Black hole spectroscopy: stability, censorship and Love

Marc Casals

Universität Leipzig (Germany)

University College Dublin (Ireland)

Centro Brasileiro de Pesquisas Físicas, CBPF (Brazil)

Niels Bohr International Academy, 7/2/2023

Outline

1. BH Perturbations

2. Stability of Kerr

3. Stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Tidal gravitational deformation (Love)

5. Conclusion

1. BH Perturbations

2. Stability of Kerr

3. Stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Tidal gravitational deformation (Love)

5. Conclusion

Kerr Black Holes

Astrophysical BHs are believed to be described by the Kerr metric:

It's an *exact* sln. of Einstein's eqs. representing a rotating BH (M, a)

mass

(intrinsic) angular momentum

It has:

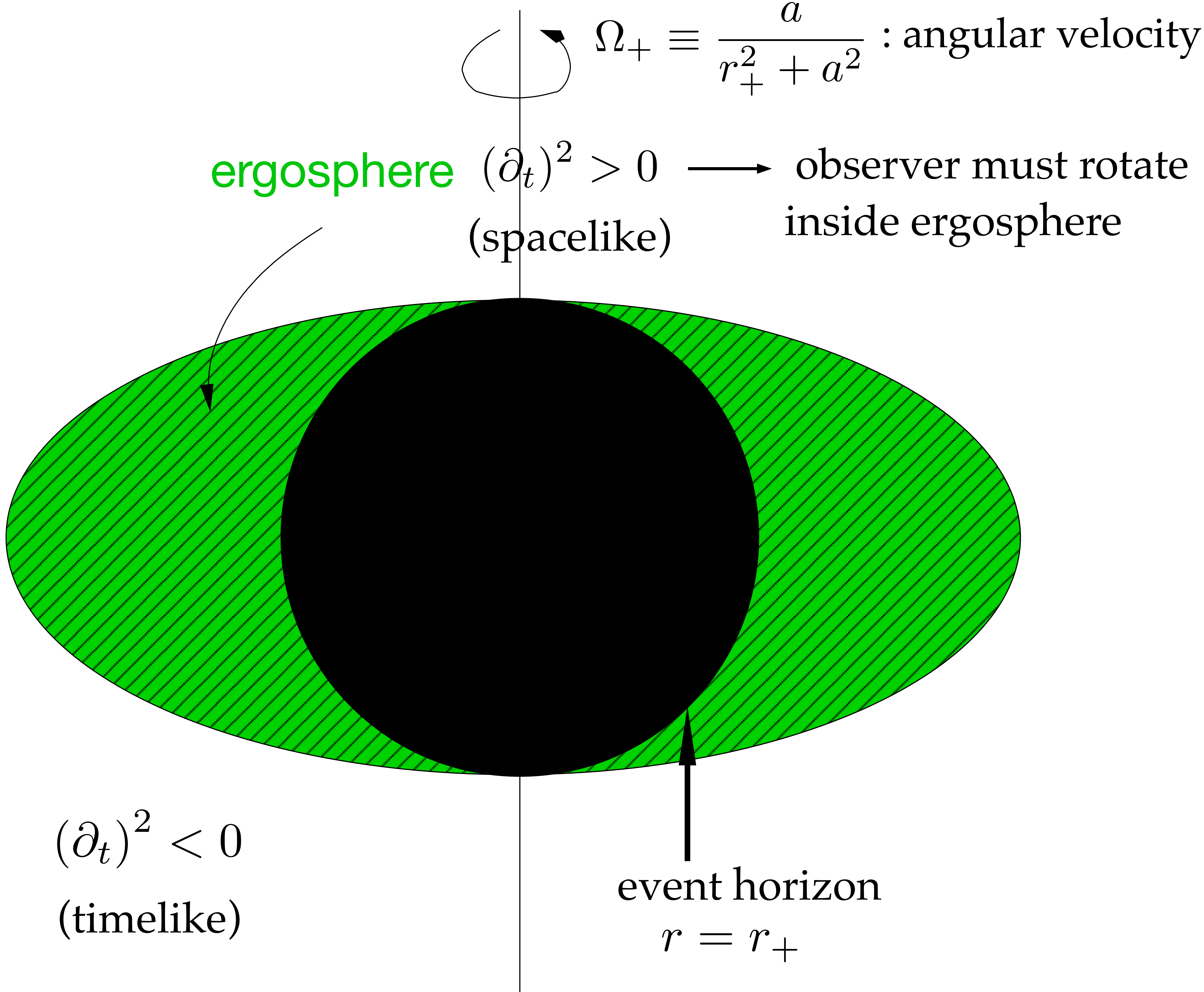
- an event horizon at radius $r = r_+ \equiv M + \sqrt{M^2 - a^2}$

maximally-rotating (extremal) is for $a = M$

- an *inner* (Cauchy horizon) at $r = r_- \equiv M - \sqrt{M^2 - a^2} \in [0, r_+]$

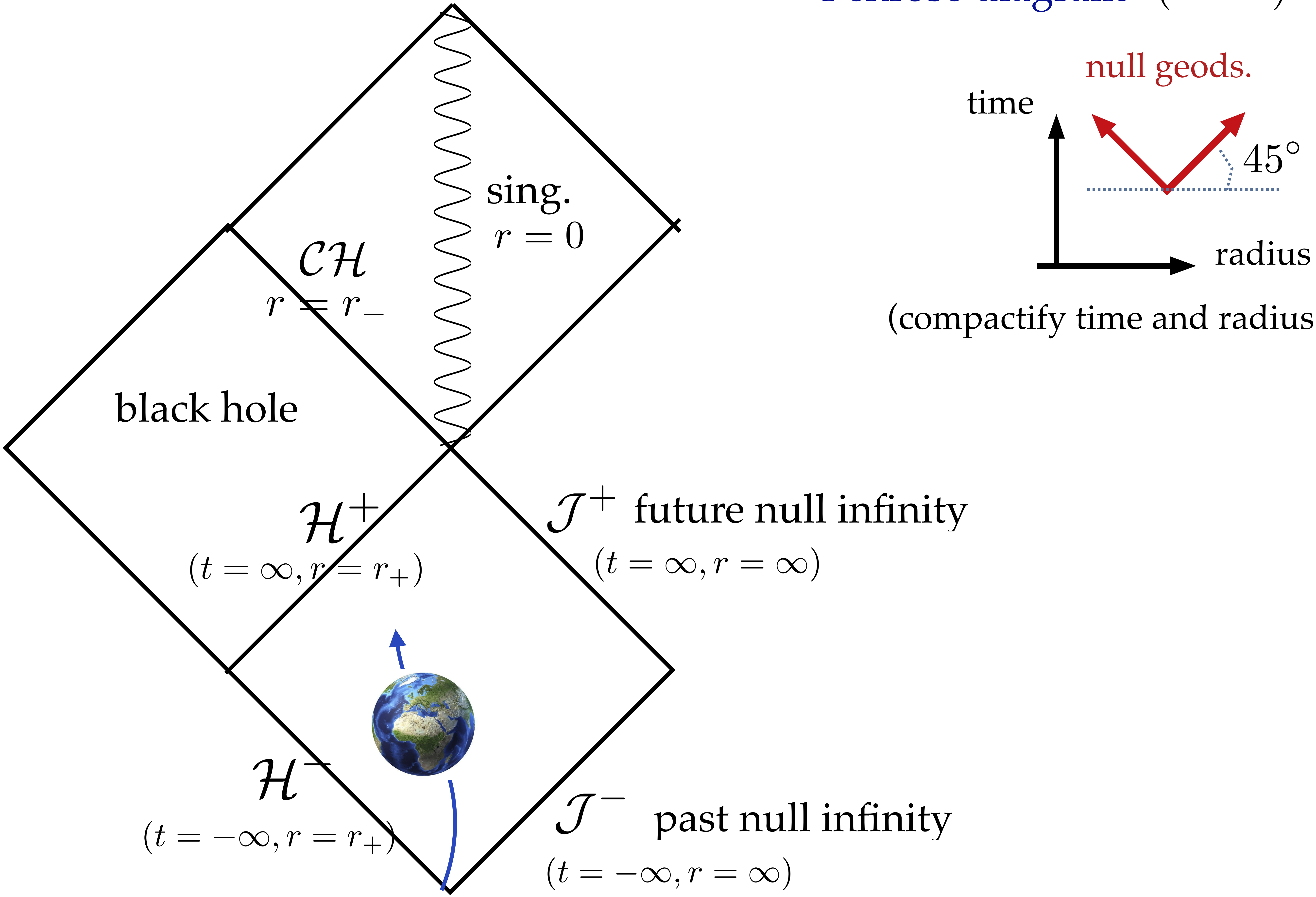
- a curvature singularity at $r = 0$

- two symmetries: stationarity (∂_t) and axi-symmetry (∂_φ)



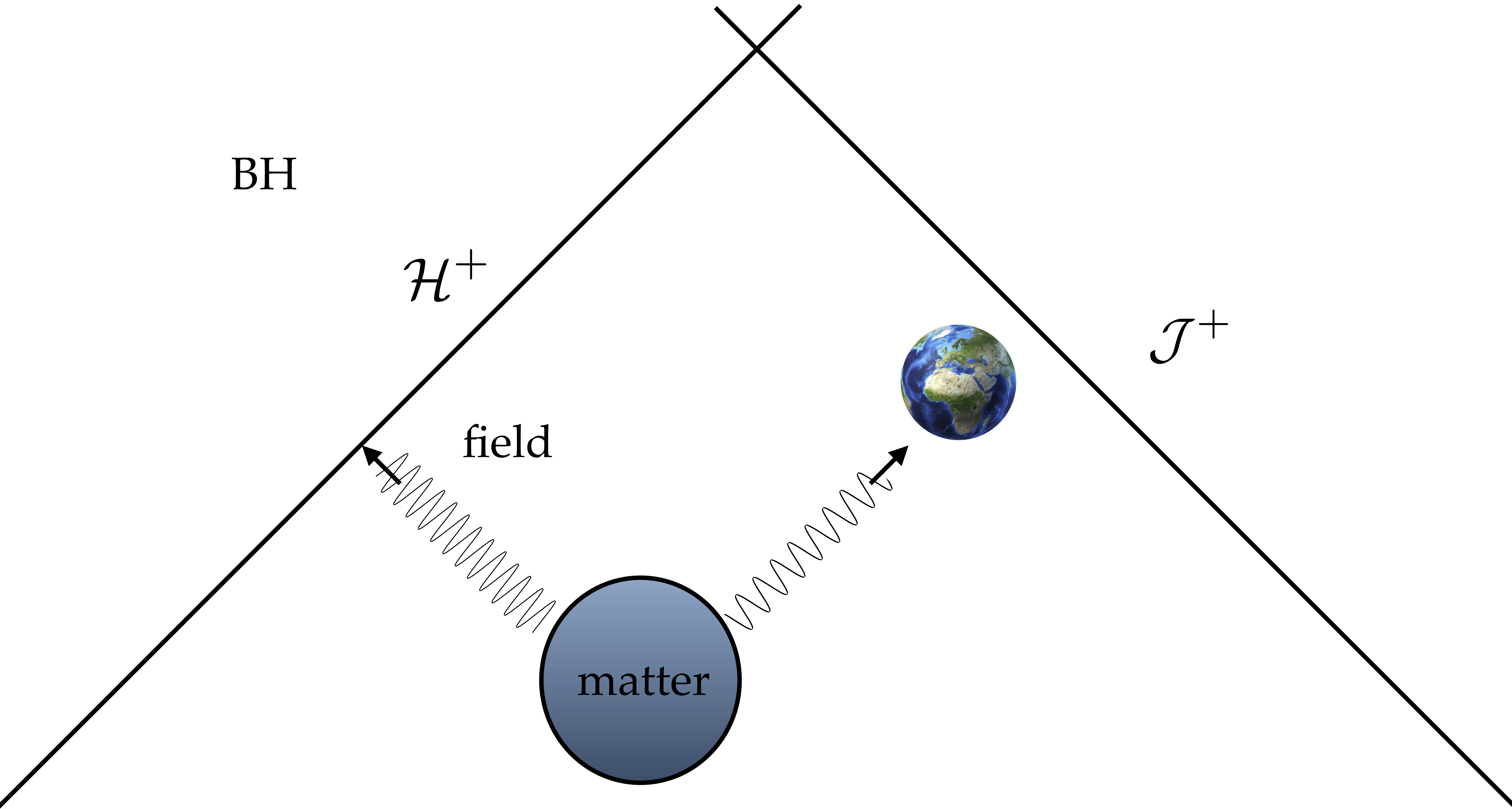
$\Omega_+ \equiv \dfrac{a}{r_+^2 + a^2}$: angular velocity
ergosphere
$(\partial_t)^2 > 0$
(spacelike)
observer must rotate inside ergosphere
$(\partial_t)^2 < 0$
(timelike)
event horizon
$r = r_+$

Penrose diagram $(c = 1)$



BH Perturbations

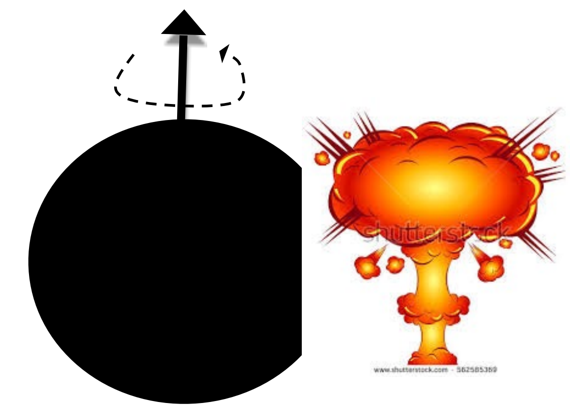
BHs are not in isolation but are ‘perturbed’ by fields (scalar, fermion, electromagnetic, gravitational…) due to neighbouring matter (eg, accretion disk, neutron star, etc) or another BH



Questions we wish to address investigating perturbations of rotating BH spacetimes:

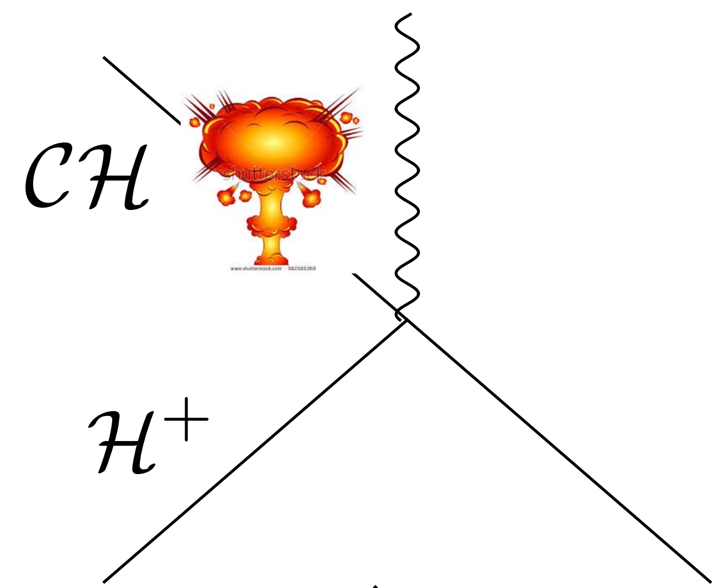
- is Kerr stable under field perturbations?

