

Probing subatomic physics with gravitational waves from neutron star binary inspirals

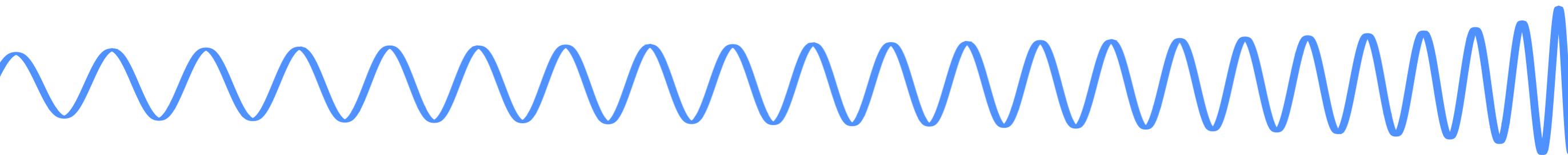
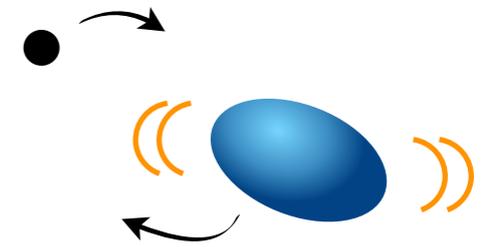
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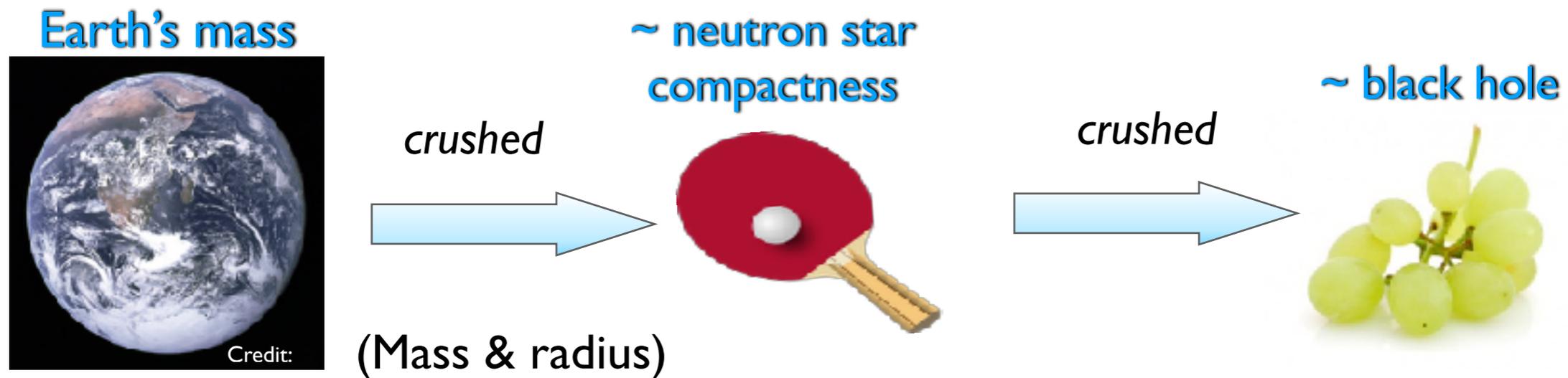
Overview

- Gravitational waves (GWs) now available as unique probes for fundamental physics
 - e.g. dense matter in neutron stars
- Extracting this information from the data requires theoretical understanding & accurate modeling
- Focus in this talk: tidal effects during a binary inspiral
 - Main characteristic parameters
 - Effects on the dynamics and GWs
- Outlook to remaining challenges and future prospects



Neutron stars (NSs)

- ▶ Gravity compresses matter to up to **several times nuclear density**
- ▶ Large extrapolation from known physics



- ▶ Thousands observed to date, some **masses** $> 2 M_{\odot}$
- ▶ **Quantum** pressure (neutron degeneracy) can only support **up to** $\sim 0.7 M_{\odot}$

- ▶ Unique window onto **strongly-interacting** subatomic matter

[Oppenheimer & Volkoff 1939]

Conjectured NS structure

NS matter ranges over nearly 10 orders of magnitude in density: rich variety of physics

[density of iron $\sim 10 \text{ g/cm}^3$]

crust $\sim km$

Lattice of neutron rich nuclei

10^{10} times stronger than steel

free neutrons

$\sim 10^6 \text{ g/cm}^3$ inverse β -decay

$\sim 10^{11} \text{ g/cm}^3$ neutron drip

outer core

uniform liquid (neutron superfluid,
superconducting protons, electrons, muons)

