Closing gaps in the GW spectrum:
Ideas to detect μHz and high f GWs

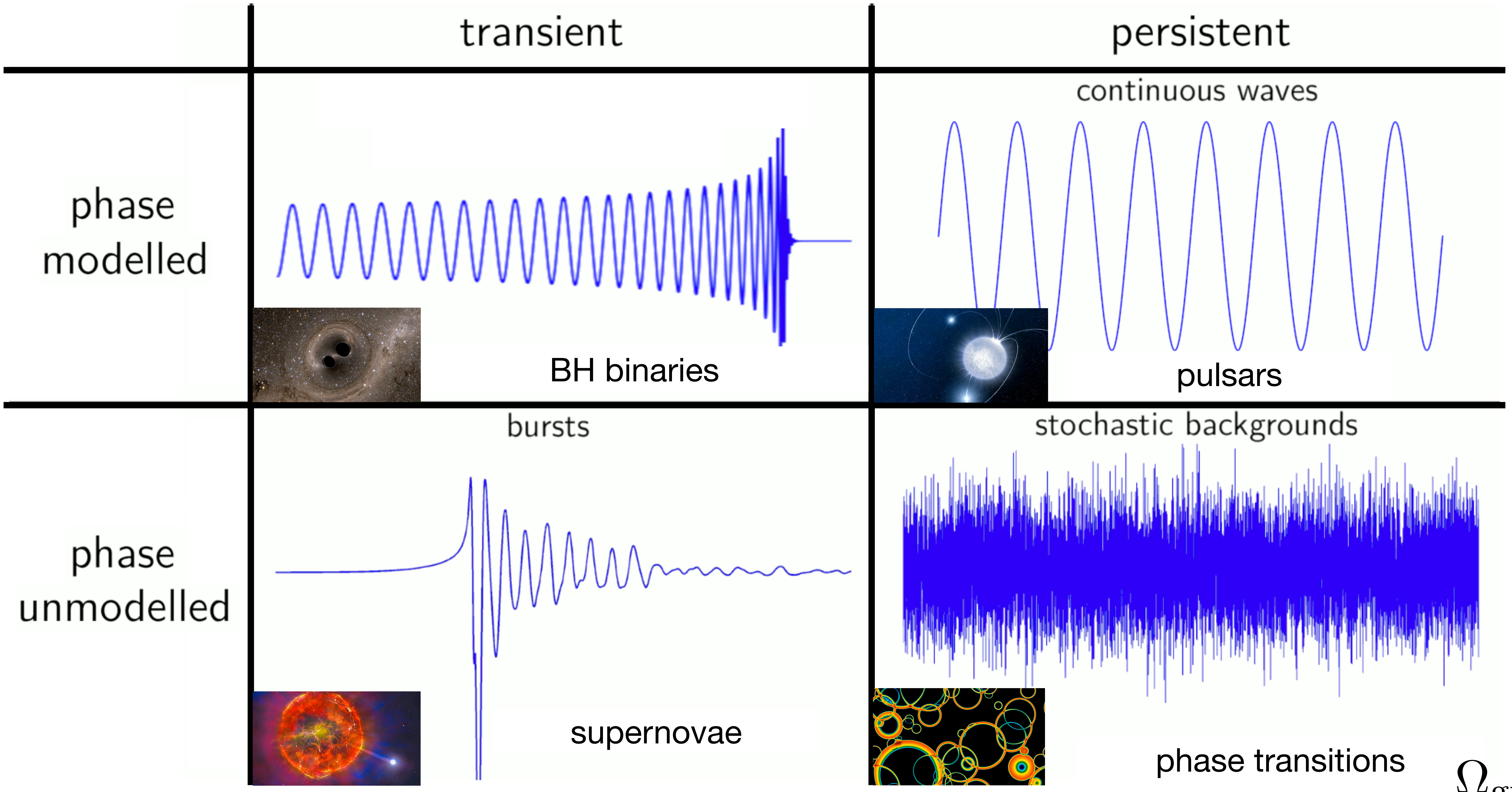
Diego Blas (ICREA/IFAE)

based on 2107.04063/2107.04601 (PRL/PRD22), 2112.11465 (PRD22) + 2303.01518 (PRD23)
(w. Alex Jenkins // A. Berlin, DB, R. T. D'Agnolo, S. Ellis, R. Harnik, Y. Kahn, J. Schütte-Engel)

ICREA
IFAE
BIST Barcelona Institute of Science and Technology
AGENCIA ESTATAL DE INVESTIGACIÓN
GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN
Generalitat de Catalunya
Departament de Recerca i Universitats

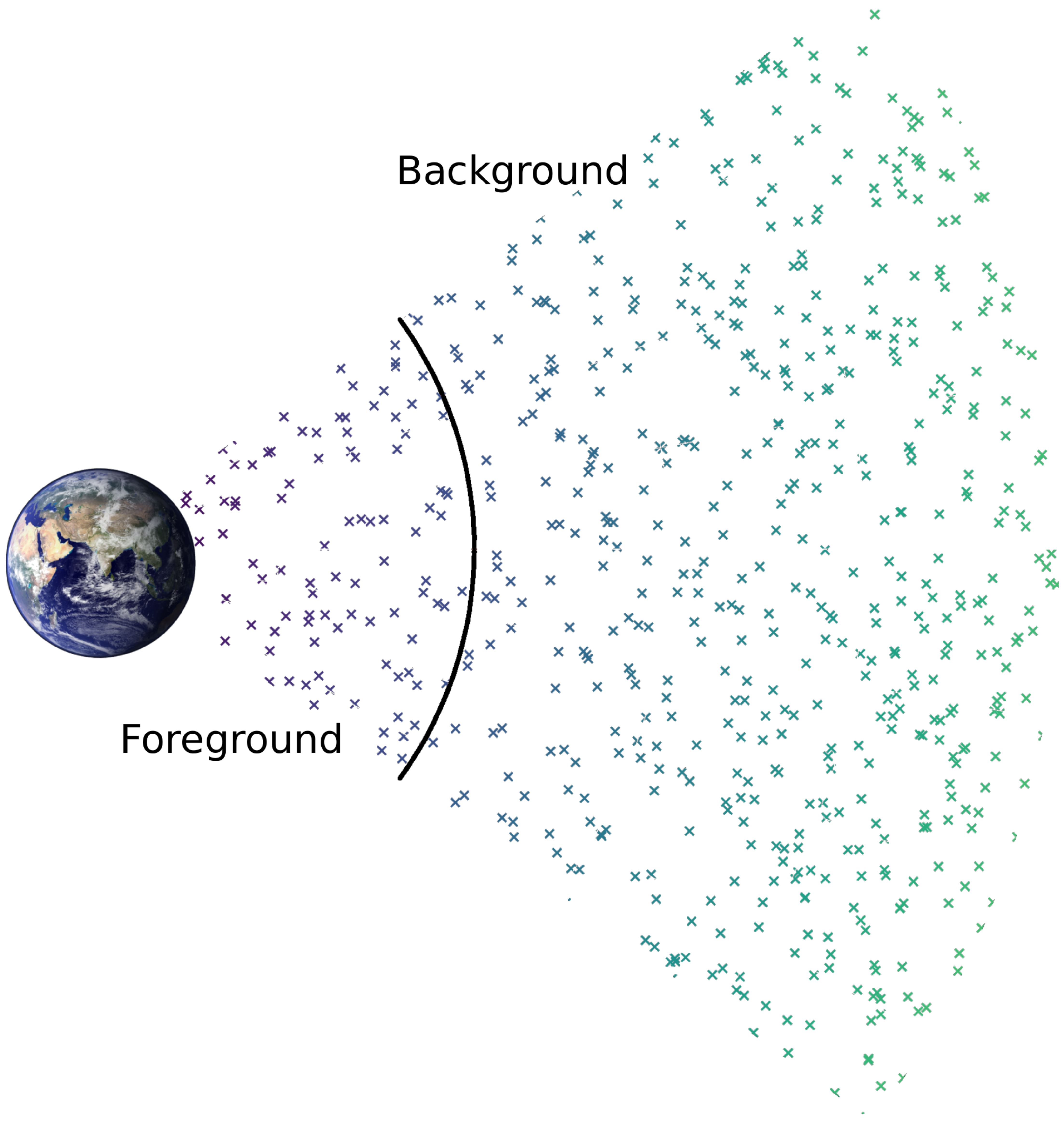
Taxonomy of GWs

$h(t)$



$\Omega_{\rm gw}(f)$

Stochastic gravitational-wave background (SGWB)



- incoherent, persistent GW signal
- faint/numerous sources
- astrophysical and cosmological
- GW density parameter:

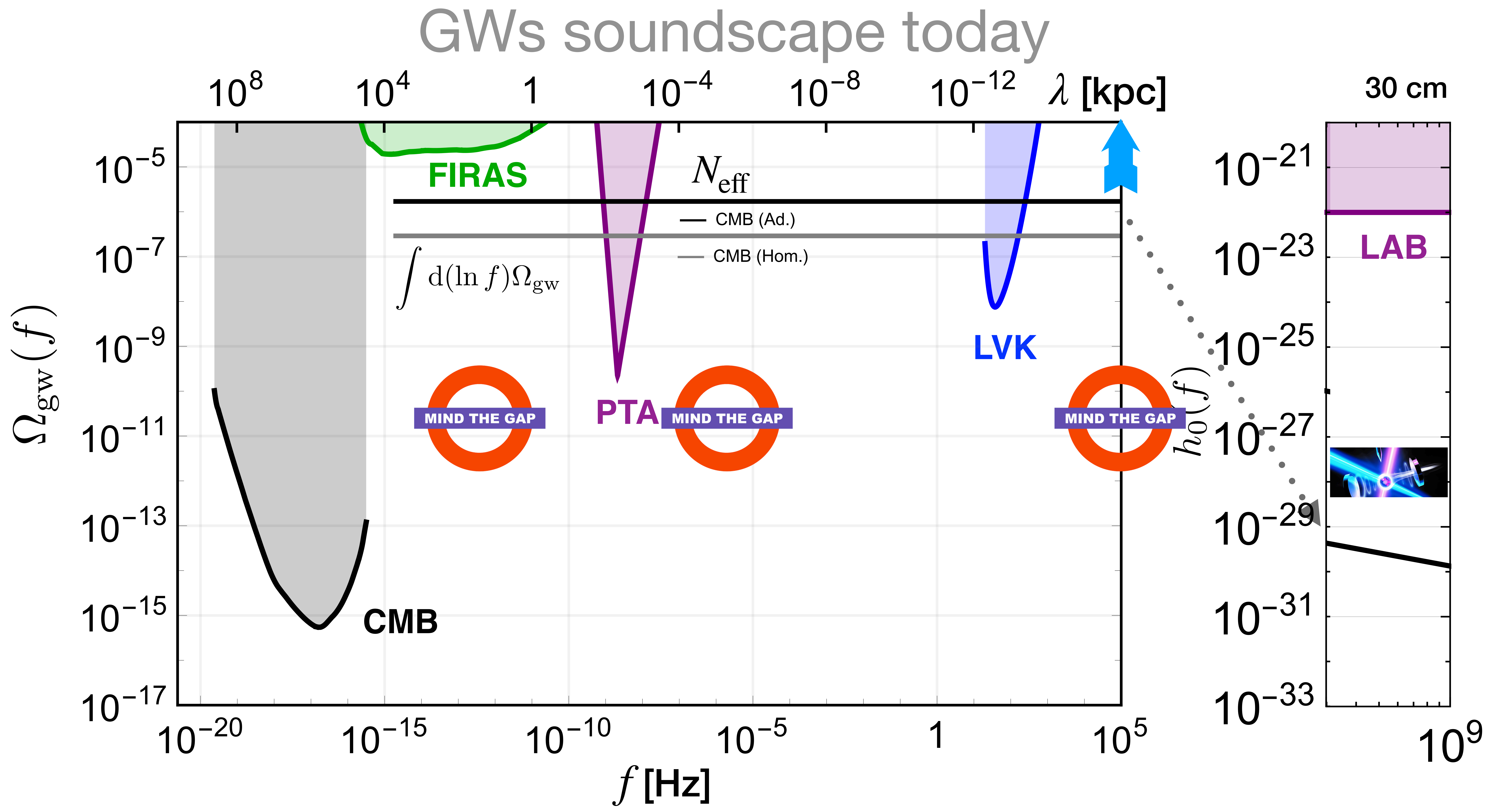
$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{crit}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}(\ln f)}$$

$$\rho_{\mathrm{GW}} \sim M_P^2 \omega^2 h_{GW}^2$$

$$\rho_c = 1.2 \times 10^{11} M_{\odot} \mathrm{Mpc}^{-3}$$
$$\sim \mathrm{keV/cm}^3$$

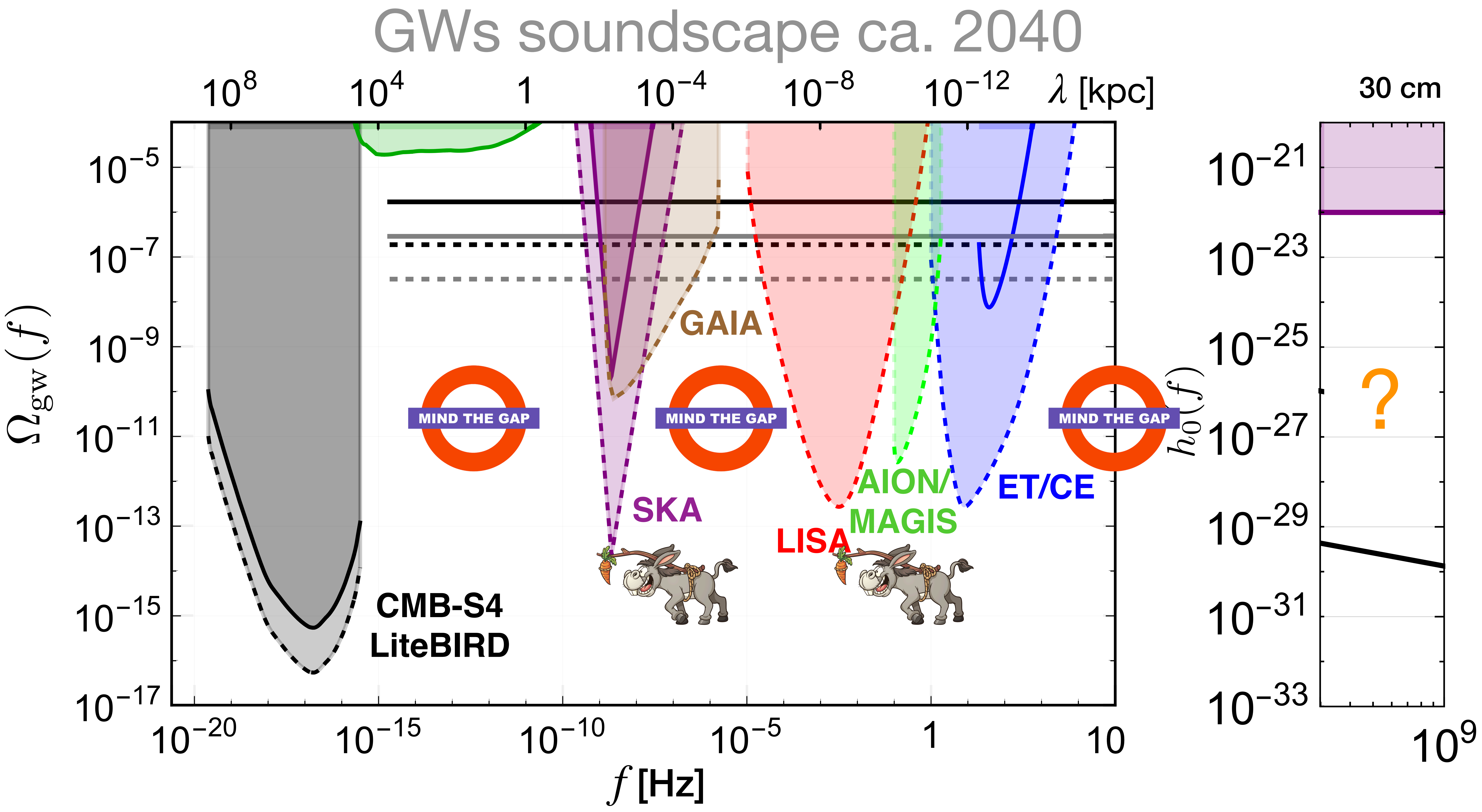
dblas@ifae.es


GWs soundscape today
λ [kpc]
30 cm
FIRAS
$N_{\rm eff}$
CMB (Ad.)
CMB (Hom.)
$\int d(\ln f)\Omega_{\rm gw}$
Kite et al., MNRAS. 505 (2021) 3, 4396
PTA
LVK
LVK PRD104, 022004 (2021)
Lasky et al PRX 6, 011035 (2016)
LAB
CMB
Clarke et al. JCAP 10 (2020) 002
$\Omega_{\rm gw}(f)$
$h_0(f)$
f [Hz]



GWs soundscape today
λ [kpc]
30 cm
FIRAS
$N_{\rm eff}$
CMB (Ad.)
CMB (Hom.)
$\int d(\ln f)\Omega_{\rm gw}$
PTA
LVK
LAB
MIND THE GAP
MIND THE GAP
MIND THE GAP
CMB
$\Omega_{\rm gw}(f)$
$h_0(f)$
f [Hz]


GWs soundscape ca. 2040
10^8
10^4
1
10^{-4}
10^{-8}
10^{-12}
λ [kpc]
30 cm
$\Omega_{\rm gw}(f)$
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
$h_0(f)$
10^{-21}
10^{-23}
10^{-25}
10^{-27}
10^{-29}
10^{-31}
10^{-33}
GAIA
MIND THE GAP
MIND THE GAP
MIND THE GAP
SKA
LISA
AION
ET/CE
?
CMB-S4
LiteBIRD
10^{-20}
10^{-15}
10^{-10}
10^{-5}
1
10
10^9
f [Hz]



GWs soundscape ca. 2040
10^8
10^4
1
10^{-4}
10^{-8}
10^{-12}
λ [kpc]
30 cm
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
$\Omega_{\rm gw}(f)$
GAIA
MIND THE GAP
MIND THE GAP
MIND THE GAP
SKA
LISA
AION/
MAGIS
ET/CE
CMB-S4
LiteBIRD
10^{-20}
10^{-15}
10^{-10}
10^{-5}
1
10
f [Hz]
10^{-21}
10^{-23}
10^{-25}
10^{-27}
10^{-29}
10^{-31}
10^{-33}
$h_0(f)$
?
10^9

GWs soundscape ca. 2040

Big Jay McNeely 1953
Stolen from V. Cardoso

MW in visible band

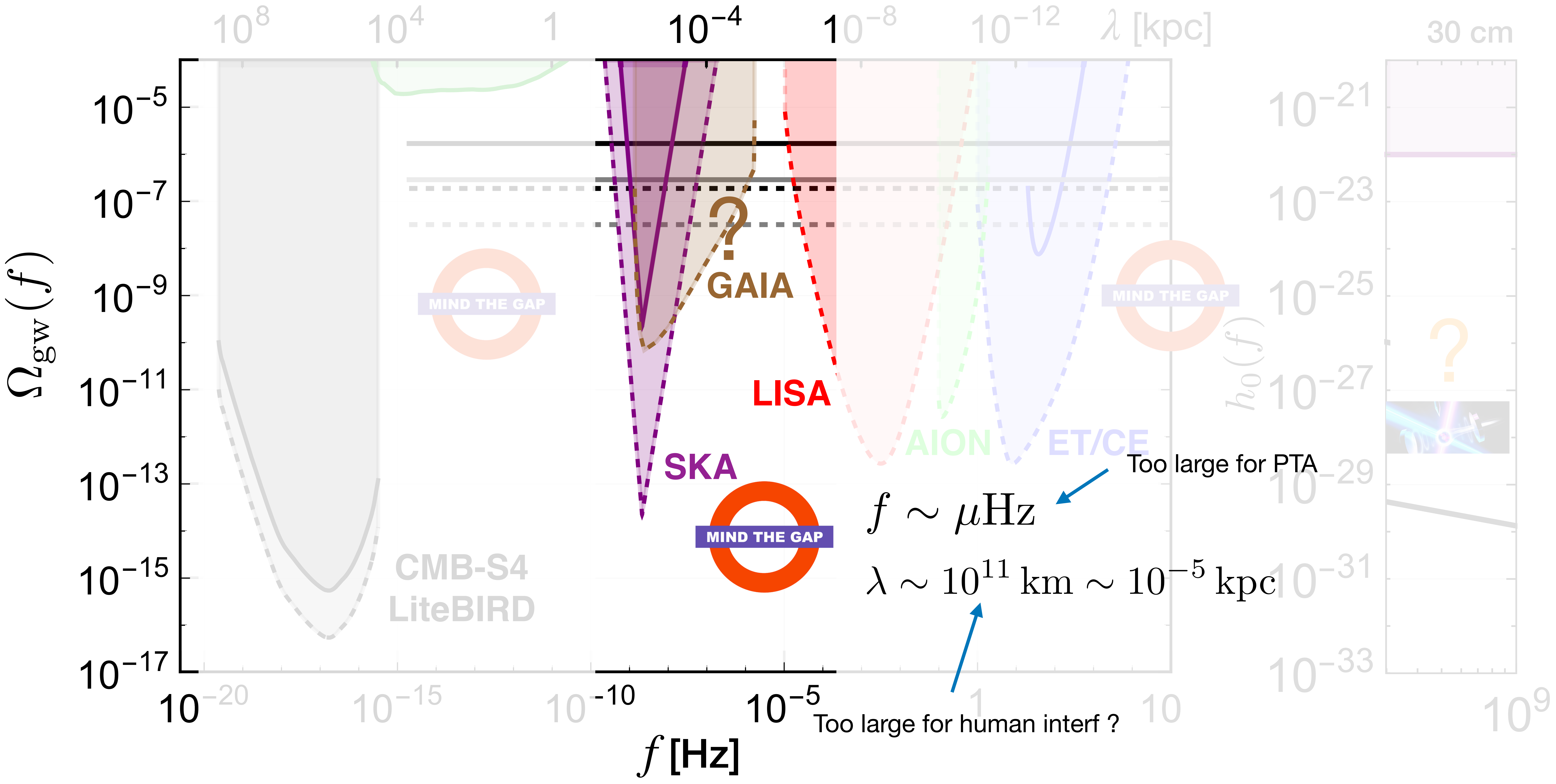
MW in X rays

Spektr-RG–eROSITA all-sky map Nature volume 588, pages 227–231 (2020).

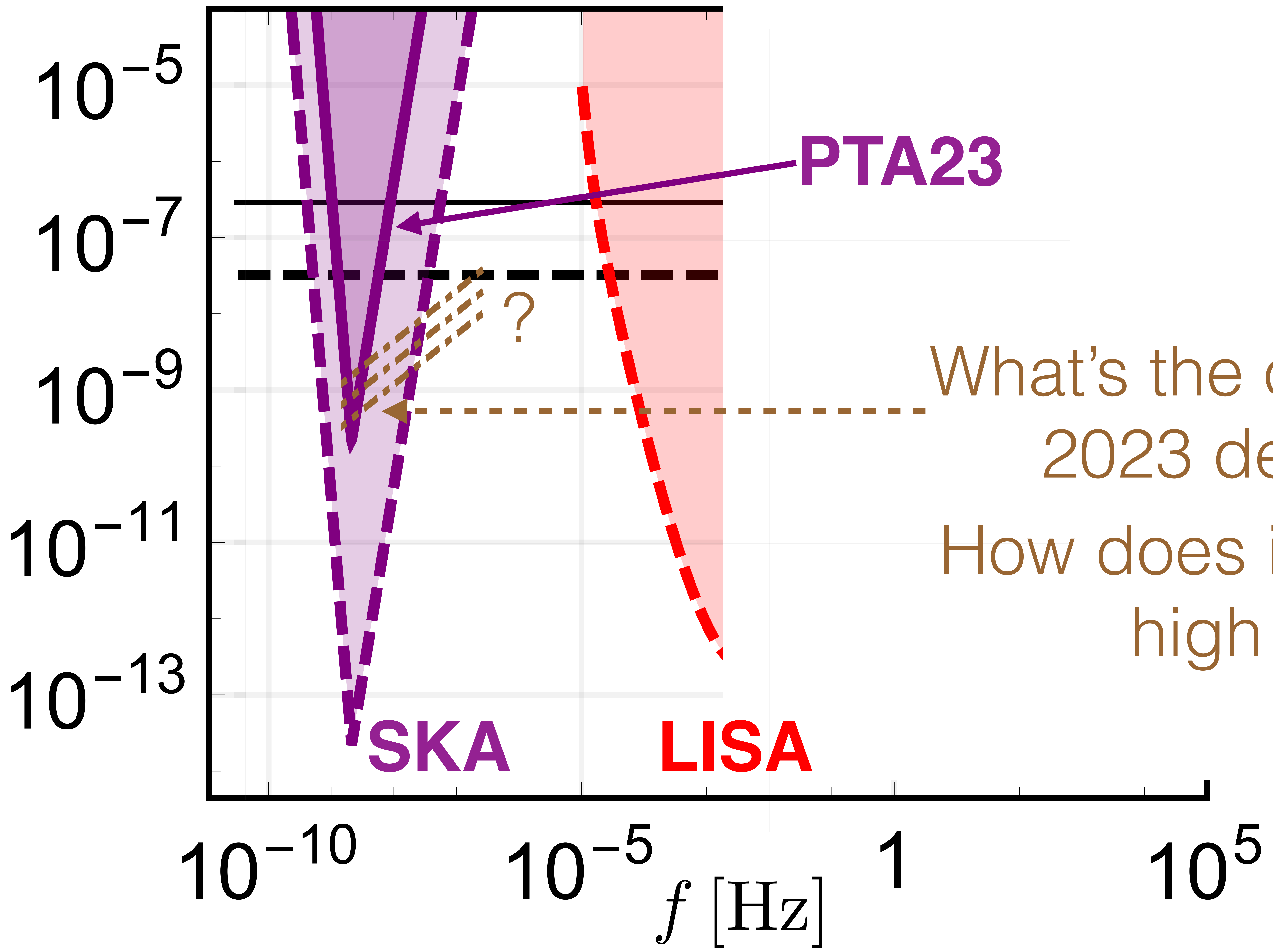
GWs soundscape ca. 2040
λ [kpc]
30 cm
$\Omega_{\rm gw}(f)$
$h_0(f)$
f [Hz]
CMB-S4
LiteBIRD
SKA
GAIA
LISA
AION
ET/CE
MIND THE GAP
MIND THE GAP
MIND THE GAP
?

Possible **backgrounds** & ideas at μHz: a rich band

Future soundscape (maybe 2040)?



Possible **backgrounds** & ideas at μHz: a rich band



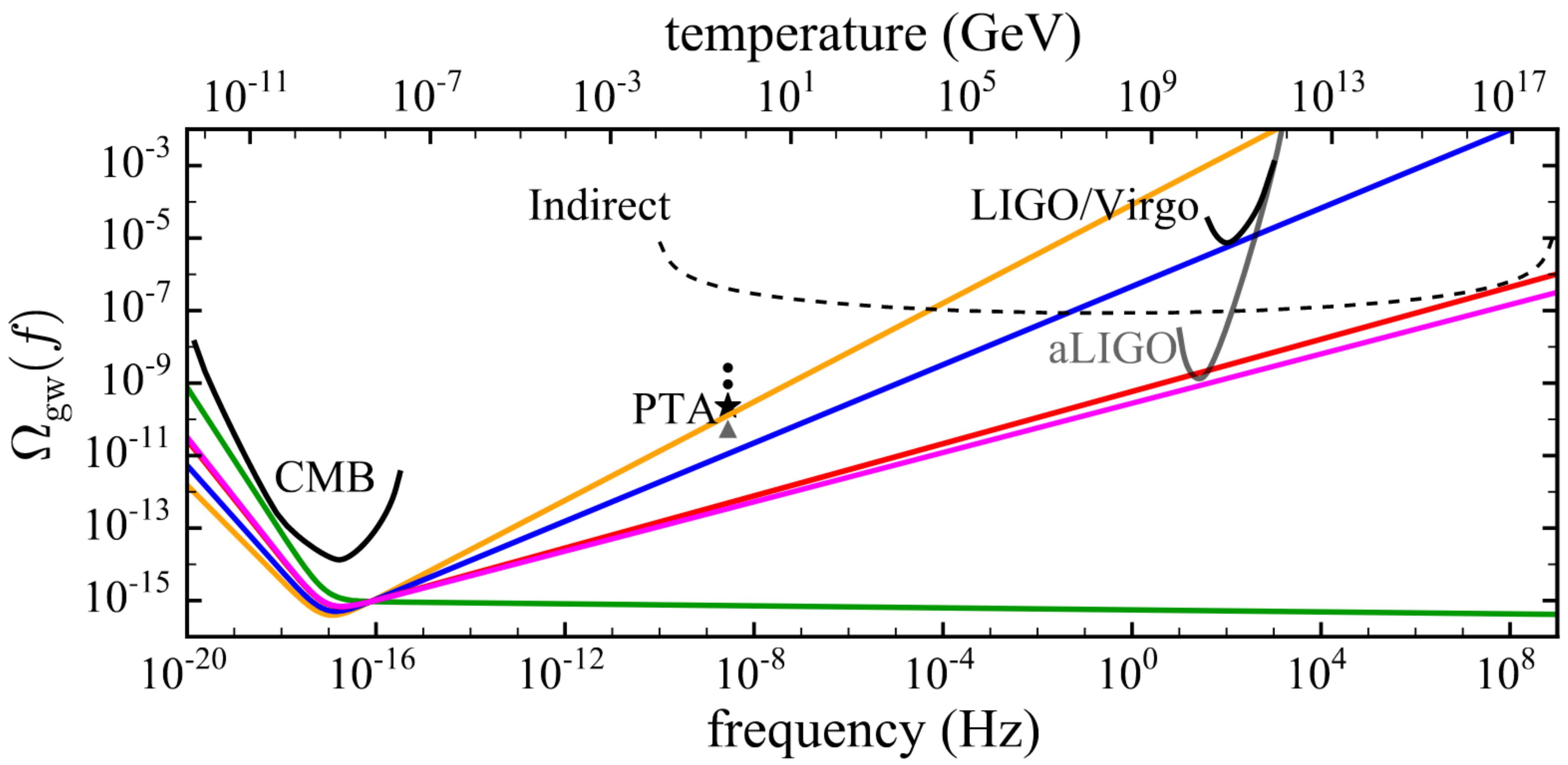
What's the origin of the 2023 detection?

How does it change at high freq?