(if not, maybe a Kerr BH is not the final stage of gravitational collapse of massive stars)

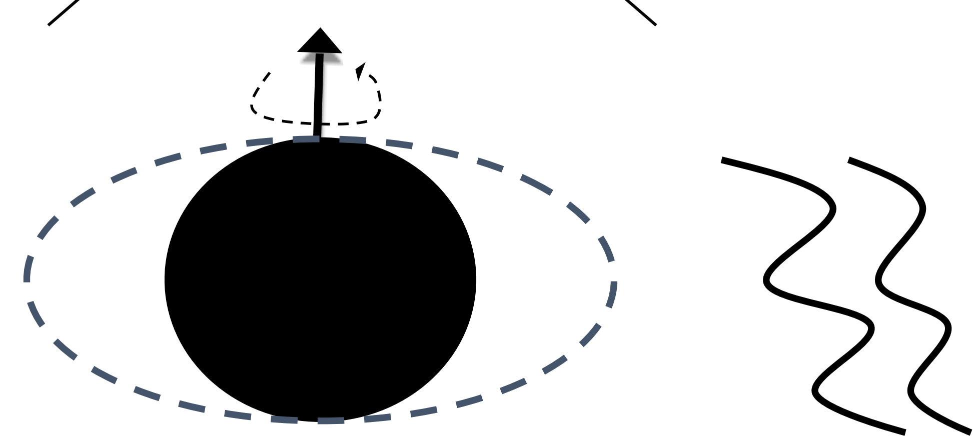


- what happens inside a BH?

(does the Cauchy hor. really exist?)



- do BHs deform under an external tidal field?



Wave Equation

We consider linear field perturbations of a *fixed* BH (ie, we do not consider the backreaction of the field on the BH) $\longrightarrow$ the fields propagate on a BH *background* $g_{\mu\nu}$

E.g., scalar field perturbations ϕ of a BH satisfy a wave eq.

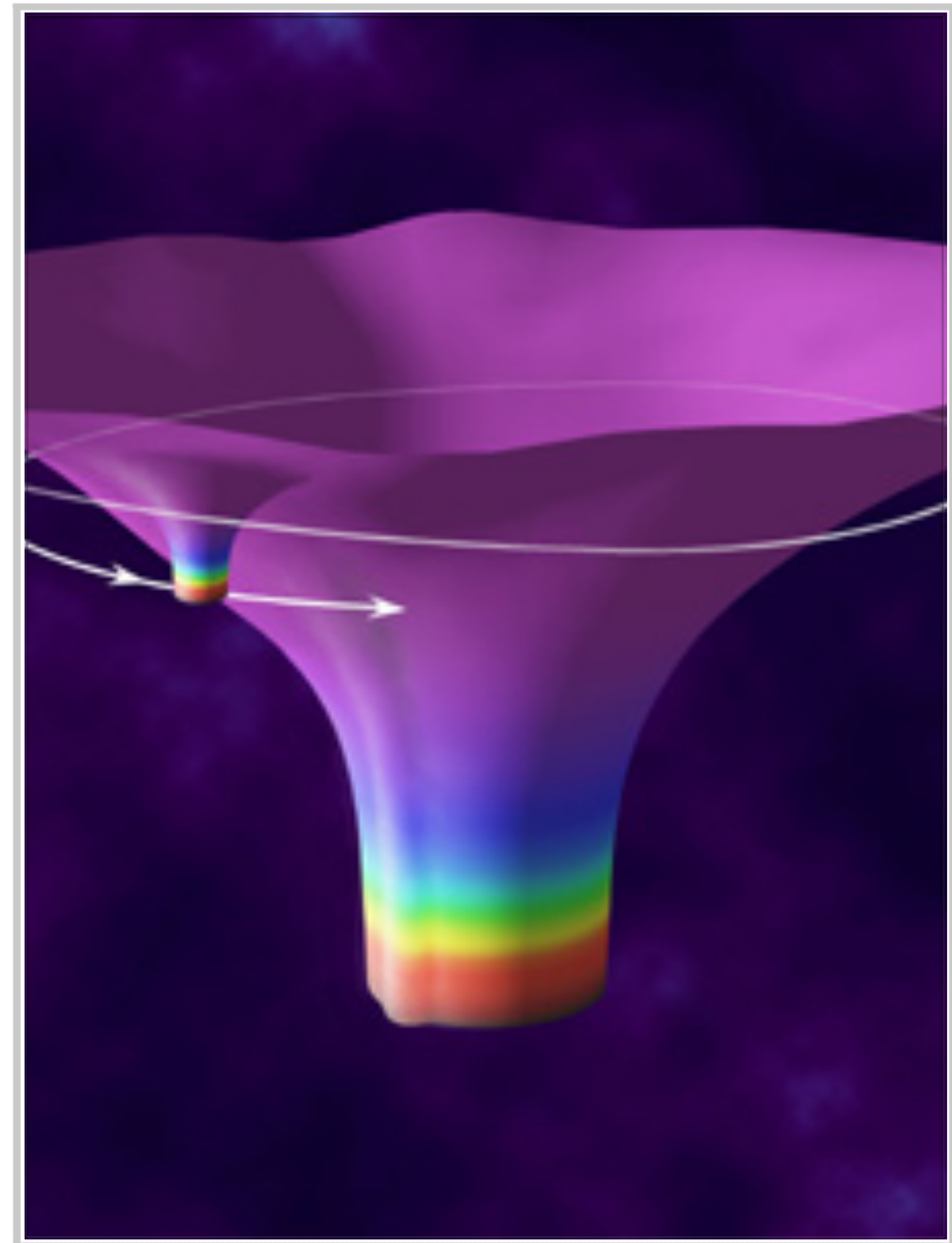
$$\underbrace{\Box\phi(x)}_{\text{spacetime point}} \equiv g_{\mu\nu}\nabla^{\mu}\nabla^{\nu}\phi(x) = \underbrace{T(x)}_{\text{source of field}}$$

Perturbations by other fields satisfy a similar wave eq.

Eg, for grav. field perturbations, *linearize* Einstein eqs.

Smaller BH (m) moving on the background metric $g_{\mu\nu}$ of a massive BH (M) causes perturbation metric $h_{\mu\nu}$

$$\text{Total metric } = \overbrace{g_{\mu\nu}}^{\text{due to M}} + \underbrace{h_{\mu\nu}}_{\text{perturbation (gravitational waves) due to m}} + O\left(\frac{m}{M}\right)^2$$



Credit: NASA

The eqs. satisfied by the different components of $h_{\mu\nu}$ do not decouple, but…

Teukolsky'73 managed to *decouple* the eqs. satisfied for combinations ψ of different components and derivatives of the various fields (spin |s|=0 scalar,=1/2 neutrino,=1 emag for Faraday tensor,=2 grav for Weyl tensor)

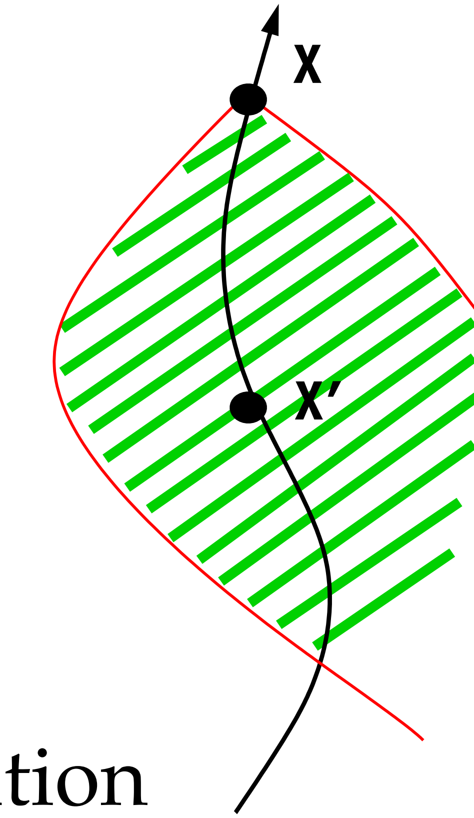
They all obey a wave-like eq.:

$$\hat{\mathcal{O}}\,\psi(x) = T(x)$$

spin-field   source of field

Green Function

A crucial object is the retarded Green function



$$\hat{\mathcal{O}}\, G_{ret}(x,x') = \delta_4(x,x') \quad \text{with causal b.c.:}$$

$$G_{ret}(x,x') = 0 \quad \text{if} \quad x' \neq J^-(x)$$

GF determines evolution in time of any initial field configuration

$$\psi(x) = \int_{t=0} \left[G_{ret}(x,x')\dot{\psi}^{ic}(\vec{x}') + \psi^{ic}(\vec{x}')\,\partial_t G_{ret}(x,x') \right] d^3\vec{x}'$$

GF can be calculated by decomposing into Fourier modes ($\leftarrow$ stationarity) and spheroidal harmonics ($\leftarrow$ axisymmetry & hidden symmetry) :

$$G_{ret} = \sum_{\ell,m} \int_{-\infty}^{\infty} d\omega\; e^{im\varphi - i\omega t} S_{\ell m\omega}(\theta) S_{\ell m\omega}(\theta') G_{\ell m\omega}(r,r')$$

$\uparrow$

Fourier modes

The GF Fourier modes satisfy a radial ODE:

$$\hat{\mathcal{O}}_r \; G_{\ell m\omega}(r,r') = \delta(r,r')$$

2nd order linear operator in r

So they can be found from two linearly independent slns. of the *homogeneous* radial ODE:

$$G_{\ell m\omega}(r,r') = \frac{R^{in}_{\ell m\omega}(r_<)R^{up}_{\ell m\omega}(r_>)}{W}$$

Wronskian

$$r_< \equiv \min{(r,r')}$$

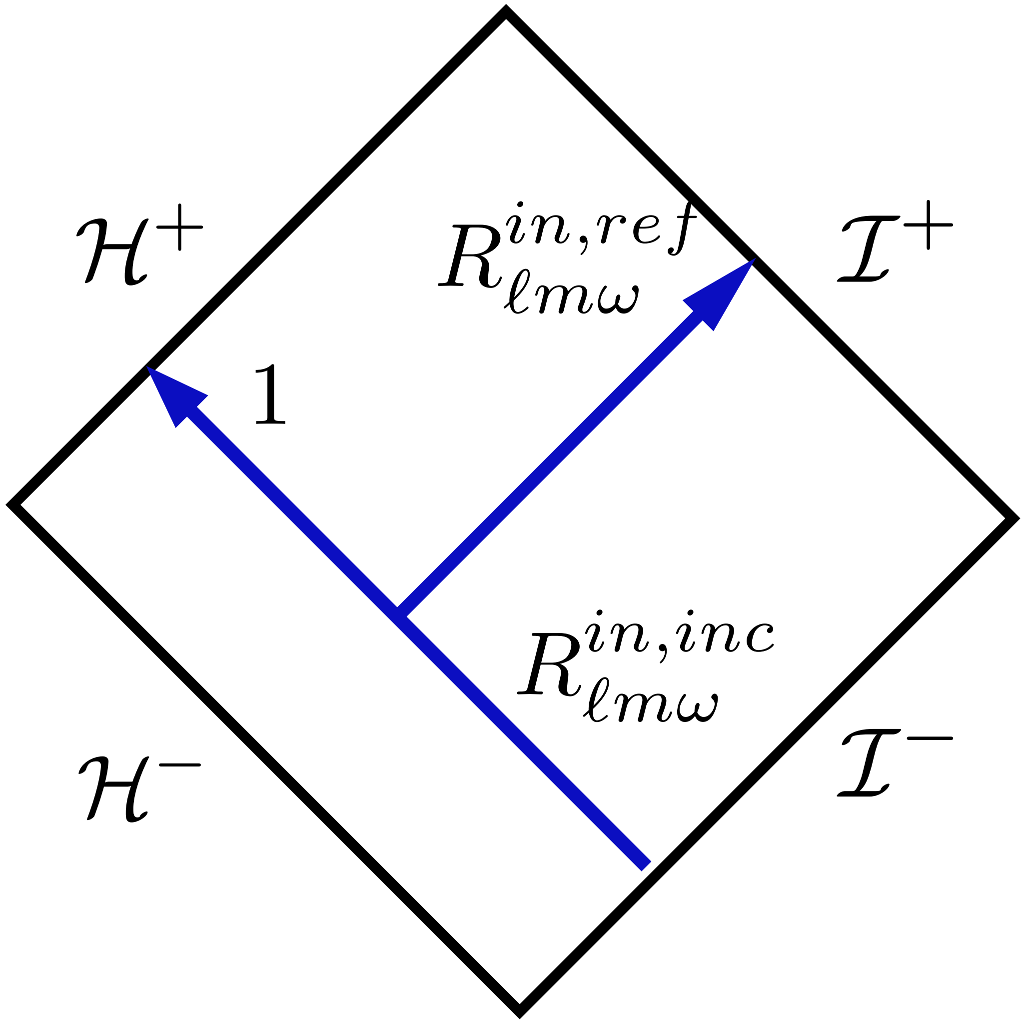
$$r_> \equiv \max{(r,r')}$$

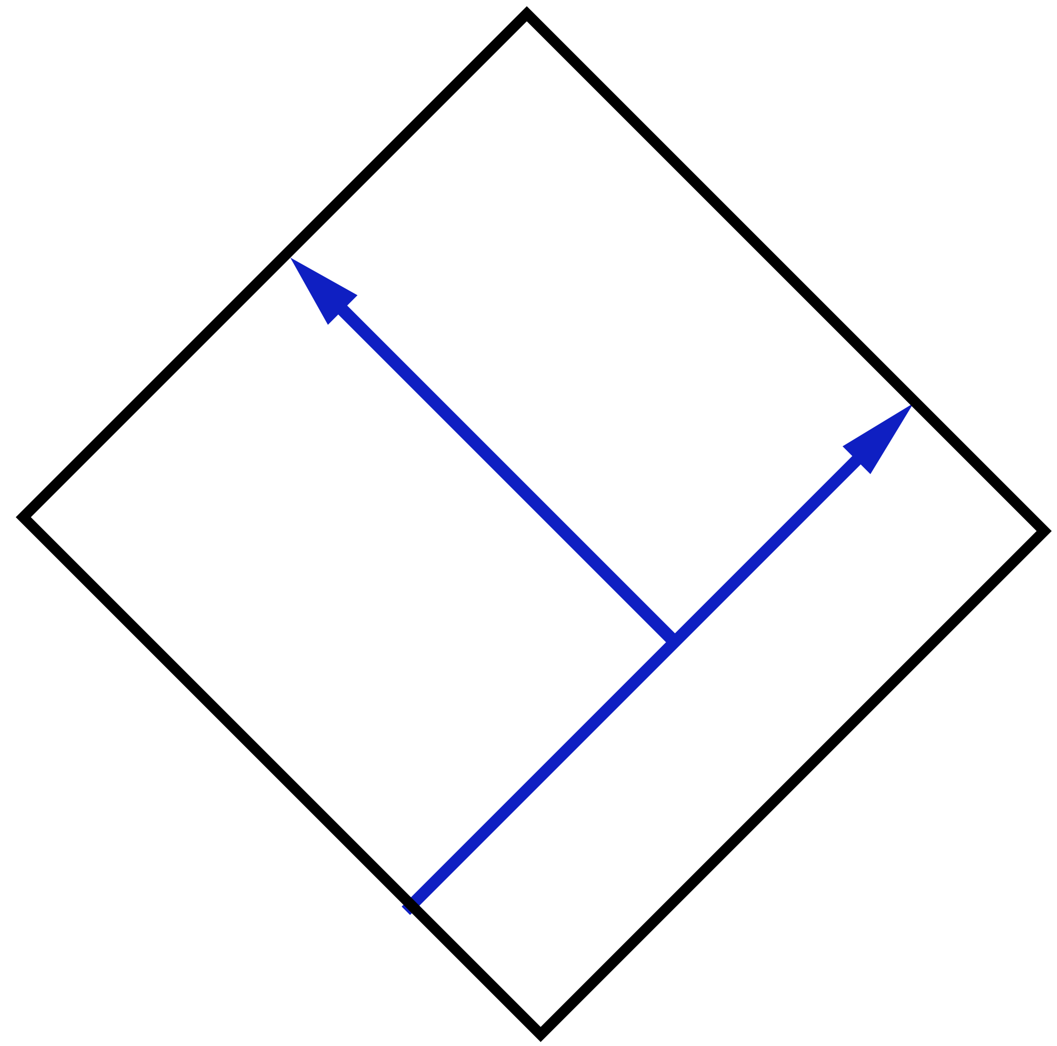
where $\hat{\mathcal{O}}_r \; R^{in/up}_{\ell m\omega}(r) = 0$