deep core

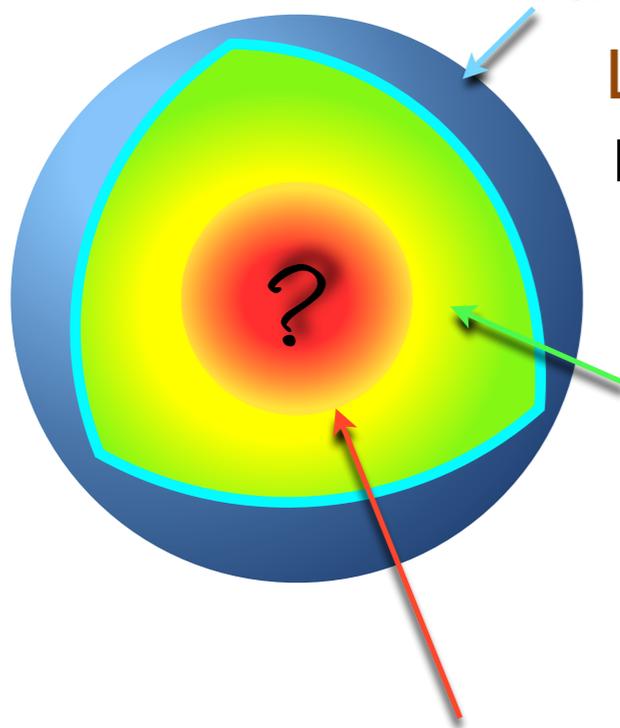
$\sim \text{few} \times 10^{14} \text{ g/cm}^3$

$\approx 2x$ nuclear density, nucleons overlap -

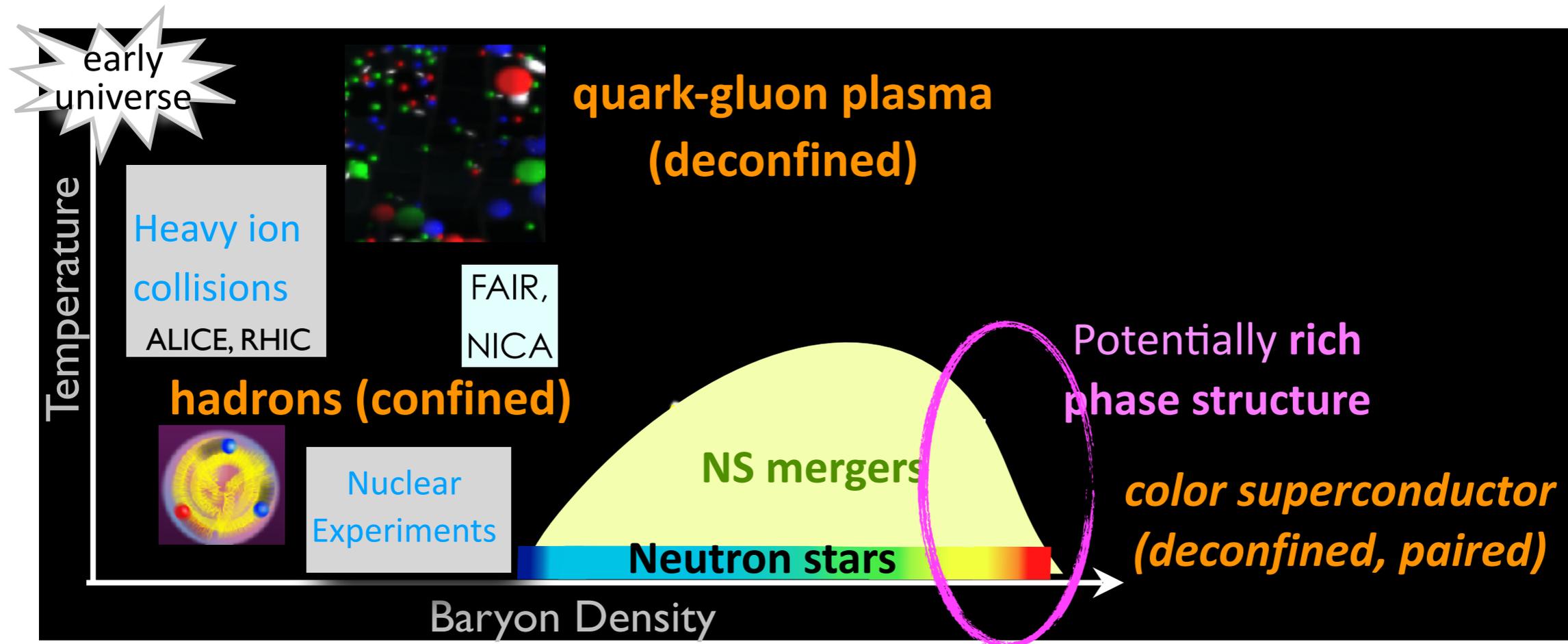
new degrees of freedom relevant

deconfined quarks?

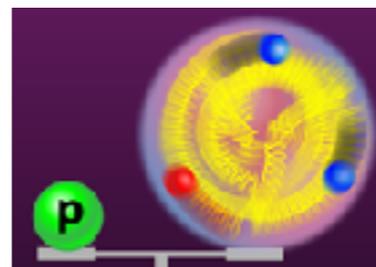
intermediate exotic condensates (hyperons, kaons, pions, ...)?