Backgrounds from fundamental physics

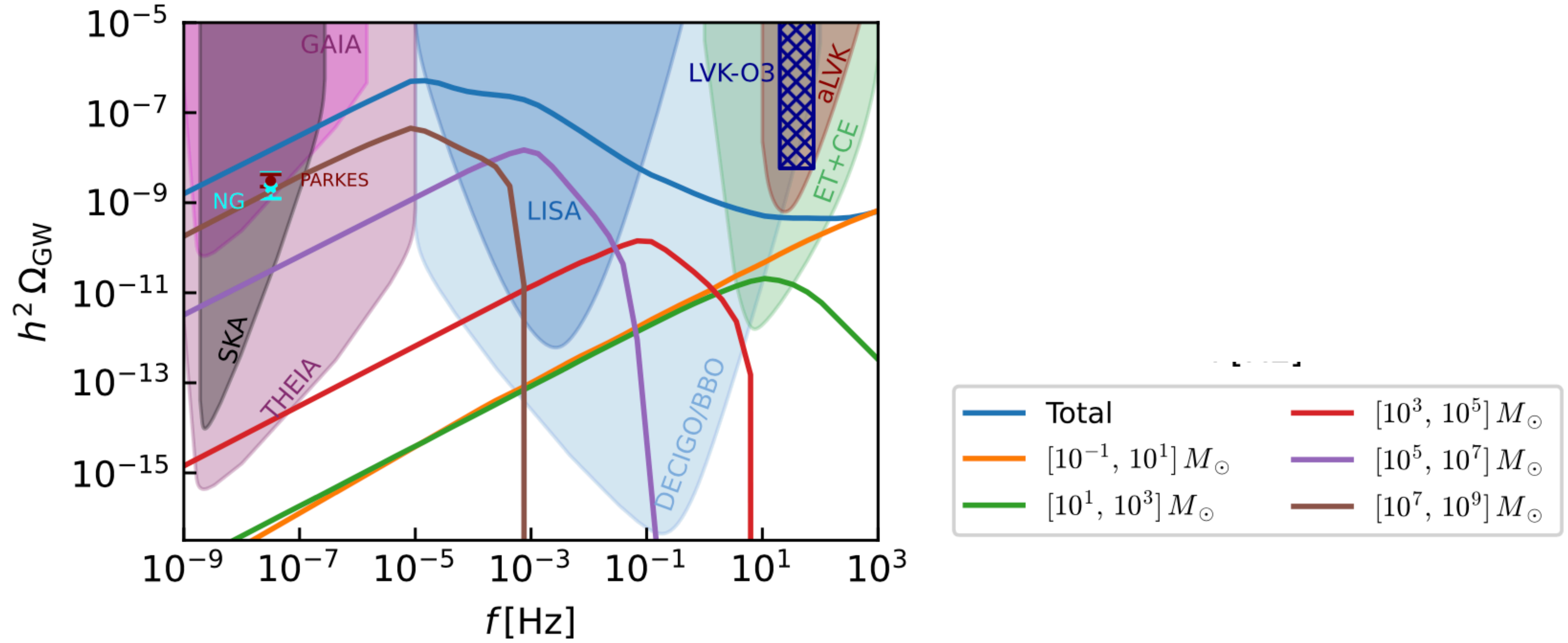
Inflation

Lasky et al PRX 6, 011035 (2016)



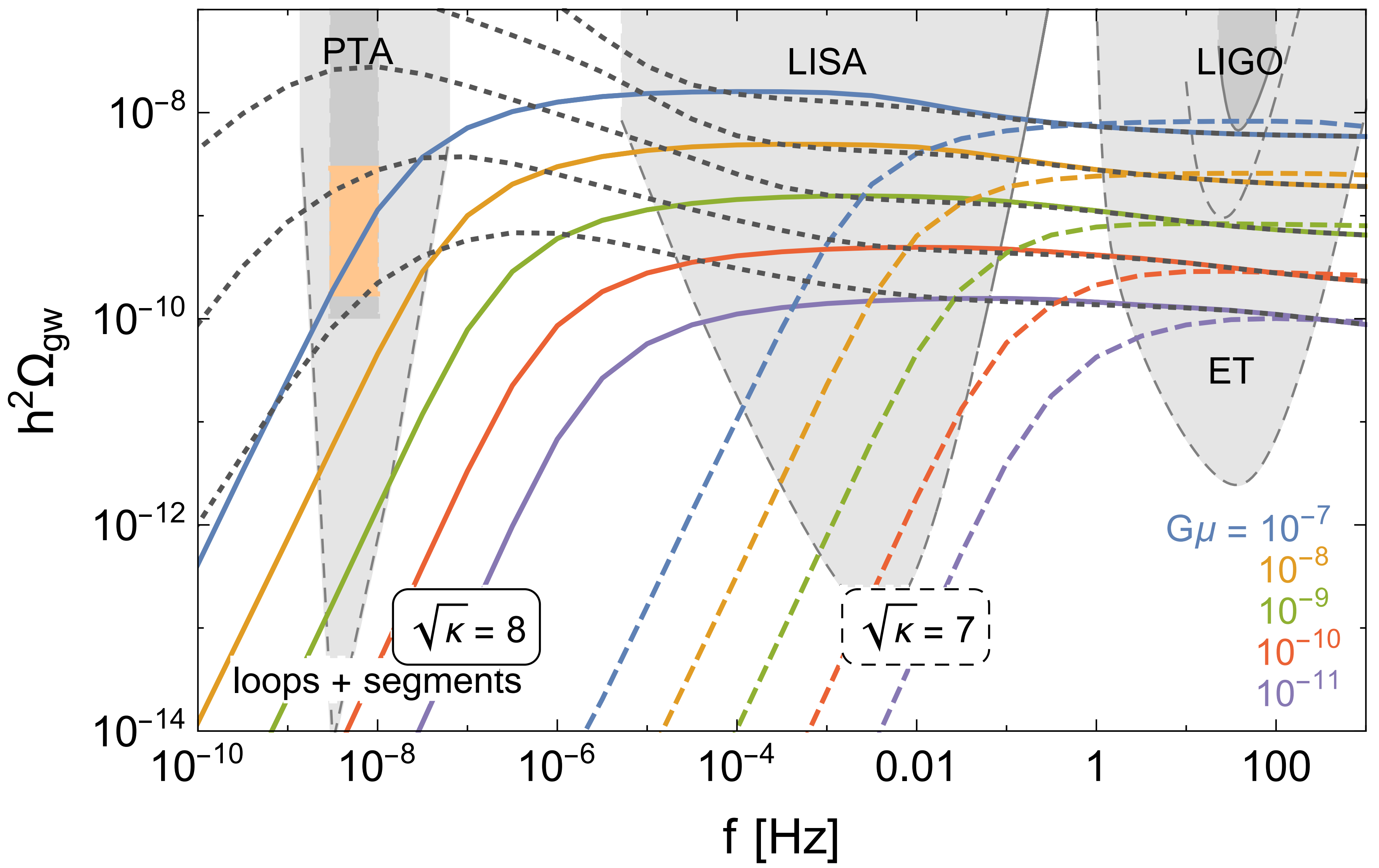
PBH

Braglia et al. JCAP 12 (2021) 12



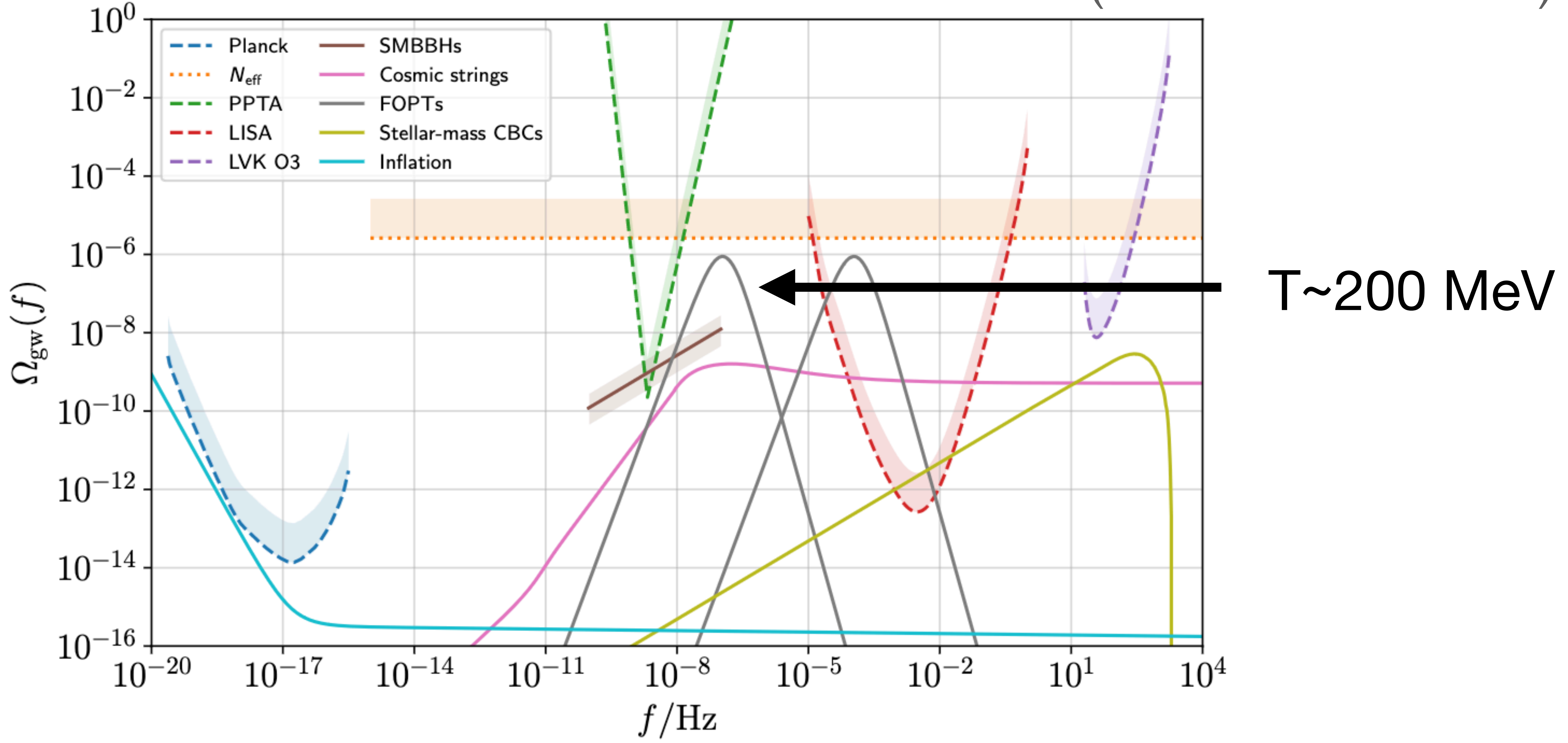
Cosmic Strings

Buchmuller et al. 2107.04578



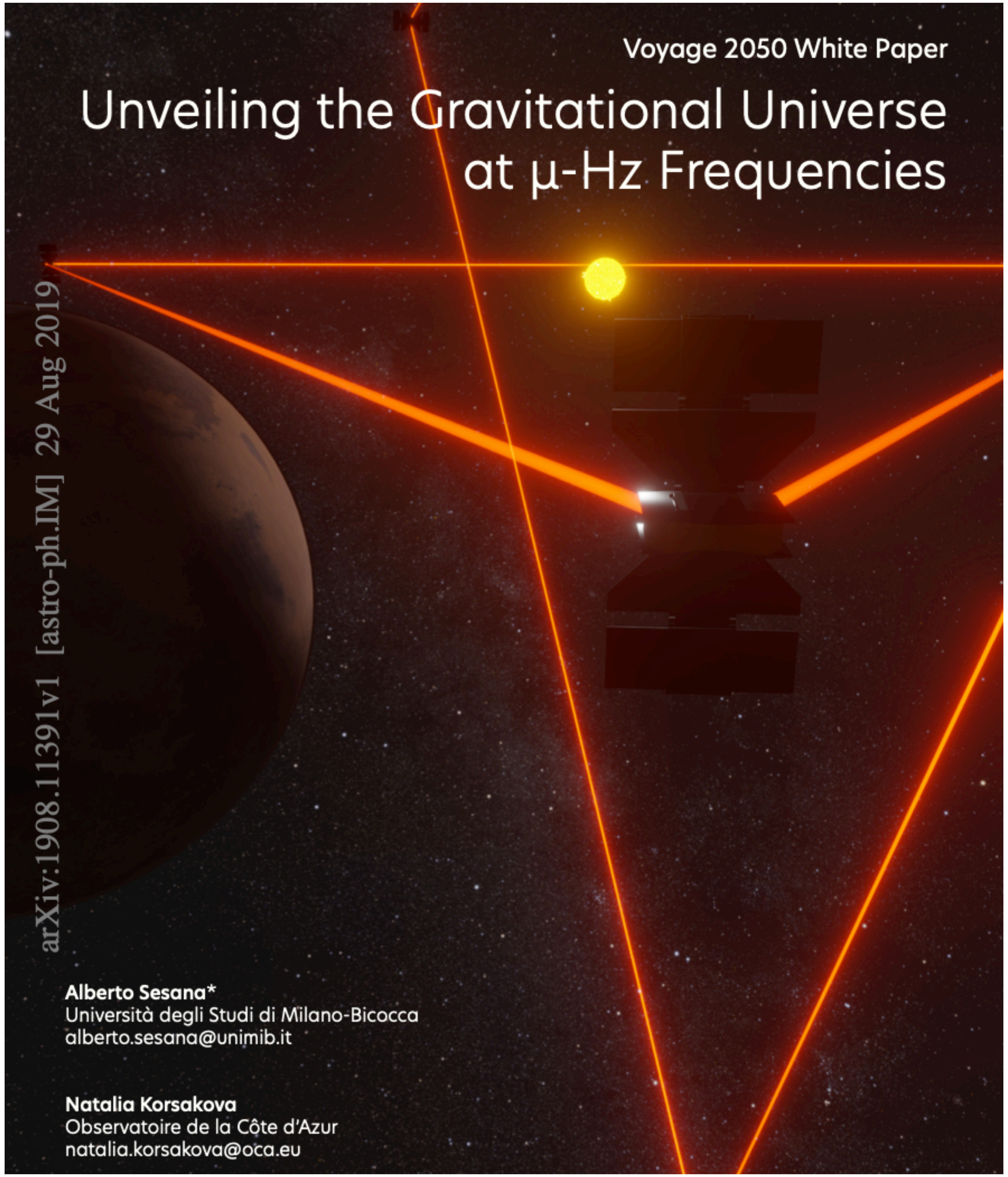
FOPT

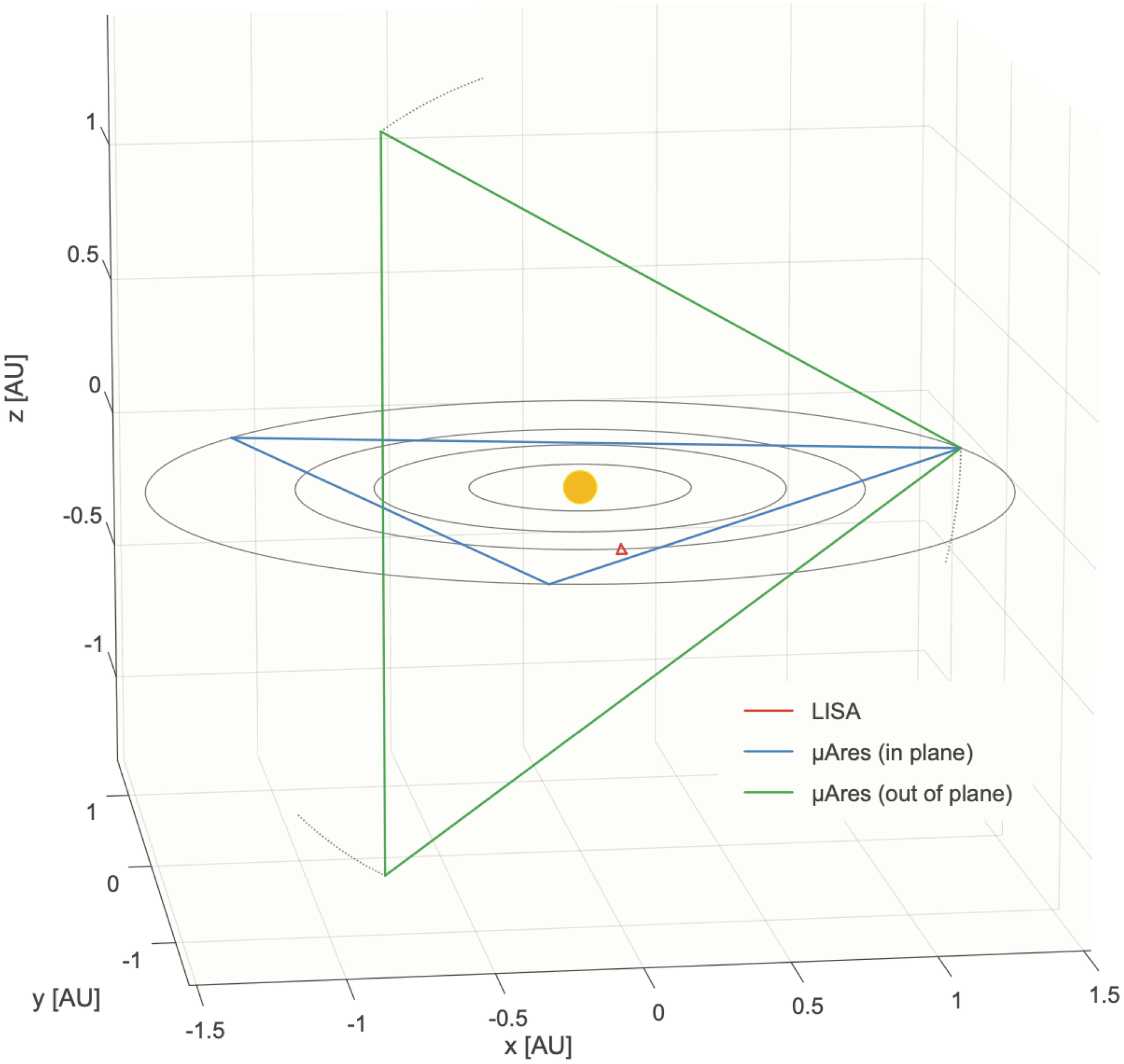
Renzini et al 2202.00178
(based on 1512.06239)



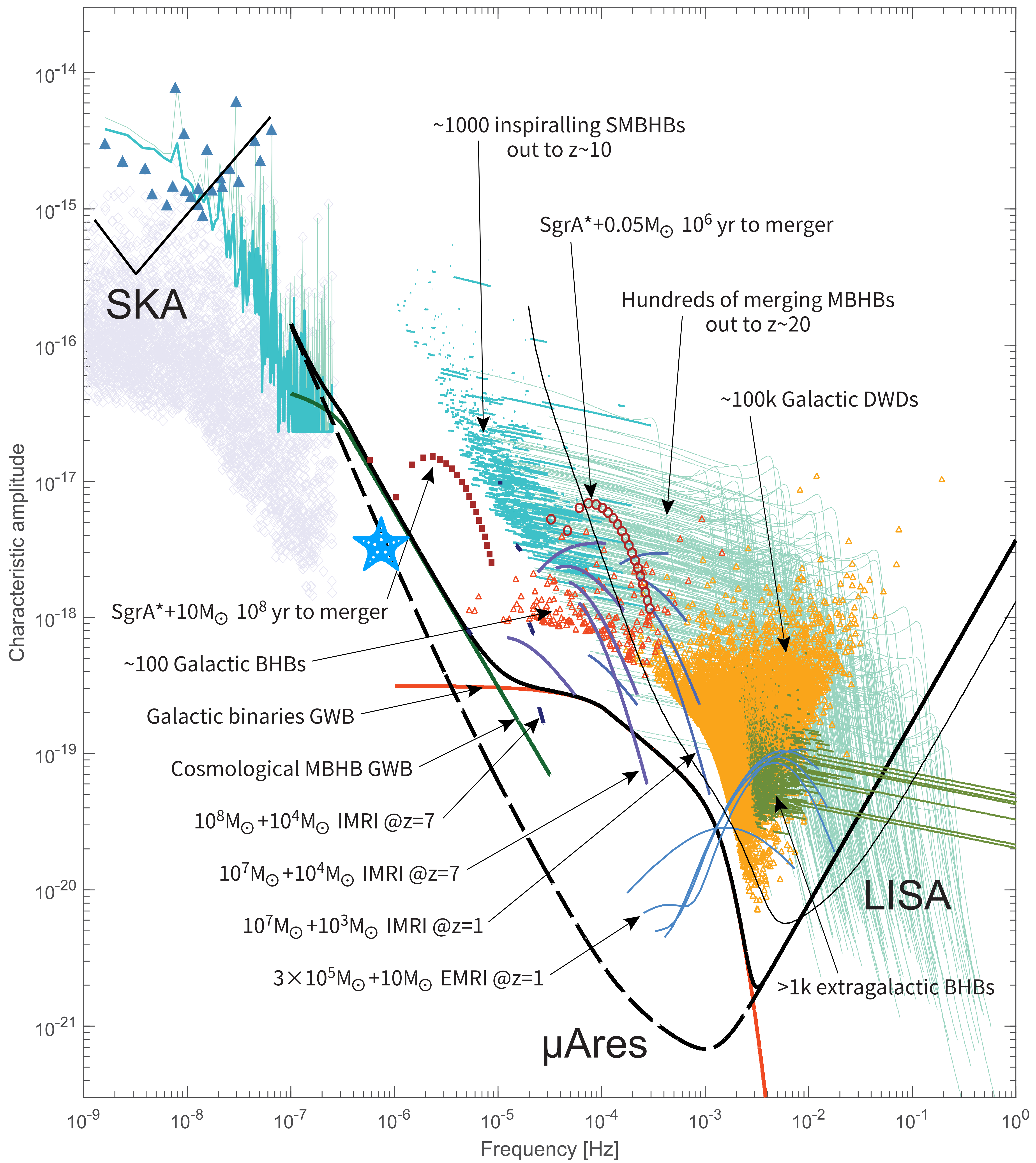
Possible backgrounds & **ideas** at μHz: a rich band

i) μAres: LISA-like concept





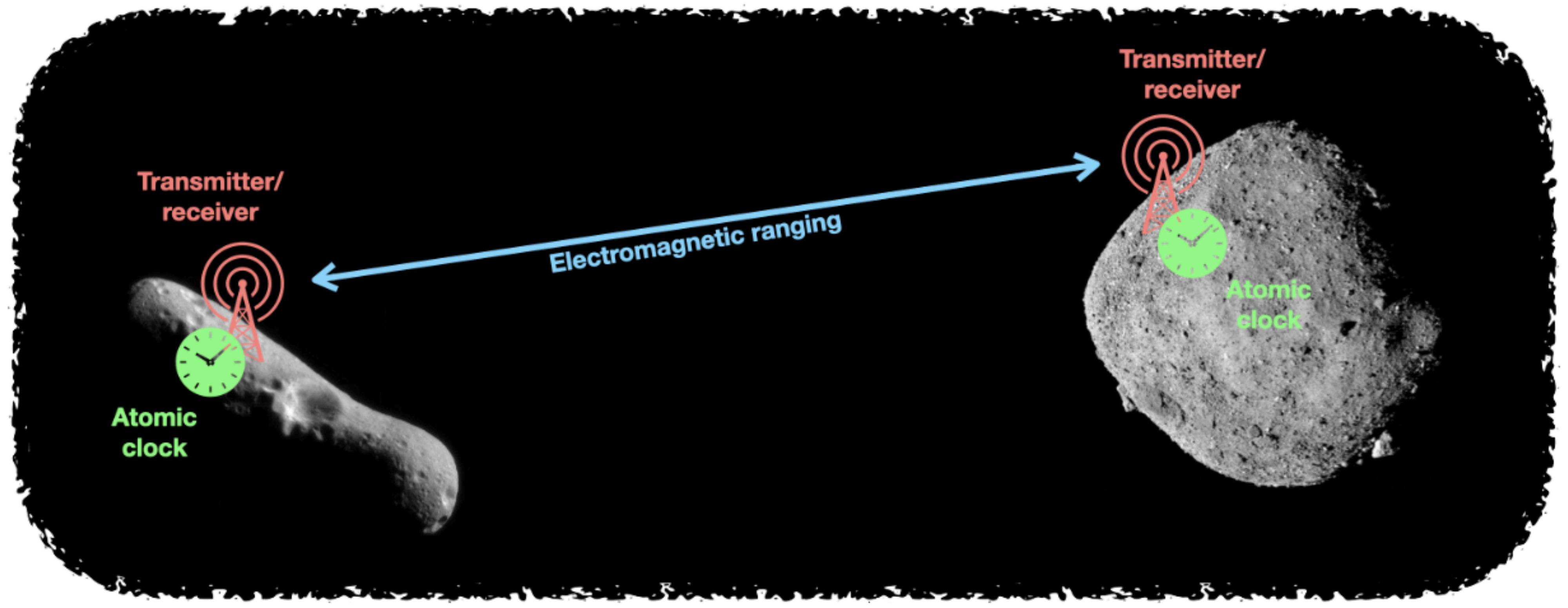
z [AU]
y [AU]
x [AU]
LISA
μAres (in plane)
μAres (out of plane)

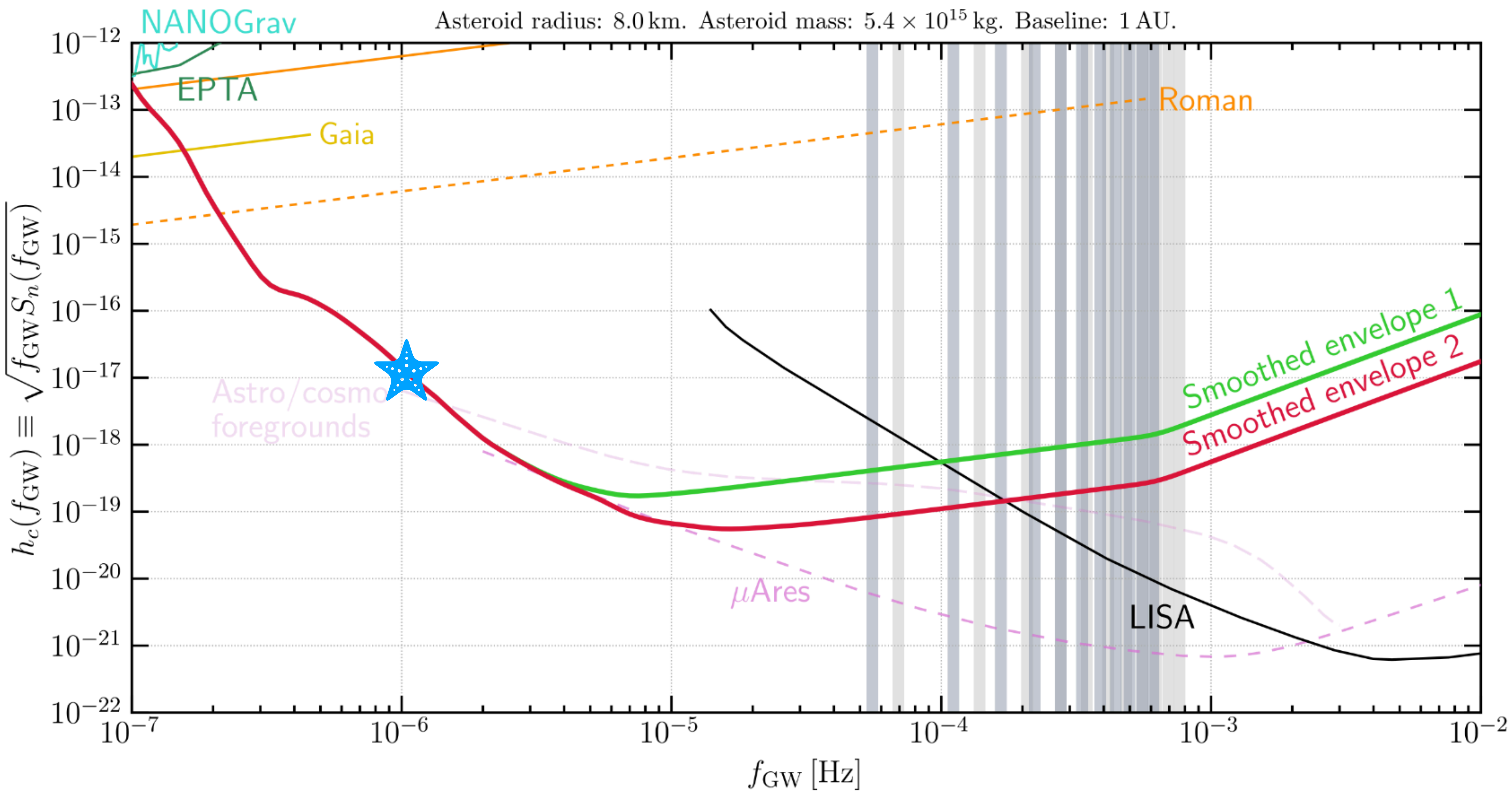


The μAres detection landscape
Characteristic amplitude
Frequency [Hz]
SKA
LISA
μAres
~1000 inspiralling SMBHBs out to z~10
SgrA*+0.05M⊙ 10^6 yr to merger
Hundreds of merging MBHBs out to z~20
~100k Galactic DWDs
SgrA*+10M⊙ 10^8 yr to merger
~100 Galactic BHBs
Galactic binaries GWB
Cosmological MBHB GWB
10^8M⊙+10^4M⊙ IMRI @z=7
10^7M⊙+10^4M⊙ IMRI @z=7
10^7M⊙+10^3M⊙ IMRI @z=1
3×10^5M⊙+10M⊙ EMRI @z=1
>1k extragalactic BHBs

ii) Ranging of asteroids?

Fedderke et al 2112.11431





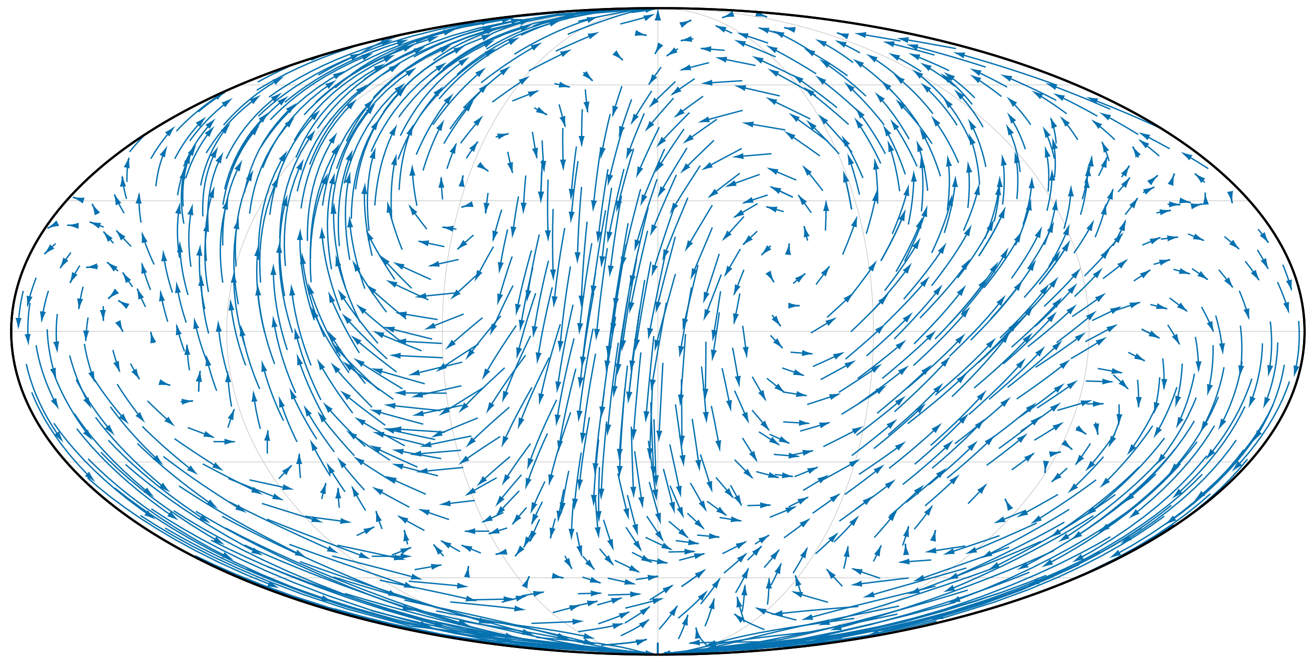
iii) Future astrometry?

