

Walking on a bed of nails: effect of dark matter discreteness on light propagation

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Beyond null geodesics



- In cosmology, light is usually taken to move on null geodesics.
- This follows from geometrical optics, which assumes that photon energy is larger than any other scale, including spacetime curvature.
- Dark matter particles give density spikes, so the curvature is not smooth.



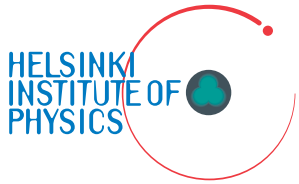
Bed of nails



- If dark matter particle of mass m is localised within a Compton wavelength, it has
 - energy density $\sim 10^{-3} m^4$
 - curvature $\sim 10^{-3} m^4/M_{Pl}^2$.
- If $m \gtrsim 10^4$ GeV, the curvature scale is larger than CMB photon energy.
- Instead of using null geodesics, we have to go back to the equation of motion.



Action and equation of motion



- Light propagation is governed by the action

$$S = -\frac{1}{4} \int d^4x \sqrt{-g} F_{\alpha\beta} F^{\alpha\beta} .$$

- Variation gives the equation of motion

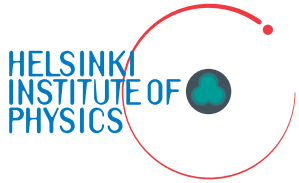
$$\square A^\alpha - \nabla^\alpha \nabla_\beta A^\beta - R^\alpha{}_\beta A^\beta = 0 .$$

- Lorenz gauge: $\nabla_\alpha A^\alpha = 0$.



Post-geometrical approximation

$$\square A^\alpha - R^\alpha{}_\beta A^\beta = 0$$



- Local plane wave form:

$$A^\alpha(x) = \sum_{n=0}^{\infty} \text{Re} \left[A_n^\alpha(x) e^{iS(x)/\epsilon} \right] \epsilon^n .$$

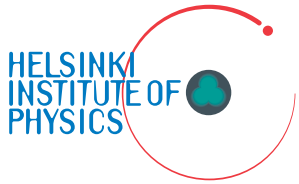
- In geometrical optics only the phase is assigned the factor $1/\epsilon$.
- We also assign $1/\epsilon^2$ to the curvature.
- As in geometrical optics, we keep terms up to second order in $1/\epsilon$.



Curvature-induced mass

$$M^2 = \frac{\rho - p}{2M_{\text{Pl}}^2}$$

$$\square A^\alpha - R^\alpha{}_\beta A^\beta = 0$$



- Assuming ideal fluid matter, we get ($k_\alpha = \partial_\alpha S$)

$$k^2 = -\frac{\rho - p}{2M_{\text{Pl}}^2} \iff E^2 = \vec{k}^2 + M^2$$

$$k^\beta \nabla_\beta k_\alpha = -\frac{1}{4M_{\text{Pl}}^2} \partial_\alpha (\rho - p) .$$

- Light gets curvature scale mass.
- Light is pushed off-geodesic by density gradient.



Clumpy matter



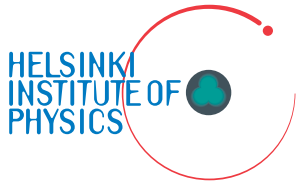
- Model dark matter particles as isolated Gaussian clumps:

$$\rho = m \sum_n \frac{1}{(2\pi)^{3/2} \lambda_c^3} e^{-\frac{r_n^2}{2\lambda_c^2}}$$

- Space inside the particles does not expand.
- Light with $E < M$ cannot enter dark matter particles, and our approximation is not valid.



Redshift



- Redshift is given by photon energy: $1 + z = \frac{E_s}{E_o}$.
- Along the light ray, we have

$$\frac{dE}{ds} = -k^\beta \nabla_\beta (u^\alpha k_\alpha) = -k^\alpha k^\beta \nabla_\beta u_\alpha - \frac{1}{2} u^\alpha \nabla_\alpha k^2 ,$$

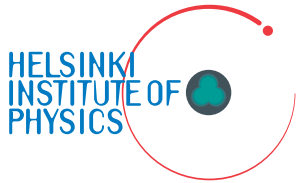
so integrating gives

$$\ln(1 + z) = \int_{s_s}^{s_o} ds E^{-1} \left[v^2 \left(\frac{1}{3} \theta + \sigma_{\alpha\beta} e^\alpha e^\beta \right) - \frac{v}{\rho + p} e^\alpha \partial_\alpha p - \frac{\dot{\rho} - \dot{p}}{4M_{Pl}^2 E^2} \right]$$

- Here v is photon velocity.