Neutron stars as QCD labs

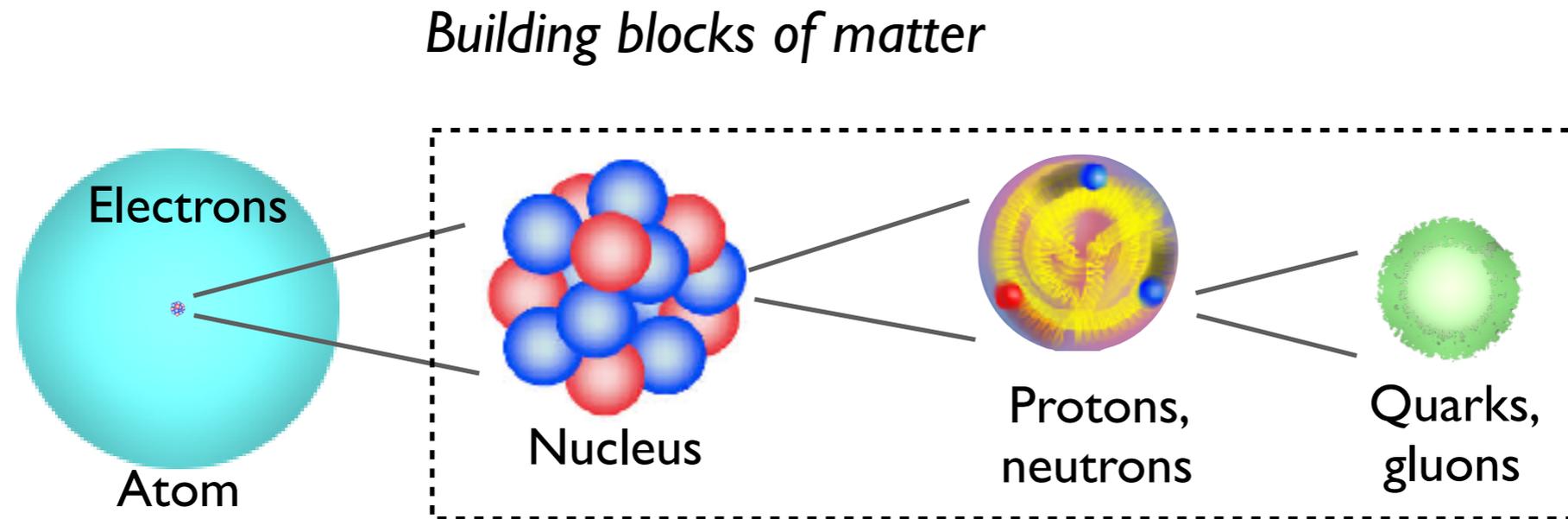


- Characterize phases of QCD, probe quark deconfinement
- Deeper understanding of strong interactions, their unusual properties
 - asymptotic freedom
 - Vacuum (condensate) effects



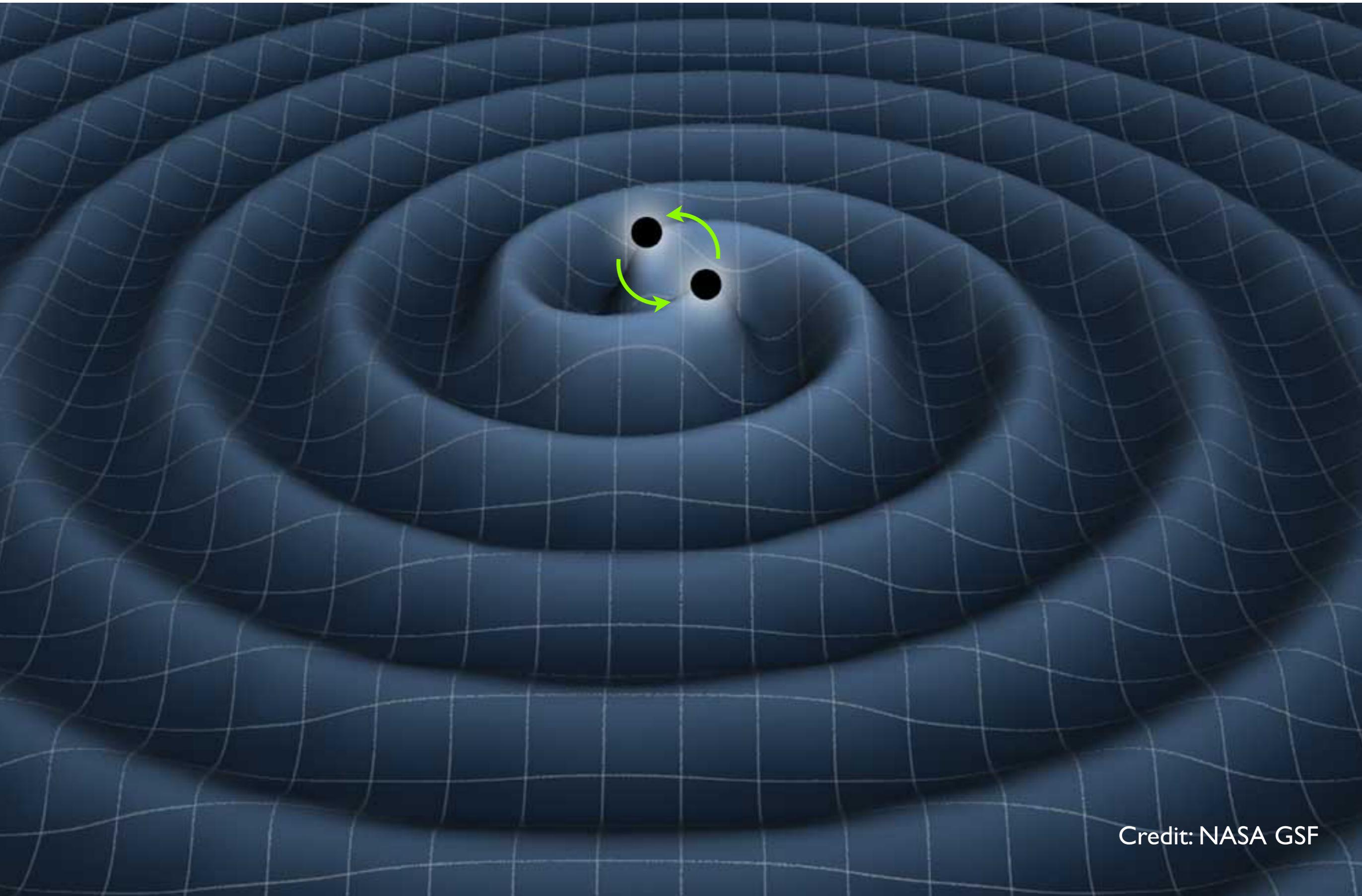
proton mass: ~ 938 MeV
only $\sim 1\%$ due to Higgs