Causal boundary conditions for the homogeneous slns.:

$$e^{-i\omega t} R^{in}_{\ell m\omega}(r)$$

$$e^{-i\omega t} R^{up}_{\ell m\omega}(r)$$



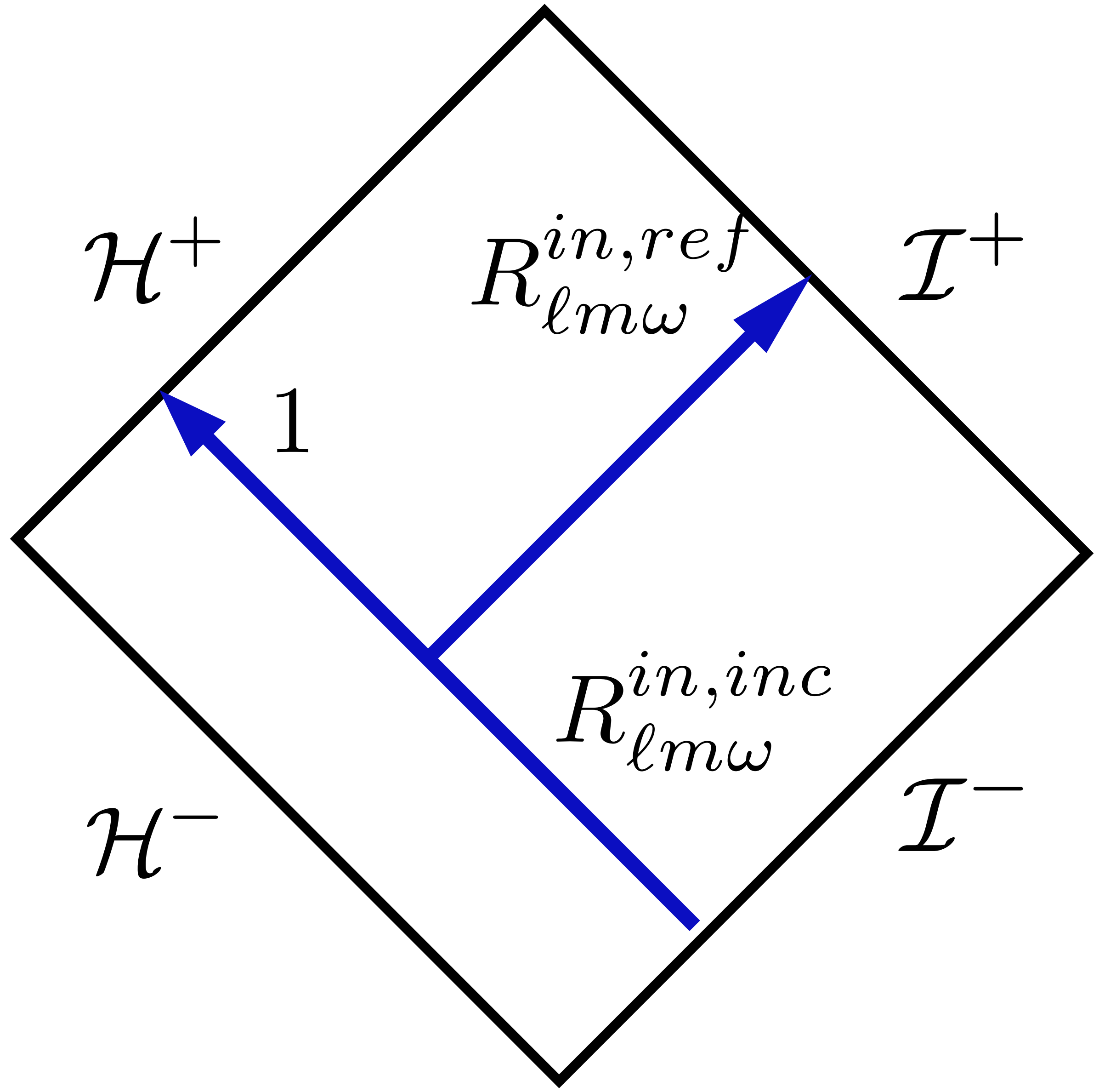


$$W = 2i\omega R^{in,inc}_{\ell m\omega}$$

Superradiance

Wronskian condition (energy conservation):

$$\left|R^{in,ref}_{\ell m\omega}\right|^2 = \left|R^{in,inc}_{\ell m\omega}\right|^2 - \frac{\omega - m\Omega_+}{\omega}$$



Superradiance: reflected wave has more energy than incident wave if

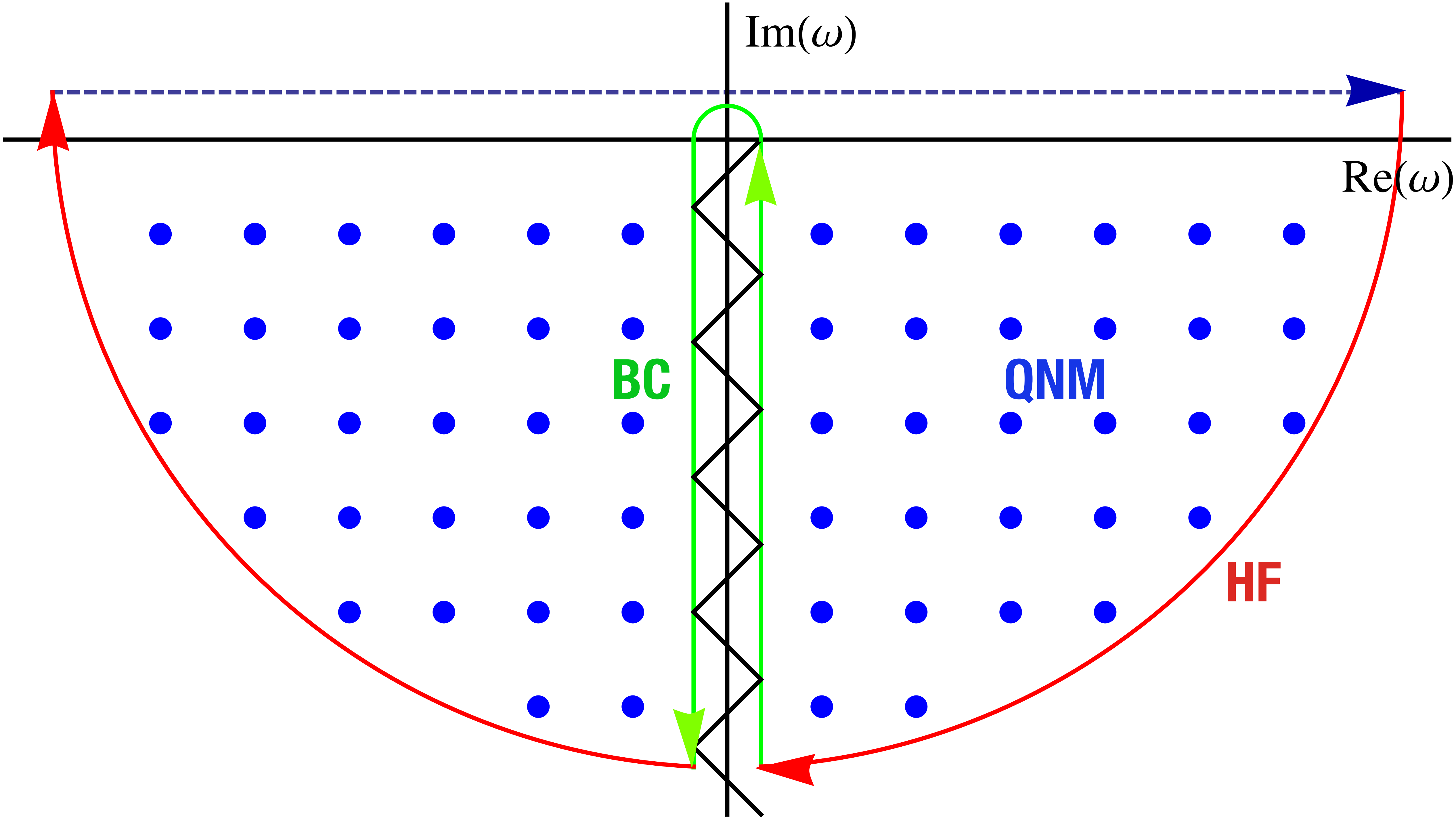
$$(\omega - m\Omega_+)\,\omega < 0$$

Field modes with those frequencies extract *rotational energy* from the BH thanks to existence of *ergosphere* (region near BH where ∂_t is spacelike)

Complex contour deformation

Instead of carrying out the Fourier integral along the real-ω axis, it's useful to deform the contour of integration into complex-ω plane

Then apply the residue th. to account for the singularities of the Fourier modes $G_{\ell m\omega}$



Then 'main' contribution to G_{ret} is from the *poles* (QNMs)

Mode solutions

Mode slns. correspond to frequencies $\omega_{\ell mn} \in \mathbb{C}$ which are *poles* of the GF modes

$$G_{\ell m\omega}(r,r') = \frac{R^{in}_{\ell m\omega}(r_<)R^{up}_{\ell m\omega}(r_>)}{W} = \infty$$

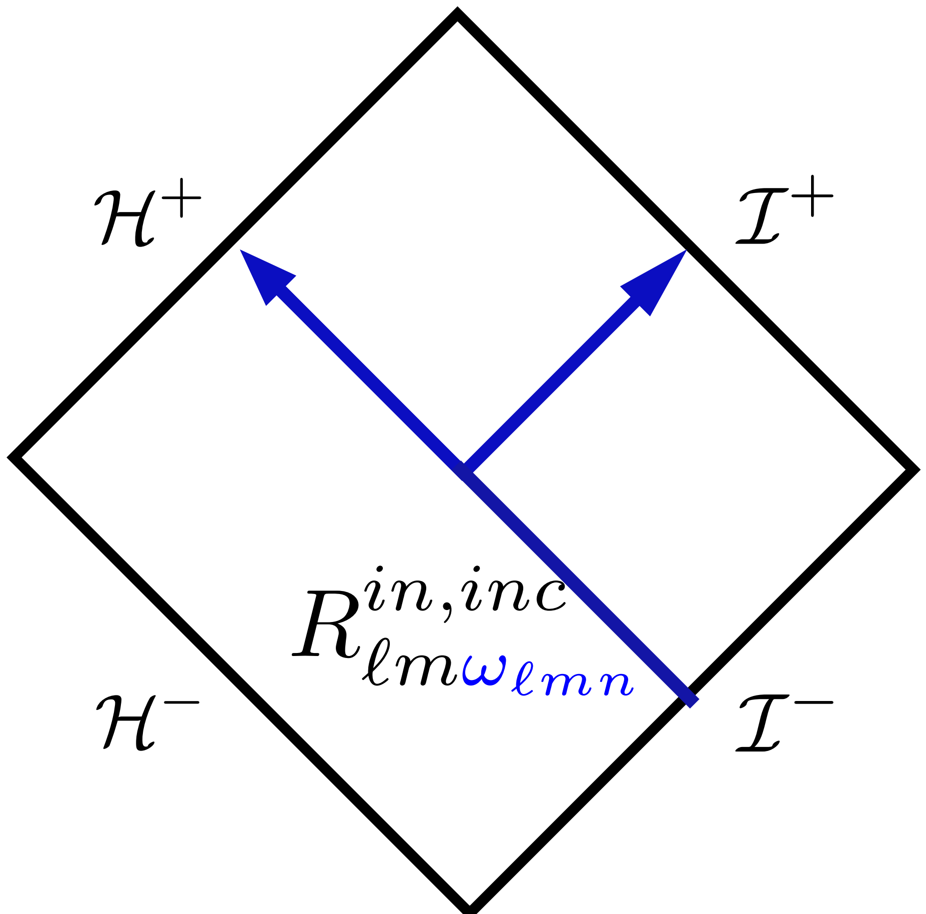
$$\omega = \omega_{\ell mn}$$

$$n = 0, 1, 2, \ldots$$

So they are zeros of the denominator:

$$W = 2i\omega R^{in,inc}_{\ell m\omega} = 0$$

Then: $e^{-i\omega_{\ell mn}t}R^{in}_{\ell m\omega_{\ell mn}}$



Mode slns. are purely ingoing waves into the horizon and purely ougoing at infinity

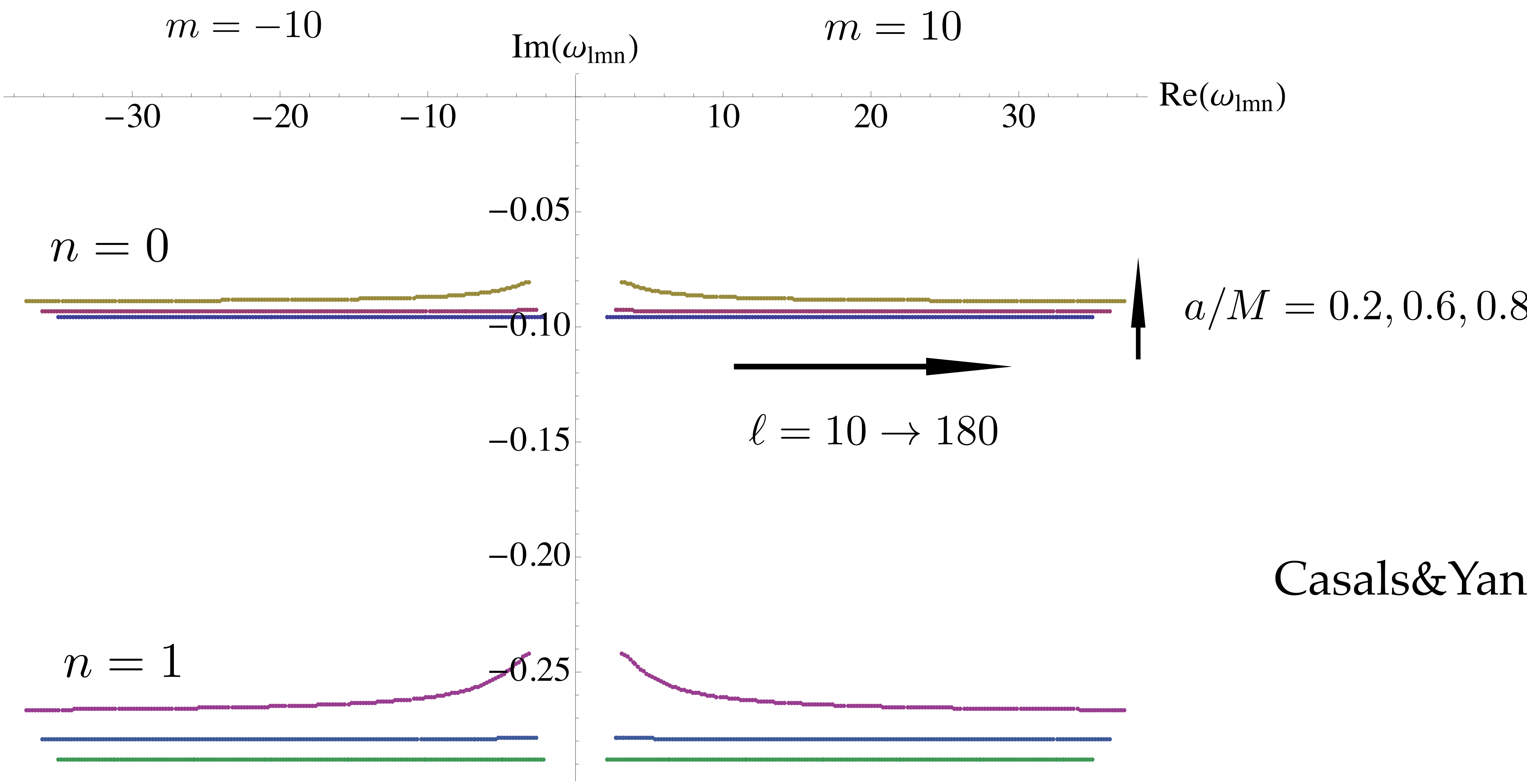
Time dependence:

$$e^{-i\omega_{\ell mn}t} \xrightarrow{t\to+\infty}$$

$\begin{cases}\text{If } Im(\omega_{\ell mn})<0 \text{: exponentially damped (\textit{quasinormal modes}, QNMs)}\\ \text{If } Im(\omega_{\ell mn})>0 \text{: exponentially growing (unstable modes)}\end{cases}$