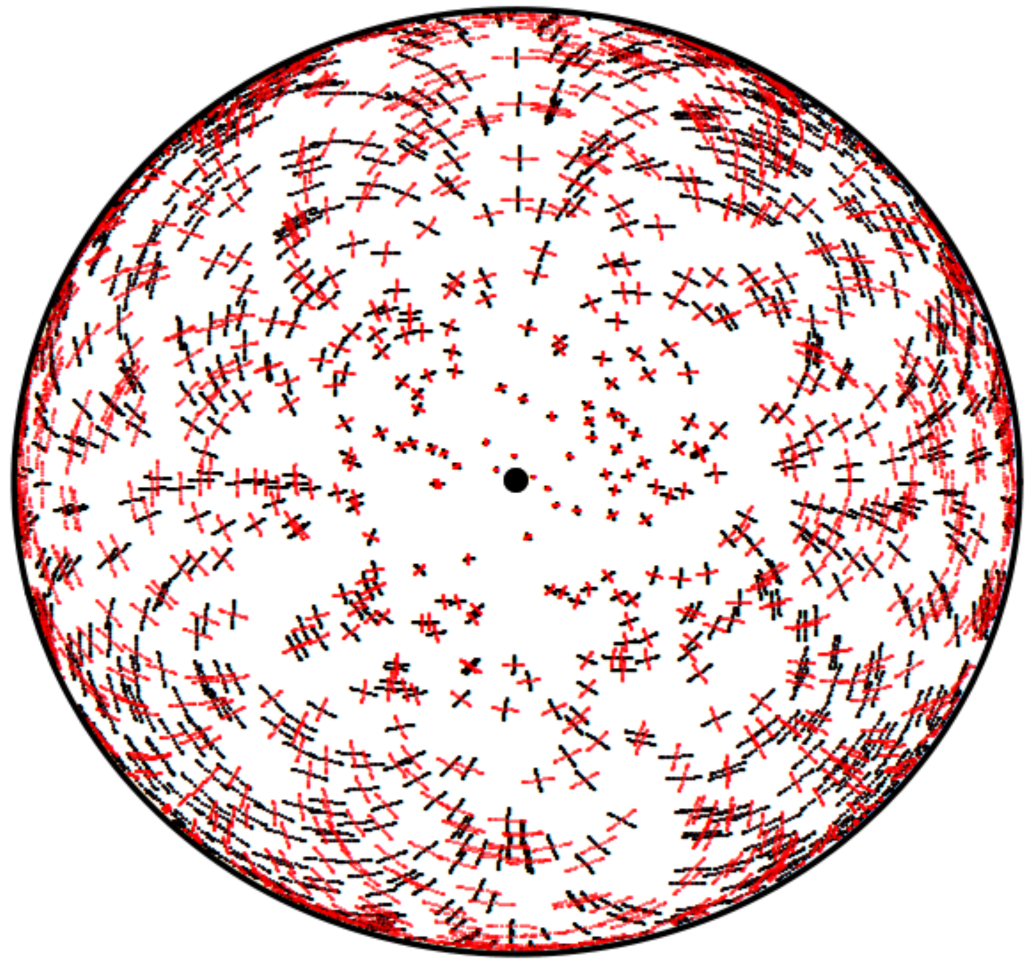
e.g. Moore et al 1707.06239
Mihaylov et al. 1804.00660

Klioner 1710.11474

Garcia-Bellido et al. 2104.04778

Monitoring many stars (GAIA or better)





Fedderke et al 2204.07677

Stellar interferometry

We evaluate the potential for gravitational-wave (GW) detection in the frequency band from 10 nHz to 1 μHz using extremely high-precision astrometry of a small number of stars

at characteristic strains around $h_c \sim 10^{-17} \times (\mu\text{Hz}/f_{\text{GW}})$. The astrometric angular precision required to see these sources is $\Delta\theta \sim h_c$ after integrating for a time $T \sim 1/f_{\text{GW}}$. We show that jitter in the photometric center of WD of this type due to starspots is bounded to be small enough to permit this high-precision, small-N approach. We discuss possible noise arising from stellar reflex motion induced by orbiting objects and show how it can be mitigated. The only plausible technology able to achieve the requisite astrometric precision is a space-based stellar interferometer. Such a future mission with few-meter-scale collecting dishes and baselines of $\mathcal{O}(100\,\text{km})$ is sufficient to achieve the target precision. This collector size is broadly in line with the collectors proposed for

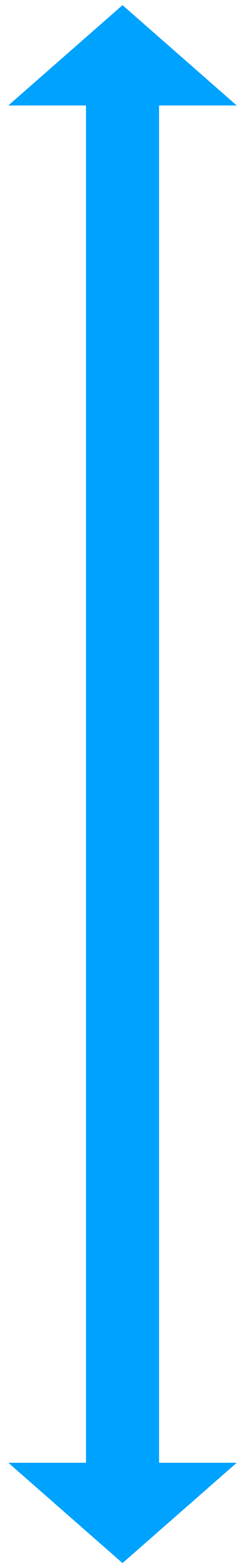
Çalışkan et al 2312.03069

iv) Atomic interferometry in space: AEDGE

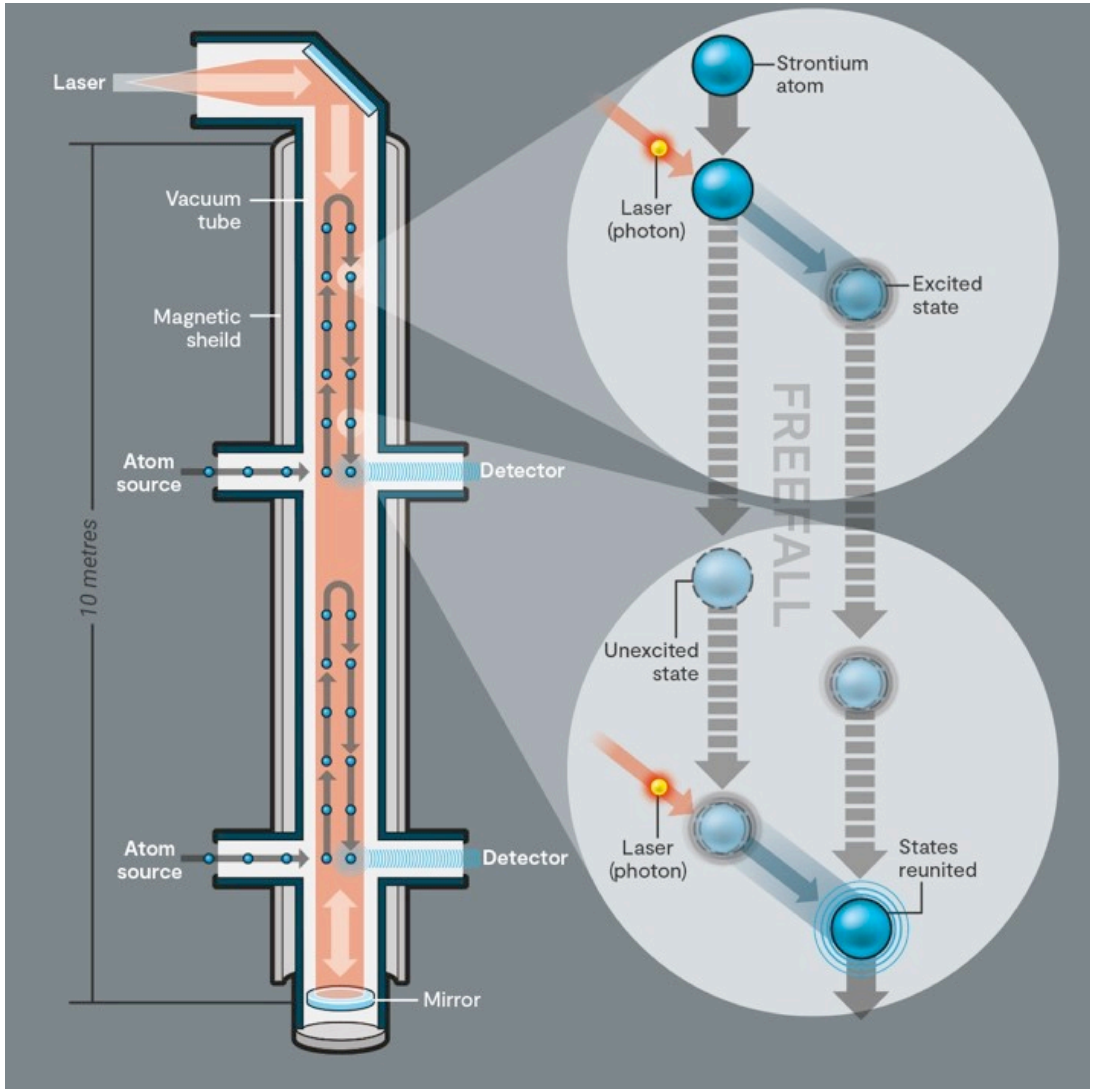
Abou El-Neaj et al 1908.00802

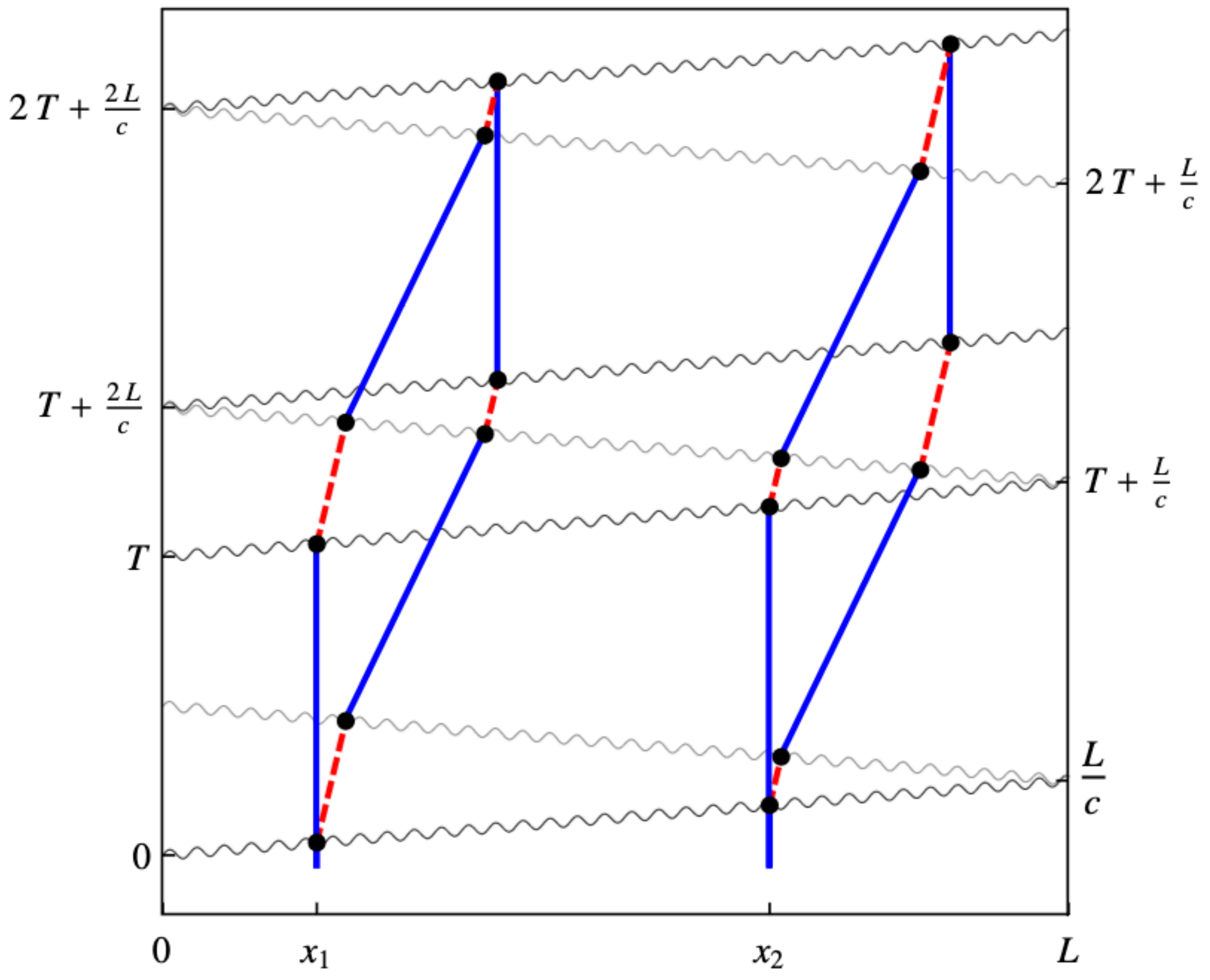
Badurina et al 2108.02468 (AION)

Graham et al 1206.0818 (MAGIS)



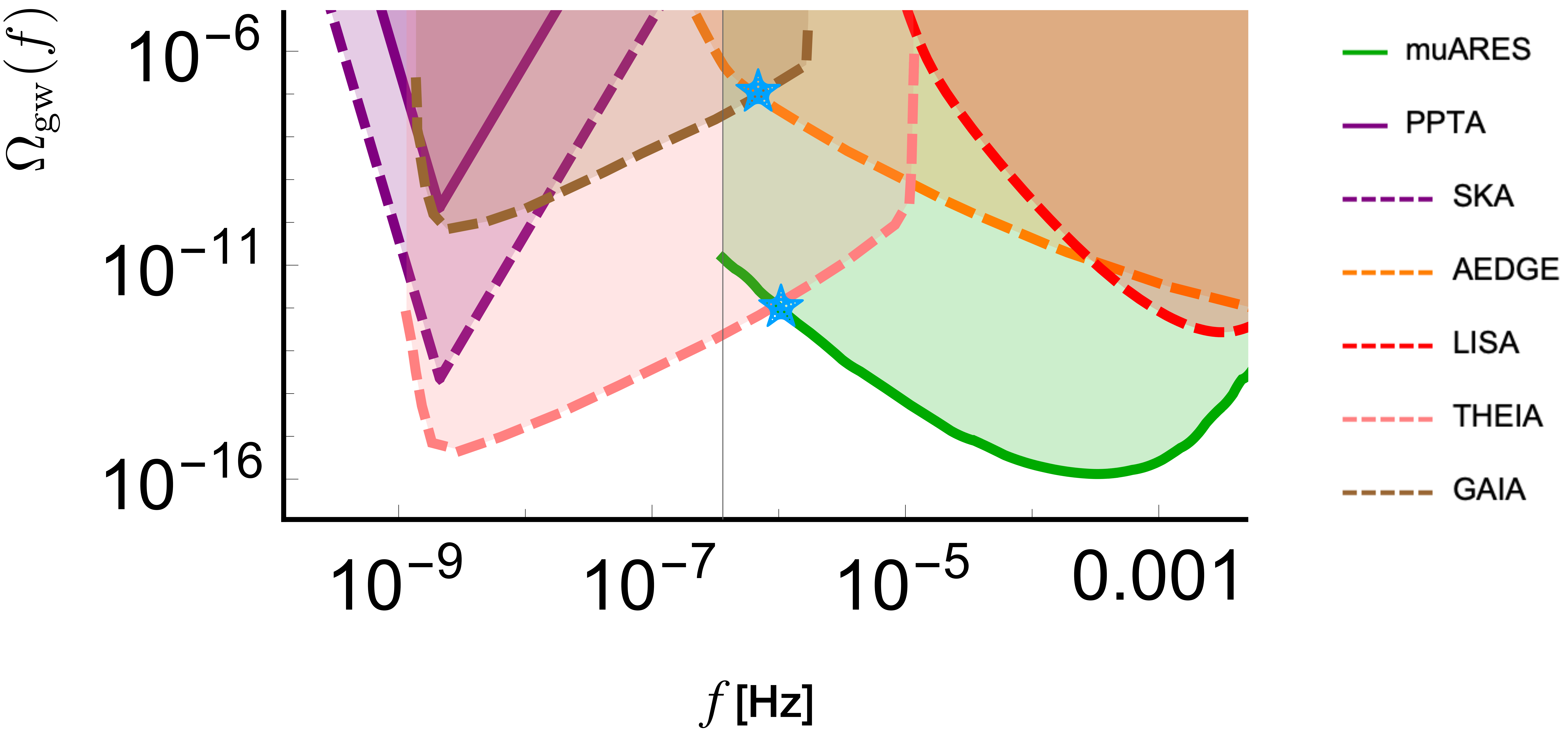
40000 km



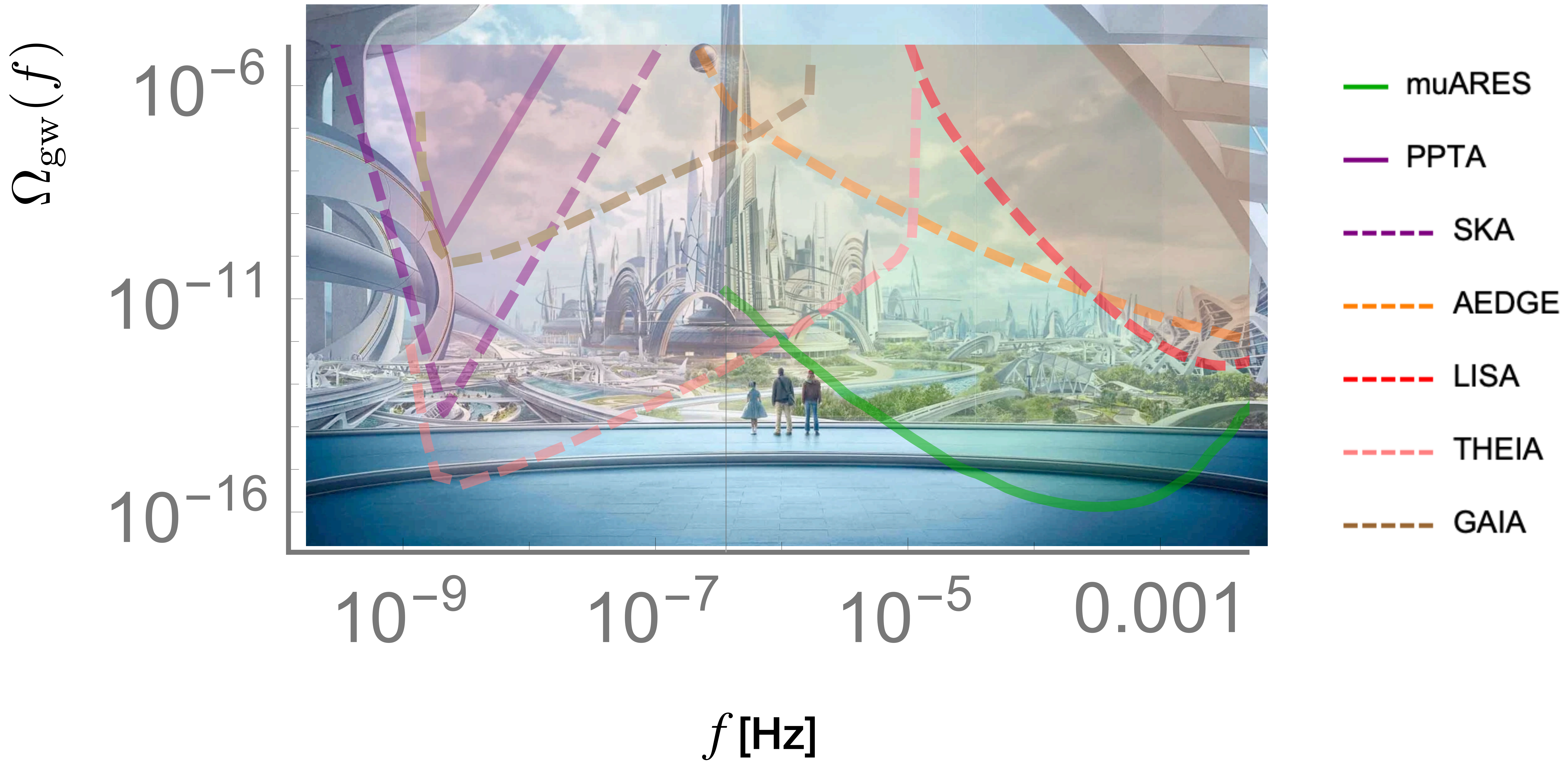


$$\Delta\phi \sim \omega L h$$

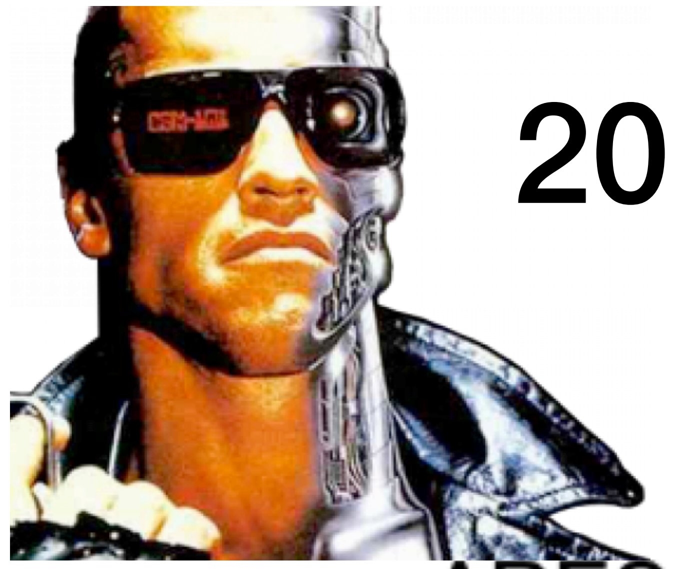
The most optimistic future…

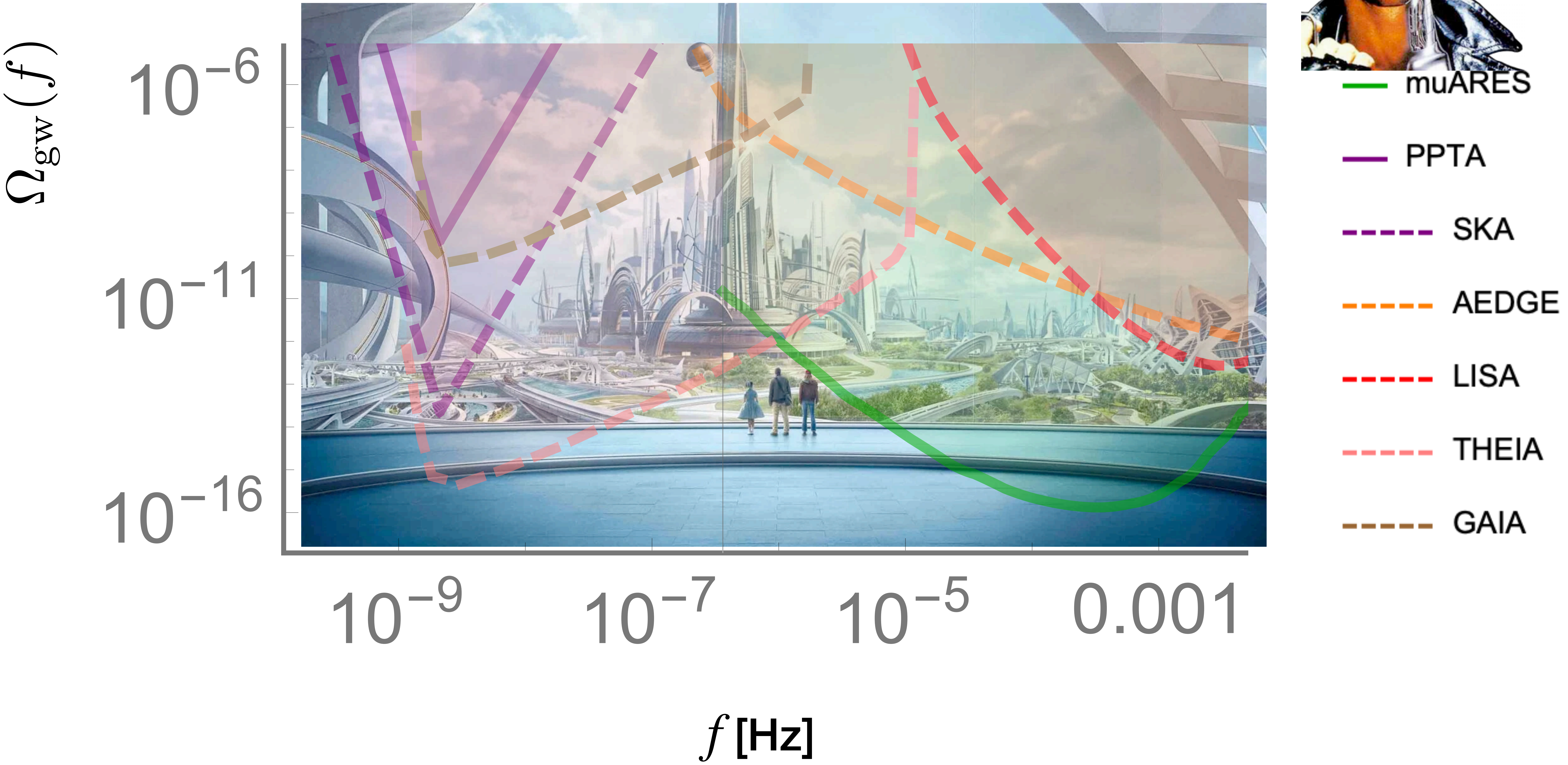


The most optimistic future…

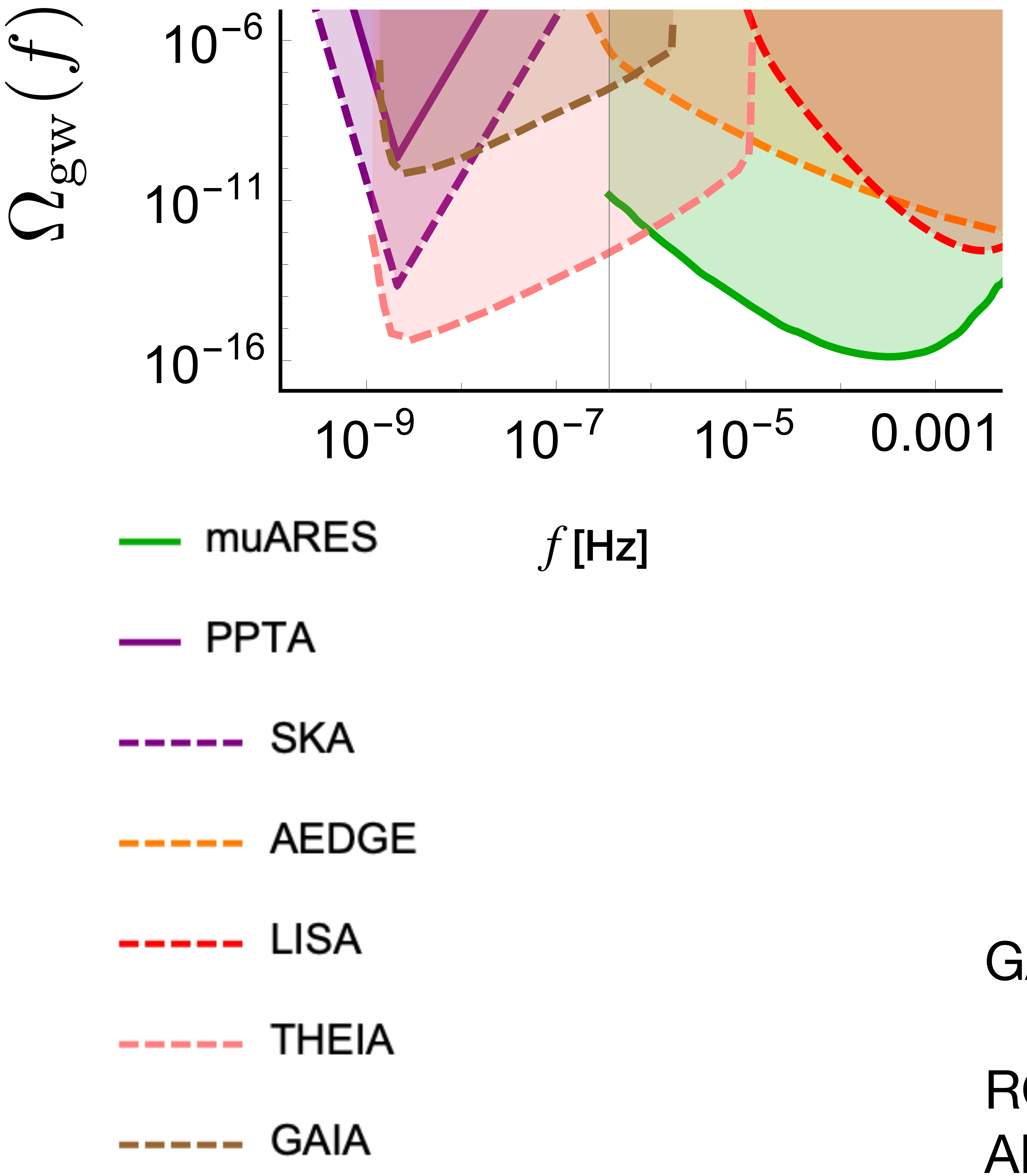


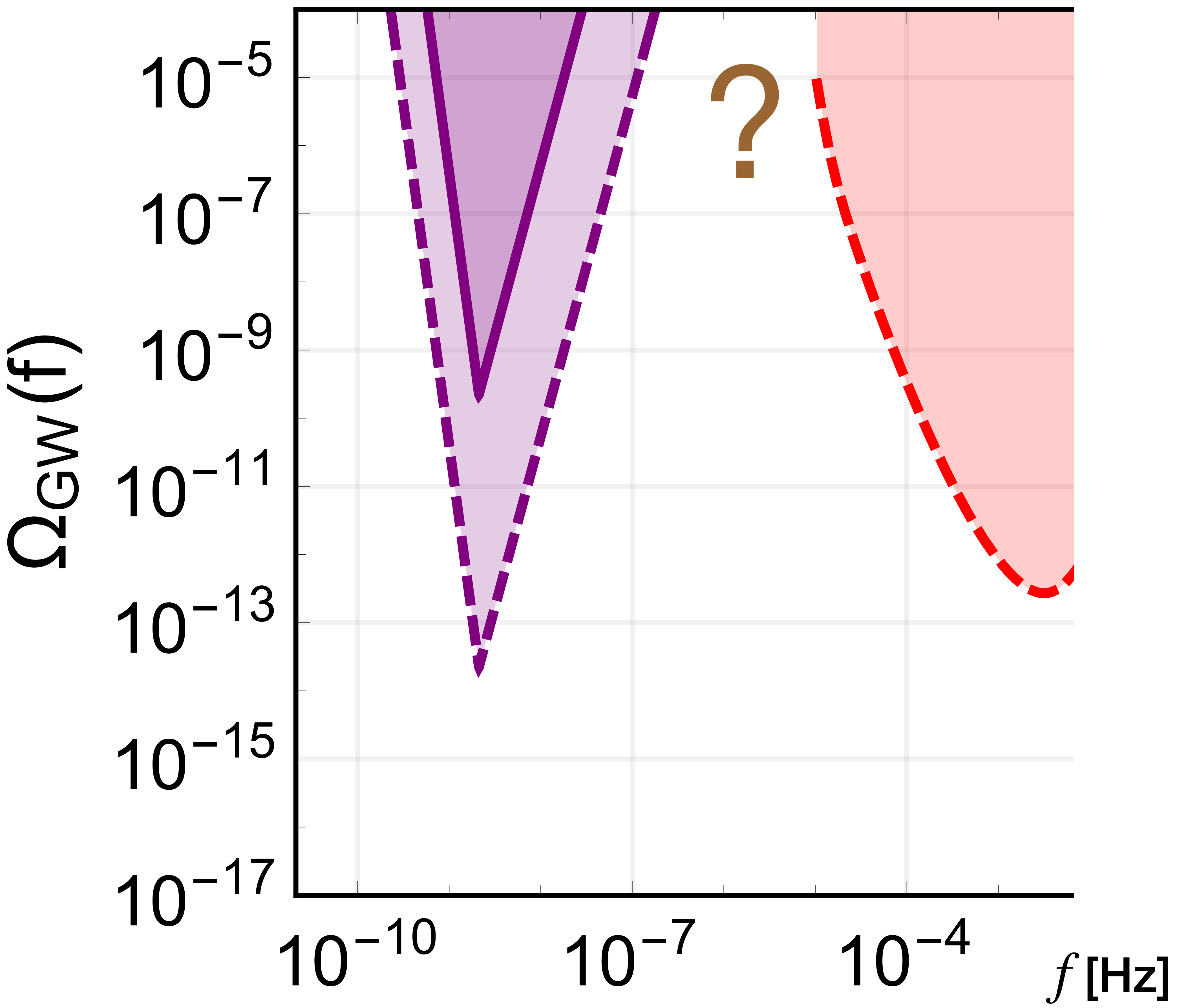
The most optimistic future…





The most optimistic future… vs 2038





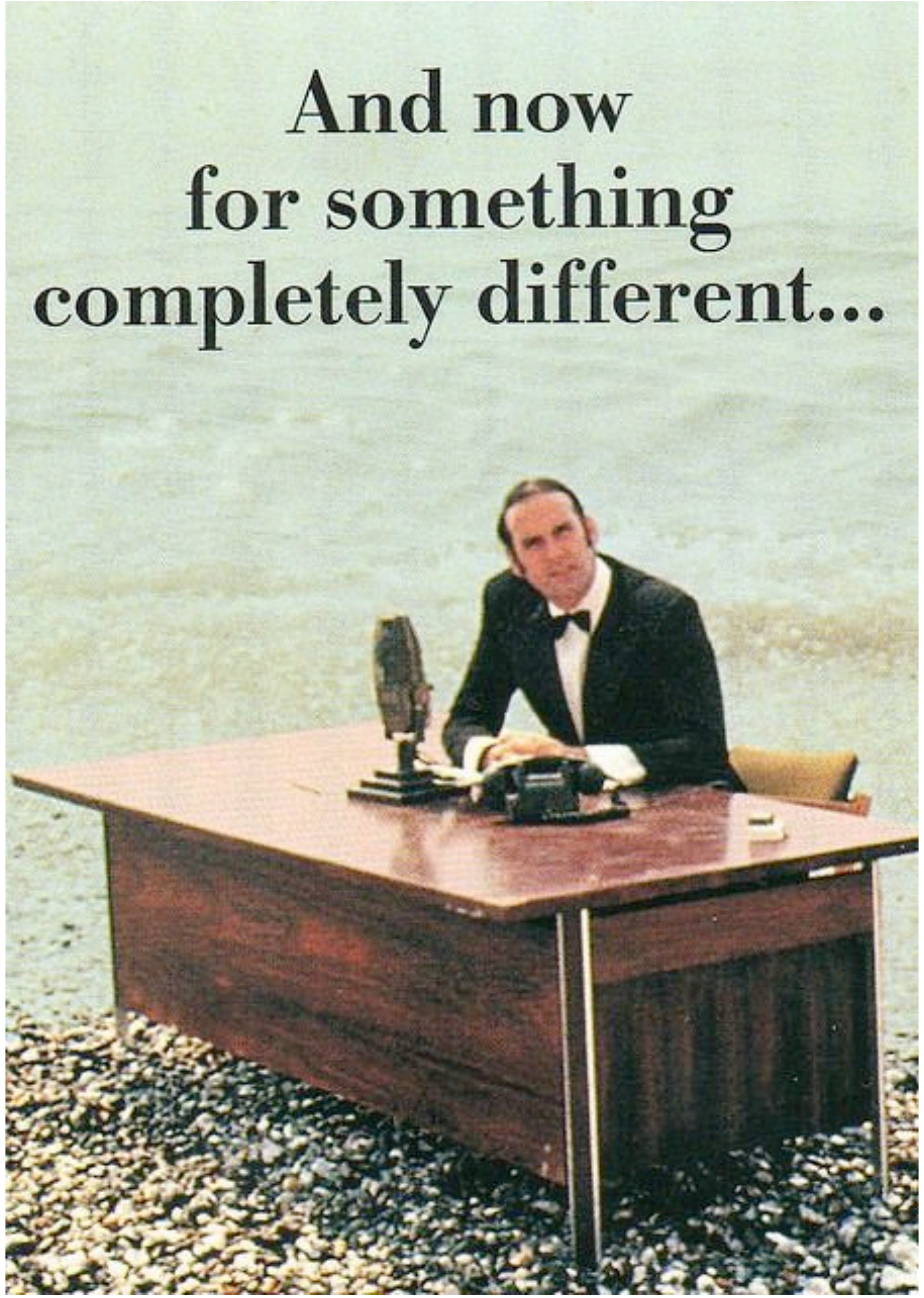
GAIA DR3 Jaraba et al 2304.06350

$$h_{70}^2\Omega_{\rm GW} \lesssim 0.087 \text{ for } 4.2\times10^{-18}\text{ Hz} \lesssim f \lesssim 1.1\times10^{-8}\text{ Hz}$$

ROMAN? Wang et al 2205.07962

AION 10m/MAGIS 100m in 2025? (small interferometers)

Is this all we can do in this band?