NSs as labs for emergent structural complexity

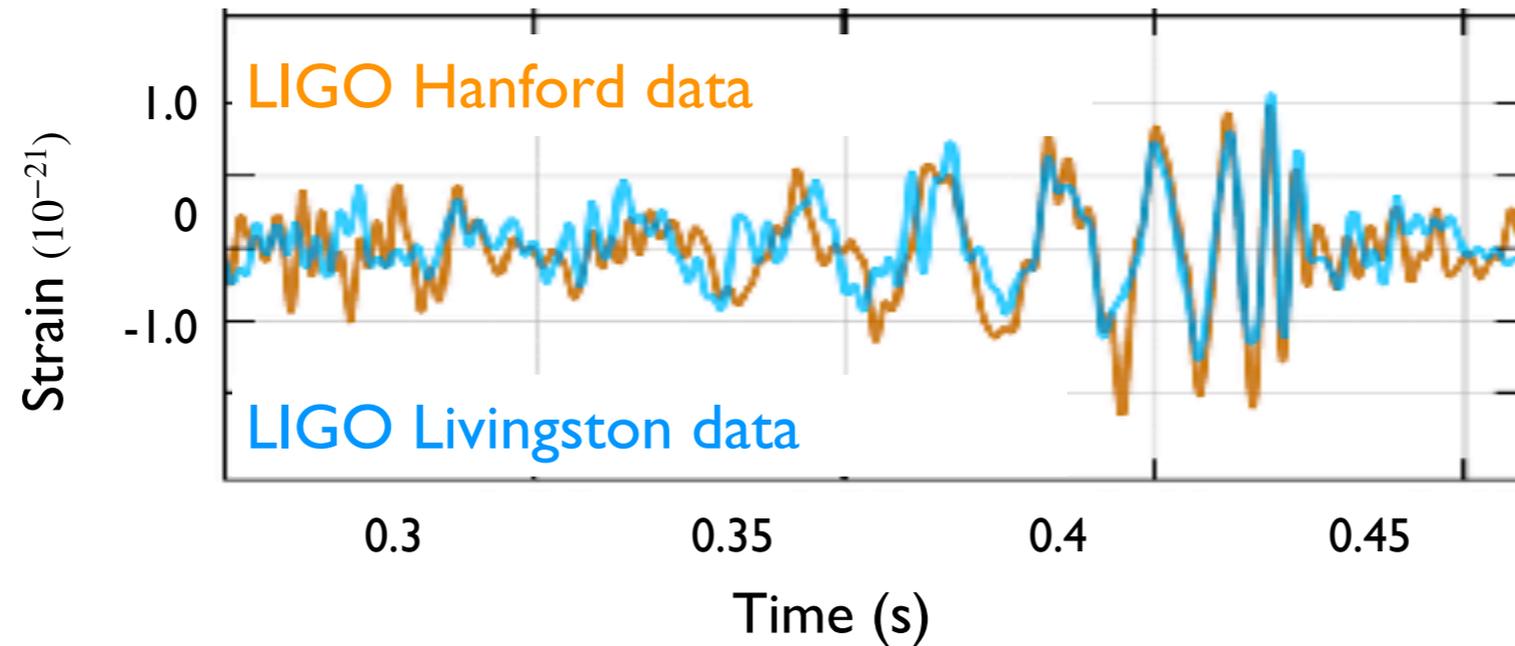


- **Collective** phenomena, **multi-body interactions**
- Effects of the **excess of neutrons** over protons (isospin asymmetry)?
- How do nucleons and their quarks and gluons **assemble and interact** to create the structure of matter?

