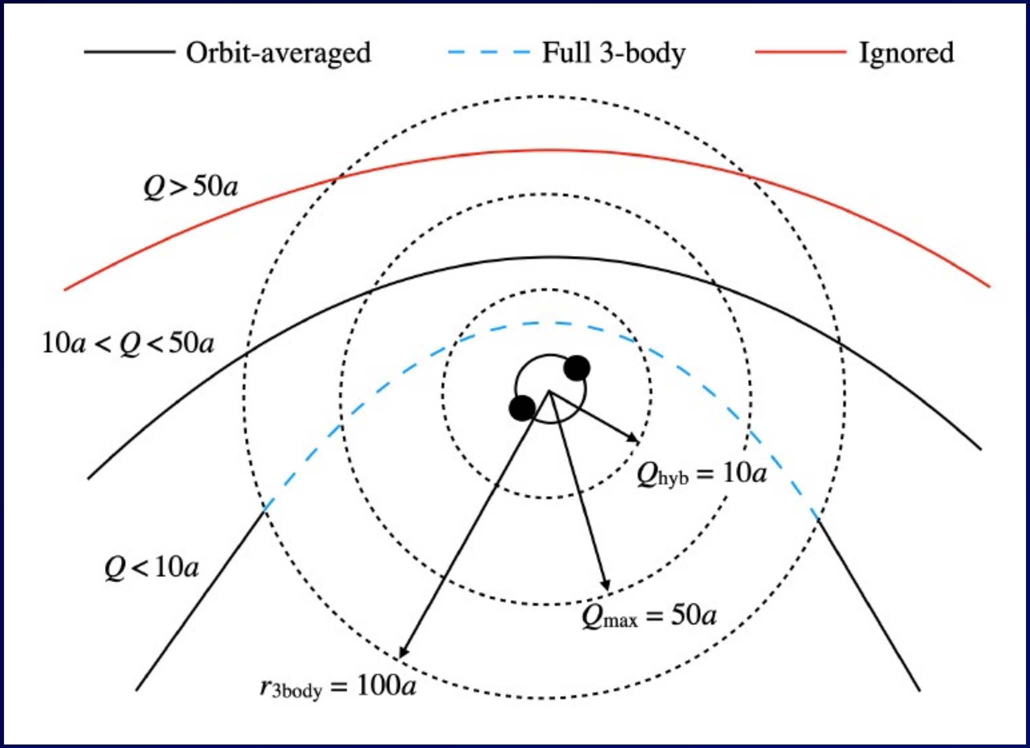
•Distant encounters – effect on eccentricity using Hamers & Samsing (2019a,b)

•Include back-reaction on the binary center of motion – directly account for dynamical friction, decay of the outer orbit

Distant encounters

- Do not change semi-major axis
- Change eccentricity and inclination
- Accounted for using Hamers & Samsing (2019) to octupole order – Orbit Averaged method
- Use hybrid version with full 3-body integration for closer encounters

Rasskazov & Rafikov (2023)



Hamers & Sammsing (2019)

$$\frac{\mathrm{d}\boldsymbol{e}}{\mathrm{d}\theta} = \epsilon_{\mathrm{SA}}(1 + E\cos\theta)\Big\{-3(\boldsymbol{j}\times\boldsymbol{e}) - \frac{3}{2}(\boldsymbol{j}\cdot\hat{\boldsymbol{R}})(\boldsymbol{e}\times\hat{\boldsymbol{R}}) + \frac{15}{2}(\boldsymbol{e}\cdot\hat{\boldsymbol{R}})(\boldsymbol{j}\times\hat{\boldsymbol{R}}) + \epsilon_{\mathrm{oct}}(1 + E\cos\theta)$$