$f \sim \mu\mathrm{Hz}$

few days

Absorption of GWs by binaries

$f \sim \mu\text{Hz}$

few days

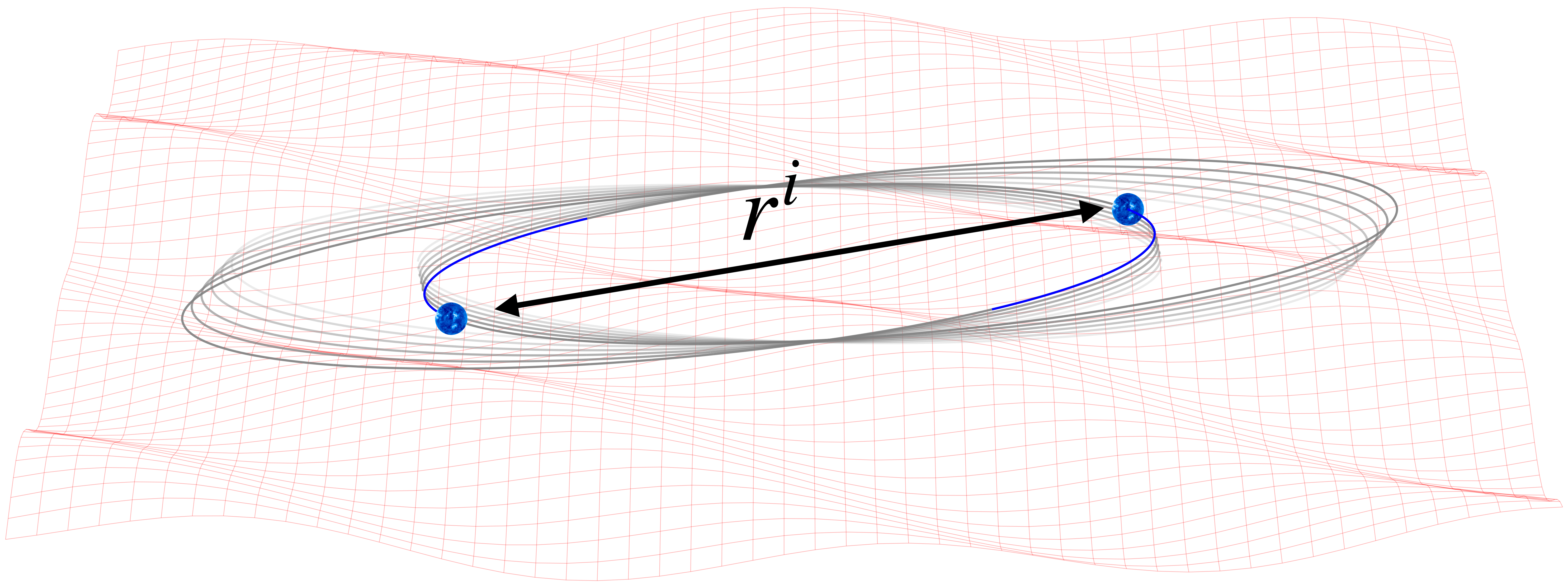
Intuitive idea (from '60s)

Influence of a GW on a binary system (e.g. non-relativistic)

$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

Newtonian potential

GW

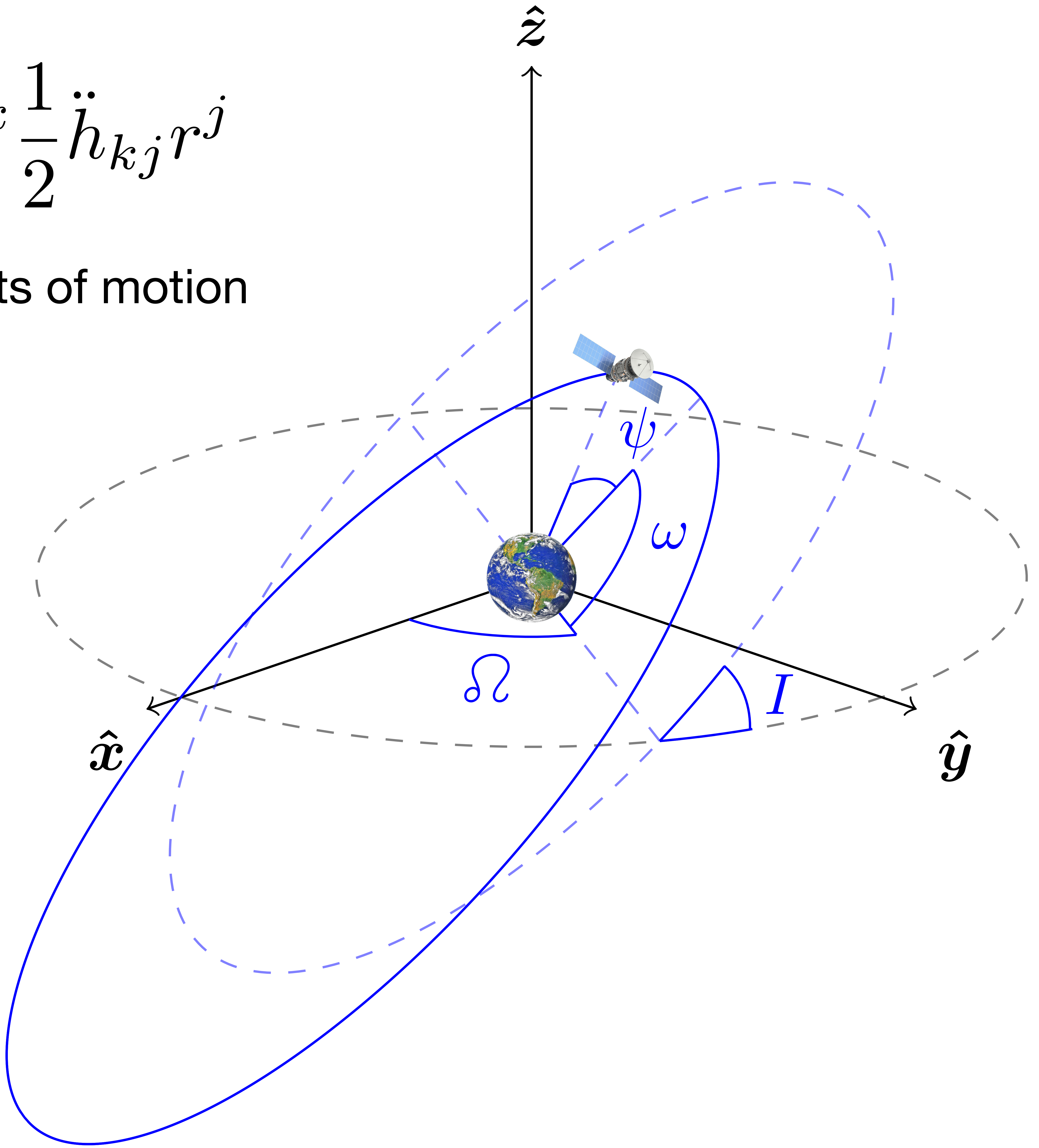


Absorption of GWs by binaries

$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

Better characterised for its 6 Newtonian constants of motion

- **period** P**, eccentricity** e:
  size and *shape* of orbit
- **inlination** I**, ascending node** Ω:
  orientation in space
- **pericentre** ω**,**
  mean anomaly at epoch ε:
  radial and angular *phases*



Absorption of GWs by binaries

$$\ddot{\boldsymbol{r}} + \frac{GM}{r^2}\hat{\boldsymbol{r}} = \delta\ddot{\boldsymbol{r}}.$$

- for generic perturbation:

$$\delta\ddot{\boldsymbol{r}} = r(\mathcal{F}_r\hat{\boldsymbol{r}} + \mathcal{F}_\theta\hat{\boldsymbol{\theta}} + \mathcal{F}_\ell\hat{\boldsymbol{\ell}}),$$

$$\dot{P} = \frac{3P^2\gamma}{2\pi}\left[\frac{e\sin\psi\mathcal{F}_r}{1+e\cos\psi} + \mathcal{F}_\theta\right],$$

$$\dot{e} = \frac{\dot{P}\gamma^2}{3Pe} - \frac{P\gamma^5\mathcal{F}_\theta}{2\pi e(1+e\cos\psi)^2},$$

$$\dot{I} = \frac{P\gamma^3\cos\theta\mathcal{F}_\ell}{2\pi(1+e\cos\psi)^2},$$

$$\dot{\Omega} = \frac{\tan\theta}{\sin I}\dot{I},$$

$$\dot{\omega} = \frac{P\gamma^3}{2\pi e}\left[\frac{(2+e\cos\psi)\sin\psi\mathcal{F}_\theta}{(1+e\cos\psi)^2} - \frac{\cos\psi\mathcal{F}_r}{1+e\cos\psi}\right] - \cos I\dot{\Omega},$$

$$\dot{\varepsilon} = -\frac{P\gamma^4\mathcal{F}_r}{\pi(1+e\cos\psi)^2} - \gamma(\cos I\dot{\Omega} + \dot{\omega}),$$

Absorption of GWs by binaries

$$\ddot{\boldsymbol{r}} + \frac{GM}{r^2}\hat{\boldsymbol{r}} = \delta\ddot{\boldsymbol{r}}.$$

- for generic perturbation:

$$\delta\ddot{\boldsymbol{r}} = r(\mathcal{F}_r\hat{\boldsymbol{r}} + \mathcal{F}_\theta\hat{\boldsymbol{\theta}} + \mathcal{F}_\ell\hat{\boldsymbol{\ell}}),$$

$$\dot{P} = \frac{3P^2\gamma}{2\pi}\left[\frac{e\sin\psi\mathcal{F}_r}{1+e\cos\psi} + \mathcal{F}_\theta\right],$$

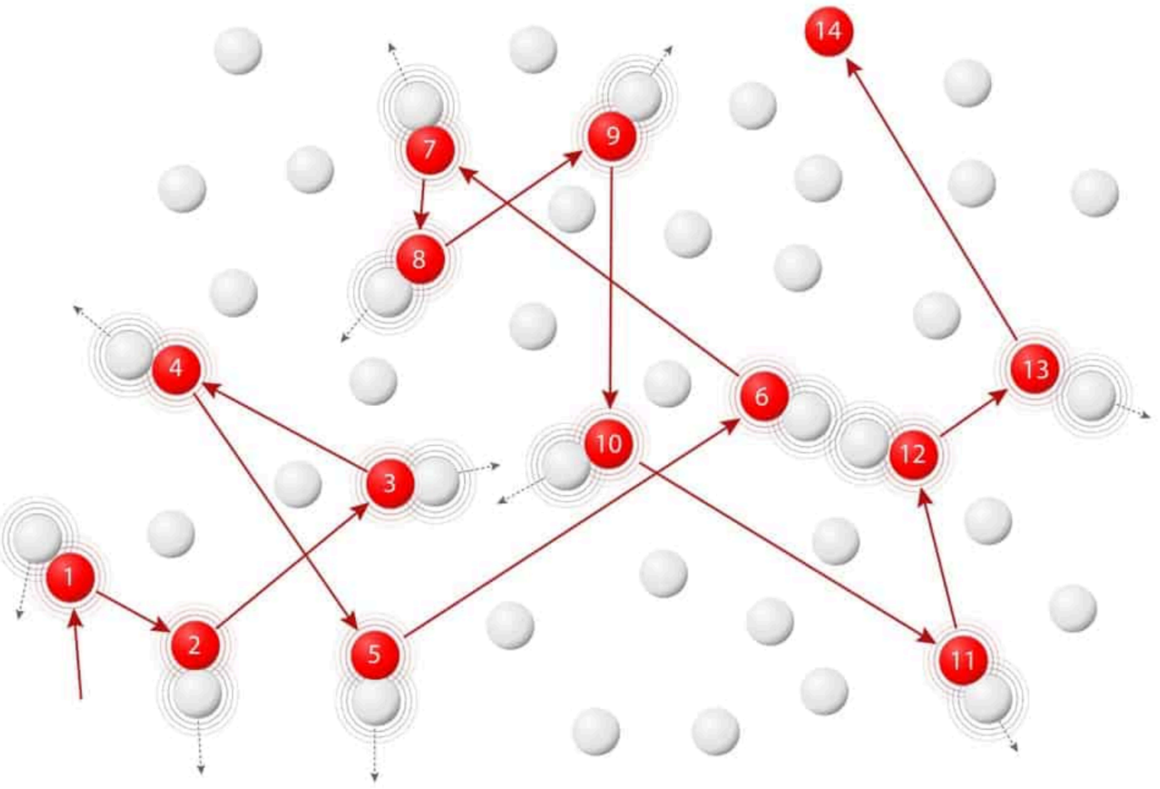
$$\dot{e} = \frac{\dot{P}\gamma^2}{3Pe} - \frac{P\gamma^5\mathcal{F}_\theta}{2\pi e(1+e\cos\psi)^2},$$

$$\dot{I} = \frac{P\gamma^3\cos\theta\mathcal{F}_\ell}{2\pi(1+e\cos\psi)^2},$$

$$\dot{\Omega} = \frac{\tan\theta}{\sin I}\dot{I},$$

$$\dot{\omega} = \frac{P\gamma^3}{2\pi e}\left[\frac{(2+e\cos\psi)\sin\psi\mathcal{F}_\theta}{(1+e\cos\psi)^2} - \frac{\cos\psi\mathcal{F}_r}{1+e\cos\psi}\right] - \cos I\dot{\Omega},$$

$$\dot{\varepsilon} = -\frac{P\gamma^4\mathcal{F}_r}{\pi(1+e\cos\psi)^2} - \gamma(\cos I\dot{\Omega} + \dot{\omega}),$$



For the SGWB… Fokker-Planck approach

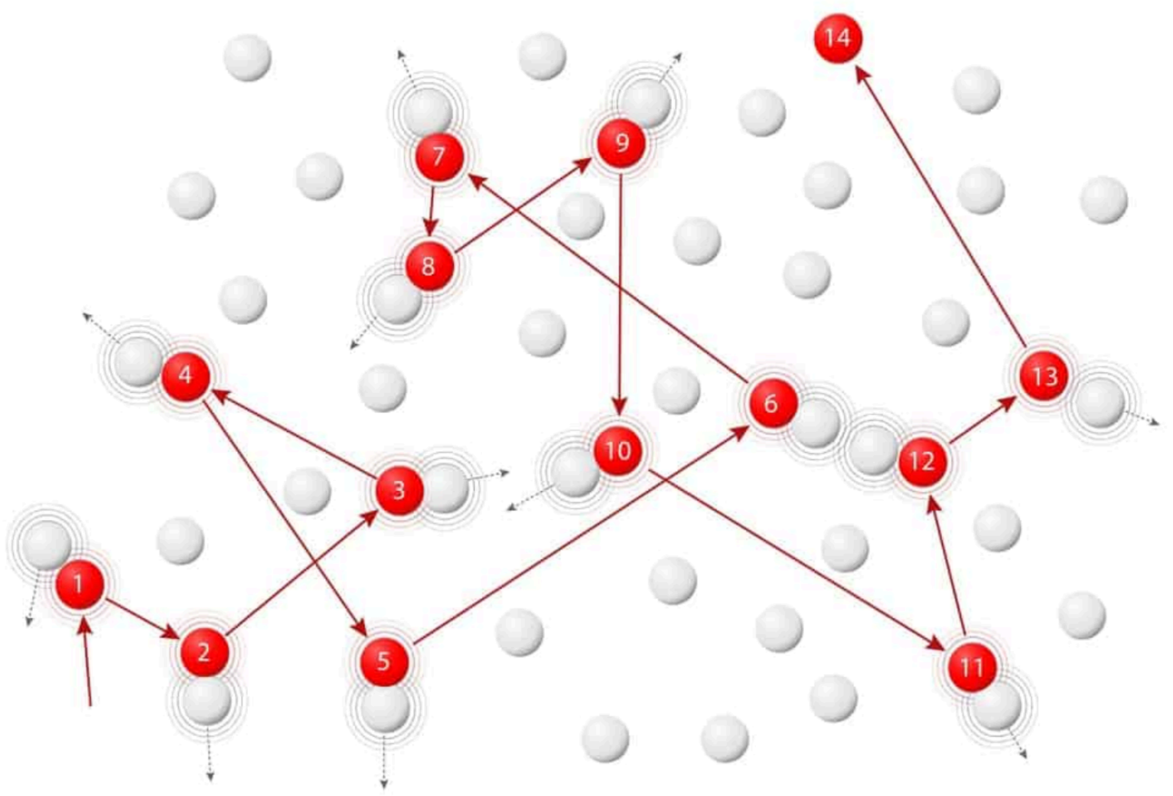
$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

deterministic

stochastic

$$\dot{X}_i(\boldsymbol{X}, t) = V_i(\boldsymbol{X}) + \Gamma_i(\boldsymbol{X}, t)$$

we move from dynamics of the variable to dynamics of the **distribution *W(X)***



$$\frac{\partial W}{\partial t} = -\partial_i \left(D_i^{(1)} W \right) + \partial_i \partial_j \left(D_{ij}^{(2)} W \right)$$

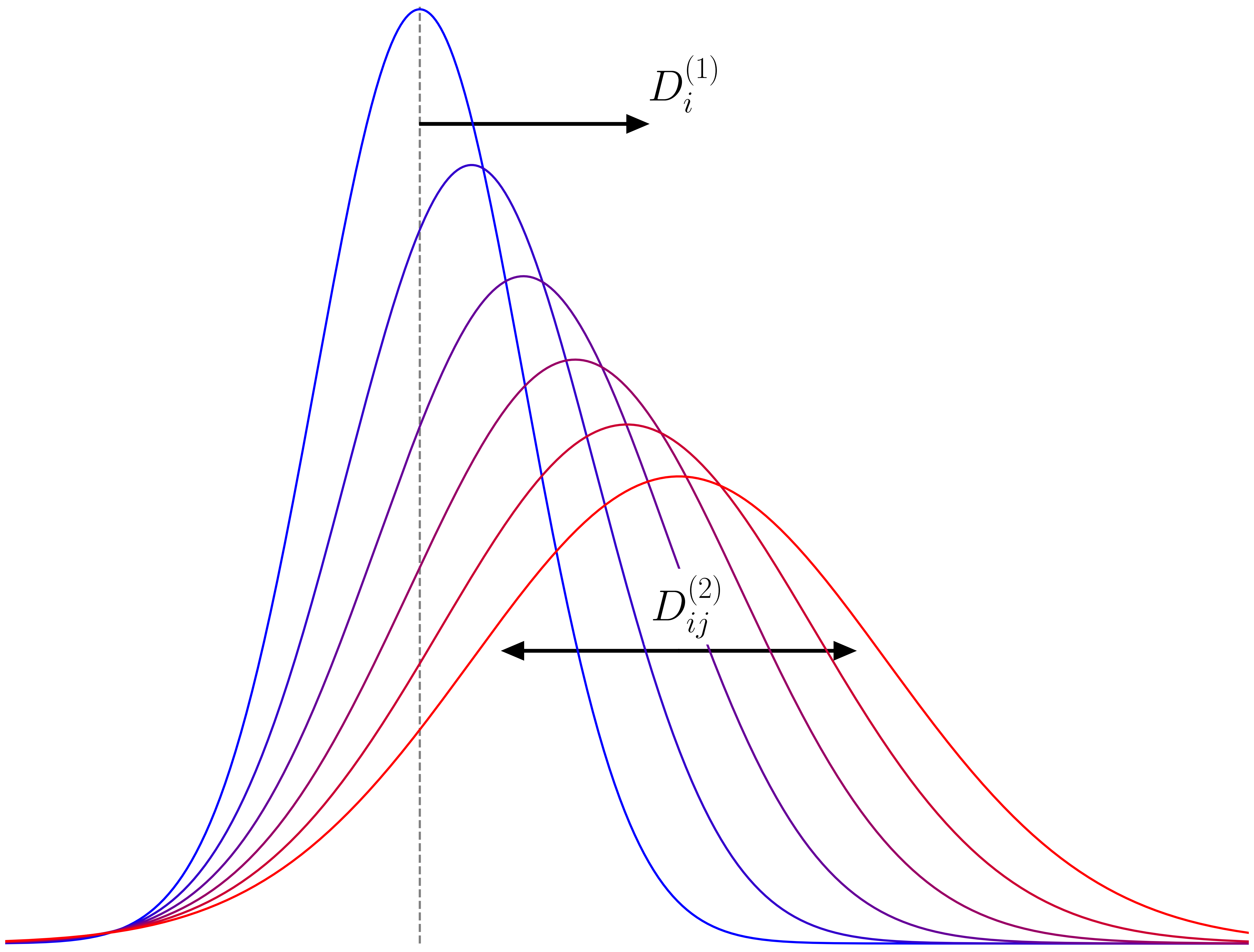
with $\partial_i \equiv \partial / \partial X_i$

$$D_i^{(1)} = V_i + \lim_{\tau \to 0} \frac{1}{\tau} \int_t^{t+\tau} \mathrm{d}t' \int_t^{t'} \mathrm{d}t'' \left\langle \Gamma_j \left(\boldsymbol{x}, t'' \right) \partial_j \Gamma_i \left(\boldsymbol{x}, t' \right) \right\rangle .$$

$$D_{ij}^{(2)} = \lim_{\tau \to 0} \frac{1}{2\tau} \int_t^{t+\tau} \mathrm{d}t' \int_t^{t+\tau} \mathrm{d}t'' \left\langle \Gamma_i \left(\boldsymbol{x}, t' \right) \Gamma_j \left(\boldsymbol{x}, t'' \right) \right\rangle .$$

Our approach to the problem

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



- track distribution function $W(\boldsymbol{X}, t)$ of orbital elements $\boldsymbol{X} = (P, e, I, \Omega, \omega, \varepsilon)$
- evolves through *Fokker-Planck eqn.*

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial X_i}\left(D_i^{(1)} W\right) + \frac{\partial}{\partial X_i}\frac{\partial}{\partial X_j}\left(D_{ij}^{(2)} W\right)$$

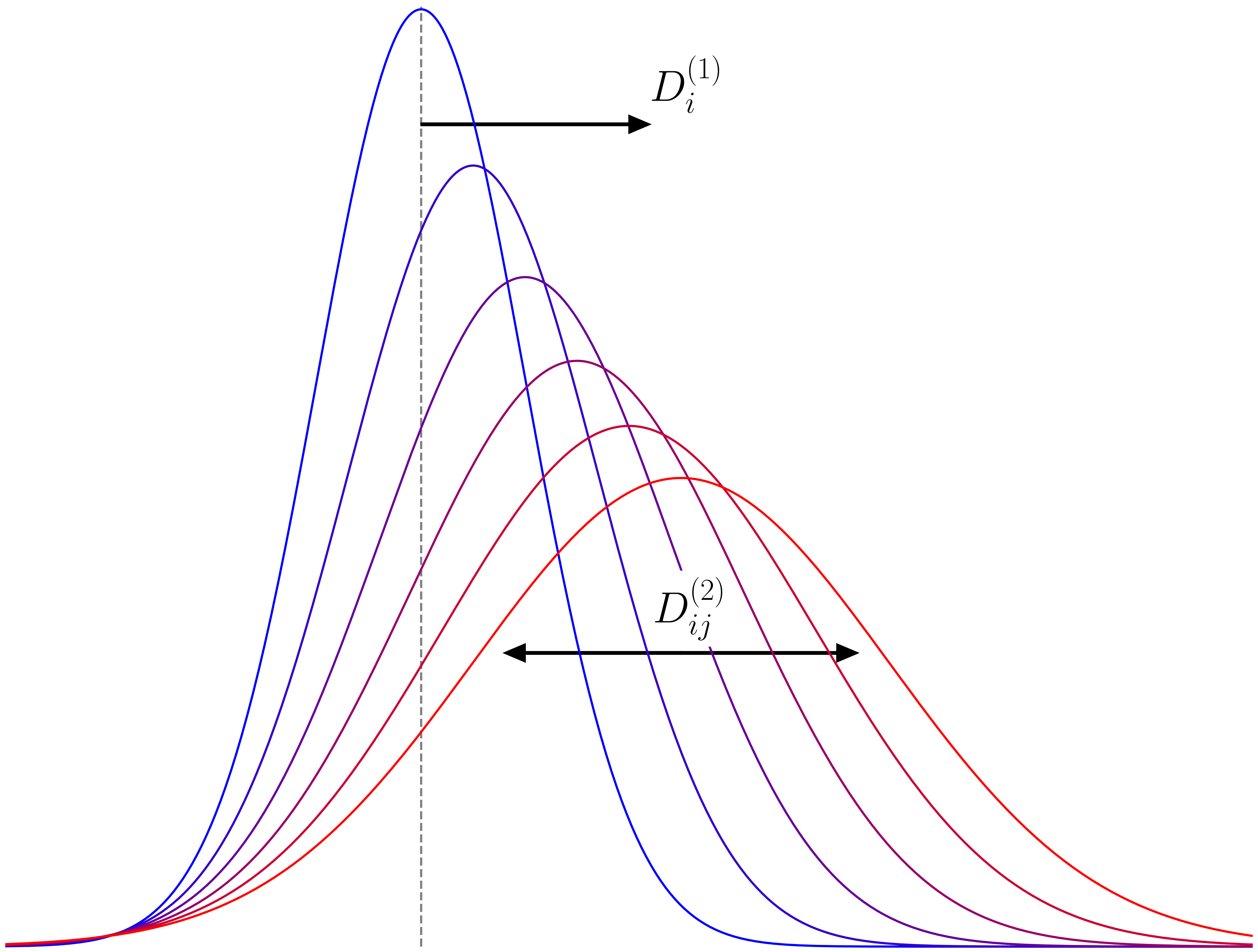
- *drift* and *diffusion* coefficients (averaged over orbits)

$$D_i^{(1)}(\boldsymbol{X}) = V_i(\boldsymbol{X}) + \sum_{n=1}^{\infty} \mathcal{A}_{n,i}(\boldsymbol{X})\Omega_{\rm gw}(n/P)$$

$$D_{ij}^{(2)}(\boldsymbol{X}) = \sum_{n=1}^{\infty} \mathcal{B}_{n,ij}(\boldsymbol{X})\Omega_{\rm gw}(n/P)$$

Our approach to the problem

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



- track distribution function $W(\boldsymbol{X}, t)$ of orbital elements $\boldsymbol{X} = (P, e, I, \Omega, \omega, \varepsilon)$
- evolves through *Fokker-Planck eqn.*

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial X_i}\left(D_i^{(1)} W\right) + \frac{\partial}{\partial X_i}\frac{\partial}{\partial X_j}\left(D_{ij}^{(2)} W\right)$$

- *drift* and *diffusion* coefficients

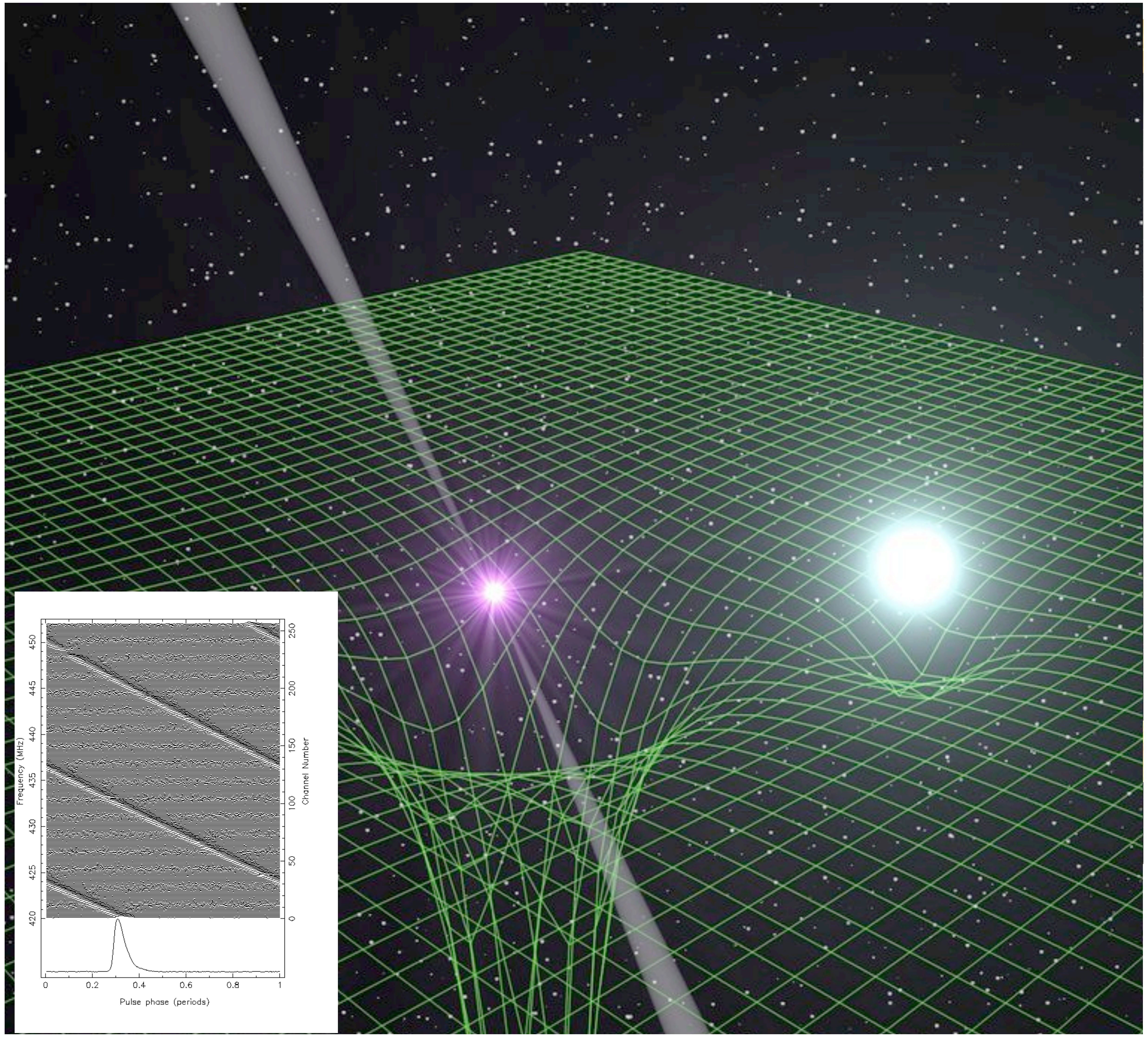
(averaged over orbits)

$$D_i^{(1)}(\boldsymbol{X}) = V_i(\boldsymbol{X}) + \sum_{n=1}^{\infty} \mathcal{A}_{n,i}(\boldsymbol{X}) \boxed{\Omega_{\mathrm{gw}}(n/P)}$$