Gravitational waves from compact-object binary systems

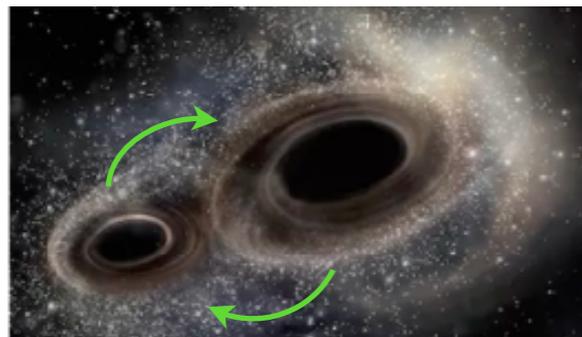


Models are essential to detect and interpret GW signals

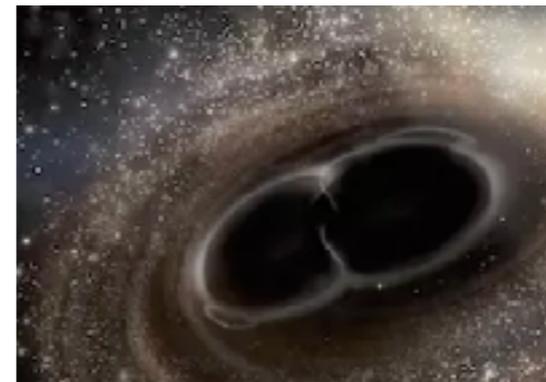


The first detection:
GW150914 data

Cross-correlating the data with **theoretical models** revealed:
it was the signal from a **black hole merger**



Two black holes of ~ 30 sun masses
Orbiting at 50,000km/sec ...

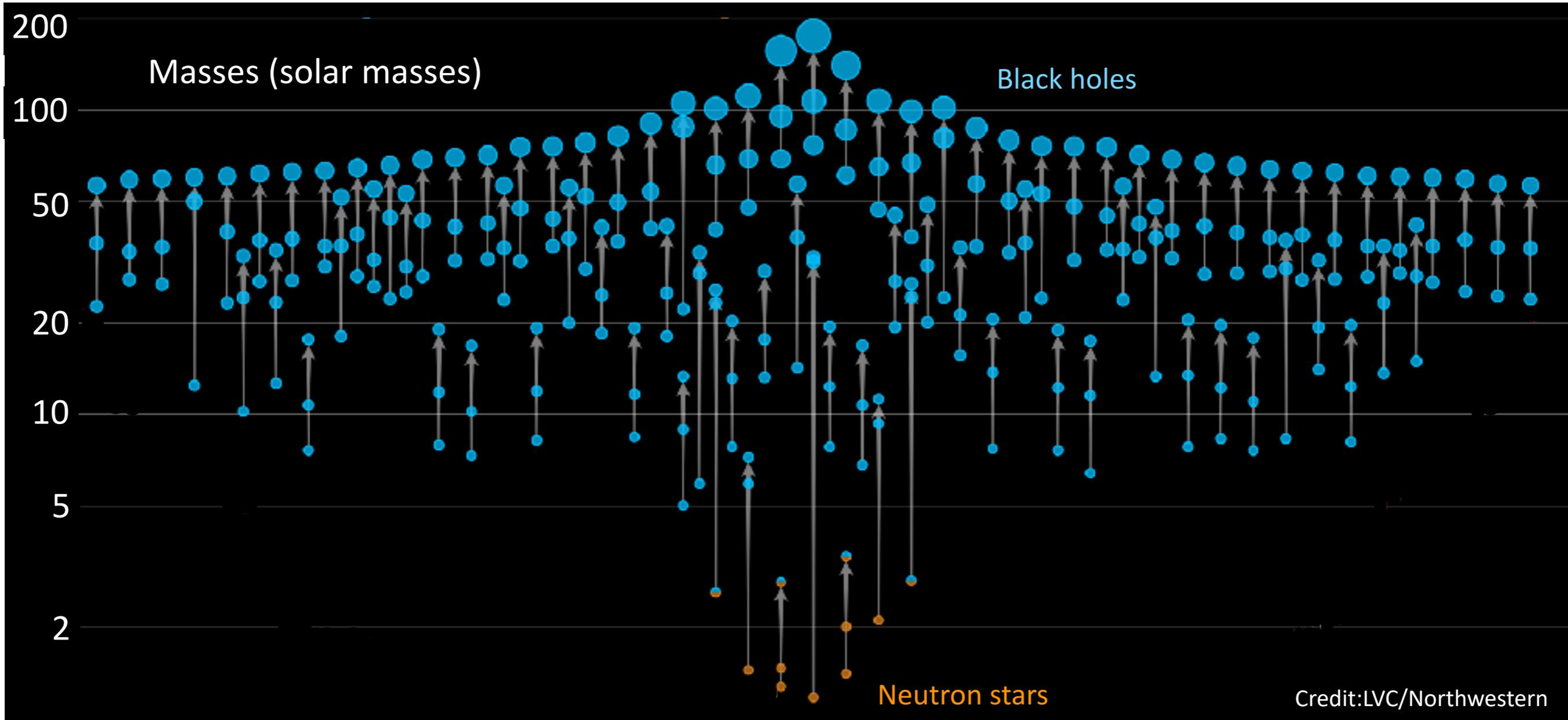


Spiral together until
they collide ...