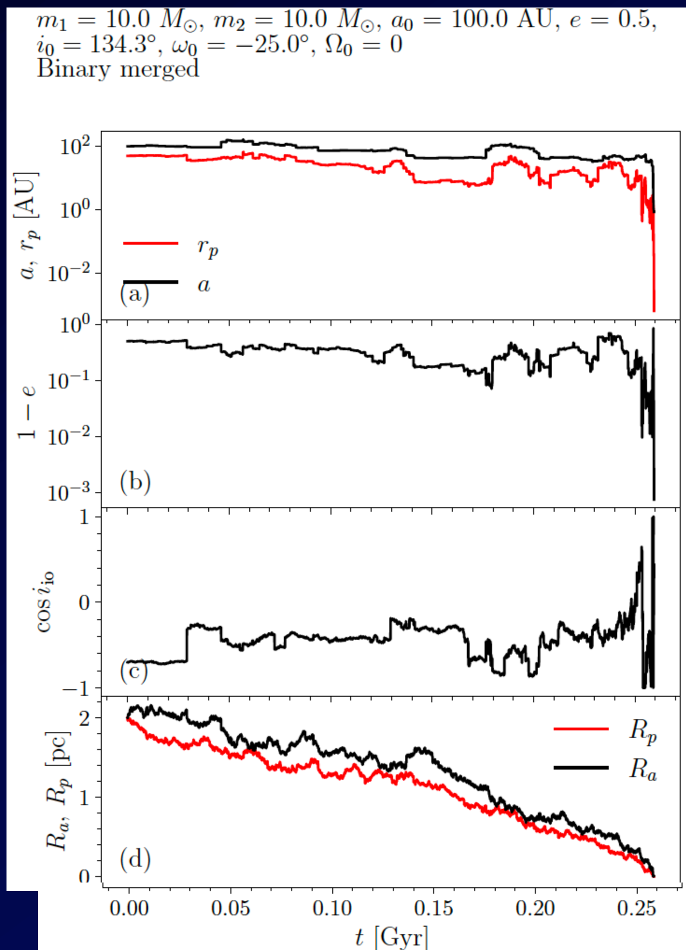
$$\times\frac{15}{16}[16(\boldsymbol{e}\cdot\hat{\boldsymbol{R}})(\boldsymbol{j}\times\boldsymbol{e}) - (1-8e^2)(\boldsymbol{j}\times\hat{\boldsymbol{R}}) + 10(\boldsymbol{e}\cdot\hat{\boldsymbol{R}})(\boldsymbol{j}\cdot\hat{\boldsymbol{R}})(\boldsymbol{e}\times\hat{\boldsymbol{R}}) + 5(\boldsymbol{j}\cdot\hat{\boldsymbol{R}})^2(\boldsymbol{j}\times\hat{\boldsymbol{R}}) - 35(\boldsymbol{e}\cdot\hat{\boldsymbol{R}})^2(\boldsymbol{j}\times\hat{\boldsymbol{R}})]\Big\};$$

$$\frac{\mathrm{d}\boldsymbol{j}}{\mathrm{d}\theta} = \epsilon_{\mathrm{SA}}(1 + E\cos\theta)\Big\{-\frac{3}{2}(\boldsymbol{j}\cdot\hat{\boldsymbol{R}})(\boldsymbol{j}\times\hat{\boldsymbol{R}}) + \frac{15}{2}(\boldsymbol{e}\cdot\hat{\boldsymbol{R}})(\boldsymbol{e}\times\hat{\boldsymbol{R}}) + \epsilon_{\mathrm{oct}}(1 + E\cos\theta)$$

