Walking on a bed of nails:
effect of dark matter discreteness on light propagation

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Beyond null geodesics

- In cosmology, light is usually taken to move on null geodesics.
- This follows from geometrical optics, which assumes that photon energy is larger than any other scale, including spacetime curvature.
- Dark matter particles give density spikes, so the curvature is not smooth.

Bed of nails

- If dark matter particle of mass m is localised within a Compton wavelength, it has
  - energy density $\sim 10^{-3}\, m^4$
  - curvature $\sim 10^{-3}\, m^4/M_{Pl}^2$.
- If $m \gtrsim 10^4$ GeV, the curvature scale is larger than CMB photon energy.
- Instead of using null geodesics, we have to go back to the equation of motion.

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Action and equation of motion

- Light propagation is governed by the action

$$S = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}F_{\alpha\beta}F^{\alpha\beta} \ .$$

- Variation gives the equation of motion

$$\Box A^\alpha - \nabla^\alpha\nabla_\beta A^\beta - R^\alpha{}_\beta A^\beta = 0 \ .$$

- Lorenz gauge: $\nabla_\alpha A^\alpha = 0$.

Post-geometrical approximation

$$\Box A^{\alpha} - R^{\alpha}{}_{\beta} A^{\beta} = 0$$

- Local plane wave form:

$$A^{\alpha}(x) = \sum_{n=0}^{\infty} \mathrm{Re}\left[A_n^{\alpha}(x) e^{iS(x)/\epsilon}\right] \epsilon^{n} \ .$$

- In geometrical optics only the phase is assigned the factor $1/\varepsilon$.
- We also assign $1/\varepsilon^2$ to the curvature.
- As in geometrical optics, we keep terms up to second order in $1/\varepsilon$.

Curvature-induced mass

$$M^2 = \frac{\rho - p}{2M_{\rm Pl}^2} \qquad \Box A^\alpha - R^\alpha{}_\beta A^\beta = 0$$

- Assuming ideal fluid matter, we get $(k_\alpha = \partial_\alpha S)$

$$k^2 = -\frac{\rho - p}{2M_{\rm Pl}^2} \quad \Longleftrightarrow \quad E^2 = \vec{k}^2 + M^2$$

$$k^\beta \nabla_\beta k_\alpha = -\frac{1}{4M_{\rm Pl}^2}\partial_\alpha(\rho - p)\ .$$

- Light gets curvature scale mass.
- Light is pushed off-geodesic by density gradient.

Clumpy matter

- Model dark matter particles as isolated Gaussian clumps:

$$\rho = m \sum_n \frac{1}{(2\pi)^{3/2} \lambda_{\mathrm{c}}^3} e^{-\frac{r_n^2}{2\lambda_{\mathrm{c}}^2}}$$

- Space inside the particles does not expand.

- Light with *E*<*M* cannot enter dark matter particles, and our approximation is not valid.

Redshift

- Redshift is given by photon energy: $1+z=\frac{E_{\rm s}}{E_{\rm o}}$.
- Along the light ray, we have

$$\frac{\mathrm{d}E}{\mathrm{d}s}=-k^\beta\nabla_\beta\left(u^\alpha k_\alpha\right)=-k^\alpha k^\beta\nabla_\beta u_\alpha-\frac{1}{2}u^\alpha\nabla_\alpha k^2 \ ,$$

so integrating gives

$$\ln\left(1+z\right)=\int_{s_{\rm s}}^{s_{\rm o}}\mathrm{d}s E^{-1}\left[v^2\left(\frac{1}{3}\theta+\sigma_{\alpha\beta}e^\alpha e^\beta\right)-\frac{v}{\rho+p}e^\alpha\partial_\alpha p-\frac{\dot\rho-\dot p}{4M_{\rm Pl}^2E^2}\right]$$

- Here *v* is photon velocity.

Negligible effect on redshift

- Redshift is given by:

$$\ln(1+z) = \int_{s_\mathrm{s}}^{s_\mathrm{o}} \mathrm{d}s E^{-1} \left[v^2 \left(\frac{1}{3}\theta + \sigma_{\alpha\beta} e^\alpha e^\beta \right) - \frac{v}{\rho + p} e^\alpha \partial_\alpha p - \frac{\dot{\rho} - \dot{p}}{4 M_{\mathrm{Pl}}^2 E^2} \right]$$

- The effect of curvature spikes is negligible, because particles occupy tiny fraction of volume.

- Distance involves (derivative of the density)^2.

Angular diameter distance

- Angular diameter distance D_A is determined by the area expansion rate $\tilde{\theta}$ as
$$D_A \propto \exp\left(\frac{1}{2}\int \mathrm{d}s\tilde{\theta}\right) \; .$$
- We get $\tilde{\theta}$ from $\nabla_\beta k_\alpha = \frac{1}{2}\tilde{\theta}\tilde{h}_{\alpha\beta} + \tilde{\sigma}_{\alpha\beta} + A_{\alpha\beta} \; .$
- Recall that $k^\beta \nabla_\beta k_\alpha = -\frac{1}{4M_{\mathrm{Pl}}^2}\partial_\alpha(\rho - p) \; .$
- As with the redshift, we integrate $\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}s} \; .$

Large effect on distance

$$M^2 = \frac{\rho}{2M_{\mathrm{Pl}}^2} \qquad D_A \propto \exp\left(\frac{1}{2}\int \mathrm{d}s\tilde{\theta}\right)$$

- The leading correction to the angular diameter distance is given by

$$\int \mathrm{d}s\tilde{\theta} = \int \mathrm{d}s \int \mathrm{d}s' \left\{ -\frac{1}{2}\tilde{\theta}^2 - 2\tilde{\sigma}^2 - E^2 \frac{\langle\rho\rangle}{M_{\mathrm{Pl}}^2} \left[1 + \frac{5}{128\sqrt{2}\pi^2} \frac{m^2}{E^2} \frac{M^2}{E^2} - \frac{13}{24\sqrt{2}} \frac{M^2}{E^2} + \frac{5}{72\sqrt{3}} \left(\frac{M^2}{E^2} \right)^2 \right] \right\}$$

- Terms on the second line are at most 0.4 and 0.04, but the last term on the first line can be large.

Upper limit on dark matter mass

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- The corrected distance equation is

$$\frac{\mathrm{d}^2 D_A}{\mathrm{d}s^2} = -4\pi G_\mathrm{N} \langle \rho \rangle \left[1 + \alpha (E/E_\mathrm{o})^{-4}\right] D_A \ ,$$

where $\alpha \equiv \frac{5}{8192\pi^{13/2}} \frac{m^6}{M_\mathrm{Pl}^2 E_\mathrm{o}^4}$.

- This could give an alternative to dark energy and/or explain the H_0 tension if α were negative.

- As it is, it gives the constraint $m < 100$ MeV.

Two caveats

- Is the post-geometrical approximation valid?
  - Intermediate region, local plane wave form.
- Is the particle size correct?
  - Due to spreading of wavefunction, particle size grows linearly in time, but decoherence localises it.
  - Need a realistic quantum mechanical treatment.

Spiky and thorny

- Dark matter particles lead to spikes in curvature.
  - Photons get a gravitational mass and are pushed off geodesics.

- No change in redshift, possibly big change in distance.
  - Constraint on dark matter mass: $m < 100$ MeV.

- Need to understand treatment of light propagation and spread of dark matter particle wavefunction.