... and form a single
black hole remnant

Census of binary mergers measured to date



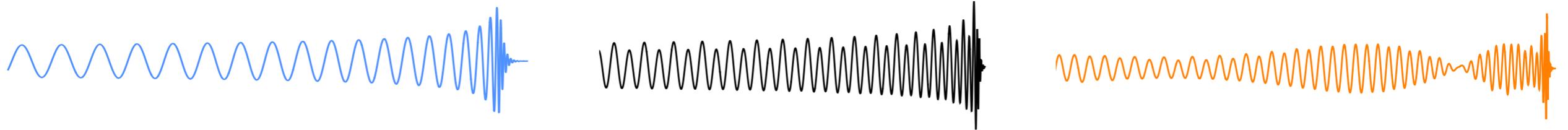
2017 Nobel prize

R. Weiss, K. Thorne, B. Barish



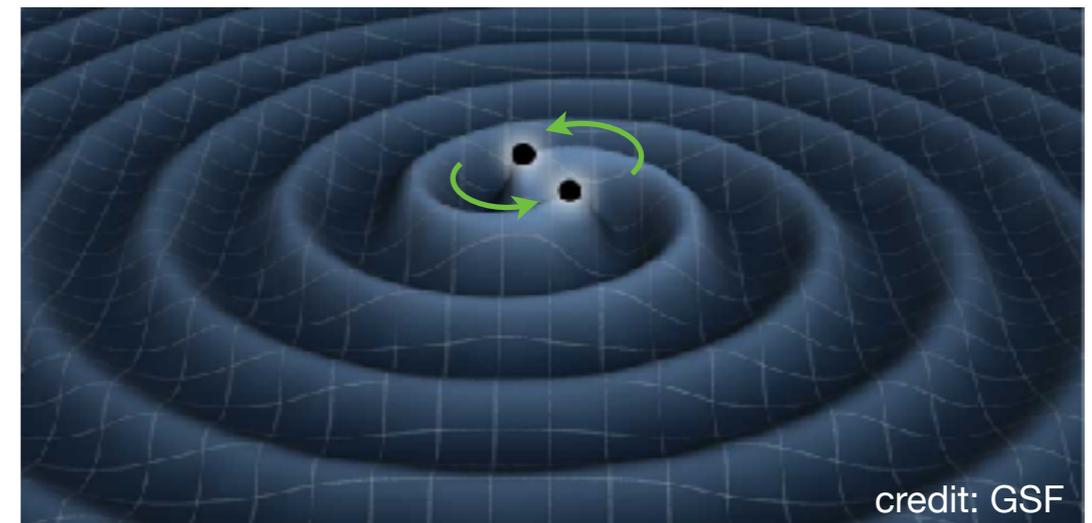
GW measurements (binary systems)

- ▶ Details of the waveforms encode fundamental source properties (masses, spins, ...)



- ▶ Measurements cross-correlate millions of **template models** with the data to determine the source parameters
- ▶ Computation of template waveforms is very challenging:

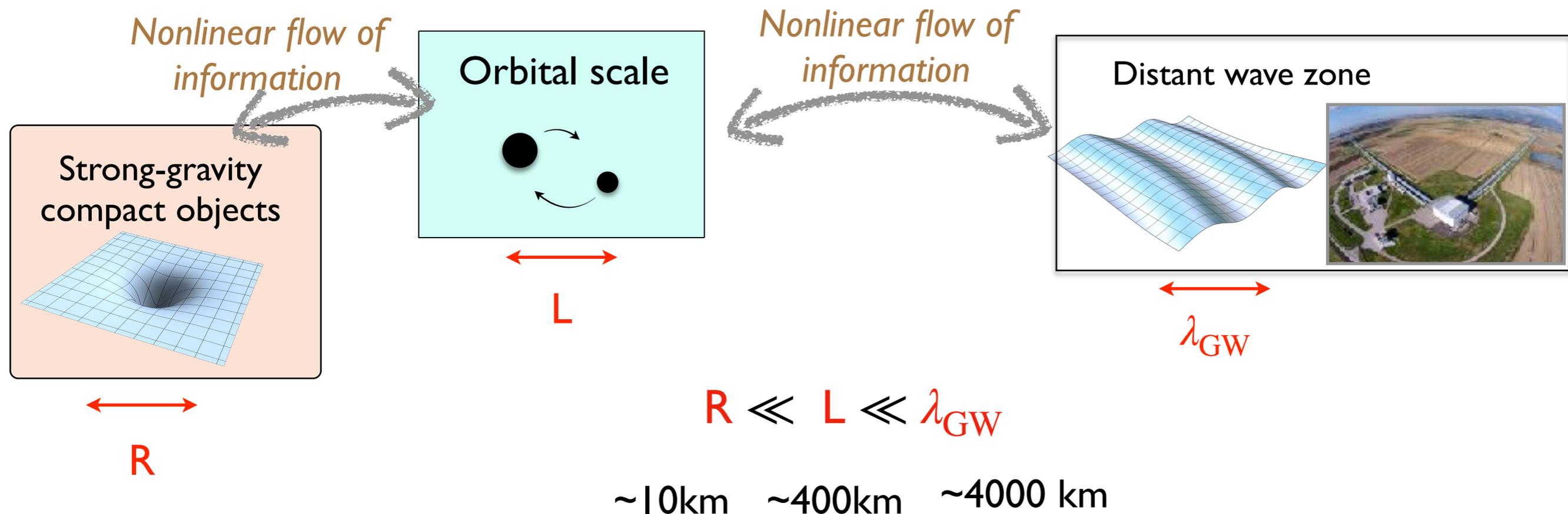
Must solve the **nonlinear Einstein Field equations** coupled with the **matter equations of motion** for the dynamical spacetime