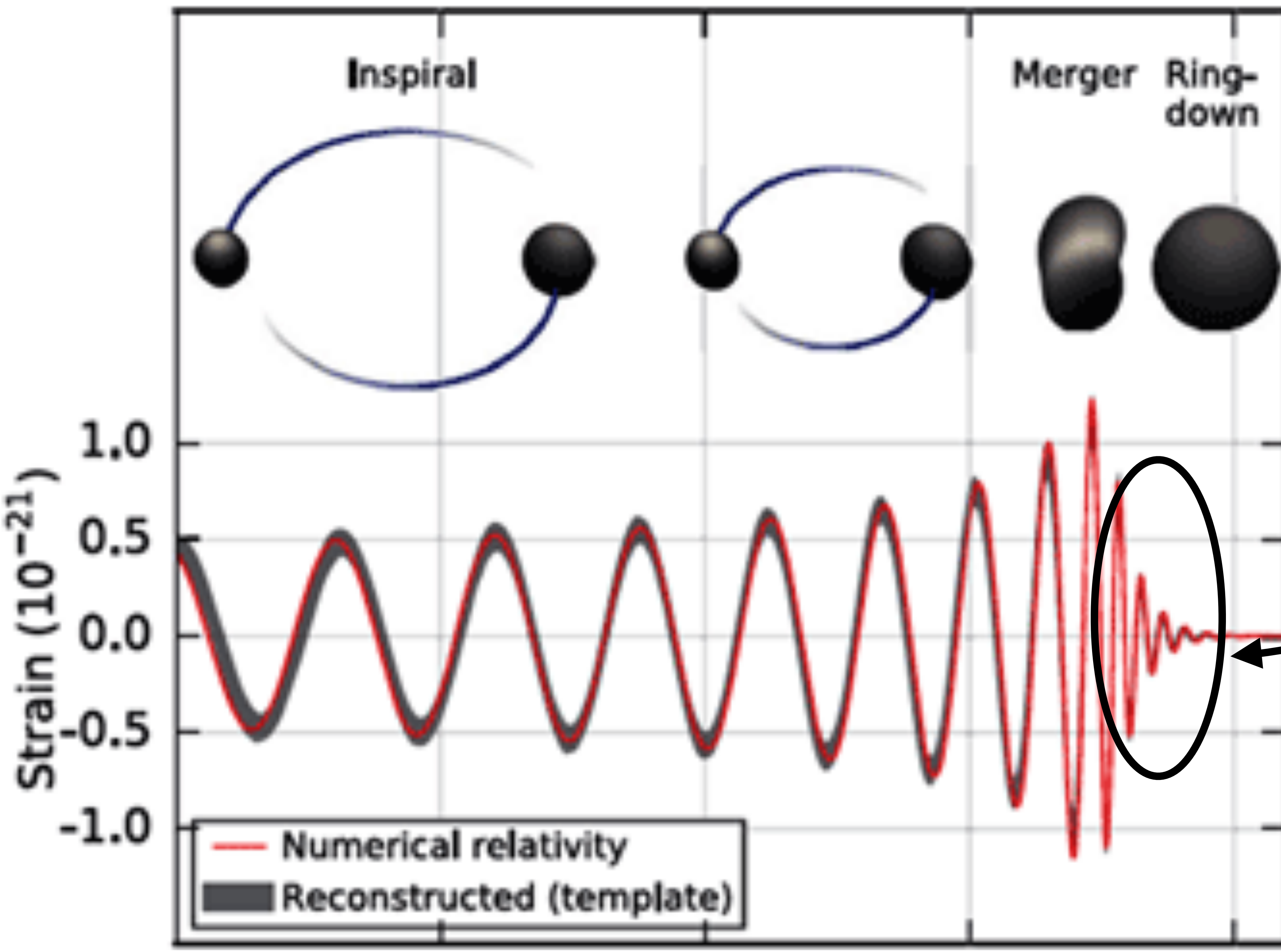
[If $Im\left(\omega_{\ell mn}\right)=0$: *marginally* unstable]

QNMs $(Im(\omega_{\ell mn}) < 0)$ of Kerr:



Casals&Yang

The last stage (*ringdown*) of a gravitational waveform can be modelled as perturbations of Kerr via QNMs:



perturbed Kerr emitting GWs

QNM measured by LIGO agrees with theoretical calculation

LIGO'16

Q: Are there any unstable modes ($Im(\omega_{\ell mn}) > 0$) in Kerr?

1. BH Perturbations

2. Stability of Kerr

3. Stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Tidal gravitational deformation (Love)

5. Conclusion

Stability of subextremal Kerr

There're no unstable modes for massless general-spin fields in subextremal ($a < M$) Kerr => Kerr is mode-stable (Whiting'89)

Note: the full linear sln. of Teukolsky eq. is obtained from *infinite* sums/integrals of frequency modes $\sum_{\ell,m} \int_{-\infty}^{\infty} d\omega$

so non-existence of unstable modes does not guarantee *full linear stability*

Kerr is fully (ie, not just modal) linearly stable under scalar perturbations (Dafermos, Rodnianski & Shlapentokh-Rothman'16)

Open question: full linear stability of Kerr under gravitational perturbations

Stability of Extremal Kerr

In *extremal* Kerr $(a = M)$:

Field (& derivatives) off the horizon $\mathcal{H}$ decays

and

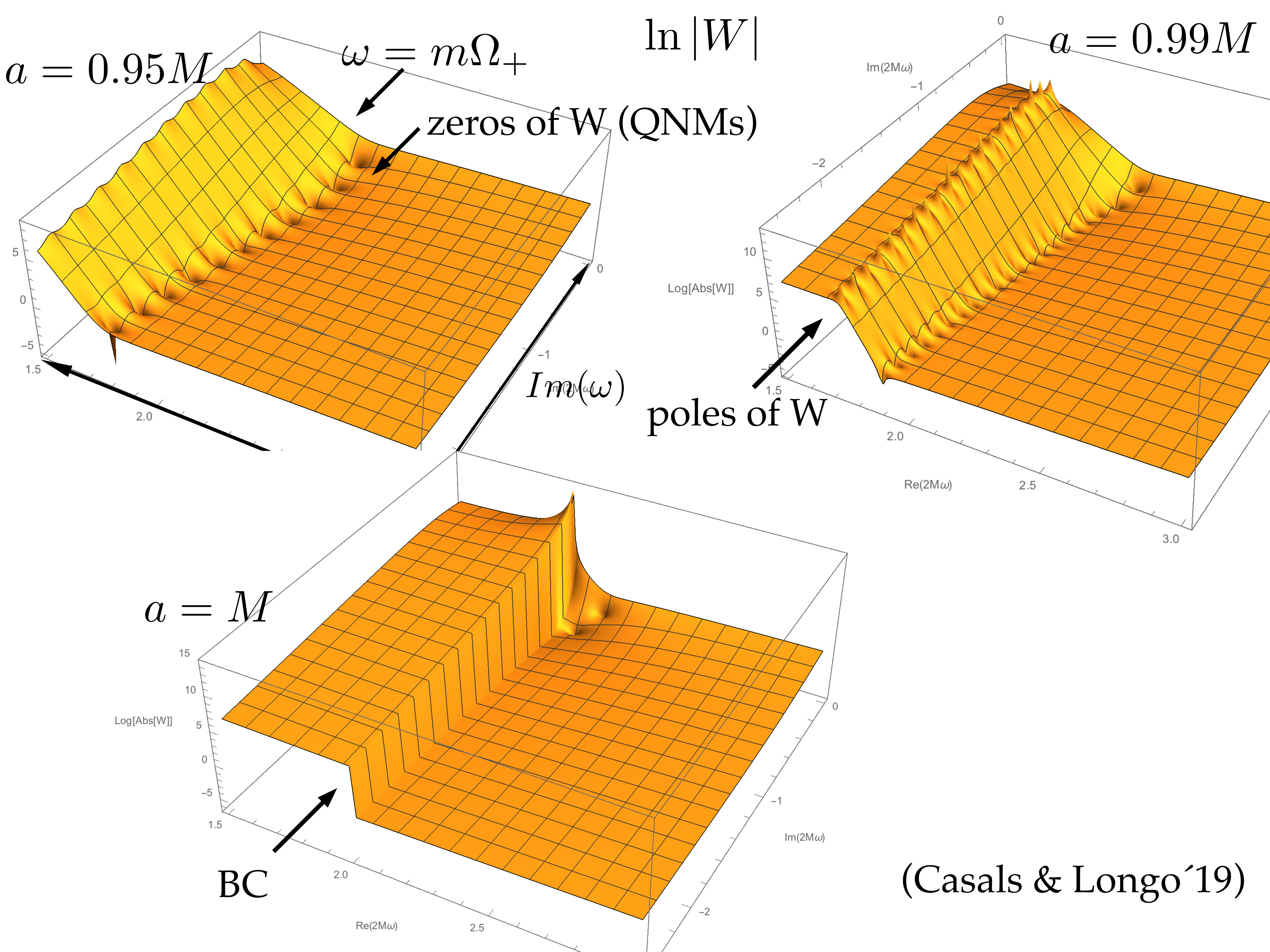
there're no unstable modes for massless general-spin fields in extremal Kerr (Teixeira da Costa'20)

But…

Transverse derivatives of axisymmetric field on the horizon of extremal Kerr grow! (Aretakis'10) (growth undetermined)

Given that there're no unstable modes, how to explain this instability?

As extremal Kerr is approached, QNMs accumulate to form a branch cut starting at the superradiant-bound frequency $\omega = m\Omega_+$ (Detweiler'80)

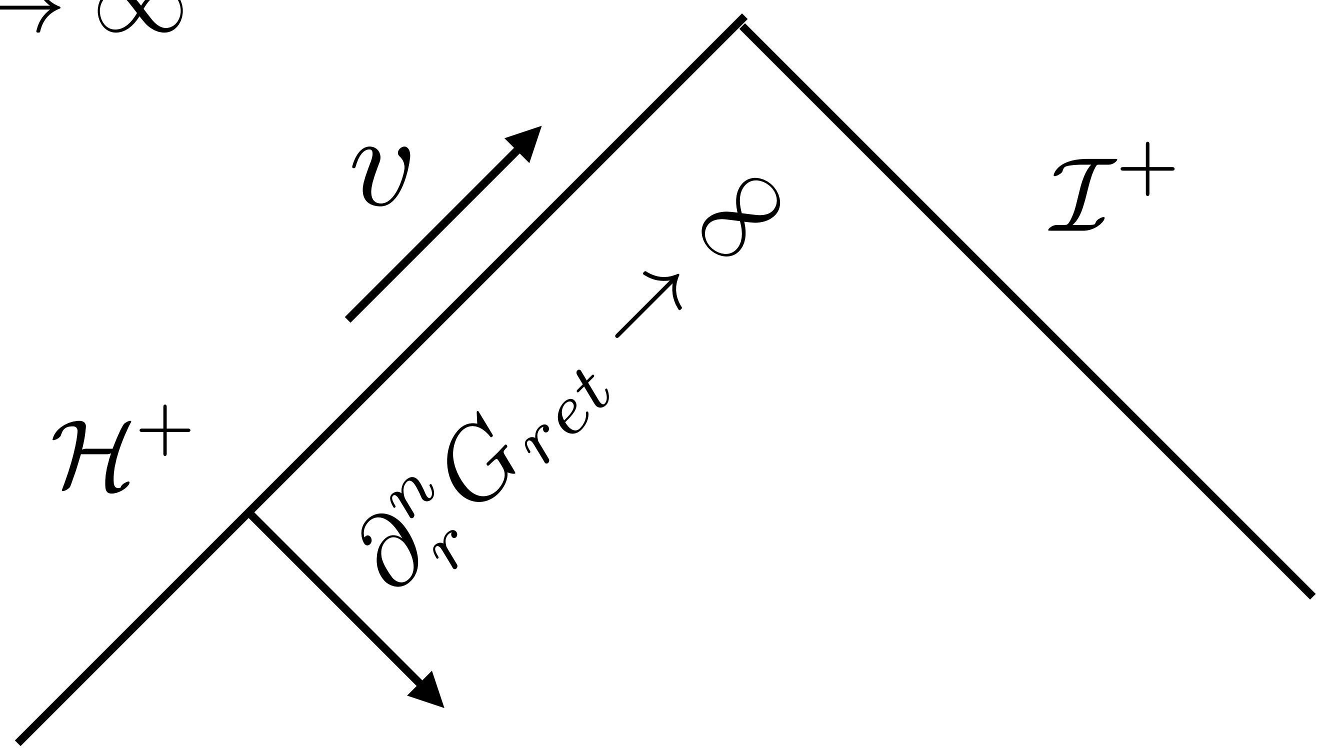


$a = 0.95M$
$\omega = m\Omega_+$
$\ln|W|$
$a = 0.99M$
zeros of W (QNMs)
$Im(\omega)$
poles of W
Log[Abs[W]]
Im(2Mω)
Re(2Mω)
$a = M$
BC
(Casals & Longo´19)

Late-time contribution from BC at $\omega = m\Omega_+$ to transverse

nth-derivative on horizon grows as (Casals, Gralla & Zimmerman'16)

$$\left(\partial_r^n G_{ret}\right)\big|_{\mathcal{H}} \sim v^{n-s-1/2} \quad \text{as} \quad v \to \infty$$



This is an *enhanced* instability for *non*-axisymmetric modes of the one found for axisymmetry by Aretakis'10

1. BH Perturbations

2. Stability of Kerr

3. Stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Tidal gravitational deformation (Love)

5. Conclusion

Kerr-Newman-de Sitter Black Holes

Kerr-Newman-de Sitter is an *electrically-charged* rotating (Kerr) BH in a *de Sitter Universe* (a Universe with accelerated expansion)