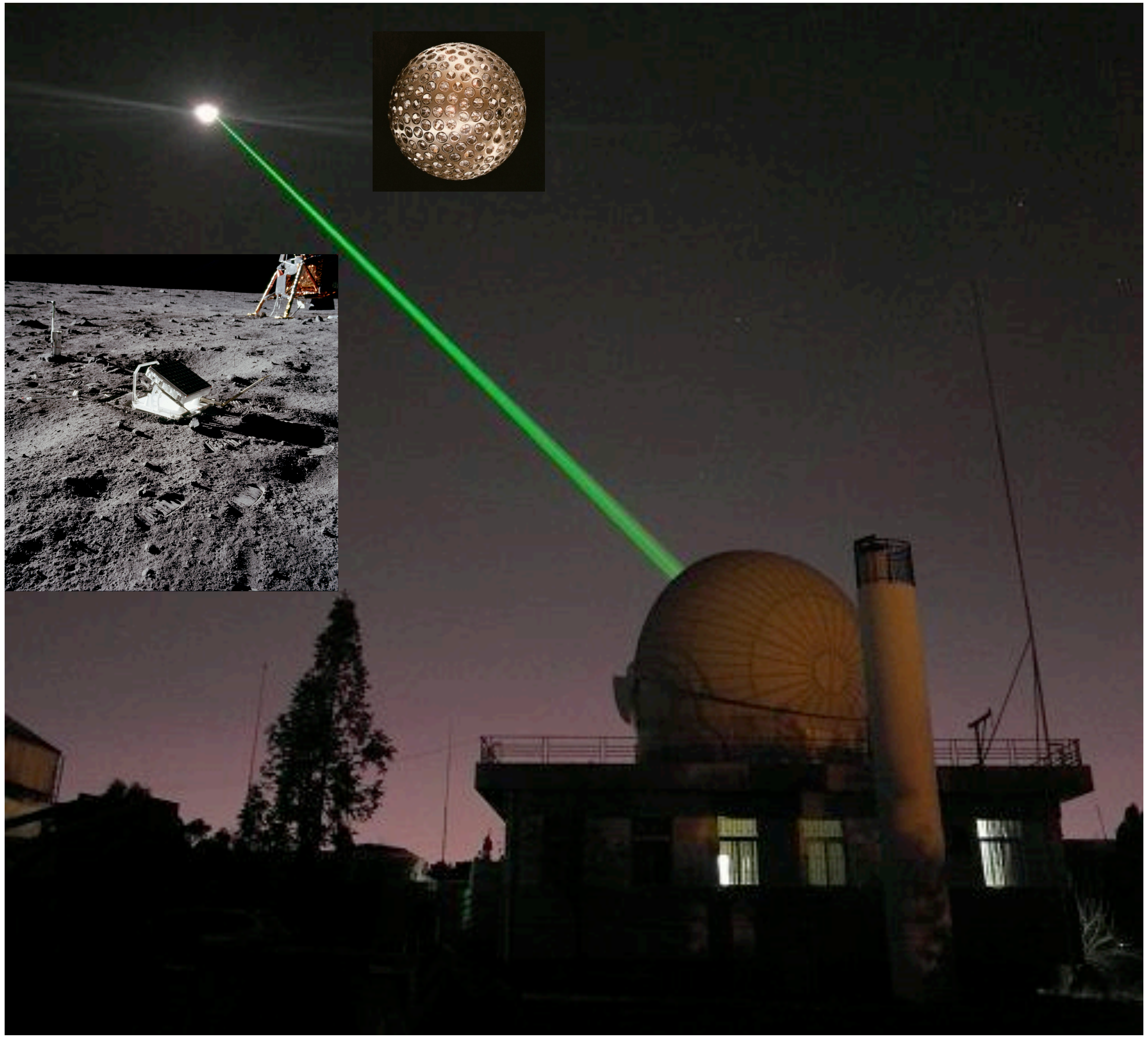
$$D_{ij}^{(2)}(\boldsymbol{X}) = \sum_{n=1}^{\infty} \mathcal{B}_{n,ij}(\boldsymbol{X}) \boxed{\Omega_{\mathrm{gw}}(n/P)}$$

Two probes

timing of binary pulsars

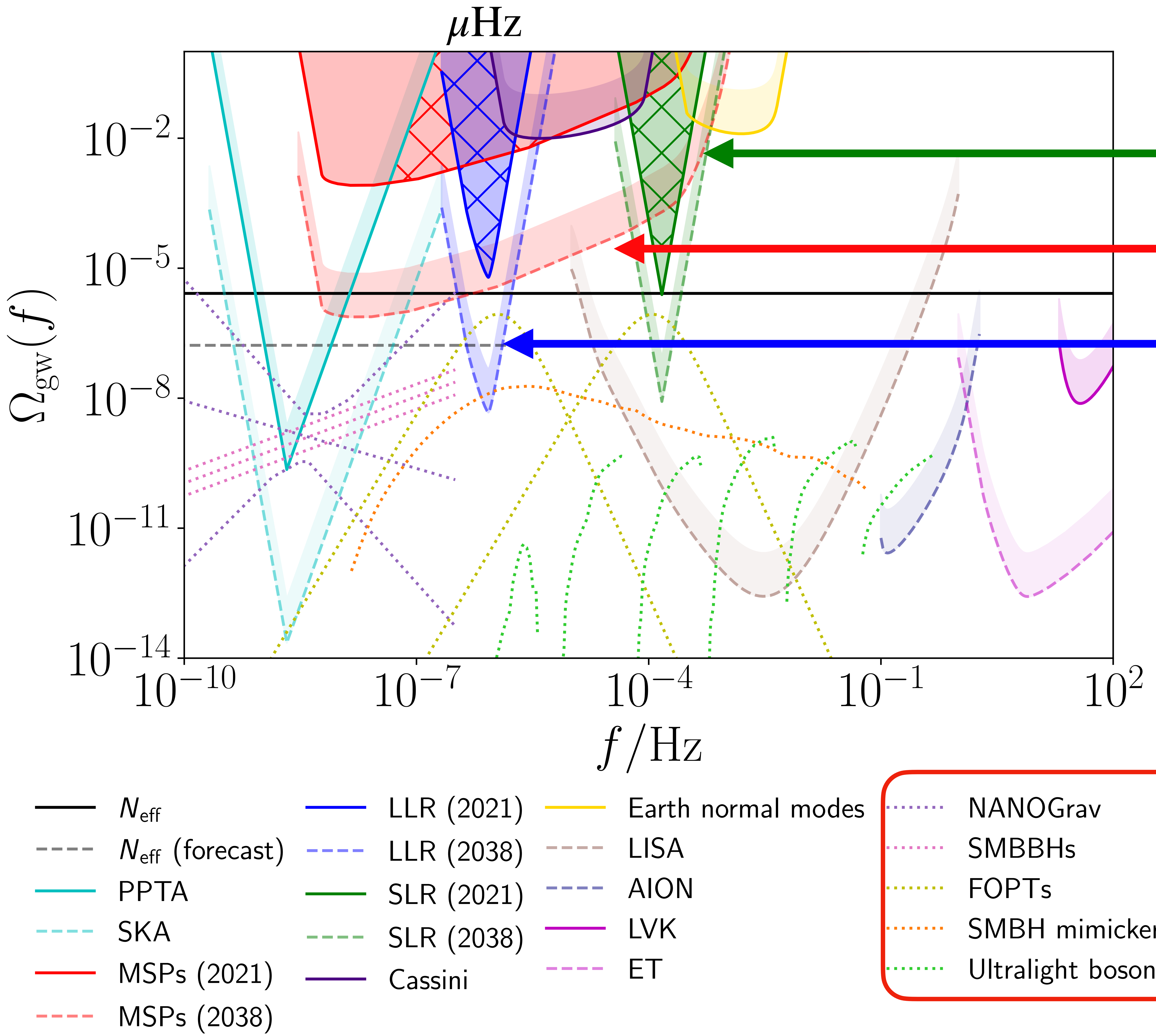


lunar and satellite laser ranging



Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



Satellites (*P*~hours)

Envelope of pulsars (*P*~hours - 100 days)

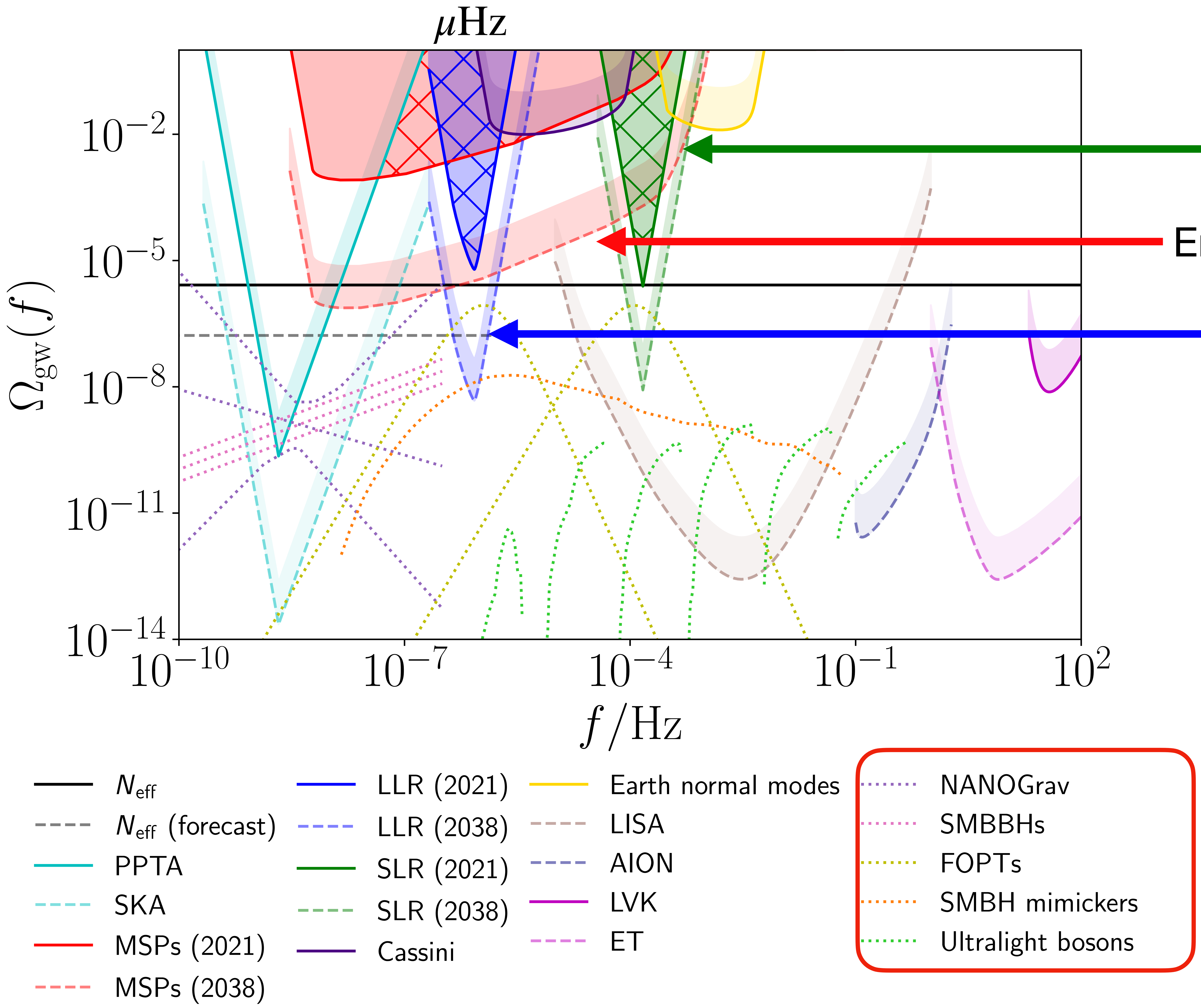
Lunar laser ranging (*P*~month)

(2038 line requires replacing the mirrors …may/will happen before 2030!)

Possible backgrounds

Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



Satellites (*P*~hours)

Envelope of pulsars (*P*~hours - 100 days)

Lunar laser ranging (*P*~month)

(2038 line requires replacing the mirrors …may/will happen before 2030!)

Murphy 1309.6294

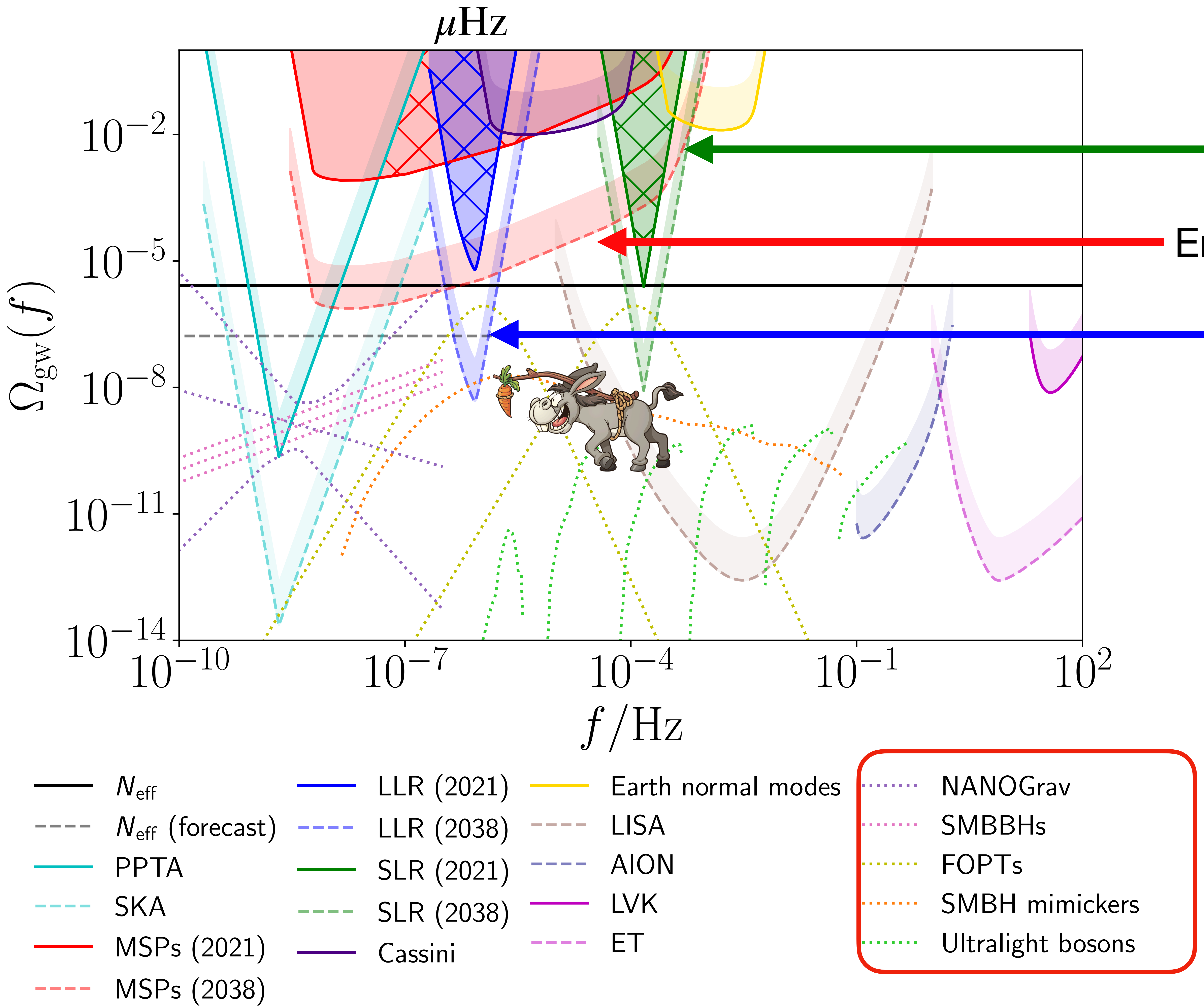
in 2050

$$\Omega \lesssim 3\times 10^{-9} \quad \text{at} \quad f\sim \mu\mathrm{Hz}$$

Possible backgrounds

Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



Satellites (*P*~hours)

Envelope of pulsars (*P*~hours - 100 days)

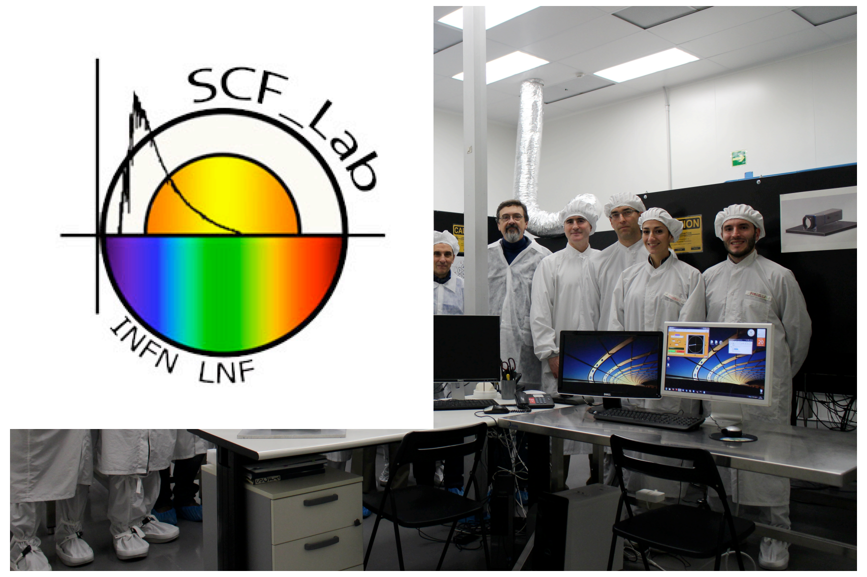
Lunar laser ranging (*P*~month)

(2038 line requires replacing the mirrors …may/will happen before 2030!)

Murphy 1309.6294

in 2050

$$\Omega \lesssim 3 \times 10^{-9} \quad \text{at} \quad f \sim \mu\text{Hz}$$



μHz GWs

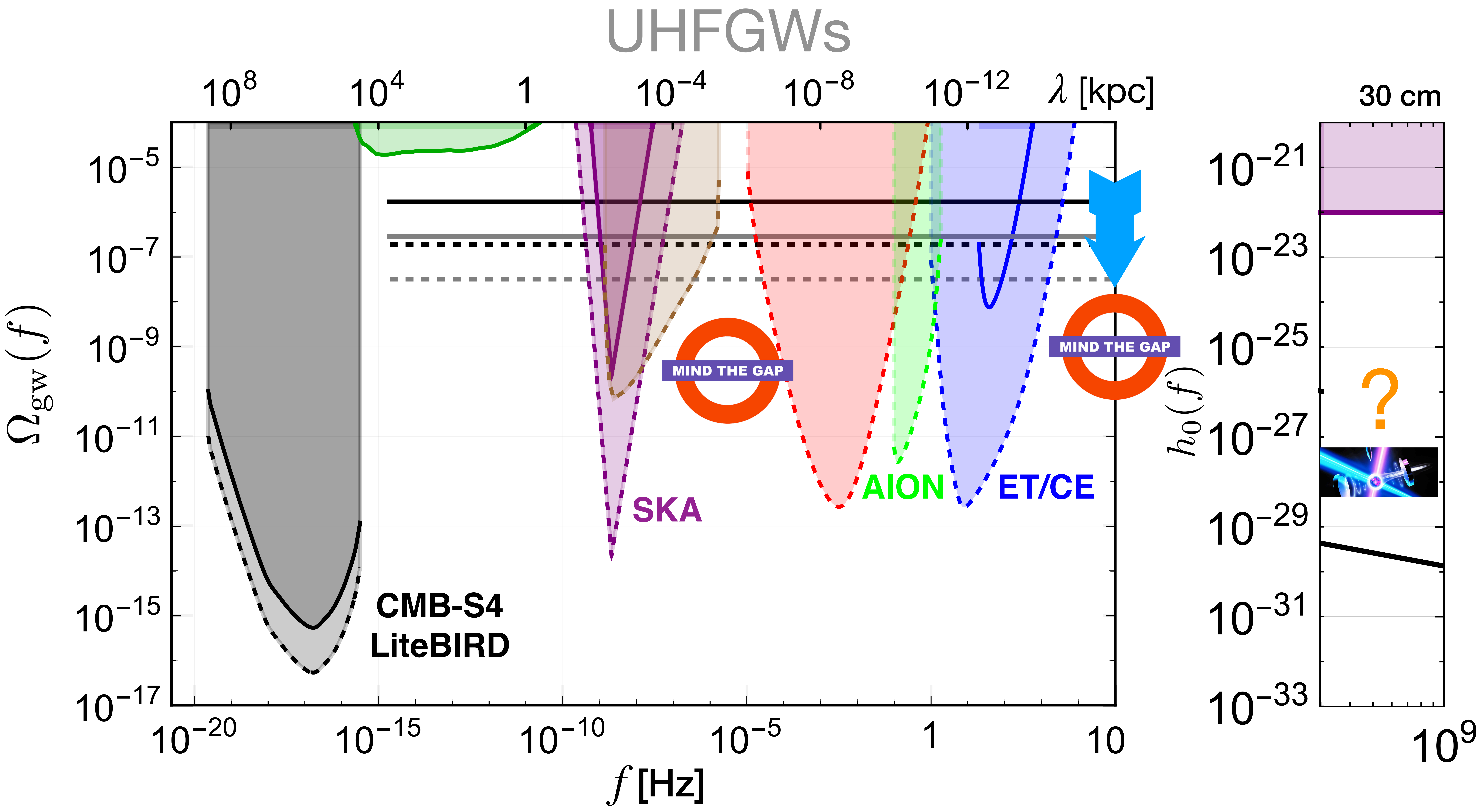
- The μHz band is very rich for **astrophysical** and **cosmological** sources

- There are **ideas** of how to access it, though **most** of them are **futuristic**

- The resonant **absorption of GWs by binaries** (LLR/SLR/pulsars) may give a handle at level (in 2038)

$$\Omega_{\rm gw} \geq 4.8 \times 10^{-9} \quad f = 0.85\,\mu{\rm Hz} \qquad\qquad \Omega_{\rm gw} \geq 8.3 \times 10^{-9} \quad f = 0.15\,{\rm mHz}$$

- **Future plans:** use LLR, SLR, pulsar ***data*** (w/ SYRTE, SCF_Lab, MPIfRA, Nanograv people…): we need/welcome new hands.
  Find ***other resonant effects***
  (e.g. in rotation, w. M de Amicis, wide binaries….which frequencies?)



UHFGWs
10^8
10^4
1
10^{-4}
10^{-8}
10^{-12}
λ [kpc]
30 cm
$\Omega_{\rm gw}(f)$
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
MIND THE GAP
MIND THE GAP
SKA
AION
ET/CE
CMB-S4
LiteBIRD
10^{-20}
10^{-15}
10^{-10}
10^{-5}
1
10
f [Hz]
$h_0(f)$
10^{-21}
10^{-23}
10^{-25}
10^{-27}
10^{-29}
10^{-31}
10^{-33}
?
10^9

UHFGWs -> Laboratory searches

GWs interact with **every** source of energy-momentum!

in the laboratory

*Interaction GWs with **light***

*Interaction GWs with **matter external dofs***

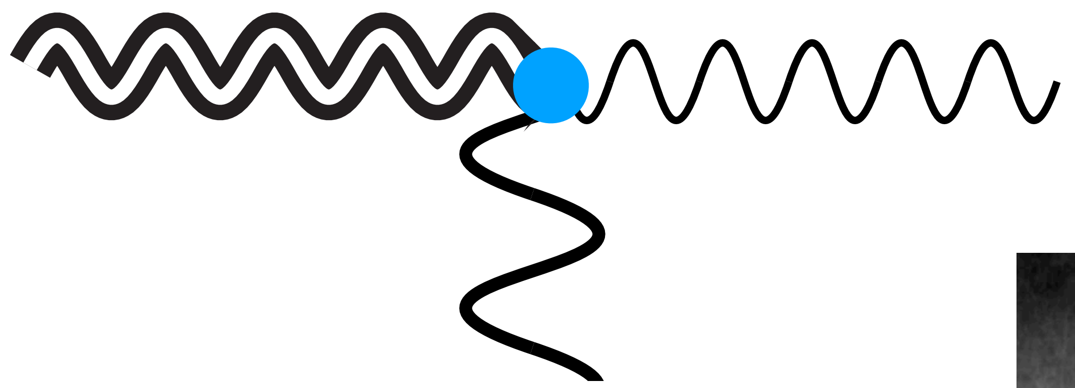
*Interaction GWs with **spin or other internal dofs***

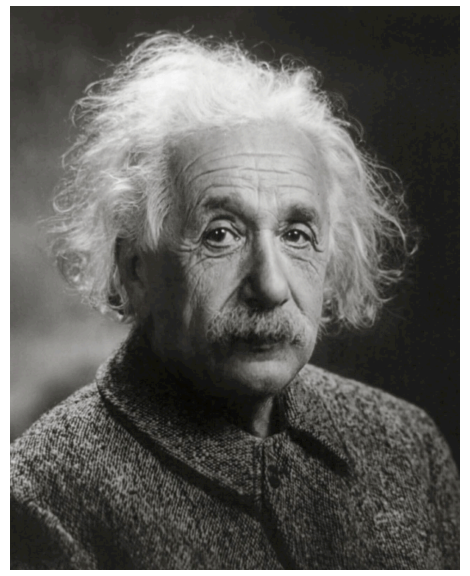
we have a lot to learn from DM searches!

GWs and EM fields

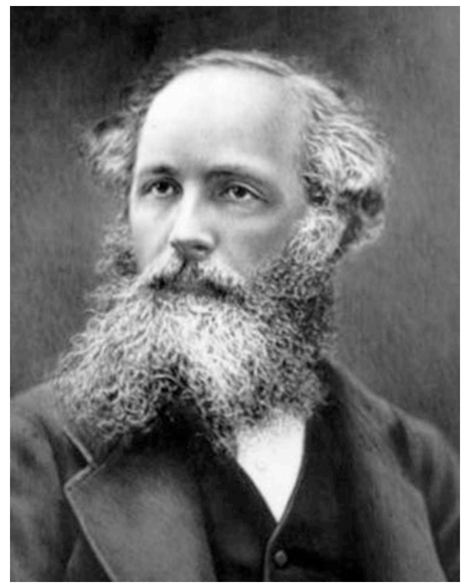
Raffelt Stodolsky 87

GWs + EM field -> EM field





$$\mathcal{L} = \sqrt{-g}\,(R + F_{\mu\nu}F^{\mu\nu}) \supset \frac{1}{2}A_\mu j^\mu_{\rm eff}(h) + \eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + O(h^2)$$

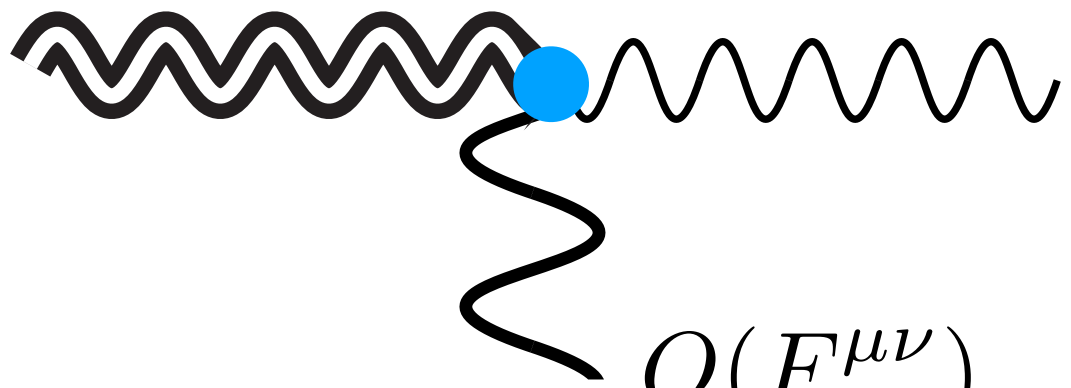


$$j^\mu_{\rm eff} = -\partial_\beta\left(\frac{1}{2}hF^{\mu\beta} + h^\beta_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\beta}\right)$$

analogy with **axions** + EM field -> EM field

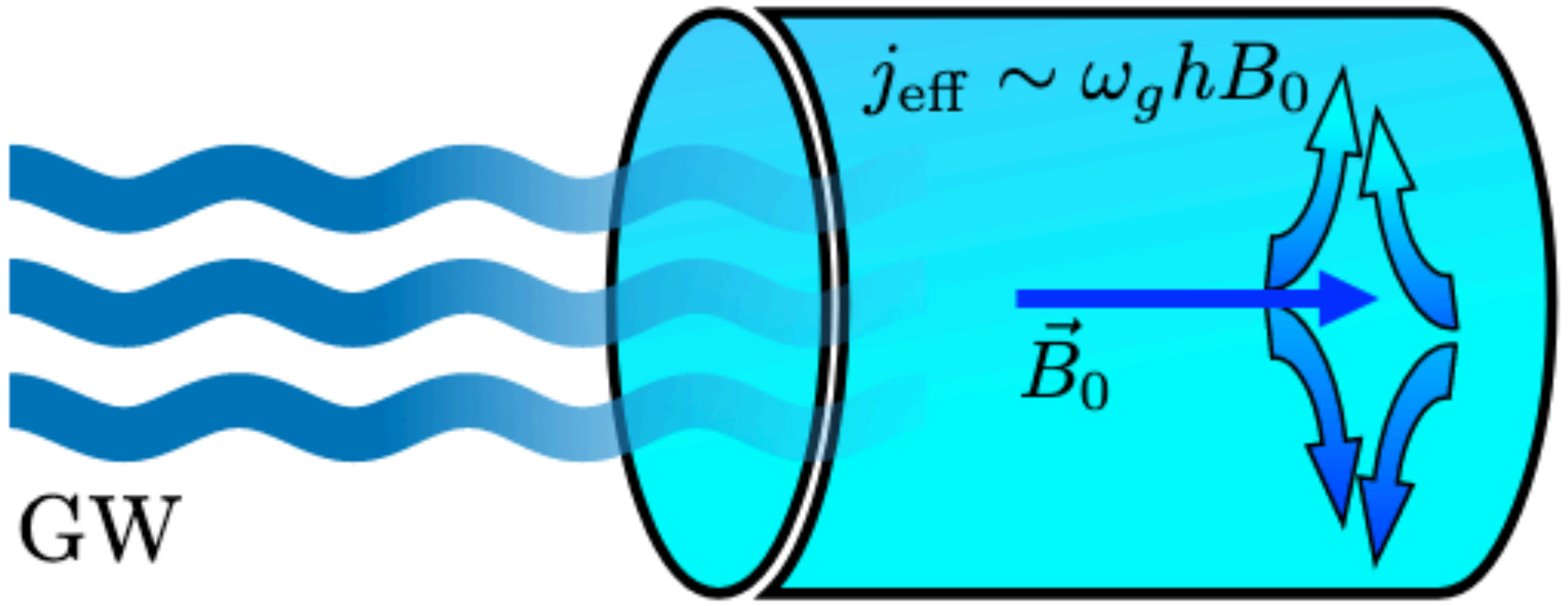
$$a\tilde{F}F$$

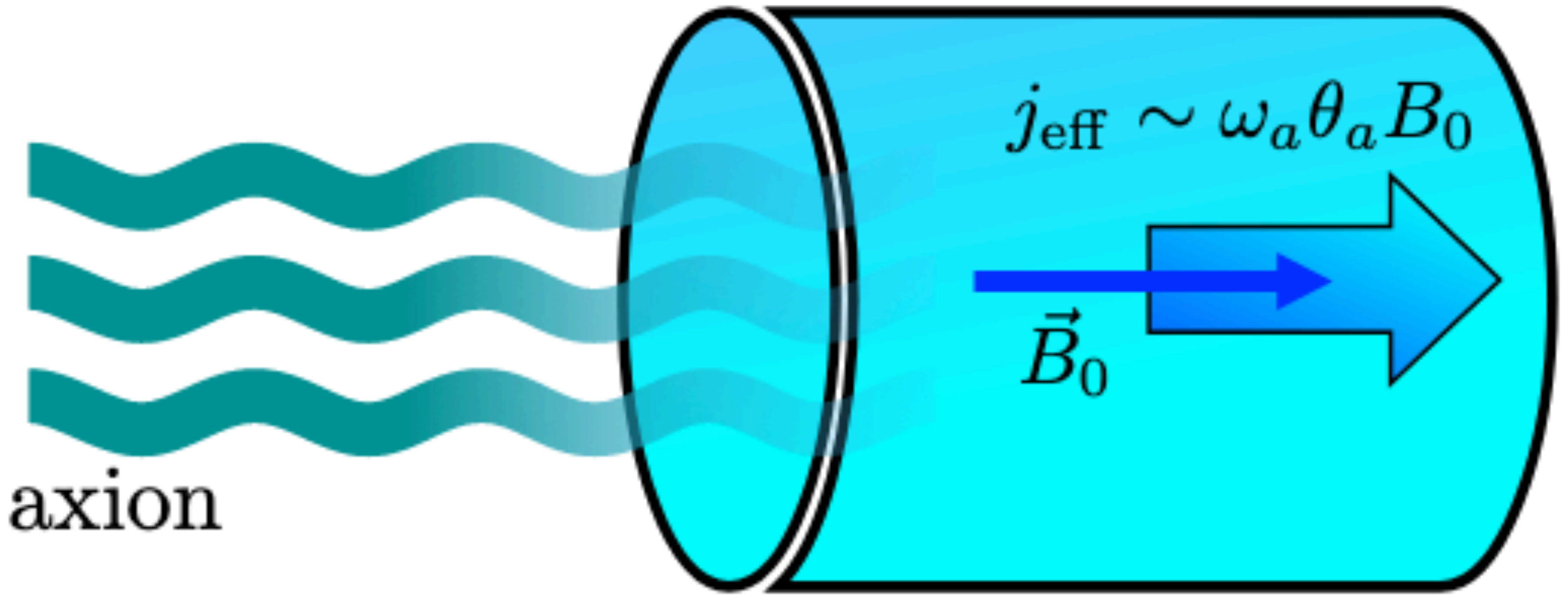
a



$O(F^{\mu\nu})$

Connection to axions



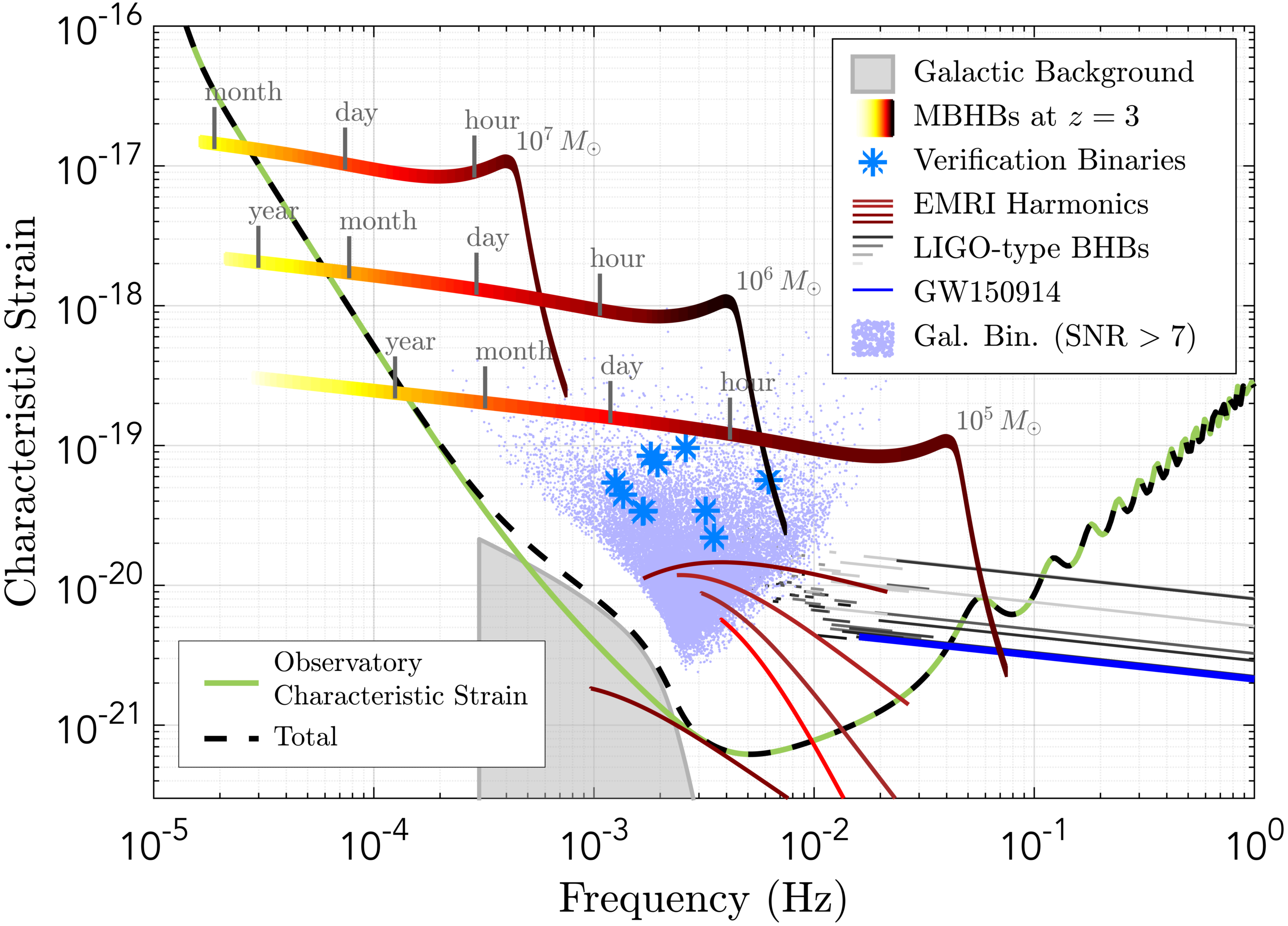


This already sets the scale of the GW we want to measure:
for resonant production (for constant $\overrightarrow{B}$) $\lambda_{gw} \approx L$