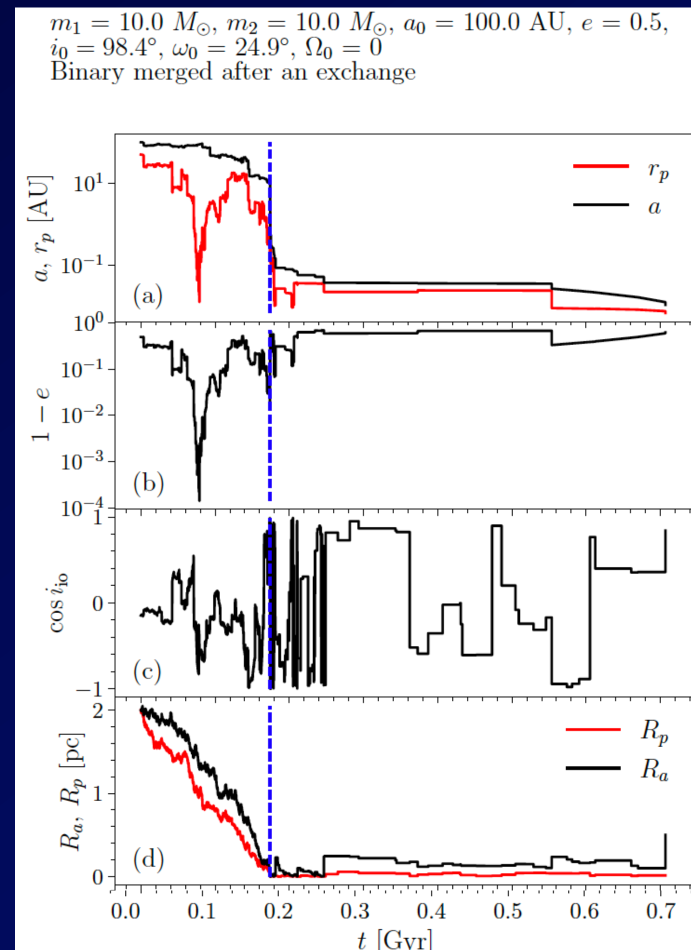
$$\times\frac{15}{16}[-(1-8e^2)(\boldsymbol{e}\times\hat{\boldsymbol{R}}) + 10(\boldsymbol{e}\cdot\hat{\boldsymbol{R}})(\boldsymbol{j}\cdot\hat{\boldsymbol{R}})(\boldsymbol{j}\times\hat{\boldsymbol{R}}) + 5(\boldsymbol{j}\cdot\hat{\boldsymbol{R}})^2(\boldsymbol{e}\times\hat{\boldsymbol{R}}) - 35(\boldsymbol{e}\cdot\hat{\boldsymbol{R}})^2(\boldsymbol{e}\times\hat{\boldsymbol{R}})]\Big\}.$$