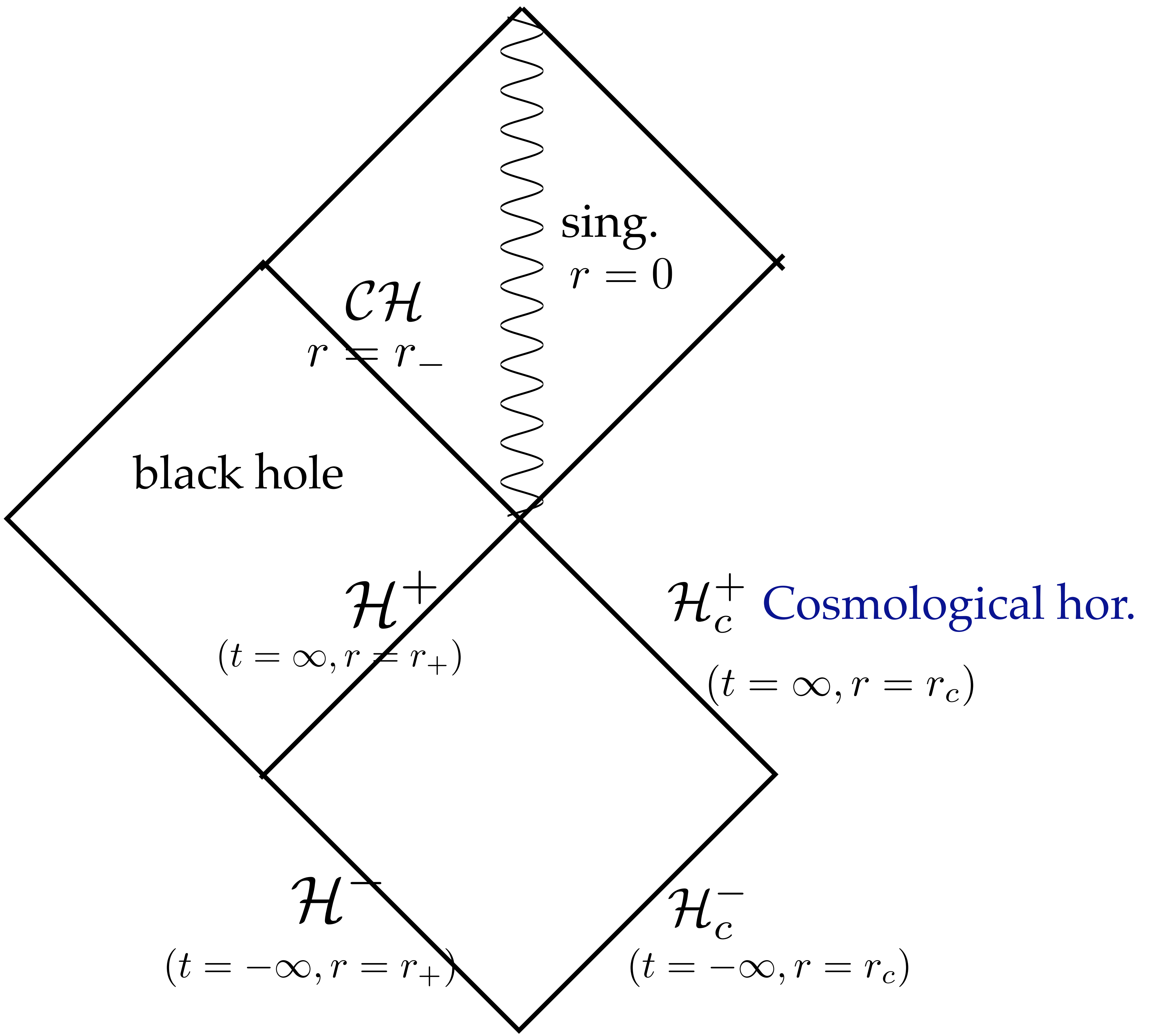
Determined by 4 parameters: (M, a, Q, Λ)

BH charge Cosmological const. > 0

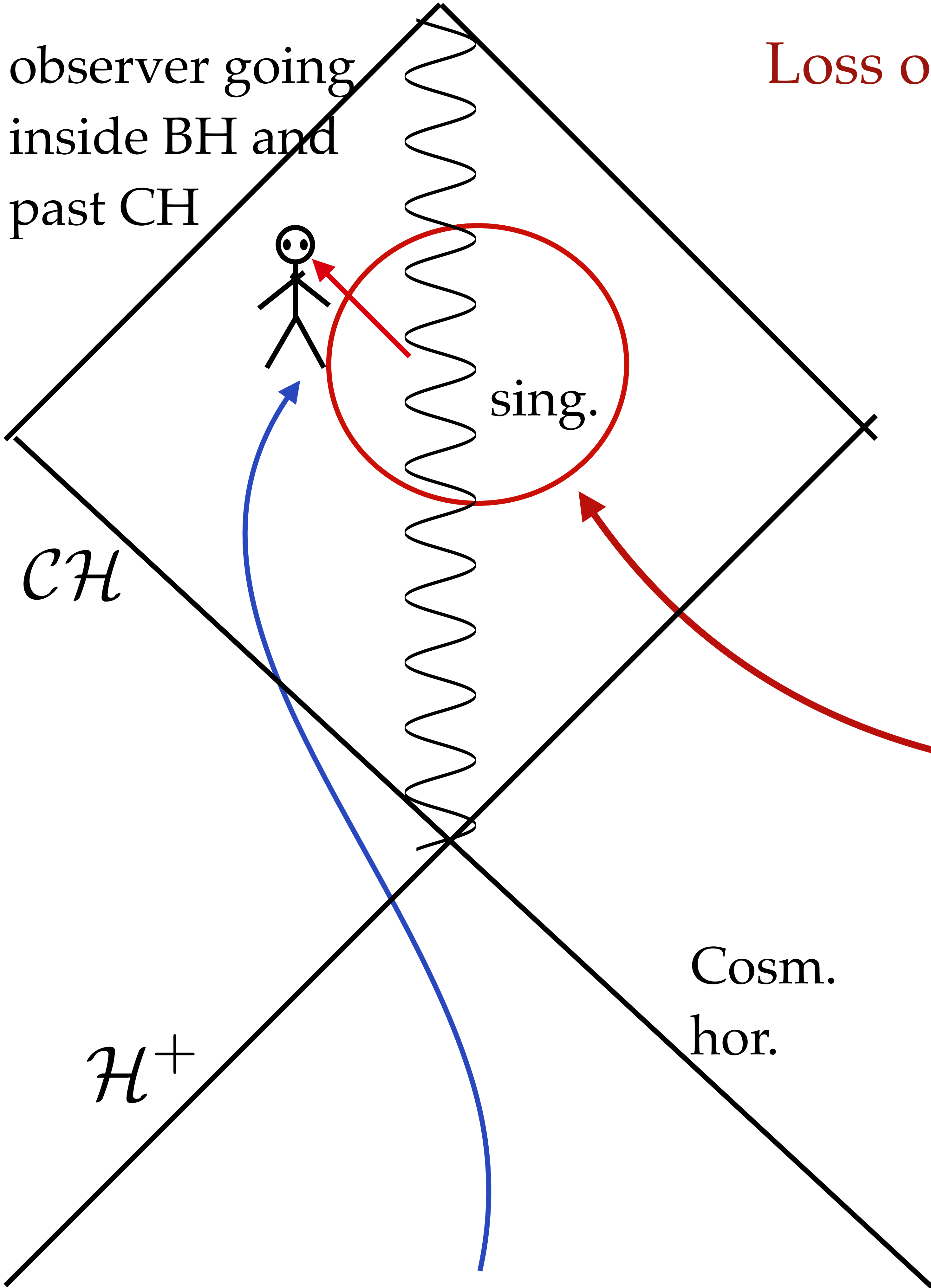
There're 3 horizons: $r_- \leq r_+ \leq r_c$

Cauchy hor. Event hor. Cosmological hor.

Penrose diagram of Kerr-Newman-de Sitter



Loss of predicability inside the BH

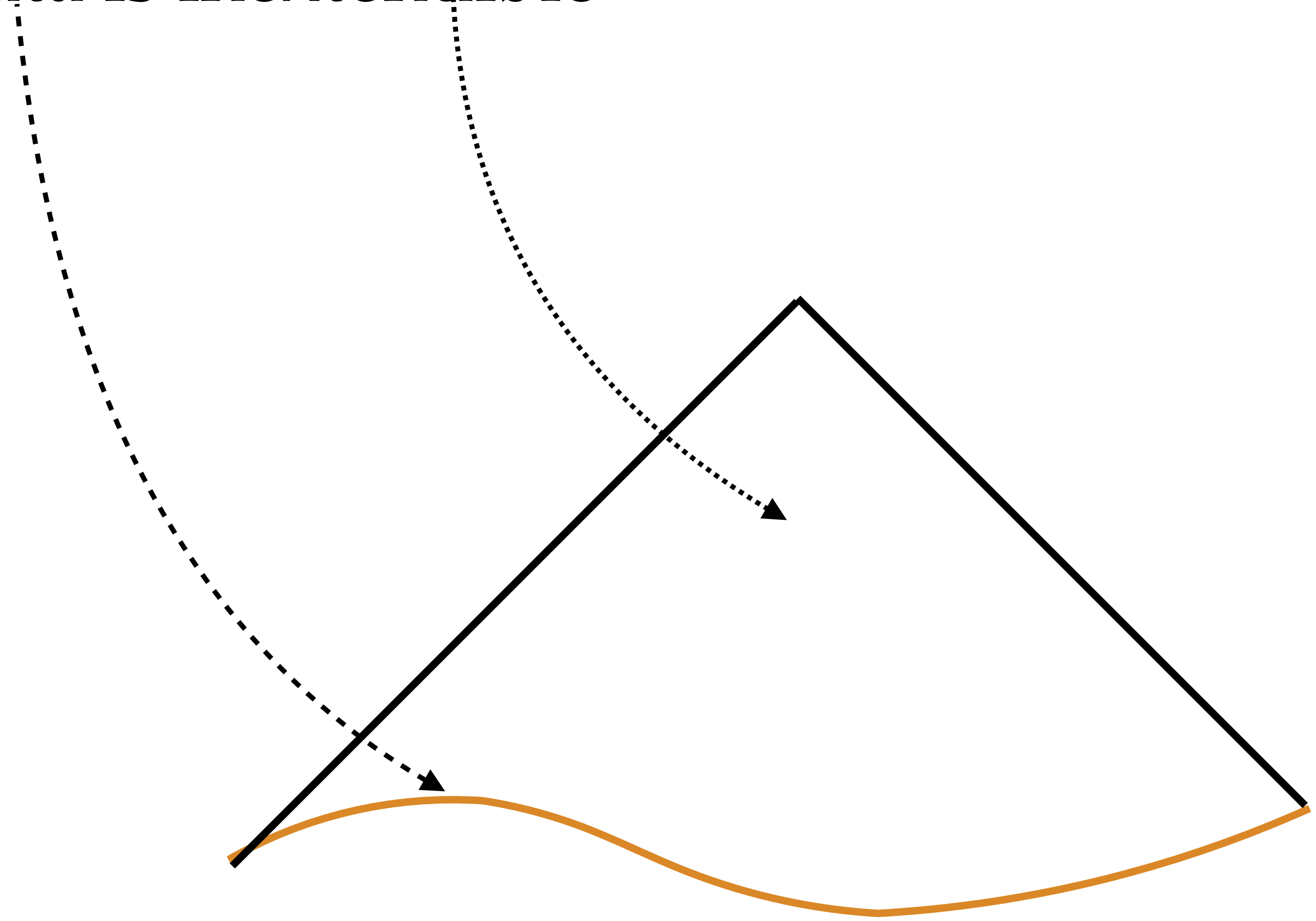


this is a *timelike* singularity, and so it's *visible* to an observer going into the BH

Unpredictability: the Cauchy ("initial") Value Problem is not well posed ('anything can come out of the sing.')

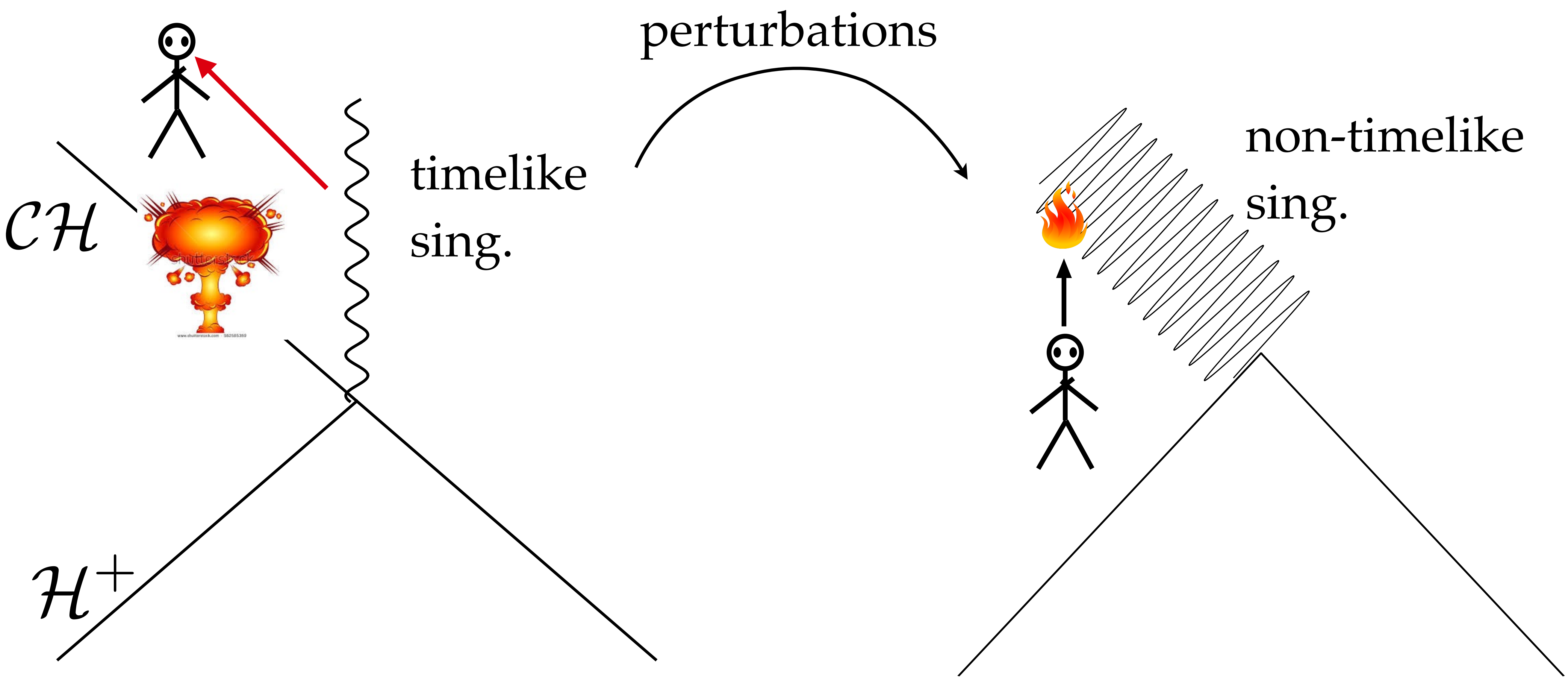
Strong Cosmic Censorship hypothesis

Strong Cosmic Censorship (SCC) Hypothesis (Penrose’72), essentially: the maximal Cauchy development via Einstein's equations of generic initial data is inextendible