waves of (a priori) laboratory size

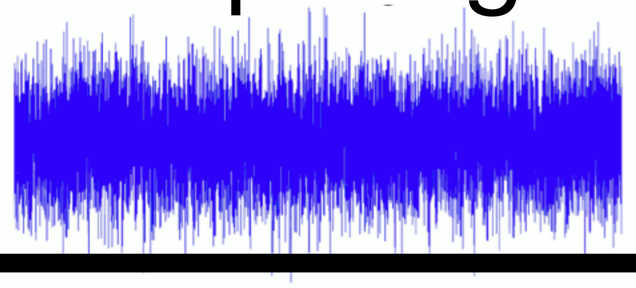
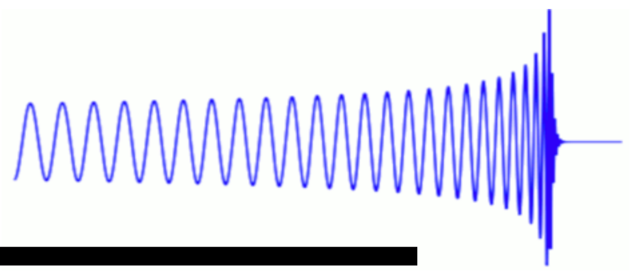
LISA Sources

Amaro-Seoane et al. 1702.00786



Sources of UHFGW ($> 10\,\mathrm{kHz}$)

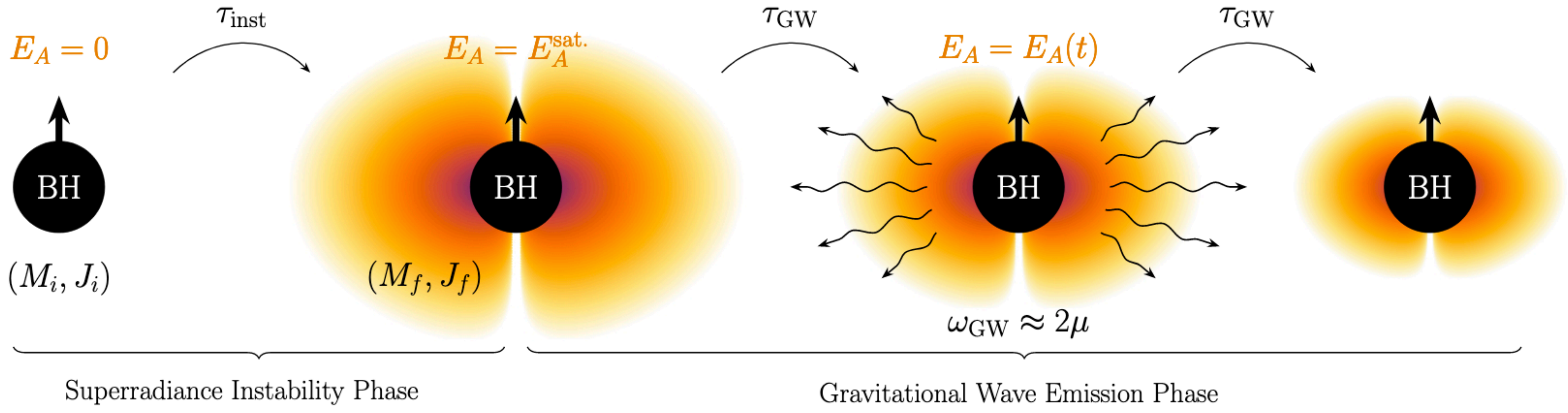
Very different picture comparing to EM spectrum

	Stochastic 		Coherent 	
standard physics	Primordial plasma thermal fluctuations Ghiglieri et al JHEP 07 (2020) 092 Ringwald et al JCAP 03 (2021) 054	✔	?	✘
	Inflation			
new physics	Phase transitions, Cosmic strings Close encounters of BHs	✔	Light PBHB Ultra-light DM Exotic objects	✔

Two words on sources

- ## SR

from Tsukada et al, '20



$$h \sim 10^{-23} \left(\frac{\Delta a_*}{0.1}\right) \left(\frac{1\mathrm{kpc}}{D}\right) \left(\frac{M_b}{1M_\odot}\right) \left(\frac{\alpha}{0.2}\right)^7$$

$$(M_b/M_\odot) \sim (10^3\mathrm{Hz}/\omega_{\rm gw})$$

$$t \sim 10^5\mathrm{yrs} \times (\mathrm{MHz}/\omega_g)^2$$

- ## BBH merger

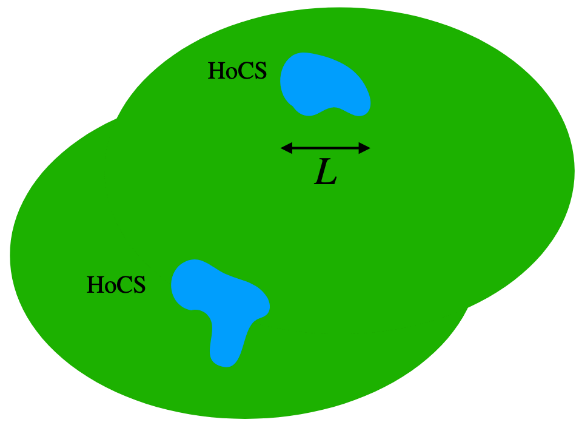
Franciolini et al 2205.02153

$$h_0 \sim 10^{-29} \times \left(\frac{1\mathrm{pc}}{D}\right) \left(\frac{M_b}{10^{-11}M_\odot}\right)^{5/3} \left(\frac{\omega_g}{1\mathrm{GHz}}\right)^{2/3}$$

$$\boxed{\tau_b \sim 10^{-3}\ \mathrm{s}} \left(\frac{10^5}{Q}\right) \left(\frac{10^{-11}M_\odot}{M_b}\right)^{5/3} \left(\frac{1\mathrm{GHz}}{\omega_g}\right)^{8/3}$$

- ## NS/NS mergers

Casalderrey et al. 2210.03171



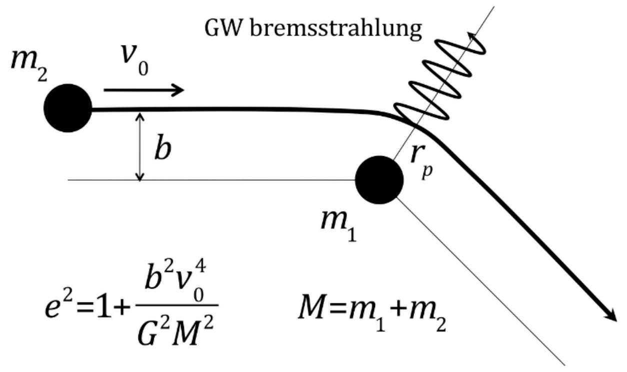
MHz

$$h_c^{\rm obs} \simeq 2.1 \times 10^{-24} v_f^2 \left(\frac{100\mathrm{Mpc}}{d}\right)$$

$$\boxed{\Delta t \simeq L \simeq 1.7 \times 10^{-2}\ \mathrm{ms}}$$

- ## Hyperbolic encounters of PBH

Garcia Bellido & S. Nesseris 1706.02111

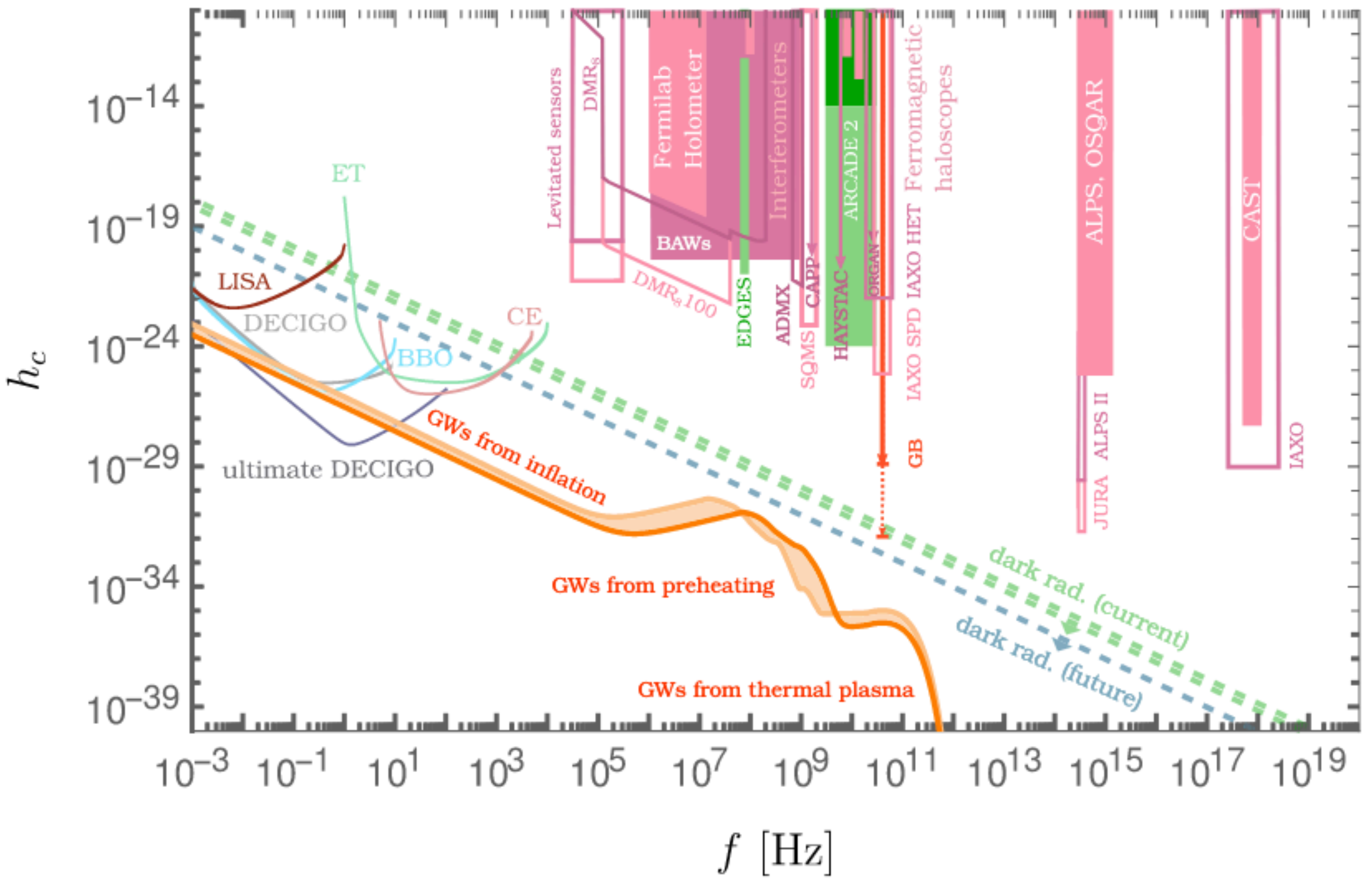


$$\boxed{t_{1/2} \simeq 1ms} \left(\frac{b}{10^{-8}\mathrm{AU}}\right) \left(\frac{0.01}{\beta}\right) (e-1) \sqrt{\frac{3\ln 2}{e+35(1+e)e}}$$

Sources of UHFGW: spoiler!

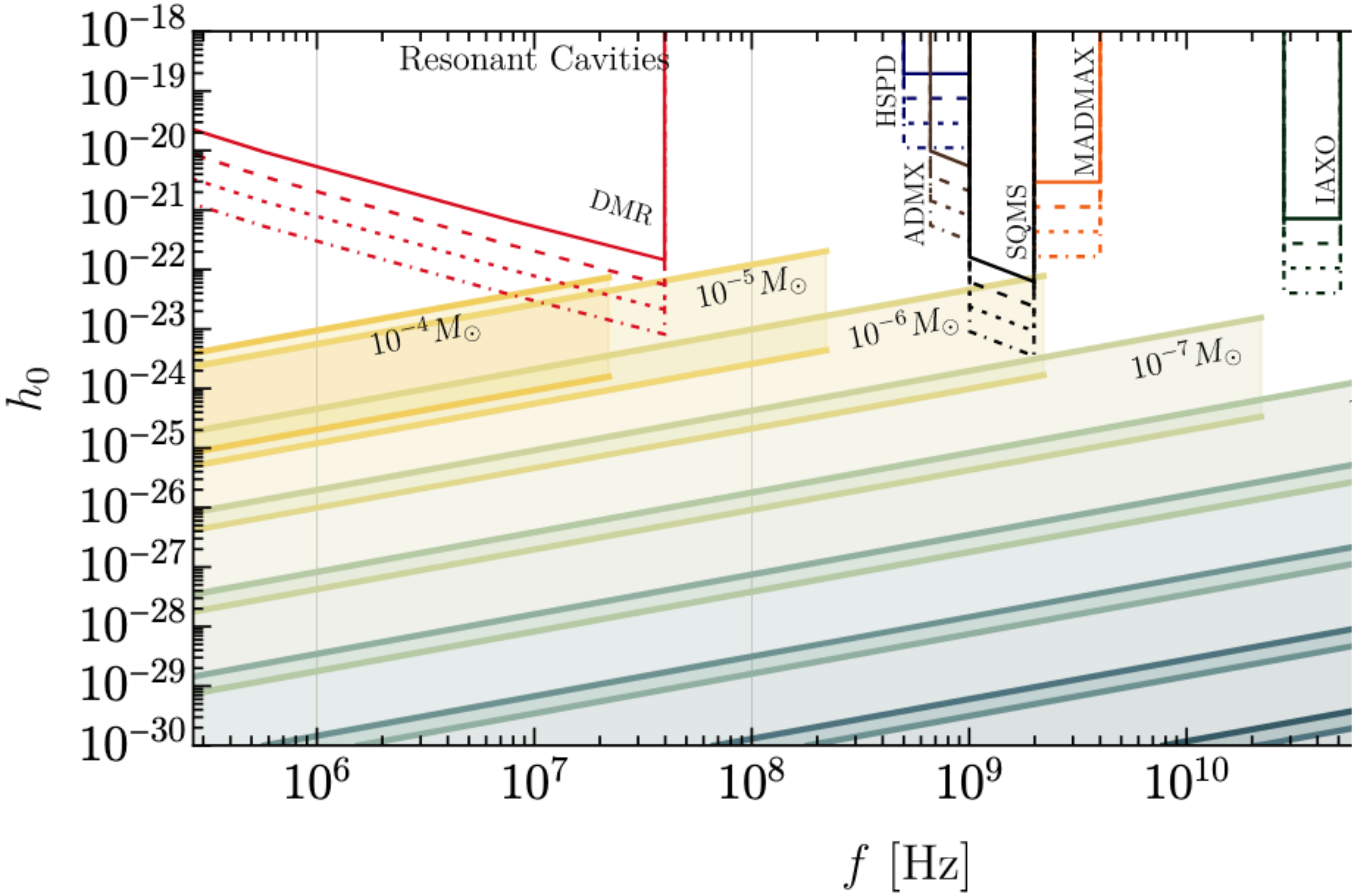
SMASH model full spectrum

Ringwald Tamarit 22



GWs from PBHs (rates 1/year)

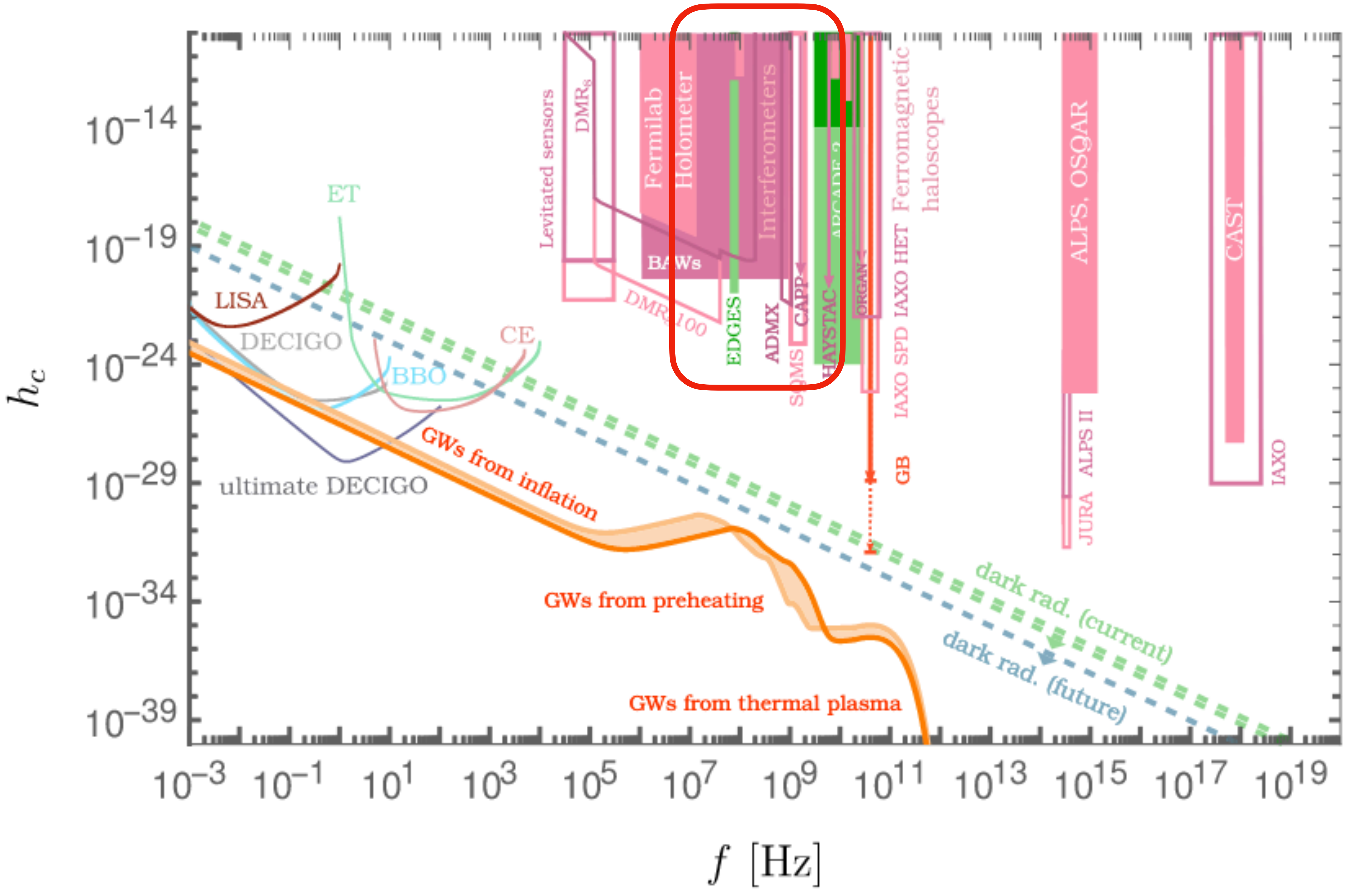
Franciolini et al 2205.02153



Sources of UHFGW: spoiler!

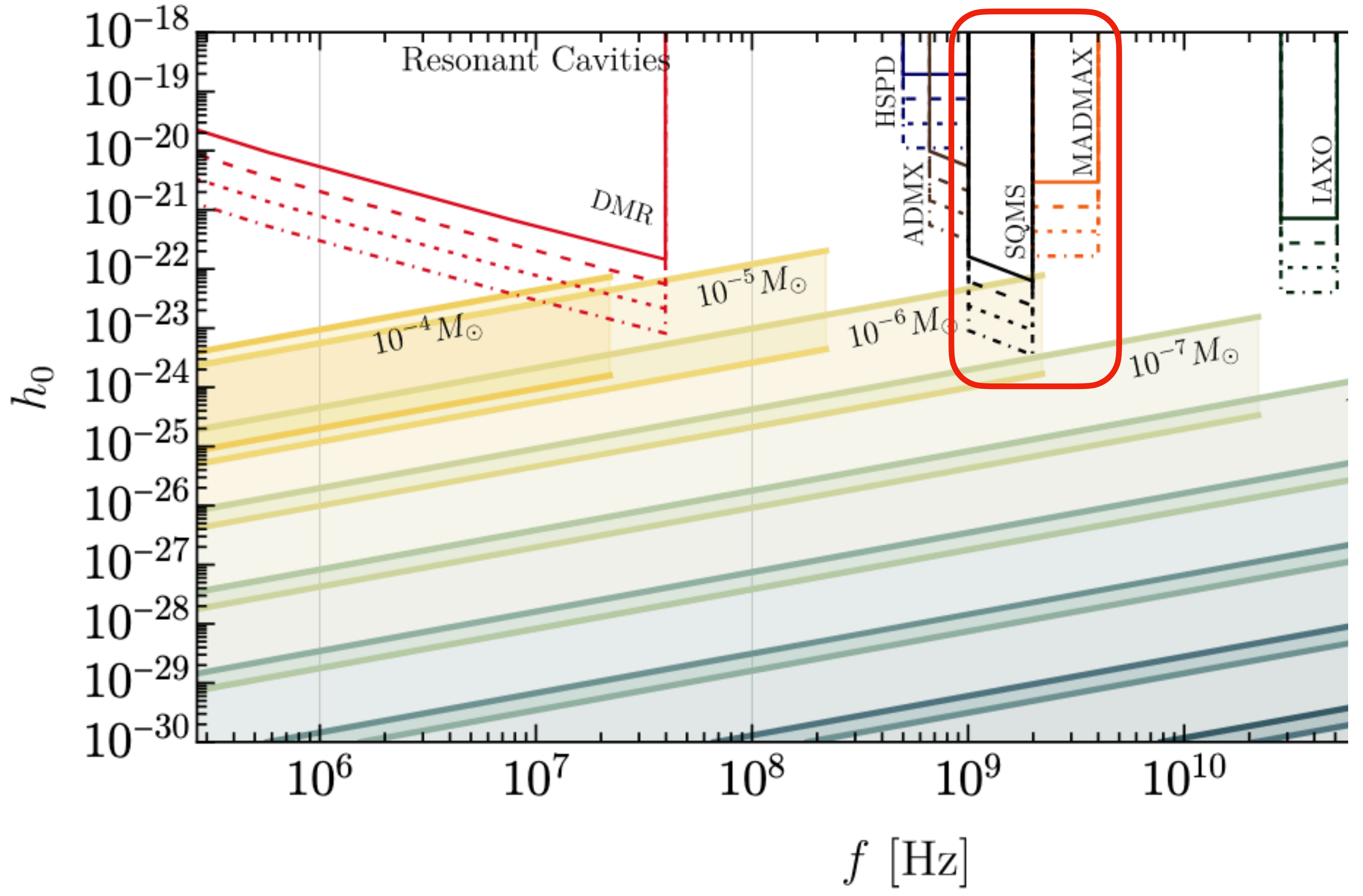
SMASH model full spectrum

Ringwald Tamarit 22

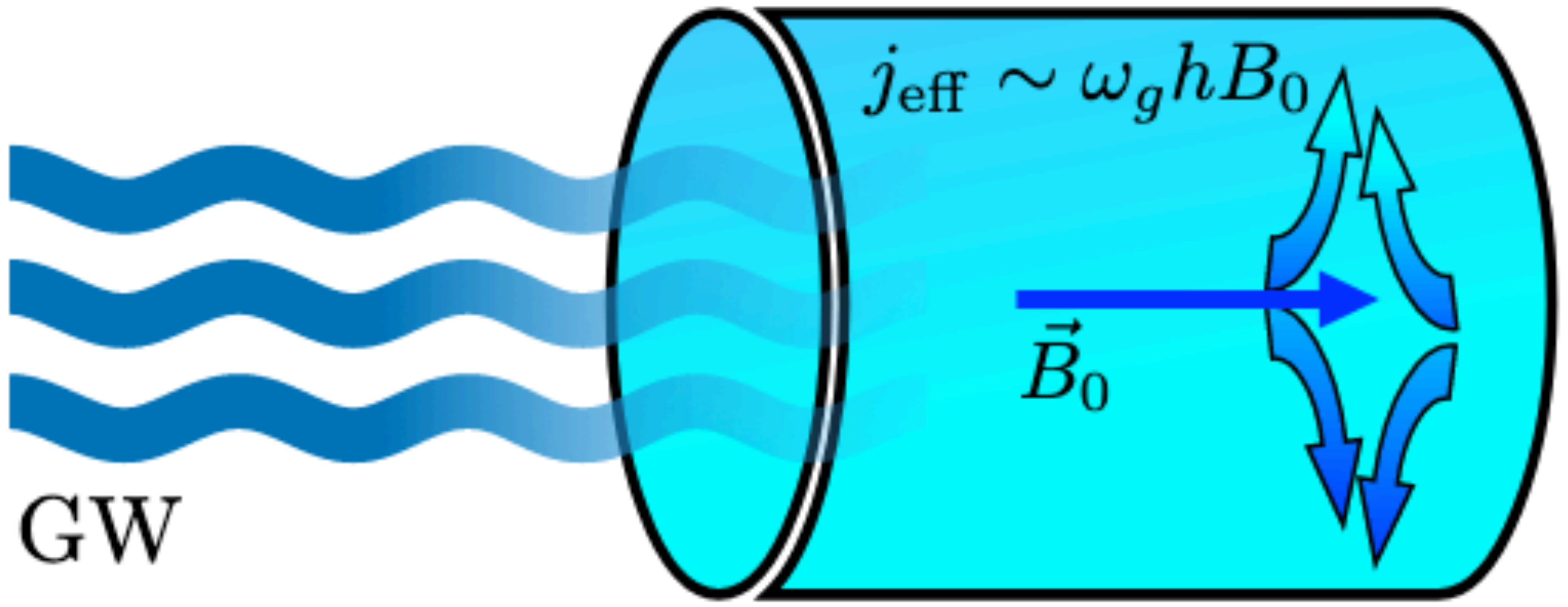


GWs from PBHs (rates 1/year)

Franciolini et al 2205.02153



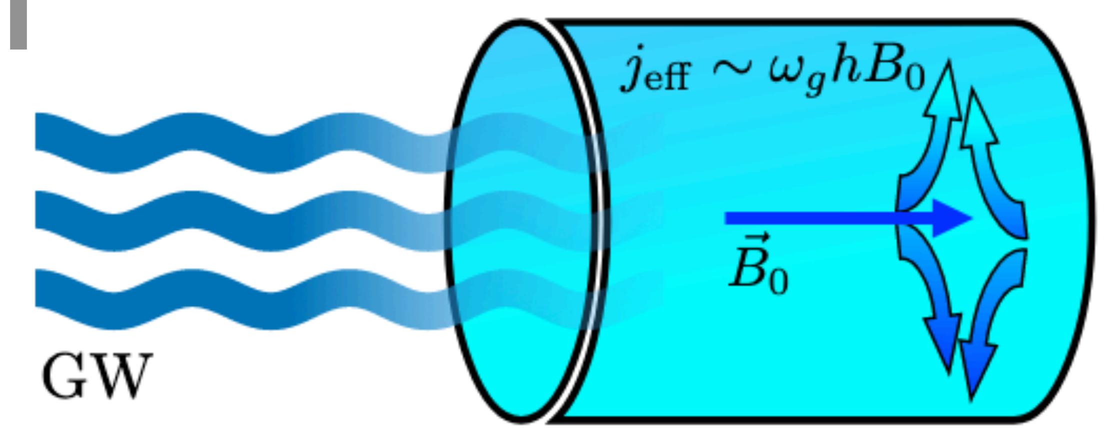
Searching for GWs with light



$$\mathcal{L} = \sqrt{-g}\,(R + F_{\mu\nu}F^{\mu\nu}) \supset \frac{1}{2}A_\mu j^\mu_{\rm eff}(h) + \eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + O(h^2)$$

$$j^\mu_{\rm eff} = -\partial_\beta\left(\frac{1}{2}hF^{\mu\beta} + h^\beta_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\beta}\right)$$