Some typical outcomes

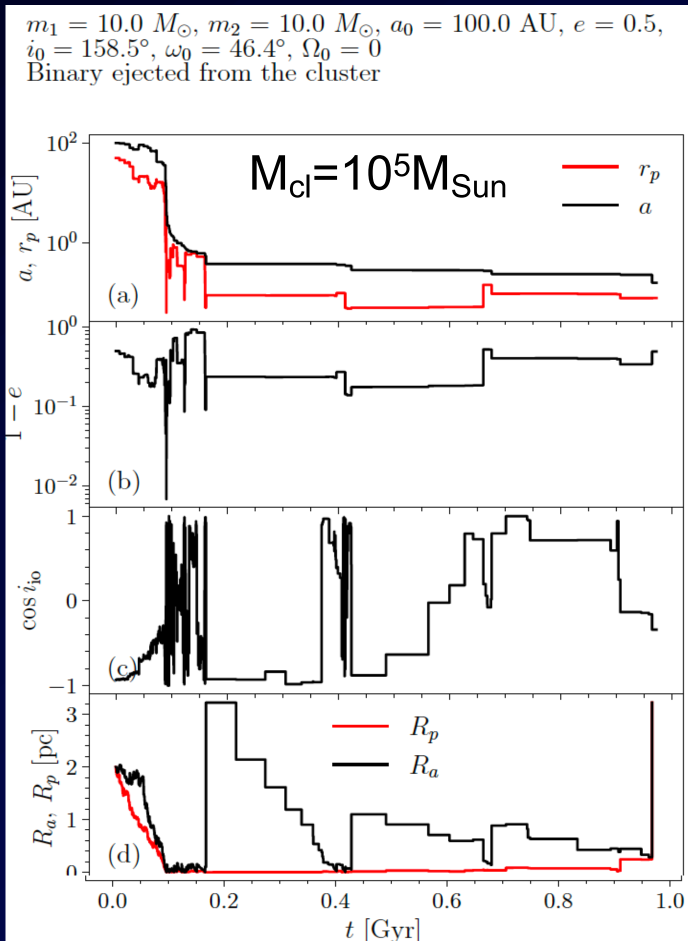
Binary merges



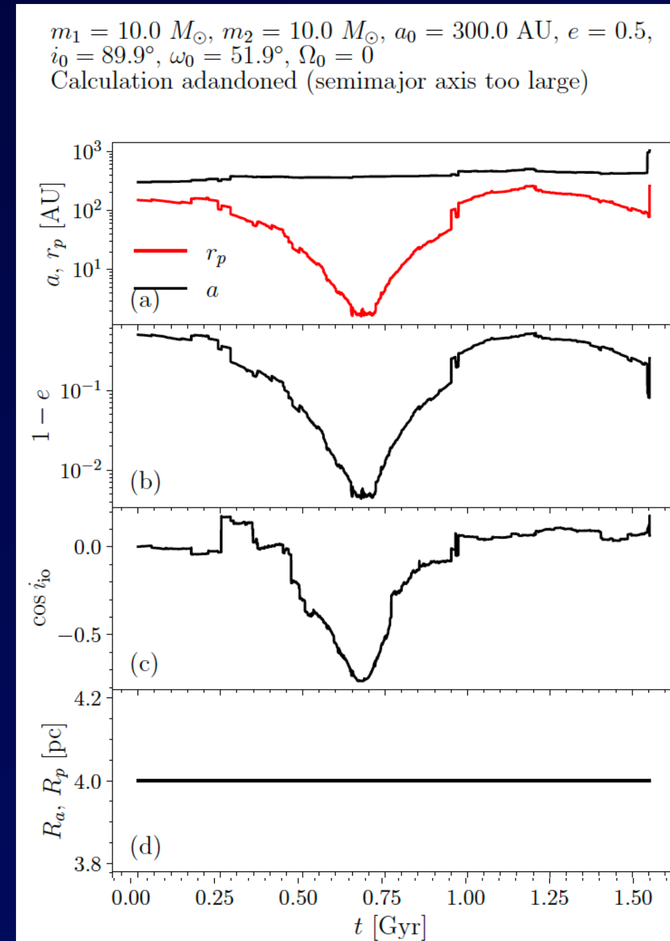
Exchange and then merger



Ejection from cluster



Secular cycles, DF disabled



- Can use BESC for statistical studies via Monte Carlo simulations for a variety of initial conditions and binary/cluster properties
- Preliminary (low number!) stats: starting with a=100 AU in $M_{cl}=10^5 M_{Sun}$ 76% (34%) of binaries merge in Hernquist (Plummer) clusters in a Hubble time

Summary

• There are many evolutionary channels possibly leading to the compact binaries – progenitors of the LIGO/Virgo GW sources

• Dynamical processes operating in massive stellar clusters is one such channel

• We studied so far unexplored secular dynamics of binaries driven by the tidal field of the parent cluster

• Phase portrait of the secular evolution is determined by a single parameter Γ, which encodes information about cluster potential and binary orbit

• High initial inclinations can result in high eccentricities, similar to Lidov-Kozai effect, resulting in mergers when assisted by the GW emission

• This route can account for several per cent of the LIGO BH-BH mergers

• Encounters with cluster stars tend to disrupt the smooth secular evolution

• Developed a numerical framework – BESC – to follow these effects simultaneously, use it for statistical studies of binary evolution in stellar clusters. Can be used for other systems: blue stragglers, hot Jupiters, X-ray binaries.

Cluster tide-driven secular evolution is an unavoidable consequence of the binary residence in the cluster. All studies of binary dynamics in clusters should consider it in general.