Negligible effect on redshift



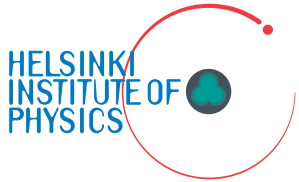
- Redshift is given by:

$$\ln(1+z) = \int_{s_s}^{s_o} ds E^{-1} \left[v^2 \left(\frac{1}{3} \theta + \sigma_{\alpha\beta} e^\alpha e^\beta \right) - \frac{v}{\rho + p} e^\alpha \partial_\alpha p - \frac{\dot{\rho} - \dot{p}}{4M_{\text{Pl}}^2 E^2} \right]$$

- The effect of curvature spikes is negligible, because particles occupy tiny fraction of volume.
- Distance involves (derivative of the density)².



Angular diameter distance



- Angular diameter distance D_A is determined by the area expansion rate $\tilde{\theta}$ as

$$D_A \propto \exp\left(\frac{1}{2} \int ds \tilde{\theta}\right) .$$

- We get $\tilde{\theta}$ from $\nabla_\beta k_\alpha = \frac{1}{2} \tilde{\theta} \tilde{h}_{\alpha\beta} + \tilde{\sigma}_{\alpha\beta} + A_{\alpha\beta}$.

- Recall that $k^\beta \nabla_\beta k_\alpha = -\frac{1}{4M_{\text{Pl}}^2} \partial_\alpha (\rho - p)$.

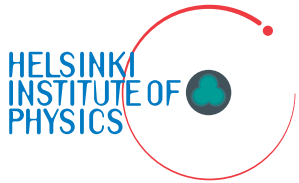
- As with the redshift, we integrate $\frac{d\tilde{\theta}}{ds}$.



Large effect on distance

$$M^2 = \frac{\rho}{2M_{\text{Pl}}^2}$$

$$D_A \propto \exp\left(\frac{1}{2} \int ds \tilde{\theta}\right)$$



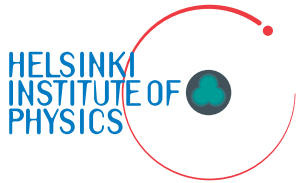
- The leading correction to the angular diameter distance is given by

$$\int ds \tilde{\theta} = \int ds \int ds' \left\{ -\frac{1}{2} \tilde{\theta}^2 - 2\tilde{\sigma}^2 - E^2 \frac{\langle \rho \rangle}{M_{\text{Pl}}^2} \left[1 + \frac{5}{128\sqrt{2}\pi^2} \frac{m^2}{E^2} \frac{M^2}{E^2} - \frac{13}{24\sqrt{2}} \frac{M^2}{E^2} + \frac{5}{72\sqrt{3}} \left(\frac{M^2}{E^2} \right)^2 \right] \right\}$$

- Terms on the second line are at most 0.4 and 0.04, but the last term on the first line can be large.



Upper limit on dark matter mass



- The corrected distance equation is

$$\frac{d^2 D_A}{ds^2} = -4\pi G_N \langle \rho \rangle [1 + \alpha (E/E_o)^{-4}] D_A ,$$

$$\text{where } \alpha \equiv \frac{5}{8192\pi^{13/2}} \frac{m^6}{M_{\text{Pl}}^2 E_o^4} .$$

- This could give an alternative to dark energy and/or explain the H_o tension if α were negative.
- As it is, it gives the constraint $m < 100$ MeV.



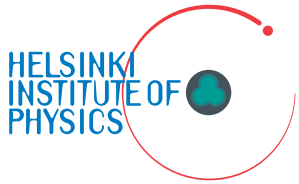
Two caveats



- Is the post-geometrical approximation valid?
 - Intermediate region, local plane wave form.
- Is the particle size correct?
 - Due to spreading of wavefunction, particle size grows linearly in time, but decoherence localises it.
 - Need a realistic quantum mechanical treatment.



Spiky and thorny



- Dark matter particles lead to spikes in curvature.
 - Photons get a gravitational mass and are pushed off geodesics.
- No change in redshift, possibly big change in distance.
 - Constraint on dark matter mass: $m < 100 \text{ MeV}$.
- Need to understand treatment of light propagation and spread of dark matter particle wavefunction.