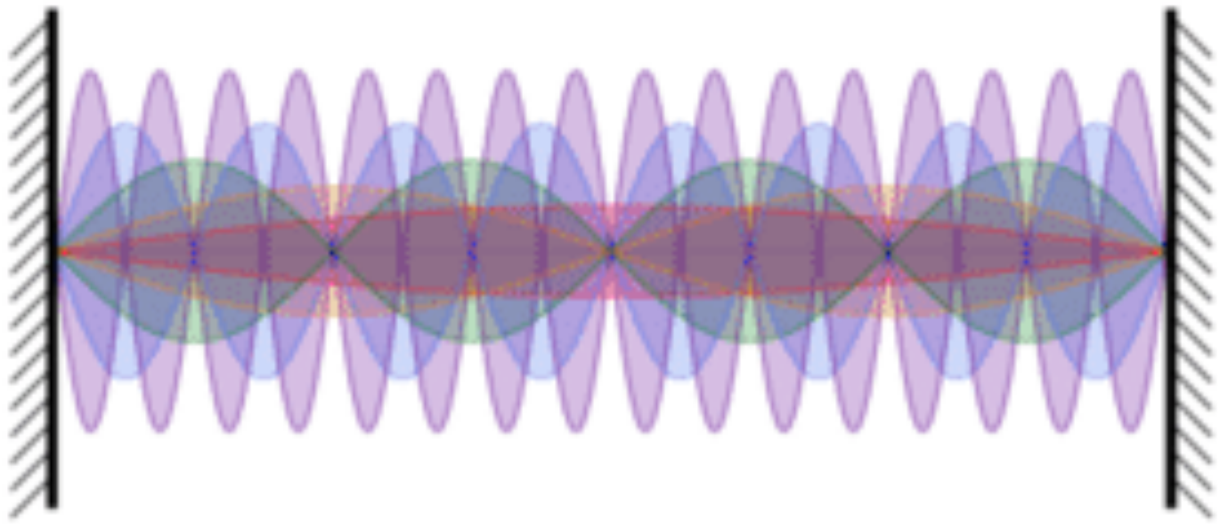
Cavity modes excitation



$$j^\mu_{\rm eff} = -\partial_\beta \left(\frac{1}{2} h F^{\mu\beta} + h^\beta_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\beta} \right)$$

$$\boldsymbol{E}(\boldsymbol{x},t) = \sum \boldsymbol{E}_{sn}(\boldsymbol{x},t) + \boldsymbol{E}_{in}(\boldsymbol{x},t)$$

solenoidal irrotational



$$\boldsymbol{E}_{sn}(\boldsymbol{x},t) = e_{sn}(t)\boldsymbol{E}_{sn}(\boldsymbol{x})$$
$$\boldsymbol{E}_{in}(\boldsymbol{x},t) = e_{in}(t)\boldsymbol{E}_{in}(\boldsymbol{x})$$

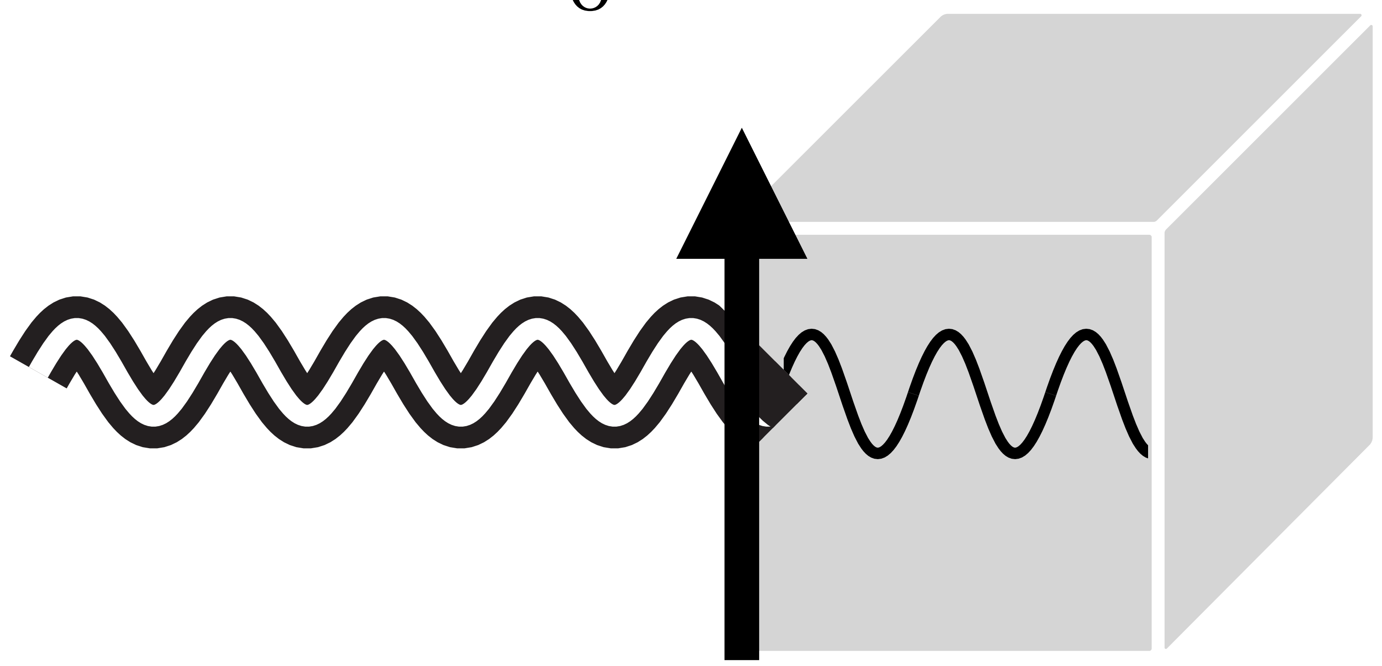
$$\left(\omega^2_{sm} + \partial^2_t + \sigma_{sm}\partial_t\right) e_{sm}(t) = e^{-i\omega_G t}\eta_{sm}$$
$$\left(\partial^2_t + \sigma_{im}\partial_t\right) e_{im}(t) = e^{-i\omega_G t}\eta_{im}$$

$$\eta \sim \int_V \mathrm{d}^3x\, E J_{eff}$$

‘source’ (here we want to maximise. It is also directional)

From axions to GWs

$P_h \sim h_0^2$



Du et al 2018

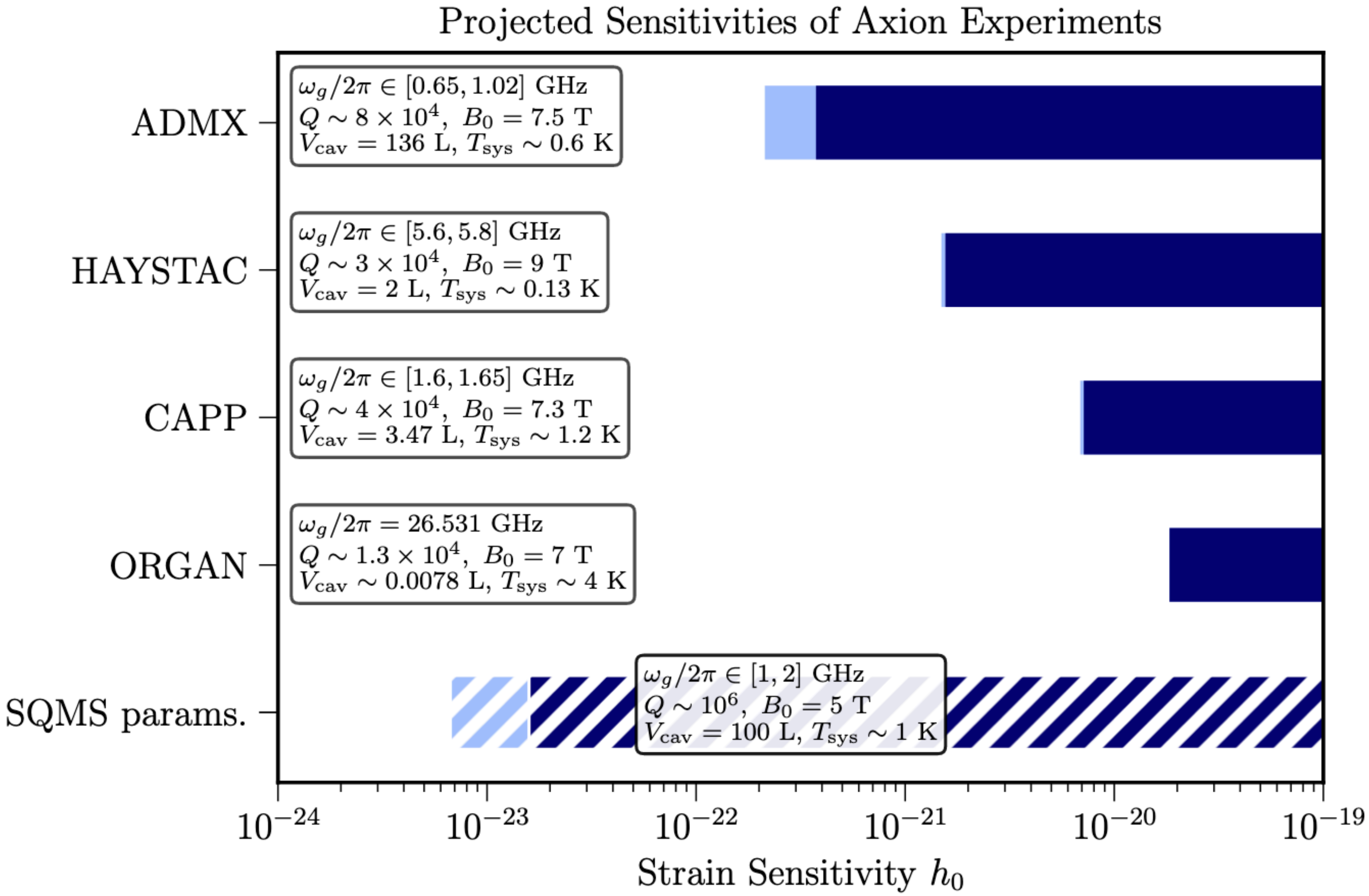
indeed: ADMX sensitivity

$$P_{\rm axion} = 1.9 \times 10^{-22}\ \mathrm{W} \left(\frac{V}{136 l}\right)\left(\frac{B}{6.8\ \mathrm{T}}\right)^2\left(\frac{C}{0.4}\right)\left(\frac{g_\gamma}{0.97}\right)^2\left(\frac{\rho_{\rm a}}{0.45 \mathrm{GeVcm}^{-3}}\right)\left(\frac{f}{650\mathrm{MHz}}\right)\left(\frac{Q}{50,000}\right)$$

quadratic in axion field and in *h*

$P_h \sim h_0^2$

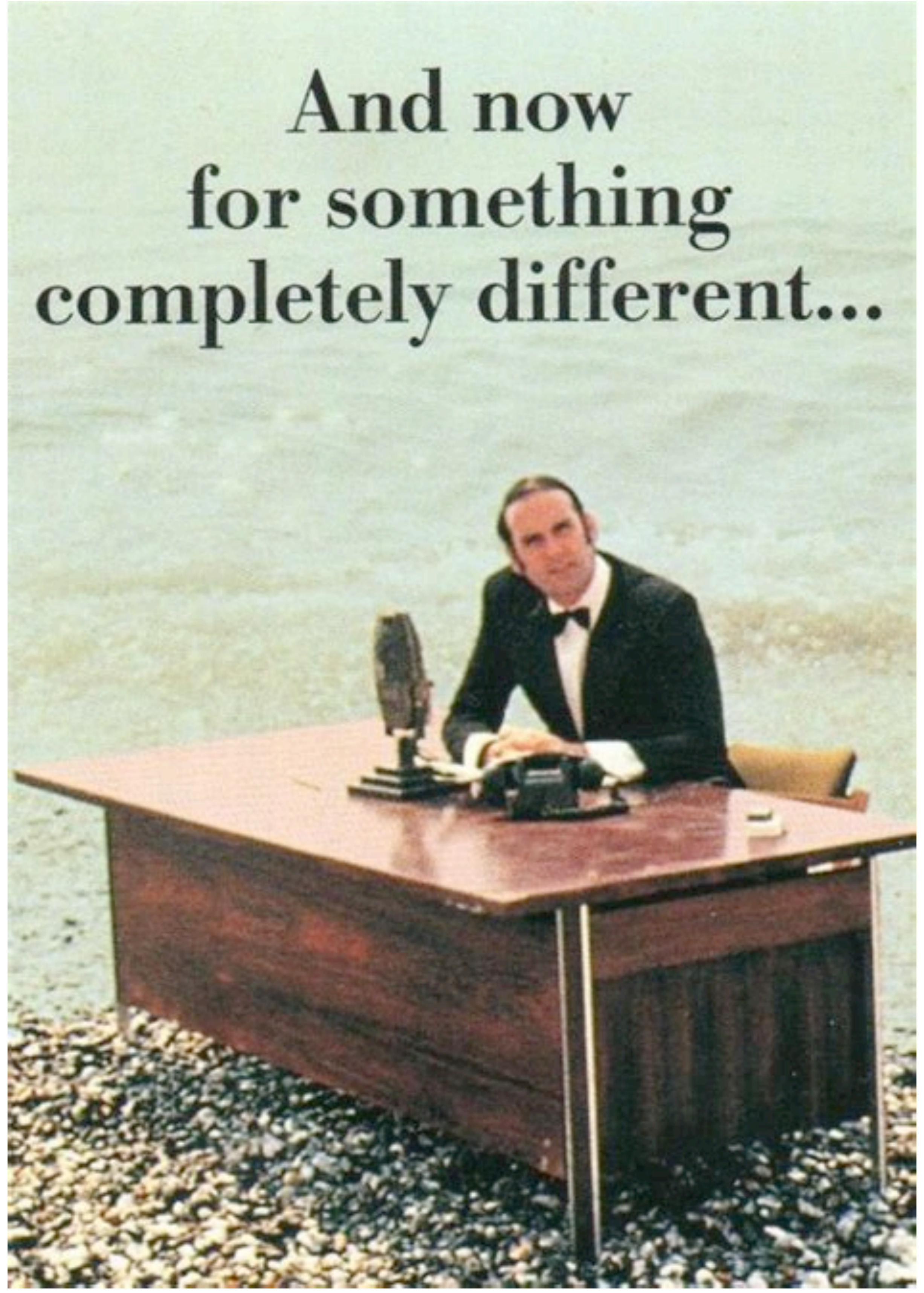
$$h_0 \gtrsim 3 \times 10^{-22} \times \left(\frac{1\ \mathrm{GHz}}{\omega_g/2\pi}\right)^{3/2}\left(\frac{0.1}{\eta_n}\right)\left(\frac{8\ \mathrm{T}}{B_0}\right)\left(\frac{0.1\ \mathrm{m}^3}{V_{\rm cav}}\right)^{5/6}\left(\frac{10^5}{Q}\right)^{1/2}\left(\frac{T_{\rm sys}}{1\ \mathrm{K}}\right)^{1/2}\left(\frac{\Delta\nu}{10\ \mathrm{kHz}}\right)^{1/4}\left(\frac{1\ \mathrm{min}}{t_{\rm int}}\right)^{1/4}$$


Projected Sensitivities of Axion Experiments
ADMX
$\omega_g/2\pi \in [0.65, 1.02]$ GHz
$Q \sim 8 \times 10^4,\ B_0 = 7.5$ T
$V_{\rm cav} = 136$ L, $T_{\rm sys} \sim 0.6$ K
HAYSTAC
$\omega_g/2\pi \in [5.6, 5.8]$ GHz
$Q \sim 3 \times 10^4,\ B_0 = 9$ T
$V_{\rm cav} = 2$ L, $T_{\rm sys} \sim 0.13$ K
CAPP
$\omega_g/2\pi \in [1.6, 1.65]$ GHz
$Q \sim 4 \times 10^4,\ B_0 = 7.3$ T
$V_{\rm cav} = 3.47$ L, $T_{\rm sys} \sim 1.2$ K
ORGAN
$\omega_g/2\pi = 26.531$ GHz
$Q \sim 1.3 \times 10^4,\ B_0 = 7$ T
$V_{\rm cav} \sim 0.0078$ L, $T_{\rm sys} \sim 4$ K
SQMS params.
$\omega_g/2\pi \in [1, 2]$ GHz
$Q \sim 10^6,\ B_0 = 5$ T
$V_{\rm cav} = 100$ L, $T_{\rm sys} \sim 1$ K
10^{-24}
10^{-23}
10^{-22}
10^{-21}
10^{-20}
10^{-19}
Strain Sensitivity h_0

A very exploratory field…

we still need to **deeply think what's better** for the future

do we move to something else ?



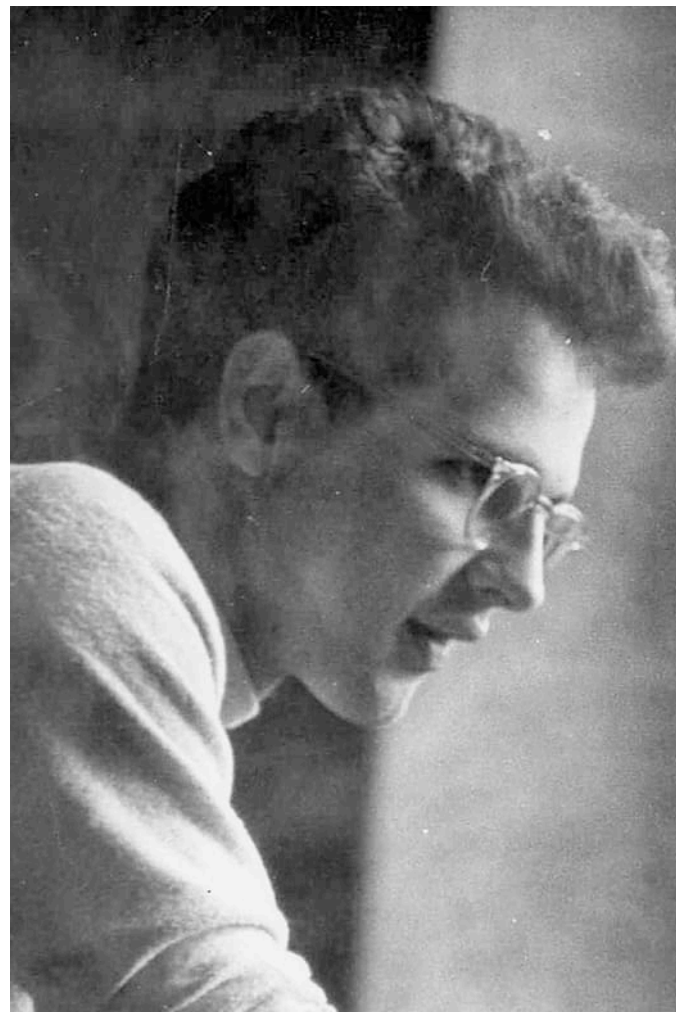
LIGO lesson

A very exploratory field…

we still need to **deeply think what's better** for the future

do we move to something else ?

Rainer Weiss, ca. 1972



LIGO concept
Weiss 1972

(V. GRAVITATION RESEARCH)

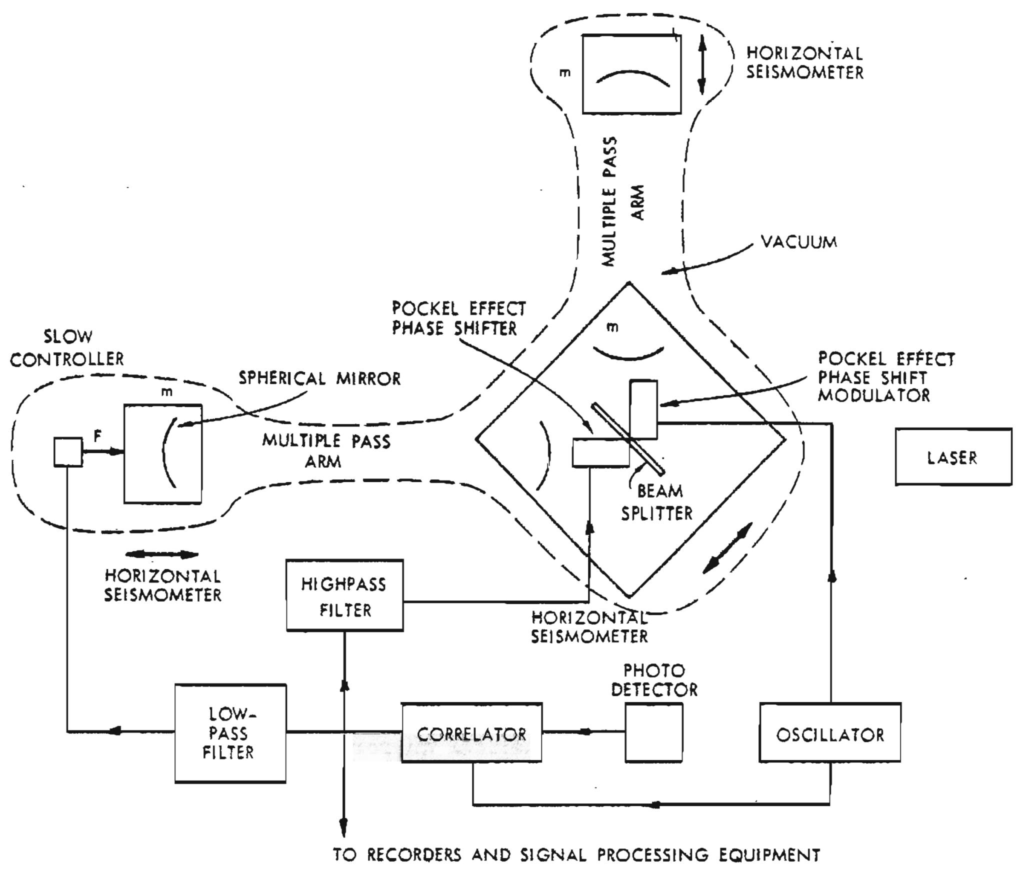


Fig. V-20 Proposed antenna.

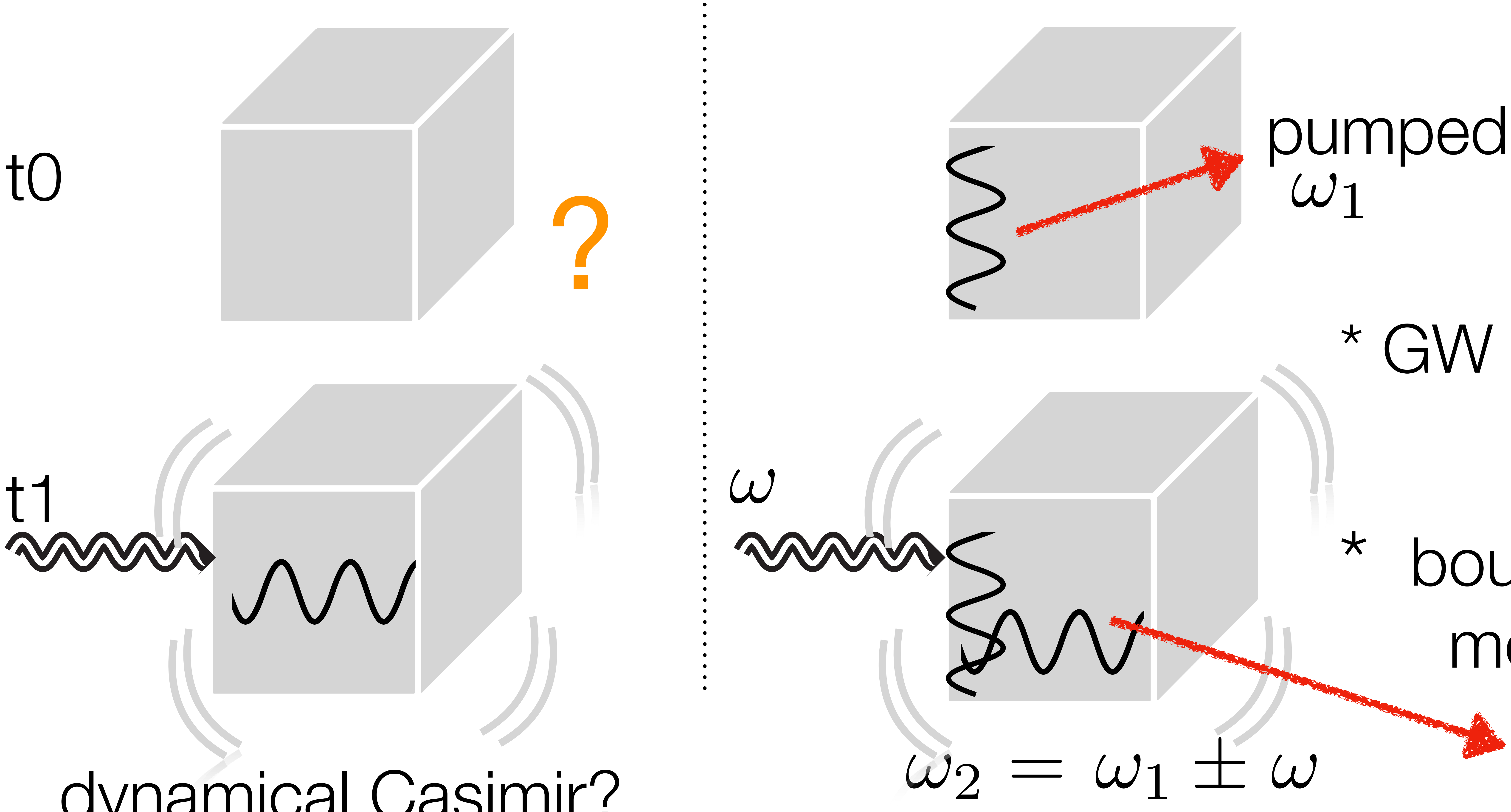
For continuous gravitational waves, the minimum detectable gravitational wave metric spectral density is then

$$h^2(f) > \frac{4}{f^2}\,\frac{\Delta x_n^2(f)}{\Delta f} \approx \frac{4\times 10^{-33}}{f^2(\mathrm{cm})} \quad \mathrm{Hz}^{-1}.$$

Recycling axion experiments II

Mechanical-coupling
(shaking the walls)

MAGO design from CERN (gr-qc/0502054)
Berlin, DB et al 2303.01518



pumped mode

* GW +EM->EM

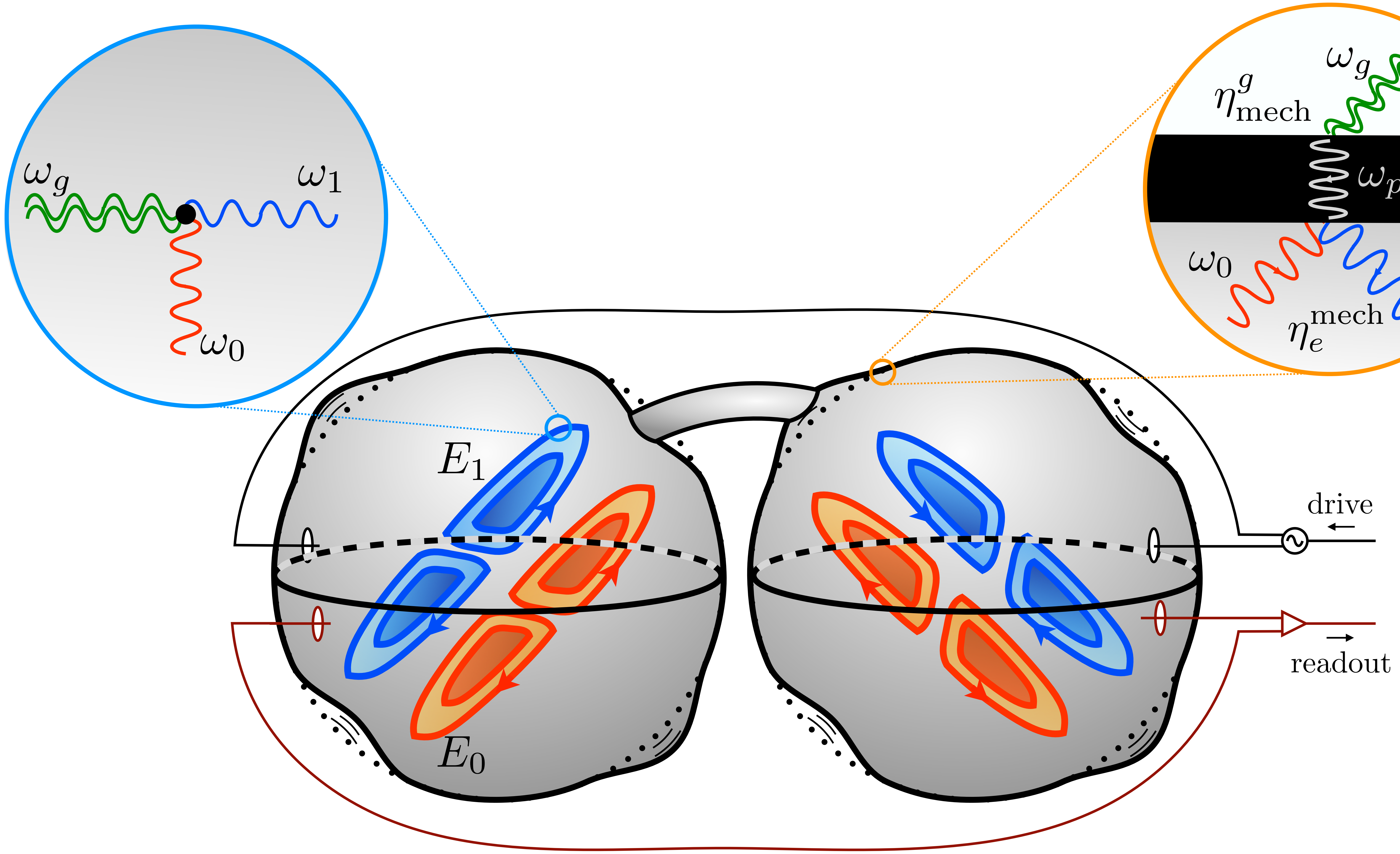
* boundaries move: mode mixing

signal mode

dynamical Casimir?