So, if BHs that exist in Nature possess singularities in their inside, then they’re not visible even to observers inside (i.e., they’re not timelike)

SCC could be upheld if the Cauchy horizon is "destroyed" by field perturbations (all results so far were for perturbations *outside* the BH)



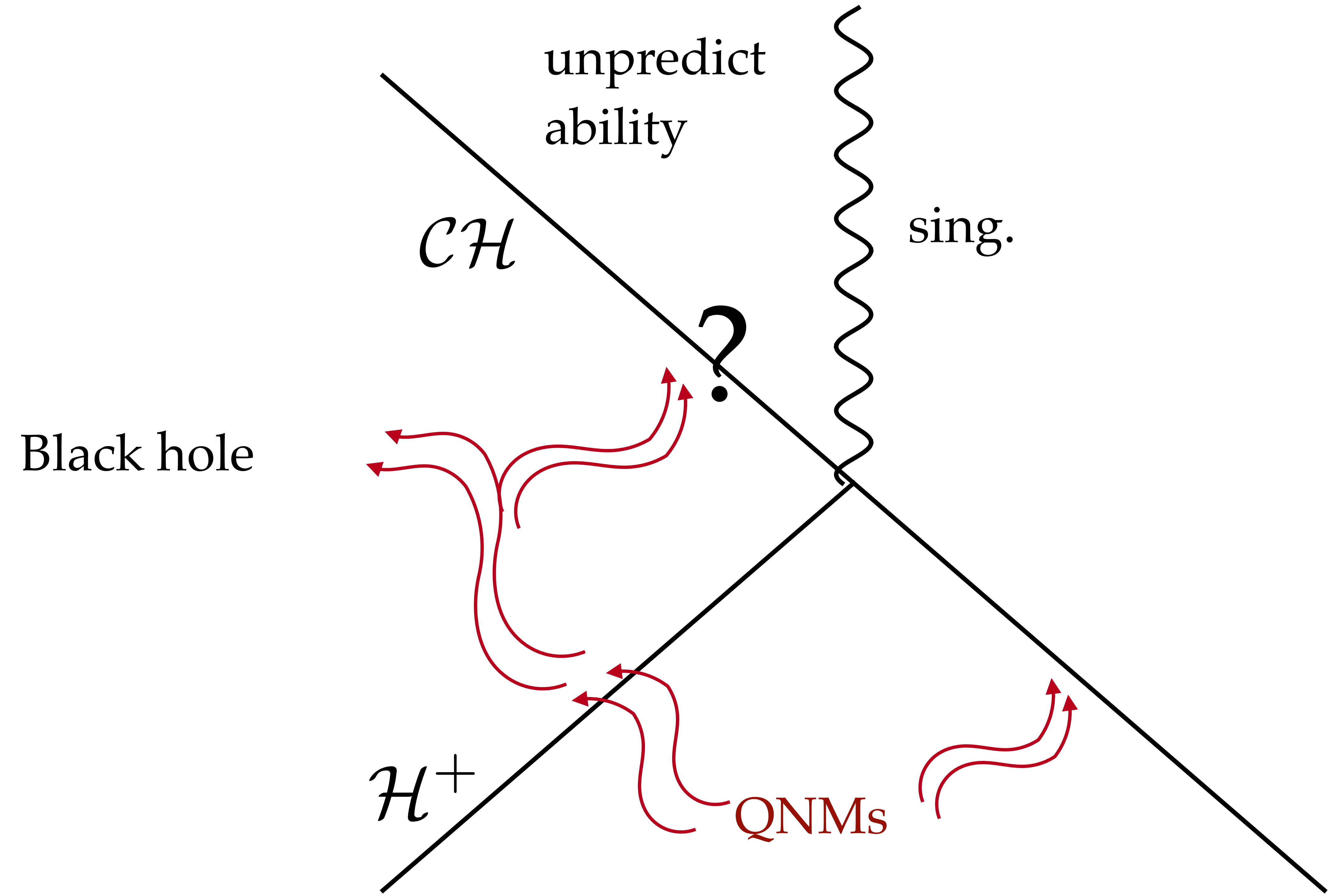
observer can see singularity

observer cannot see singularity and crashes into it in the future

But it's a hypothesis - it needs to be verified!

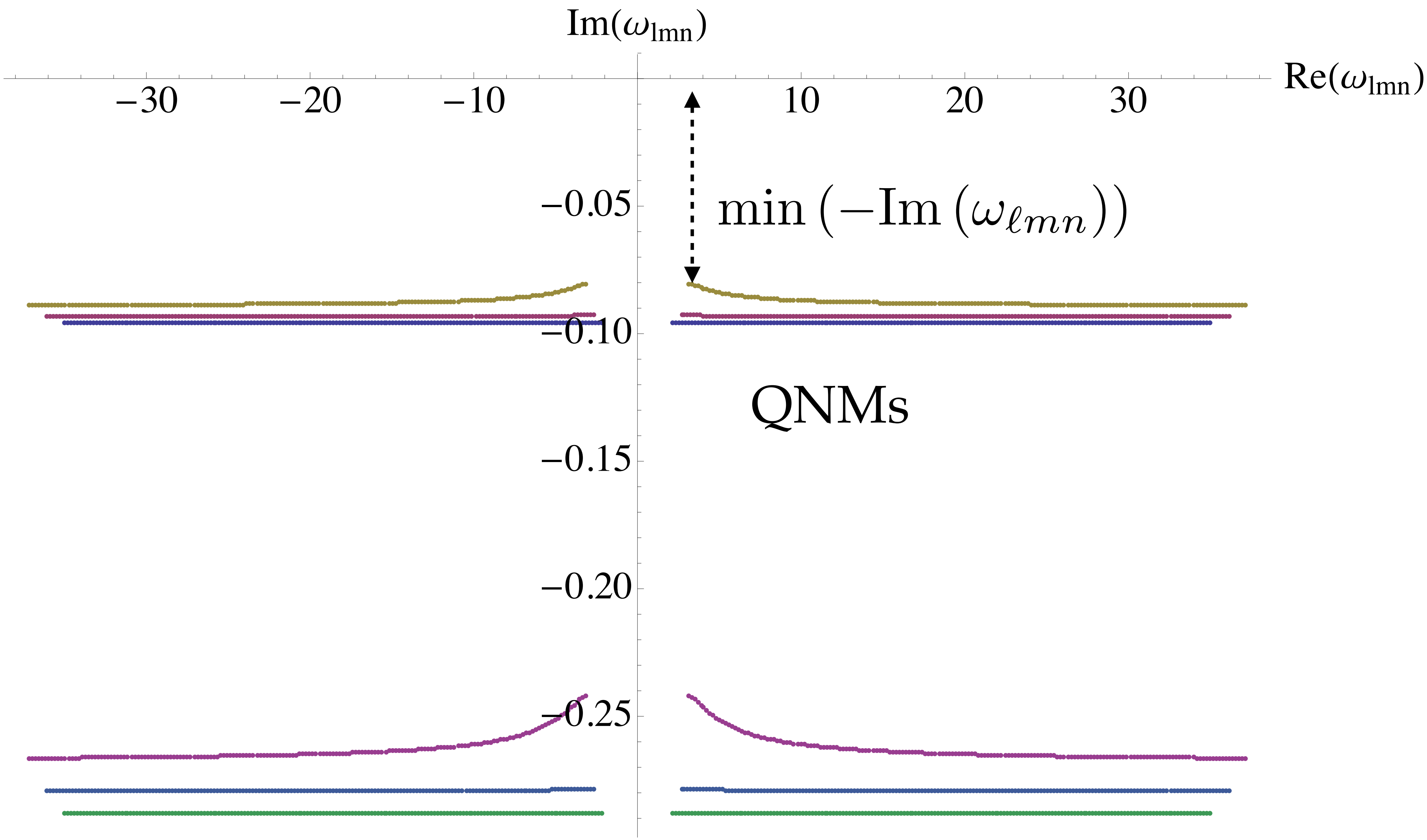
Regularity of Cauchy Horizon?

Even if the (exponentially-decaying) QNMs do not destabilize the outside of the BH, as they go inside and reach the CH, are they strong enough to destroy the CH?



If $\beta \equiv \frac{\min\left(-\mathrm{Im}\left(\omega_{\ell mn}\right)\right)}{\kappa_-} > \frac{1}{2}$, then QNM waves are too weak

when they arrive at the CH in order to destroy it => CH remains (the stress-energy tensor of field is locally integrable -> existence of *weak solutions* to Einstein's equations) and there's violation of SCC



Regularity of CH of BH when $\Lambda = 0$ or non-rotating ?

Case $\Lambda = 0$ (eg, Kerr):

CH & region of unpredictability are "destroyed" by the perturbation (stress-energy of field is not locally integrable), ie, SCC holds (Ori'92, Dafermos & Luk'17)

😀

Case $\Lambda > 0$ but $a = 0$:

CH & region of unpredictability are stable ($\beta > 1/2$, ie, violation of SCC) for an electrically-charged *non*-rotating BH (Reissner-Nordstrom-de Sitter) since the Cosmic acceleration "weakens" the field (Cardoso et al.'18)

😱

Stability of CH of *rotating* BH in de Sitter Universe?

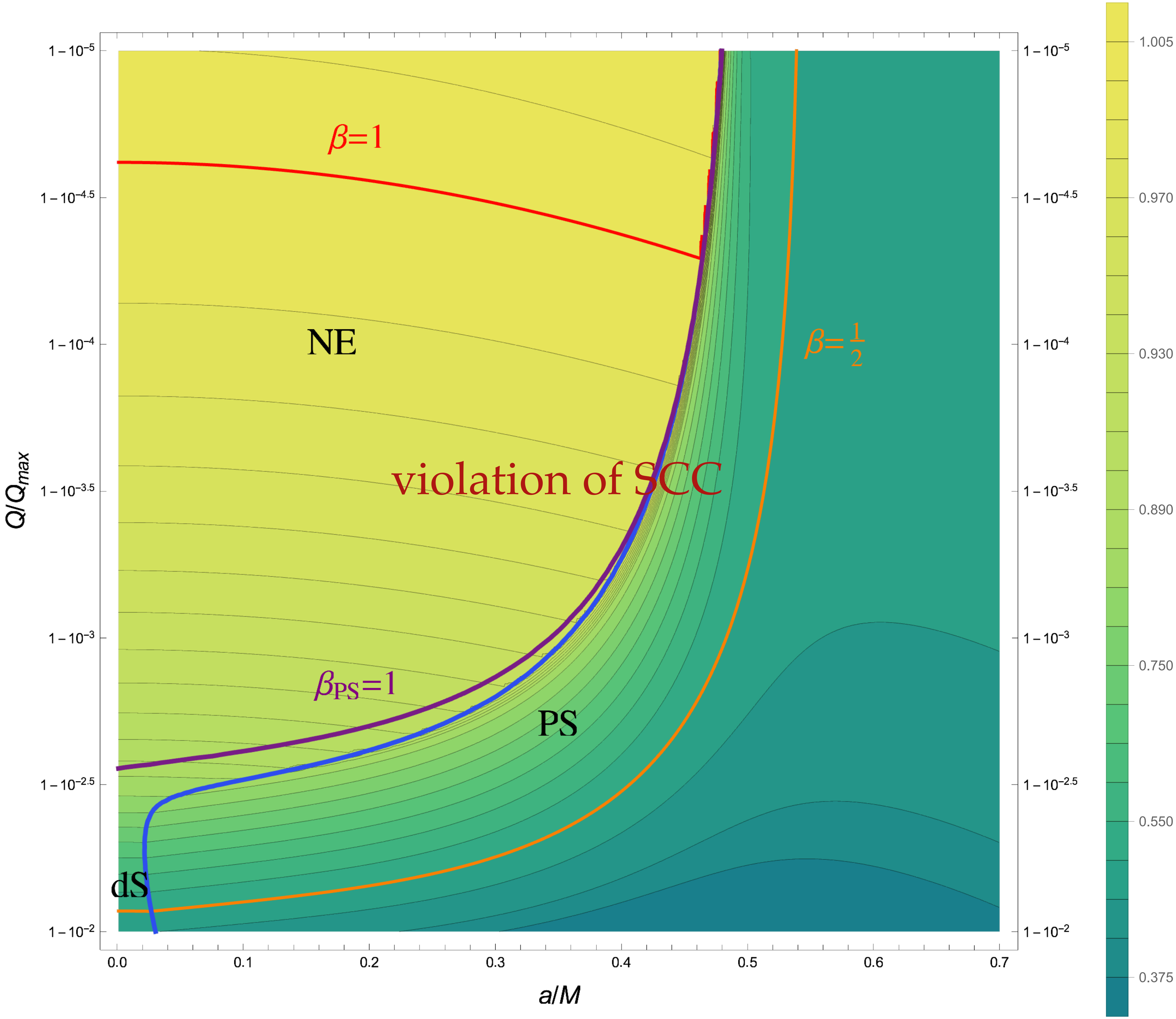
But all BH's in the Universe are rotating. What happens when we also include rotation?

Kerr-de Sitter: it seems that $\beta < \dfrac{1}{2}$ and so SCC is preserved (Dias et al.´18)

Kerr-Newman-de Sitter: investigations by Hod'18 and Rahman et al.'19 also found $\beta < \dfrac{1}{2}$ and so preservation of SCC

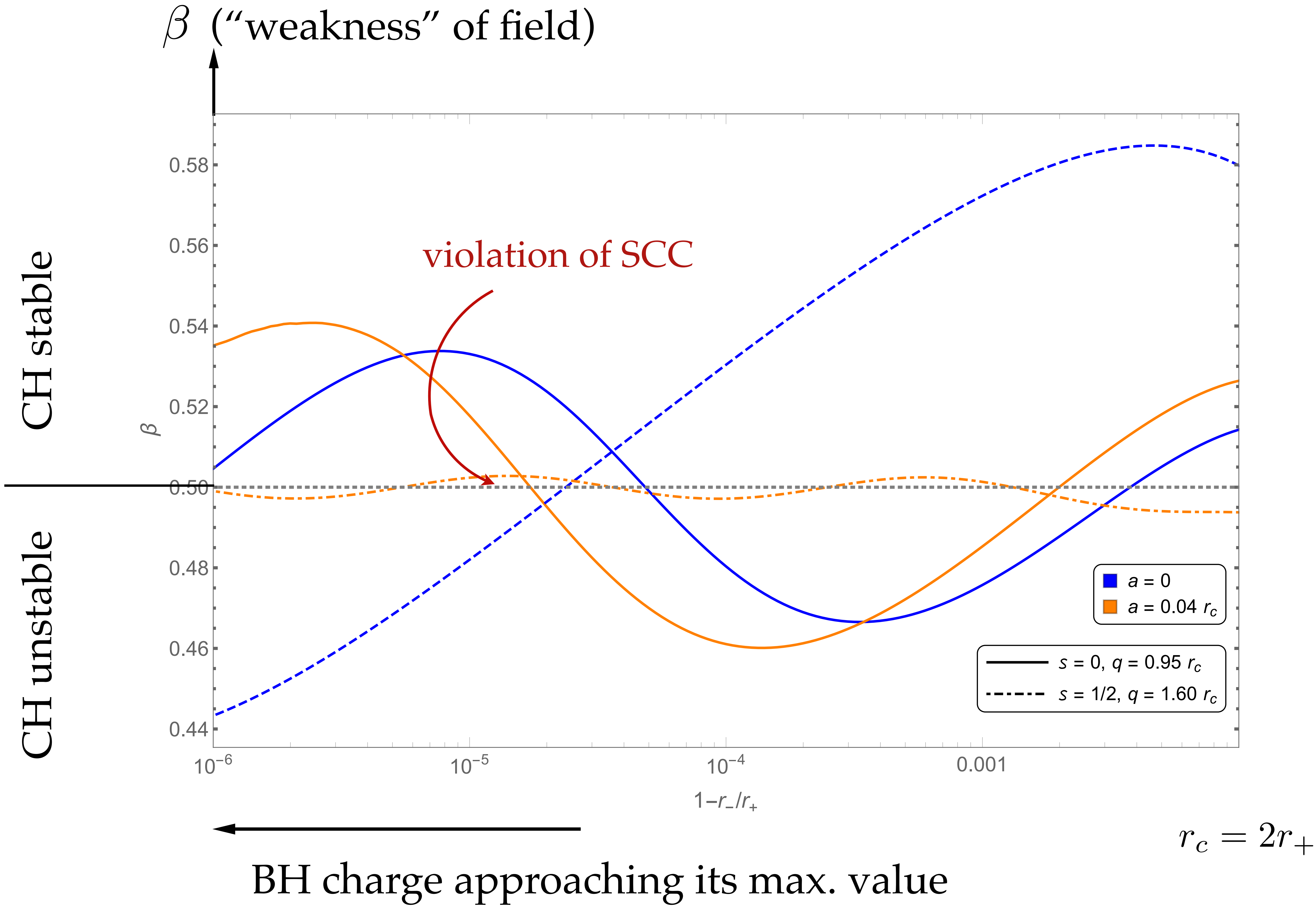
But these investigations in Kerr-Newman-de Sitter were only for a *limited* region of parameter space…