MAGO set-up

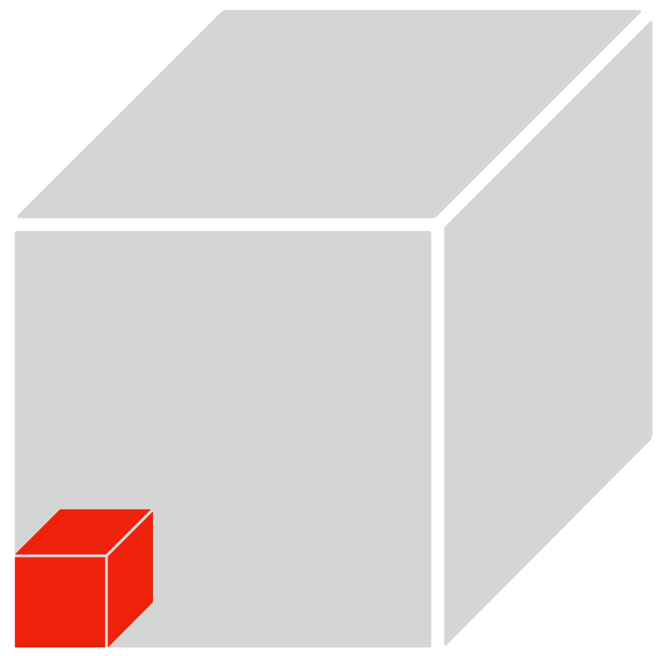
(Microwave Apparatus for Gravitational Waves Observation)



GWs exciting solids

searched for many years (Weber bars)

a solid affected by a external source (e.g. *x* direction)



$dm(x + u(x,t))$

$$dm\left(\frac{\partial^2 u}{\partial t^2} - v_s^2 \frac{\partial^2 u}{\partial x^2}\right) = dF_x(t,x),$$

$$dF_i = \frac{1}{2}\ddot{h}_{ij}^{TT} x^j dm$$

In terms of eigenmodes:

$$\mathbf{u}(\mathbf{x},t) = u_p(t)\mathbf{u}_p(\mathbf{x})$$

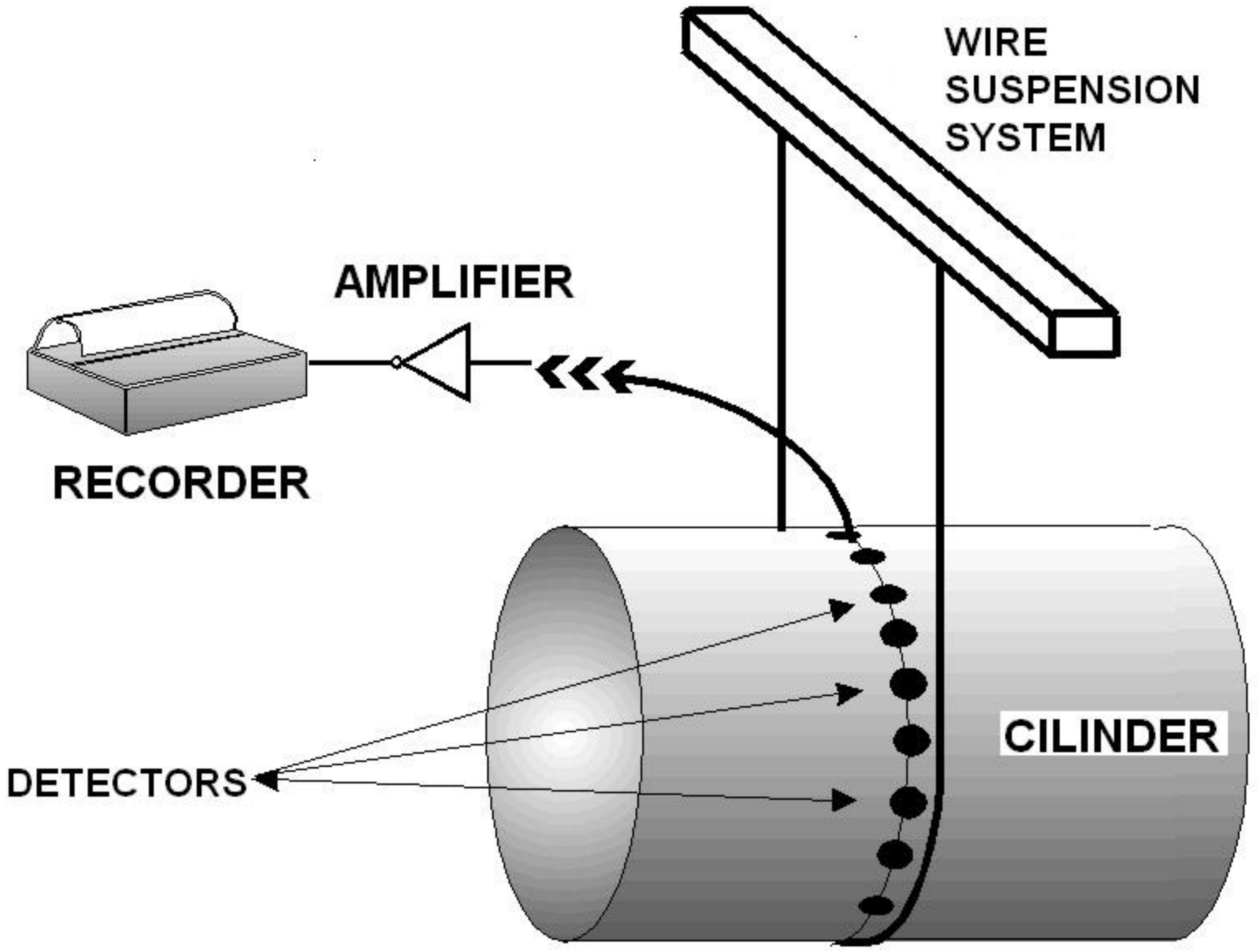
GWs exciting solids

$$\mathbf{u}(\mathbf{x},t) = u_p(t)\mathbf{u}_p(\mathbf{x})$$

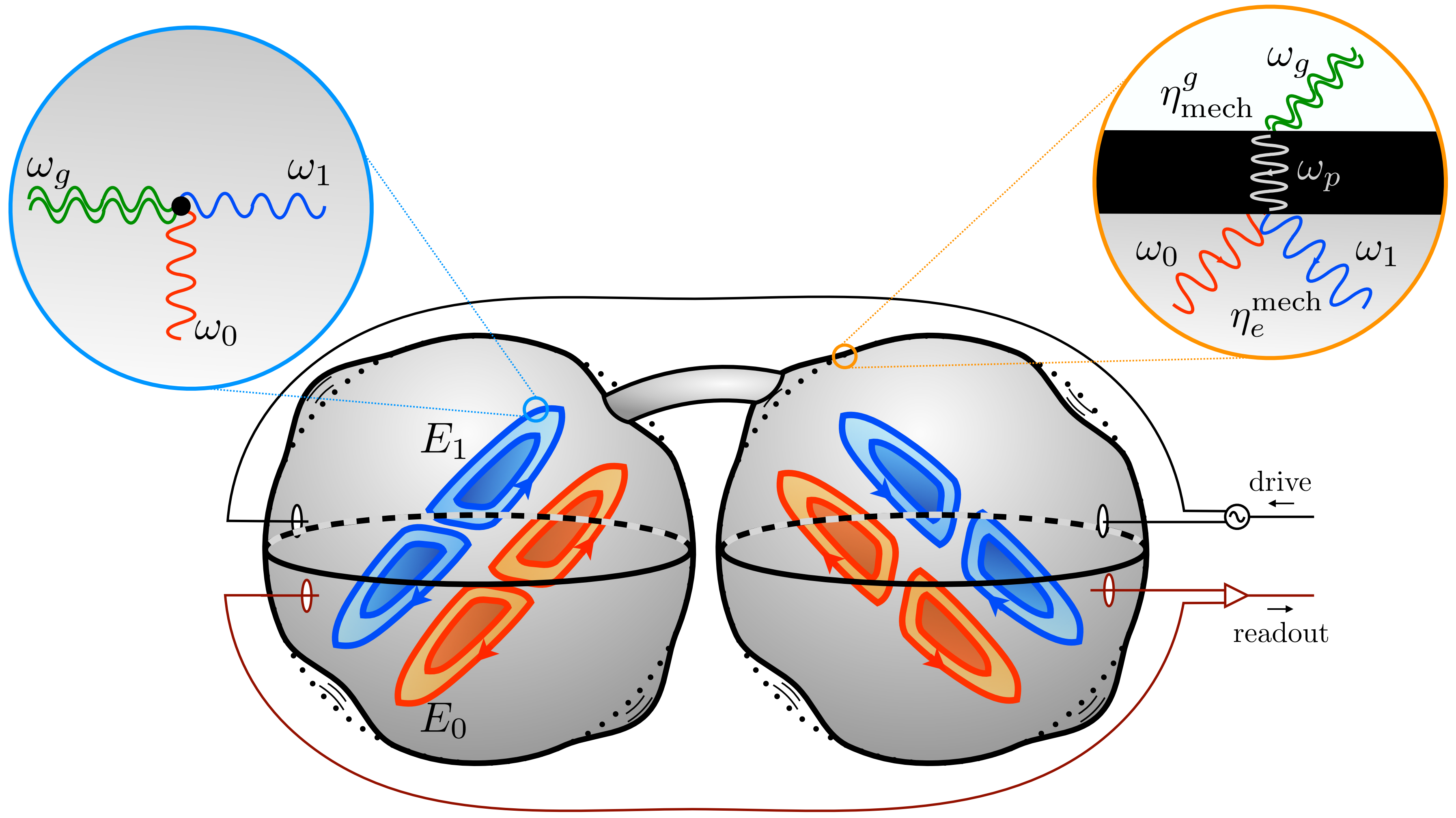
$$\ddot{u}_p + \frac{\omega_p}{Q_p}\dot{u}_p + \omega_p^2 u_p \simeq -\frac{1}{2}\omega_g^2 V_{\rm cav}^{1/3}\eta^g_{\rm mech}\; h_0 e^{i\omega_g t}$$

$$\eta^g_{\rm mech} = \frac{\hat{h}^{\rm TT}_{ij}}{V_{\rm cav}^{1/3}V_{\rm shell}}\int_{V_{\rm shell}} d^3\mathbf{x}U_p^{*i}x^j$$

this rings the solid (Weber bars)



MAGO set-up: read-out from mode mixing



$$\boldsymbol{E}_{sn}(\boldsymbol{x},t) = e_{sn}(t)\boldsymbol{E}_{sn}(\boldsymbol{x})$$

$$\left(\partial_t^2 + \omega_n^2\right) e_n \simeq -\omega_n e_m \frac{\int_{\Delta V} d^3\mathbf{x}\,(\omega_n \boldsymbol{E}_m \cdot \boldsymbol{E}_n^* - \omega_m \boldsymbol{B}_m \cdot \boldsymbol{B}_n^*)}{\int_{V_0} d^3\mathbf{x}\,|\boldsymbol{E}_n|^2}$$

mixing

modes are not orthogonal in ΔV

$$P_{\rm sig} \propto h^2_{+,\times} \left|\eta_u^{(\rm mech.\ )}\right|^2 \left|\tilde{\eta}_e^{(\rm mech.\ )}\right|^2 P_{in} \times \begin{cases} \frac{\omega_h^4}{\left(\omega_h^2 - \omega_m^2\right)^2} & (|\omega_h - \omega_m| \gg \omega_m/Q_m) \\ Q_m^2 & (|\omega_h - \omega_m| \ll \omega_m/Q_m) \end{cases}$$

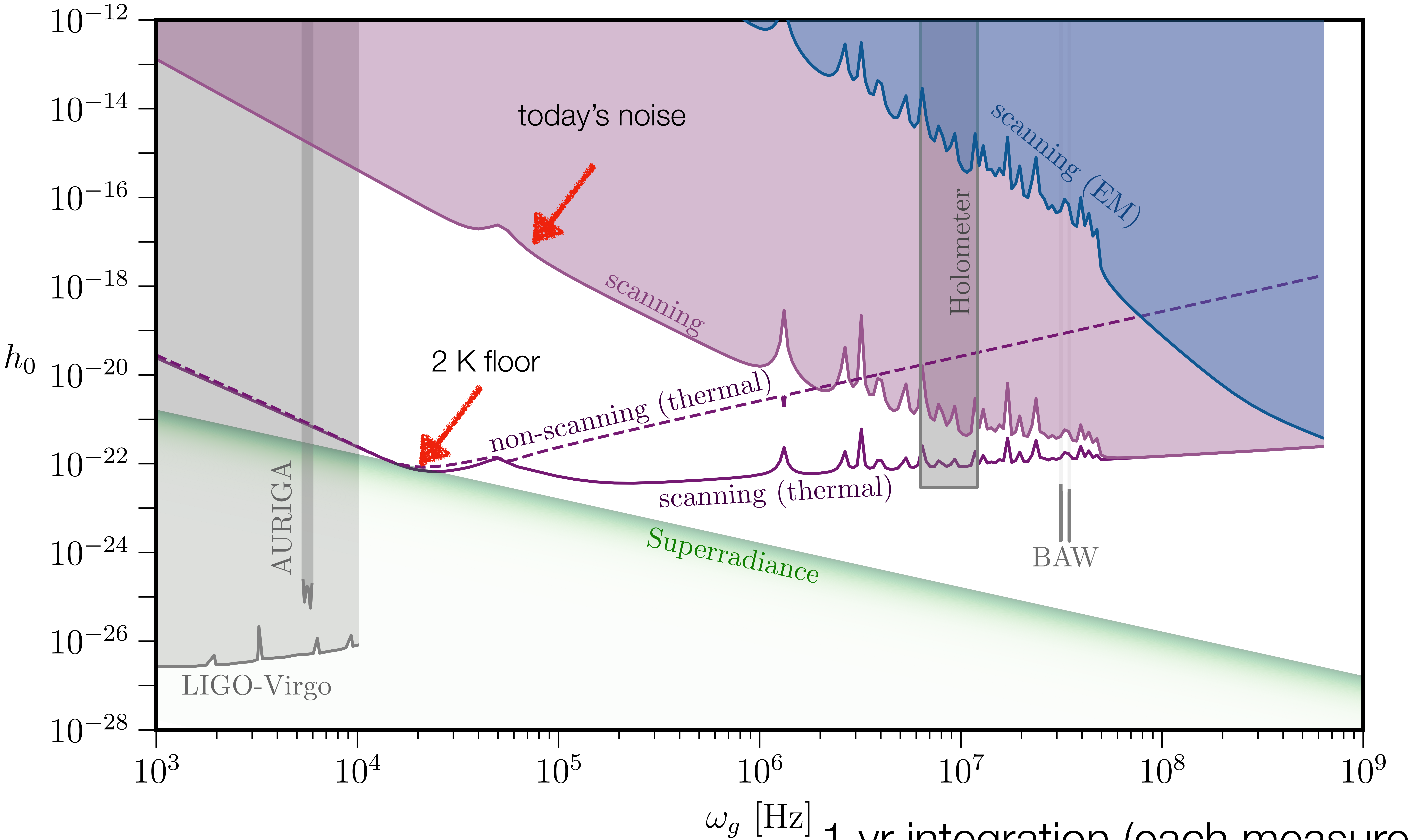
off-resonance

on-resonance

$Q_m \sim 10^6$

Estimates

A. Berlin, DB, R.T. D'Agnolo, S. Ellis, R. Harnik,
Y. Kahn, J. Schütte-Engel, M.Wentze 2303.01518



1 yr integration (each measurement ms)!

A word on sources GW coupling to spin

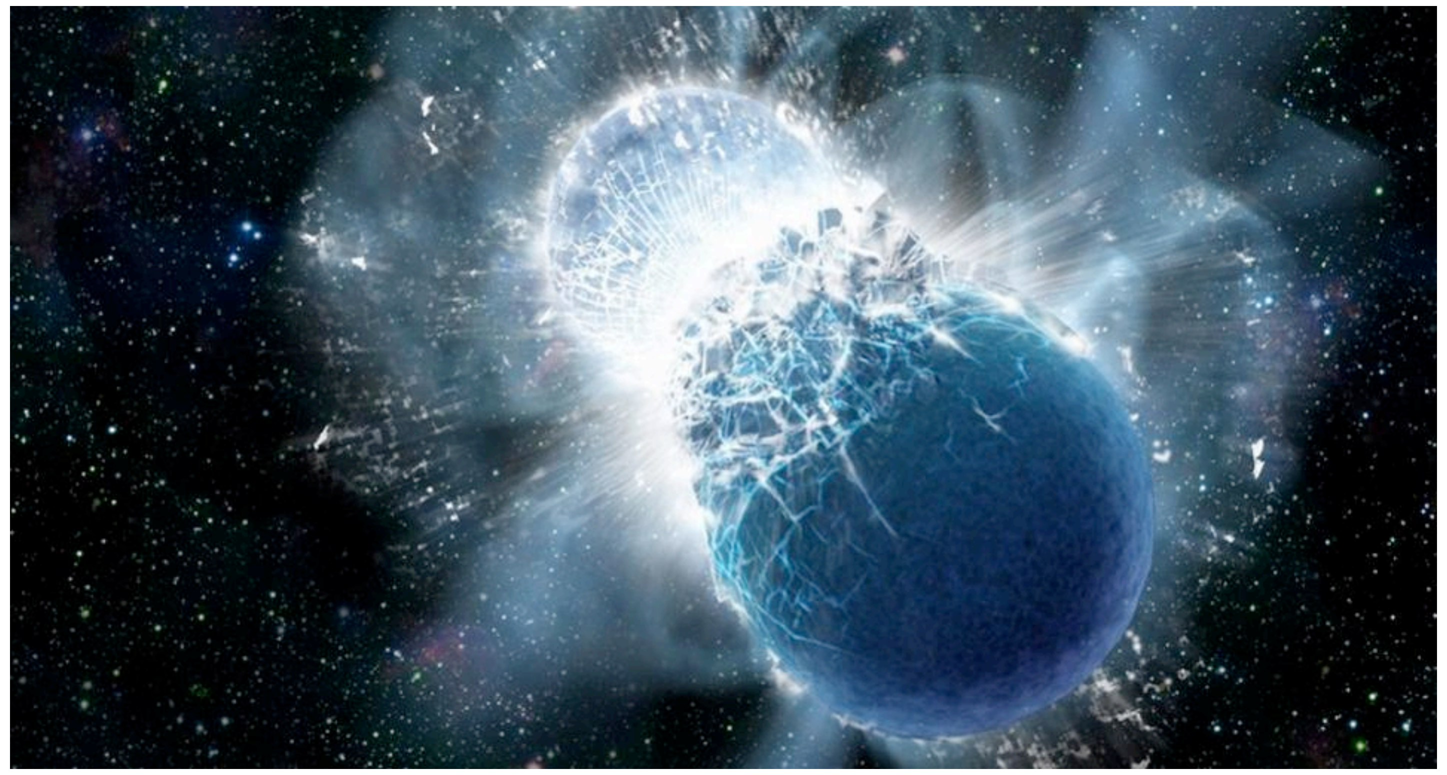
e.g. Ito and Soda 20

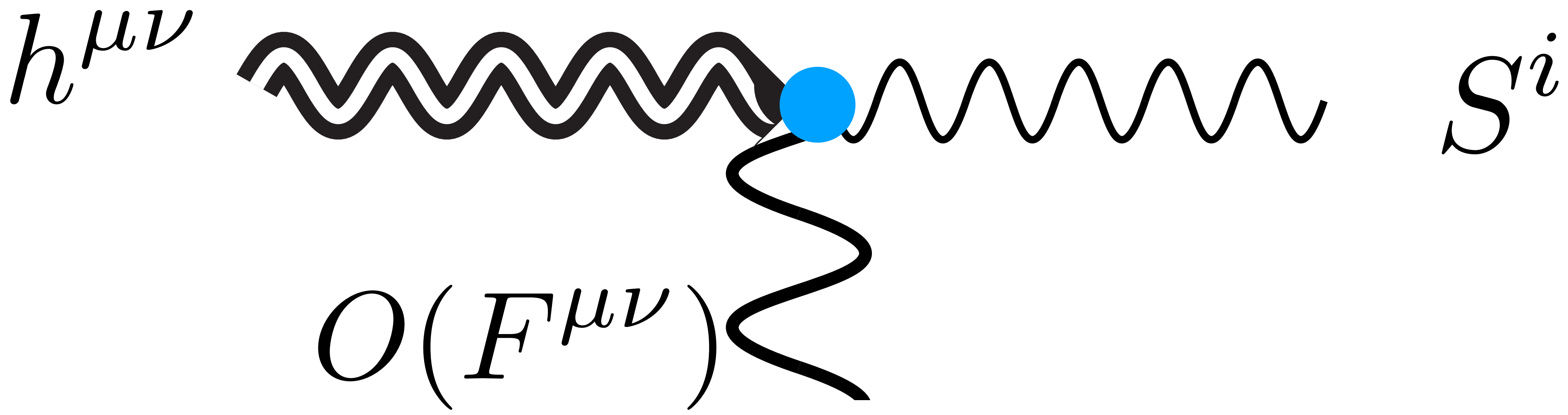
Dirac equation in GR $i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu - \Gamma_\mu - ieA_\mu\right)\psi = m\psi$,

NR limit

$$H_{\rm tot} = -\mu_B \left(2\delta_{za} + Q_{za}\right) B_z \sum_i \hat{S}^a_{(i)} - \sum_{i,j} J_{ij} \hat{\boldsymbol{S}}_{(i)} \cdot \hat{\boldsymbol{S}}_{(j)}$$

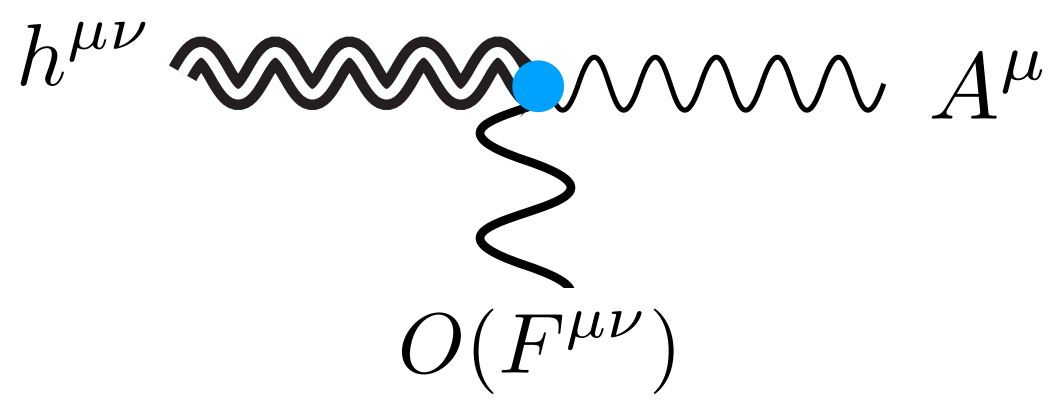
Linear in GW $Q_{ij} \supset -\frac{2}{3}\delta_{ij}\ddot{h}_{kl}\Big|_{\boldsymbol{x}=0} x^k x^l$





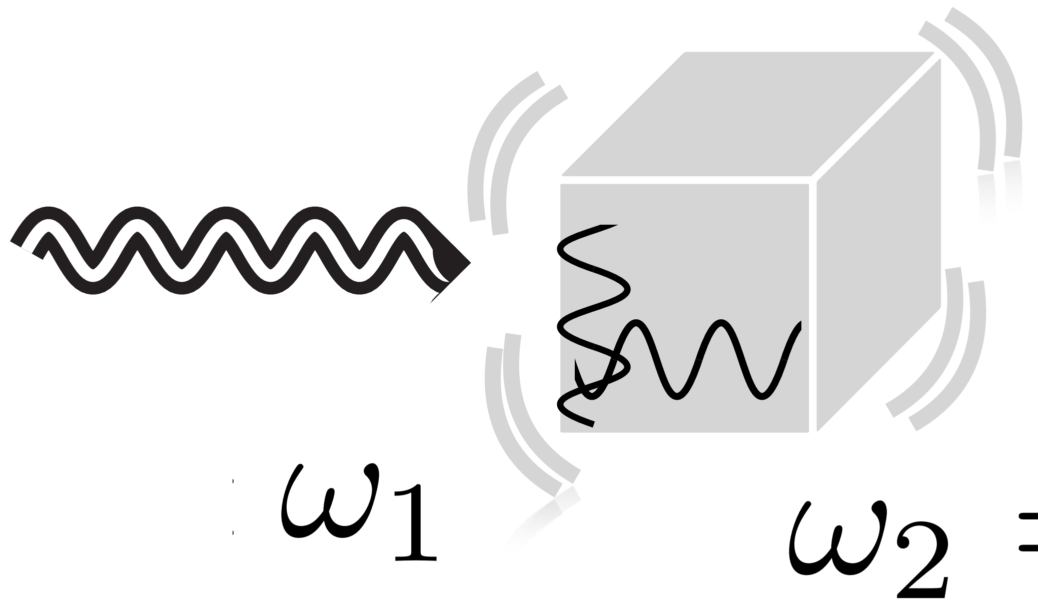
Summary and outlook

- SRF cavities are a mature technology to look for GWs at GHz either
    - Coupling to photons in a cavity



- 'ADMX' like $\omega = \omega_g$
- Heterodyne $\omega_2 = \omega_g \pm \omega_1$

    - Ringing the cavity and generating mode mixing



$$\omega_2 = \omega \pm \omega_1$$

- Heterodyne
- Casimir?

✳ Reach of $h_0 \sim 10^{-23}$ possible (100 kHz-GHz), though far from known signals

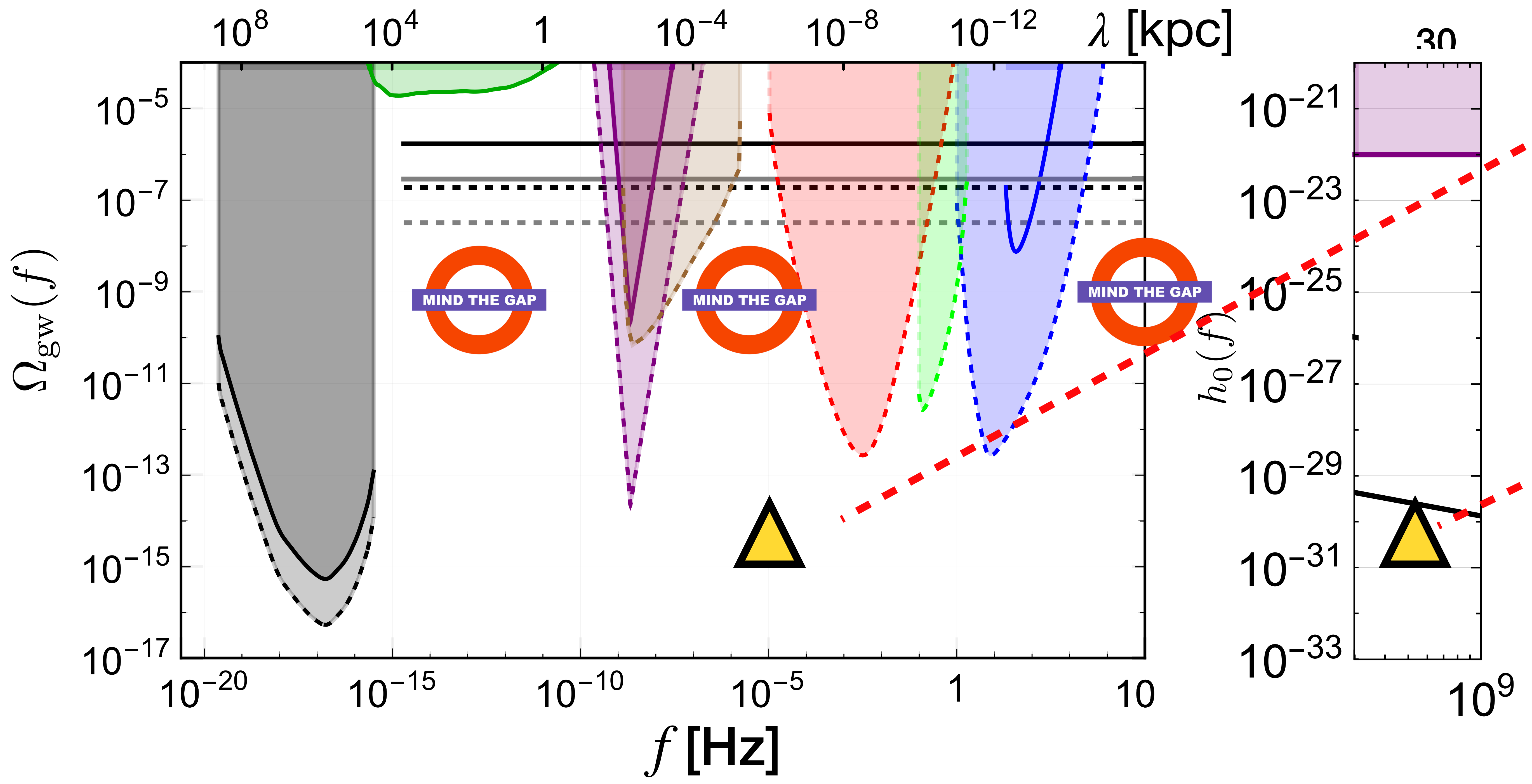
Conclusions

Multiband approach to GWs: great potential to transform both astrophysics and BSM

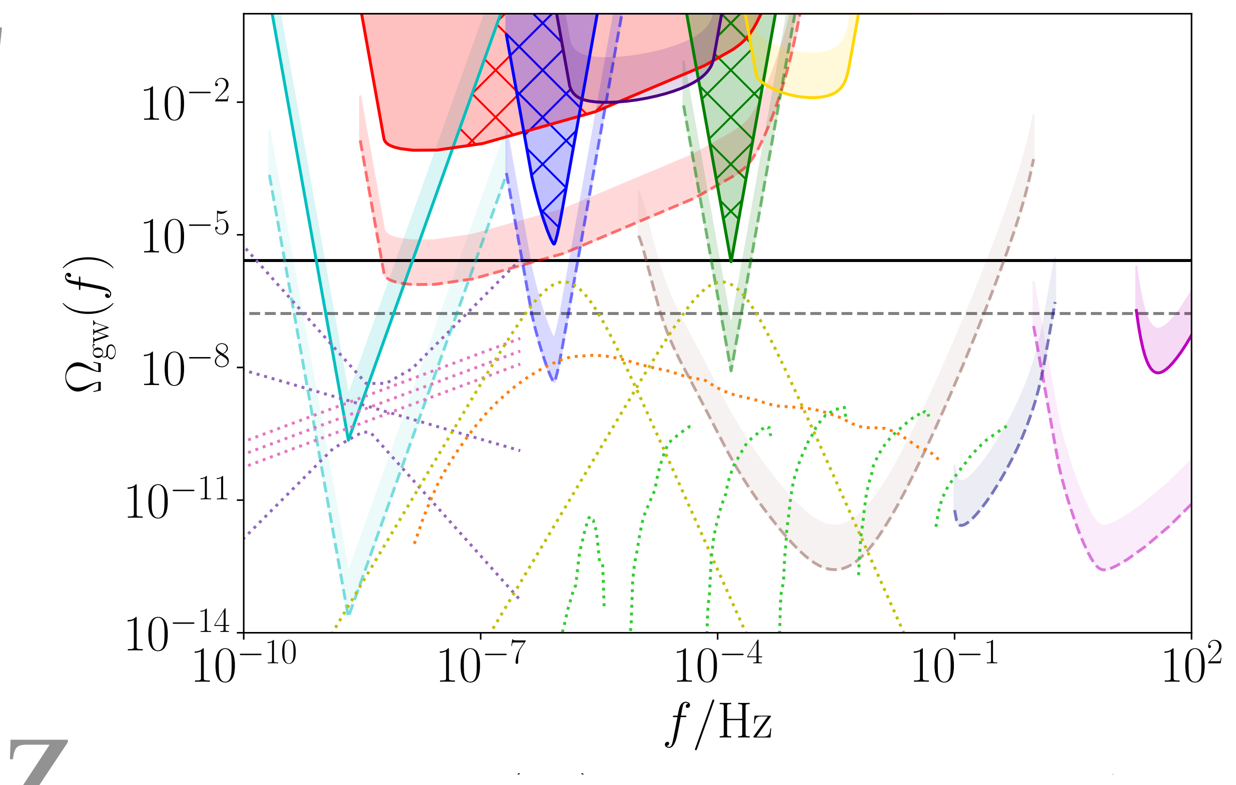
For this:

i) there is vigours effort to in four regions (CMB/PTA/LISA/LVK)

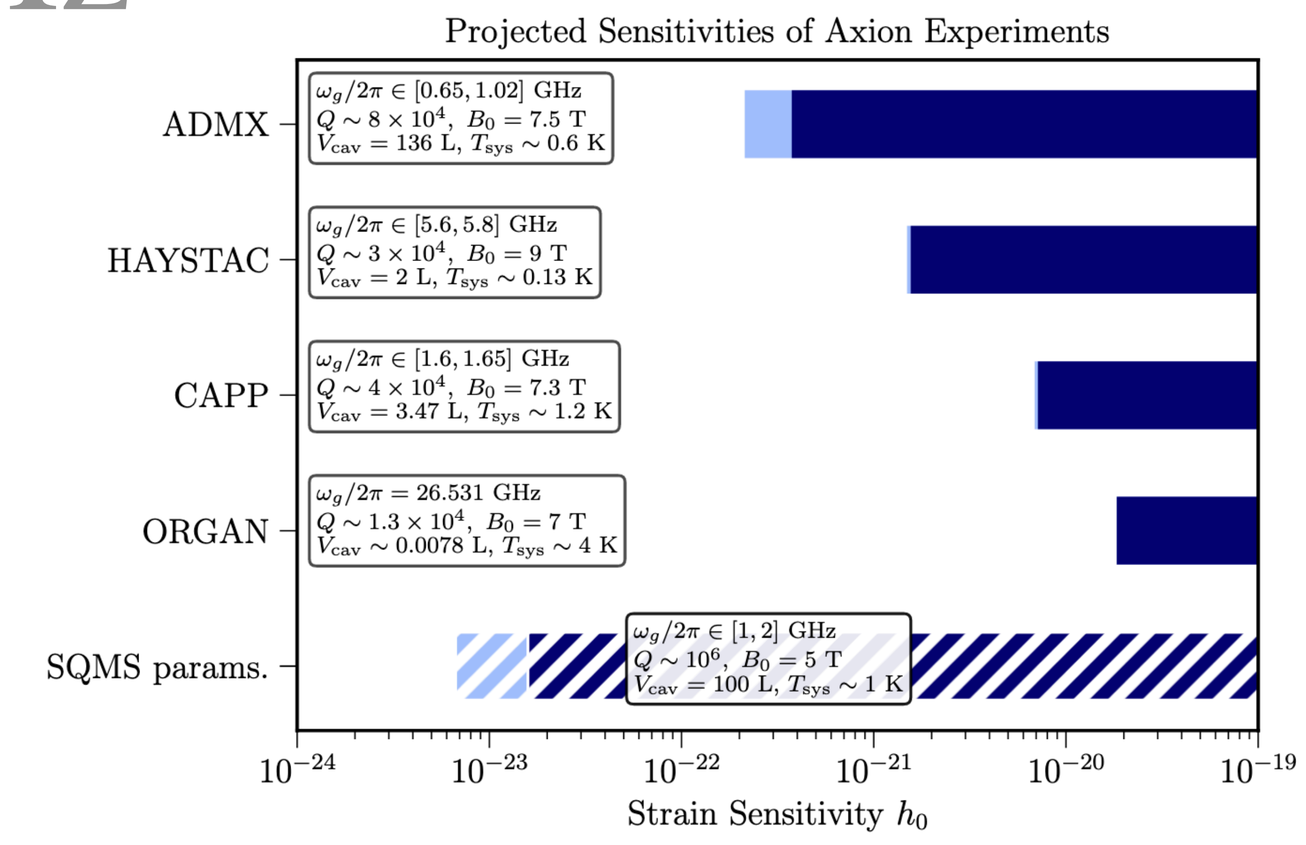
ii) new ideas are needed in other phenomenologically rich regions



μHz



GHz



Multiband

Over the rainbow