Casals & Marinho'22 for neutral scalar field in Kerr-Newman-de Sitter



$\Lambda M^2 = 0.02$

and for *charged* scalar and fermion fields:



So we find *violation* of SCC (ie, unpredictability inside BH) for a rotating BH but for unphysical values of parameters (BH charge and Cosmological const. too large)

1. BH Perturbations

2. Stability of Kerr

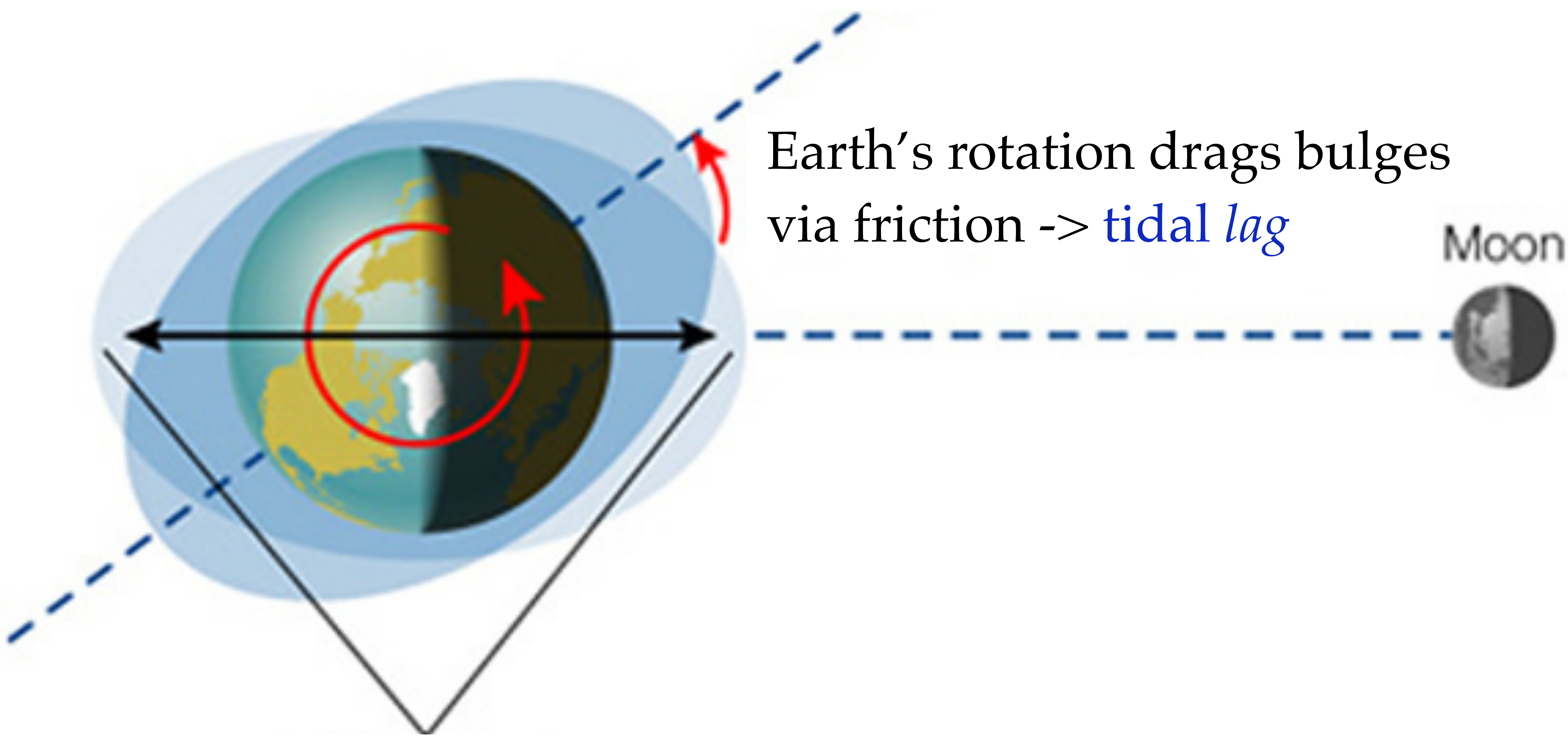
3. Stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Tidal gravitational deformation (Love)

5. Conclusion

Earth & Moon – Love in *Newtonian* gravity

Augustus Love (1909) introduced numbers characterizing the Earth´s tides in its response to the Moon´s gravitational field

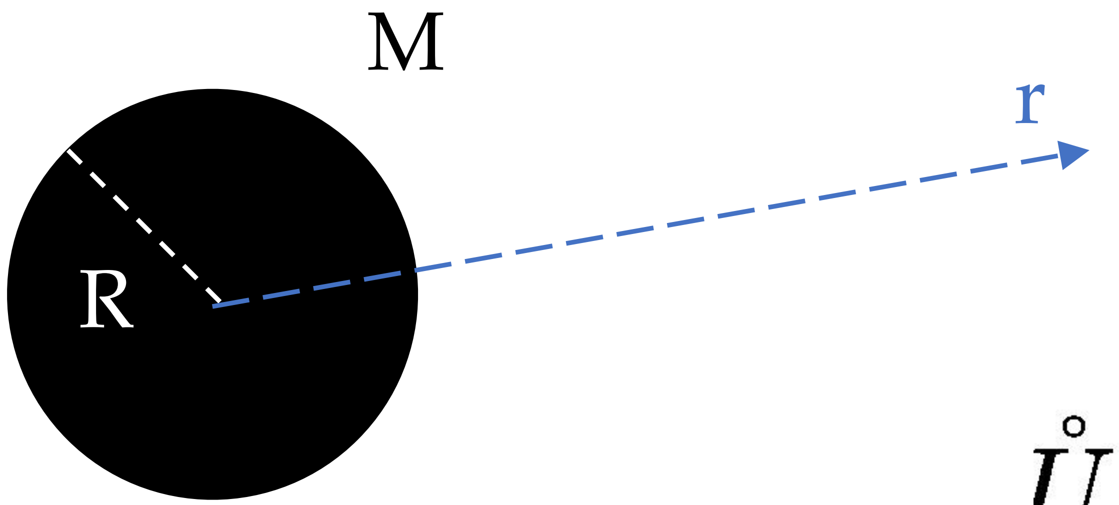


Earth's rotation drags bulges via friction -> tidal *lag*

Ocean tides due to Moon if Earth didn't rotate

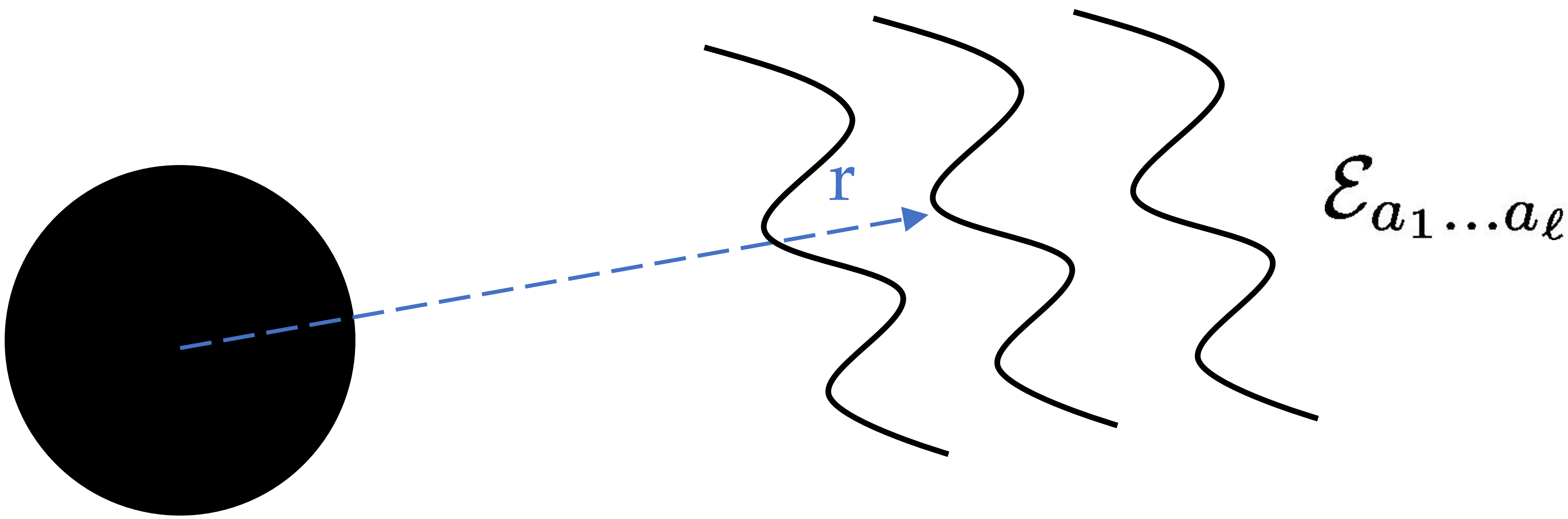
Tidal deformation in *Newtonian* Gravity

Gravitational potential of a compact body in *isolation*



$$\mathring{U} = \frac{M}{r}$$

Gravitational potential of some (weak & slowly-varying) external *tidal* field

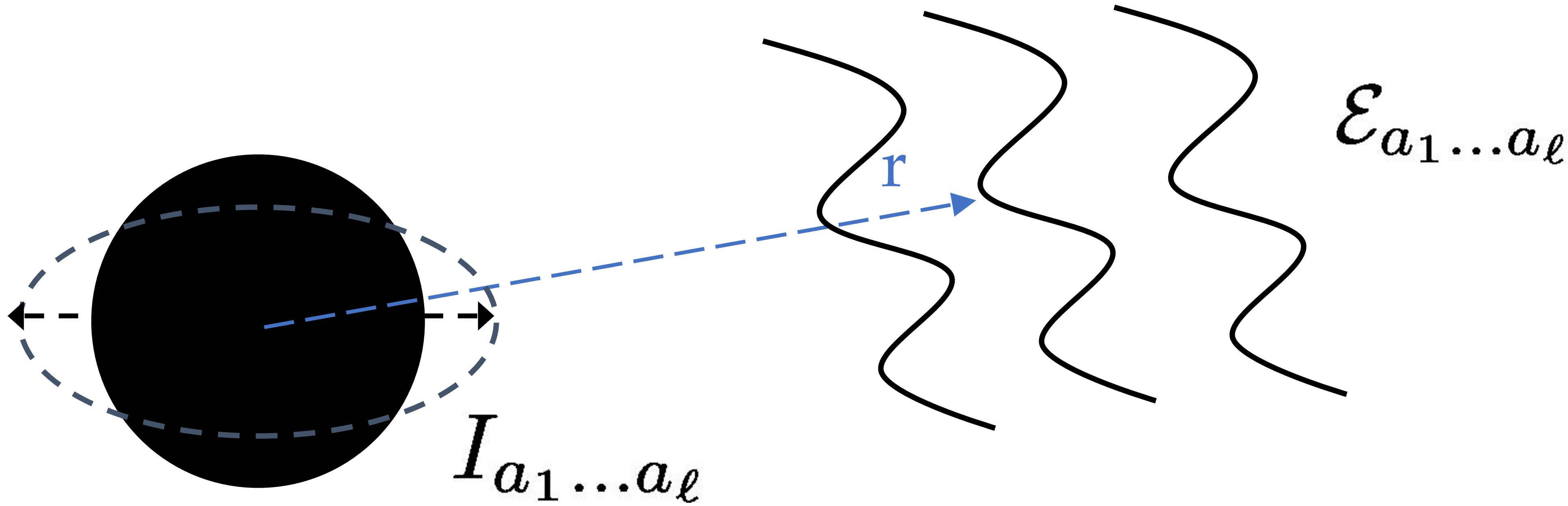


$$U^{\text{tidal}} = - \sum_{\ell=2}^{\infty} \frac{(\ell - 2)!}{\ell!} \, x^{a_1} \cdots x^{a_\ell} \; \mathcal{E}_{a_1 \dots a_\ell}(t)$$

Taylor series about origin

tidal moments

Deformation of non-rotating compact body in an external tidal field



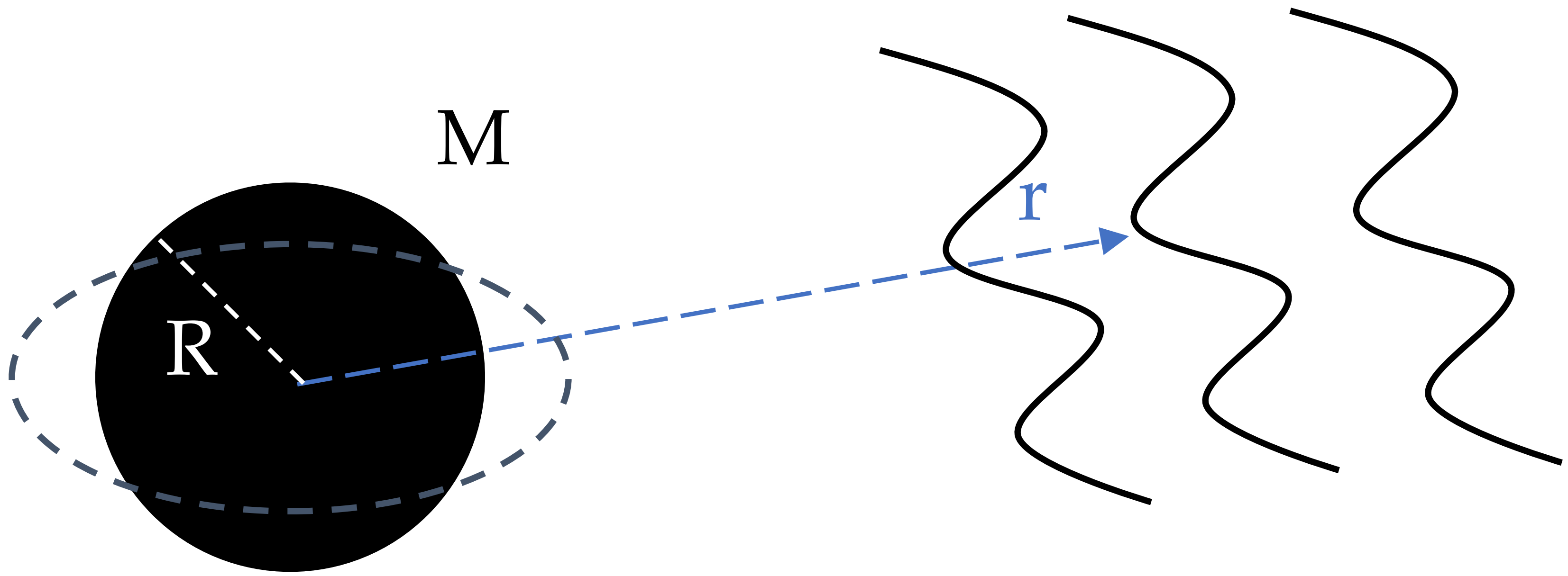
induced moments

Response potential:

$$U^{\rm resp} = \sum_{\ell=2}^{\infty} \frac{(2\ell-1)!!}{\ell!} \, \frac{x_1^a \cdots x_\ell^a \, I_{a_1\ldots a_\ell}(t)}{r^{2\ell+1}}$$

$$I_{a_1\ldots a_\ell} = \lambda_\ell \, \mathcal{E}_{a_1\ldots a_\ell}$$

tidal Love numbers (TLNs) of the compact body



Total gravitational potential by linearity (and decomposing into spherical harmonic $Y_{\ell m}(\theta,\varphi)$ modes)

$$U = \mathring{U} + U^{\text{tidal}} + U^{\text{resp}} =$$

$$\underbrace{\frac{M}{r}}_{\text{isolated}} - \sum_{\ell\geq 2}\sum_{m\leq|\ell|} \frac{(\ell-2)!}{\ell!}\,\mathcal{E}_{\ell m}\, r^{\ell}\left[\underbrace{1}_{\text{tidal}} + 2k_\ell \underbrace{\left(\frac{R}{r}\right)^{2\ell+1}}_{\text{response}}\right] Y_{\ell m}$$

dimensionless TLNs: $k_\ell \equiv -\dfrac{(2\ell-1)!!}{2(\ell-2)!}\,\dfrac{\lambda_\ell}{R^{2\ell+1}}$

It´s convenient to use a curvature Weyl scalar

$$\psi_0 = C_{\alpha\beta\gamma\delta}\ell^\alpha m^\beta \ell^\gamma m^\delta = \sum_{\ell,m} \psi_0^{\ell m}$$

(projection of Weyl tensor $C_{\alpha\beta\gamma\delta}$ on some null vectors ℓ^α and m^β)

In the Newtonian limit,

$$\lim_{c\to\infty} c^2\psi_0 = \text{2nd order differential operator on U}$$

$$\propto \mathcal{E}_{\ell m}\, r^{\ell-2}\left[1 + 2k_\ell \left(\frac{R}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}$$

tidal $\sim r^{\ell-2}$

response $\sim r^{-\ell-3}$

spin-weighted
spherical harm.

Love nums. in General Relativity

What about the TLNs in *General Relativity*?

For neutron stars, the TLNs: are not zero; depend on the eq. of state; LIGO constrains them

For BHs, the Weyl scalar ψ_0 satisfies the Teukolsky eq. for spin-2

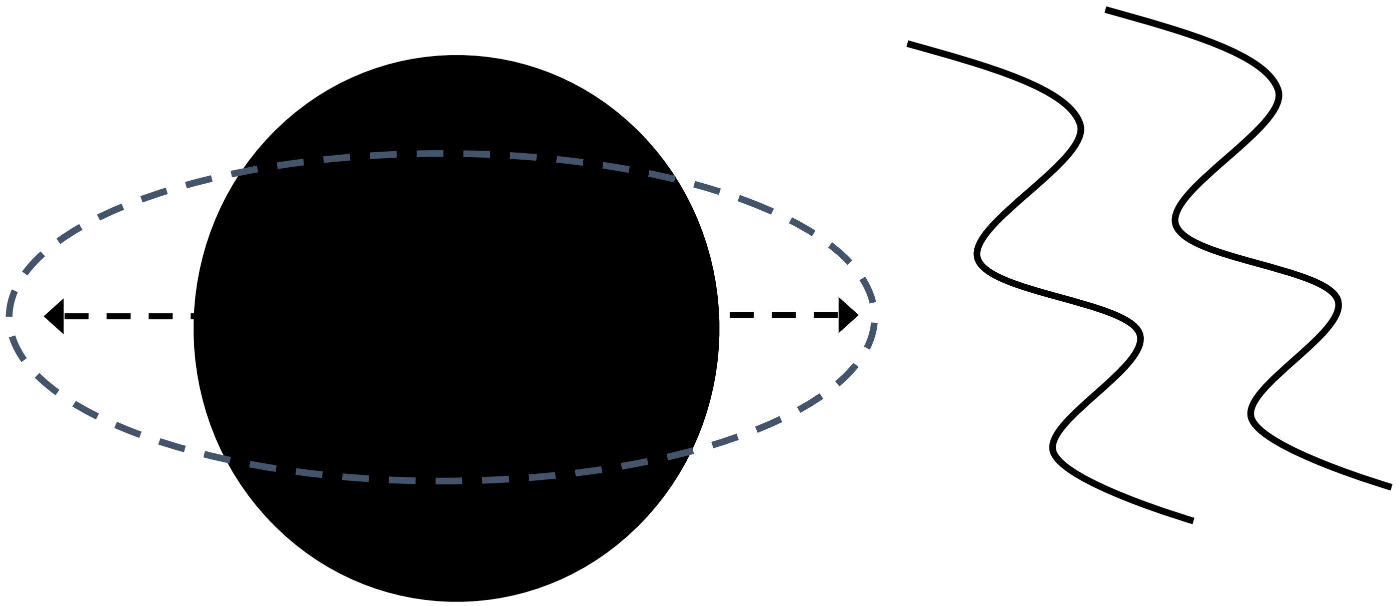
$$\hat{\mathcal{O}}\psi_0 = 0$$

and, in the case here of a *static* tidal field, its radial part satisfies the radial Teukolsky ODE for *zero-frequency* $\omega = 0$

$$\hat{\mathcal{O}}_r R_{\ell m \omega = 0} = 0$$

Tidal and induced moments on BHs

Two types of moments for the BH and for the tidal environment



electric-type:

$$\mathcal{E}_{i_1 \dots i_\ell} \propto C_{0\langle i_1|0|i_2;i_3\cdots i_\ell\rangle} \longrightarrow$$

magnetic-type:

$$\mathcal{B}_{i_1 \dots i_\ell} \propto \varepsilon_{jk\langle i_1} C_{i_2|0jk|;i_3\cdots i_\ell\rangle}$$

Weyl scalar of BH: $\psi_0 = \mathring{\psi}_0 + \psi_0^{\mathrm{resp}} \longrightarrow g_{\alpha\beta} \equiv \mathring{g}_{\alpha\beta} + h_{\alpha\beta}^{\mathrm{resp}} \longrightarrow$

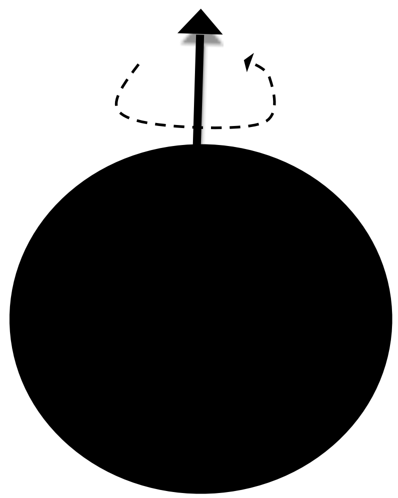
Geroch-Hansen moments (*coordinate independent*):

mass-type: $$M_{i_1 \dots i_\ell} = \underbrace{\mathring{M}_{i_1 \dots i_\ell}}_{\text{background}} + \underbrace{\delta M_{i_1 \dots i_\ell}}_{\text{induced}}$$

current-type: $$S_{i_1 \dots i_\ell} = \mathring{S}_{i_1 \dots i_\ell} + \delta S_{i_1 \dots i_\ell}$$

Moments of Kerr Black Hole

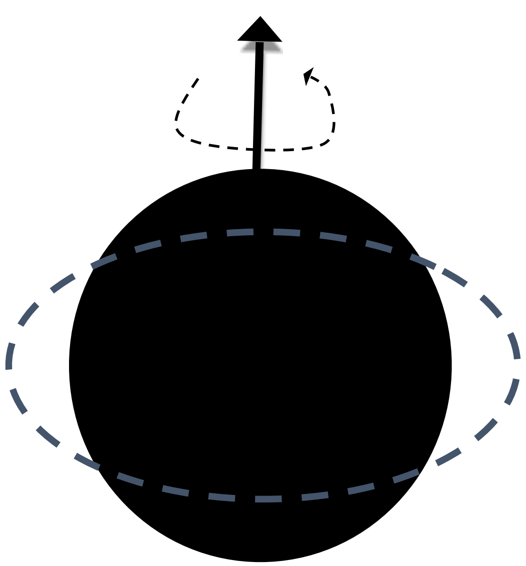
Modes of the Geroch-Hansen moments of (*isolated*) Kerr BH:

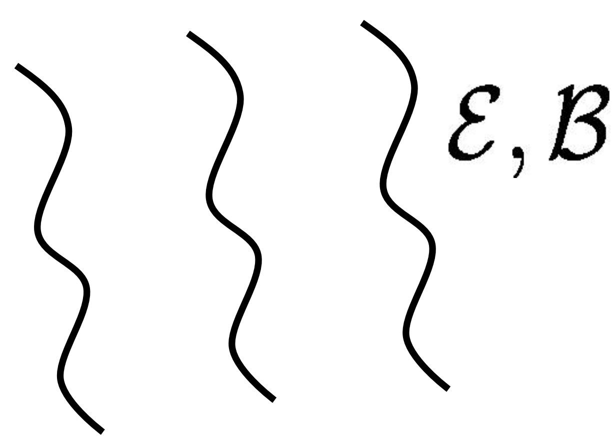


$$\mathring{M}_\ell + i\,\mathring{S}_\ell = M\,(i\,a)^\ell$$

mass   ang. mom. (per unit mass)

What about the moments of Kerr in an external *tidal environment*?





[Le Tiec, MC, Franzin' 21]

Kerr TLNs

We calculated the Weyl tensor modes of perturbed Kerr. For large-r:

$$\psi_0^{\ell m} \underset{r\to\infty}{\sim} \left[\mathcal{E}_{\ell m} + i\,\frac{\ell+1}{3}\,\mathcal{B}_{\ell m}\right] r^{\ell-2}\left[1 + 2k_{\ell m}\left(\frac{2M}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}$$

tidal

response

"Newtonian" TLNs

$$\gamma \equiv \frac{a}{2\sqrt{M^2-a^2}}$$

$$k_{\ell m} \equiv -i\,m\,\gamma\,\left(1-(a/M)^2\right)^{\ell+1/2}\,\frac{(\ell-2)!(\ell+2)!}{2(2\ell)!(2\ell+1)!}\prod_{n=1}^{\ell}(n^2+4m^2\gamma^2)$$

Cf. the modes in the Newtonian theory:

$$\lim_{c\to\infty} c^2\psi_0^{\ell m} \propto \mathcal{E}_{\ell m}\, r^{\ell-2}\left[1 + 2k_\ell\left(\frac{R}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}$$

"Newtonian" TLNs:

$$k_{\ell m} \equiv -i\, m\, \gamma\, \left(1-(a/M)^2\right)^{\ell+1/2} \frac{(\ell-2)!(\ell+2)!}{2(2\ell)!(2\ell+1)!} \prod_{n=1}^{\ell} (n^2+4m^2\gamma^2)$$

$$\gamma \equiv \frac{a}{2\sqrt{M^2-a^2}}$$

(1) They are *zero* for:

(i) rotating BH in axisymmetric tidal field ($m=0$)

(ii) Schwarzschild BH ($a=0$) in agreement with Damour & Nagar´09

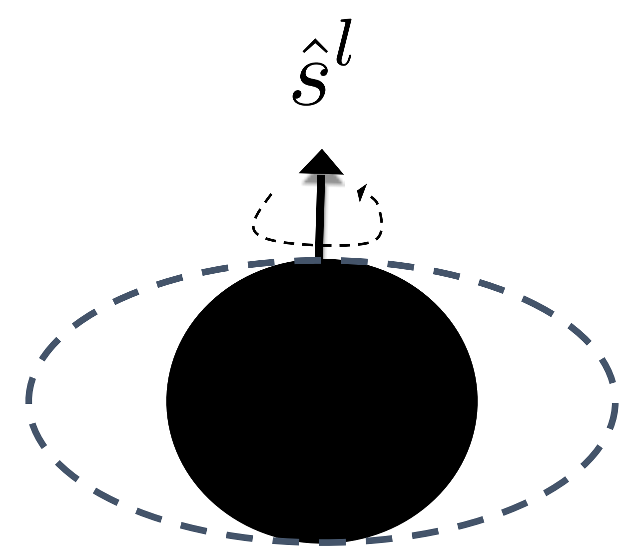
(2) For, e.g., $a=0.1M$, the (dimensionless) TLNs are

$|k_{2,\pm 2}| \approx 2\times 10^{-3}$ $\longrightarrow$ Kerr BHs are "rigid"

(3) TLNs purely *imaginary* $\longrightarrow$ the BH tidal bulge is rotated by 45° in relation to the tidal perturbation (tidal *lag*)

Induced moments

We calculated the tidally-induced quadrupole moments to linear order in a/M :



$$\delta M_{ij} \doteq \lambda_{ijkl}\,\mathcal{E}^{kl} \doteq \frac{16}{45}\,a\,M^4\,\mathcal{E}^k{}_{(i}\,\varepsilon_{j)kl}\,\hat{s}^l$$

$$\delta S_{ij} \doteq \lambda_{ijkl}\,\mathcal{B}^{kl} \doteq \frac{16}{45}\,a\,M^4\,\mathcal{B}^k{}_{(i}\,\varepsilon_{j)kl}\,\hat{s}^l$$

where the tidal Love *tensor* is $\lambda_{ij\langle kl\rangle} \doteq -\dfrac{16}{45}\,a\,M^4\,\delta_{(i|\langle k}\,\varepsilon_{l\rangle|j)q}\,\hat{s}^q$

So rotating (as opposed to non-rotating) BHs *deform* in a (static) tidal field!

This effect is purely dissipative (Chia´21; Goldberger, Li & Rothstein´21):

we showed that the above induced moments yields the *tidal torquing* (ie, change of BH angular momentum) in Kerr (Poisson'04)

1. BH Perturbations

2. Stability of Kerr

3. Mode stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Tidal gravitational deformation (Love)

5. Conclusion

Conclusion

- Kerr: mode stability proven; in *extremal* case, blow-up at late times of normal derivatives on the EH (due to a new branch cut)

  Open question: prove *full* linear stability under grav. perturbations

- SCC: violated for unphysical parameters in Kerr-Newman-dS

  Open questions: can SCC be saved in Kerr-Newman-de Sitter (e.g., by nonlinearities or quantum effects)? can SCC be generally proven?

- Love: rotating BHs *deform* in a static tidal field

Mange tak!