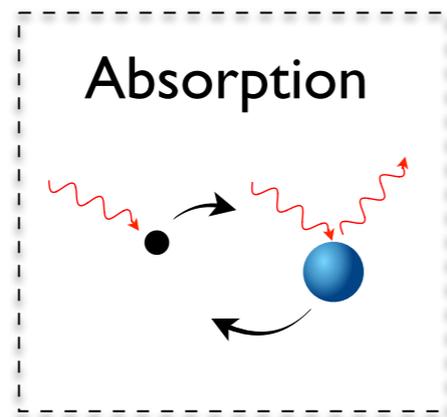
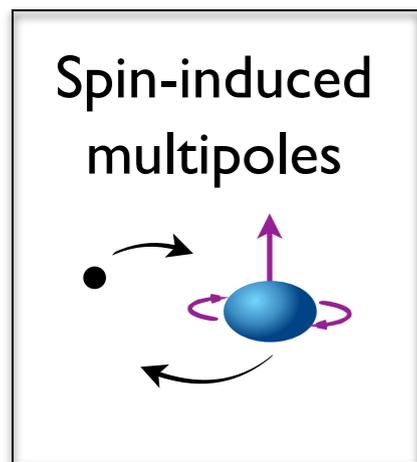
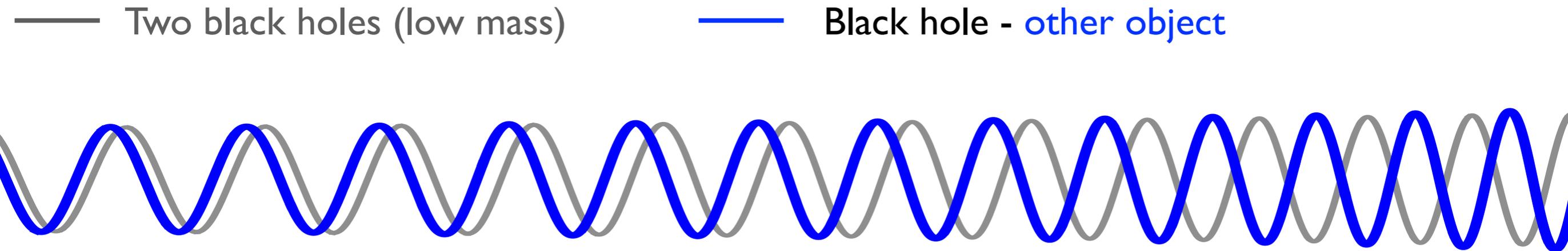
Approaches to computing waveforms

- Numerical relativity simulations: access to complex merger regimes ... limited in parameter space. sometimes difficult to identify fundamental physics parameters based solely on numerical outputs
- When **different physics dominate at different scales**:
 - **tapestry** of approximation schemes in different patches of spacetime
 - Can be further re-summed, e.g. into the effective one body framework

Example for comparable-mass **inspirals**:



GW signatures of interior structure during inspiral



Various tidal effects

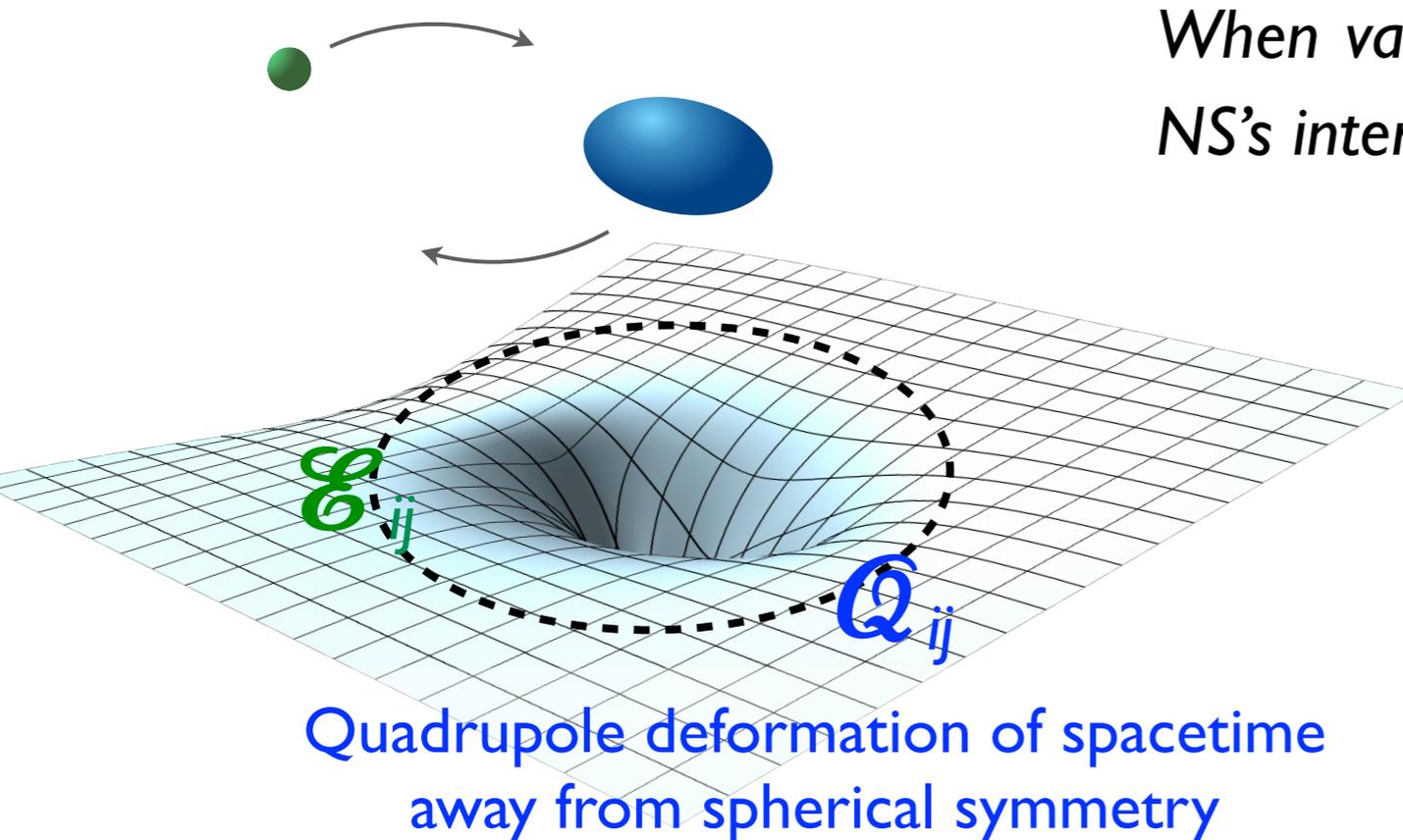
Resonant & non-resonant excitation of isolated characteristic modes

GW spectroscopy of NS interiors

- Generic phenomena, effects are small but clean and cumulative, accessible with current detectors
- associated **characteristic parameters** (e.g. Love numbers, quasi-normal mode frequencies) set the size of the GW signatures, encode object's internal structure

Dominant tidal effects

- In a binary: tidal field $\mathcal{E}_{ij} = R_{0i0j}$ due to spacetime curvature from companion



When variations in tidal field are much faster than NS's internal timescales (adiabatic limit):

Induced deformation:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

tidal deformability parameter

[TH 2008, Flanagan, TH 2008]

=0 for a black hole

[Kol, Smolkin '11, Pani+, Chia '20, Casals, LeTiec '20, ...]

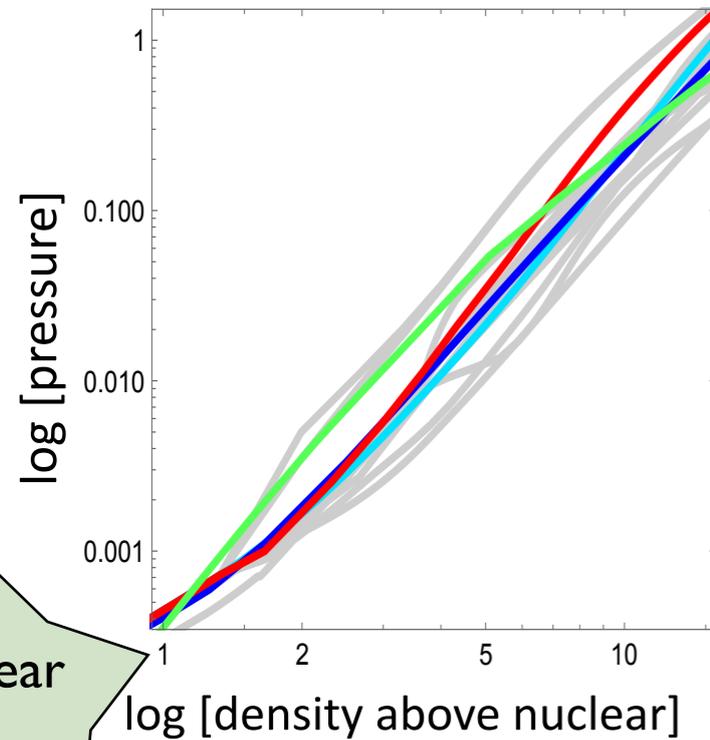
Similarly for higher multipoles

[Damour & Nagar, Binnington & Poisson 2009]

+ much recent work on intriguing connections with symmetries etc

Properties of NS matter reflected in tidal deformability

NS matter models
(equations of state EoS)



Perturbative QCD

Nuclear EFT, ...

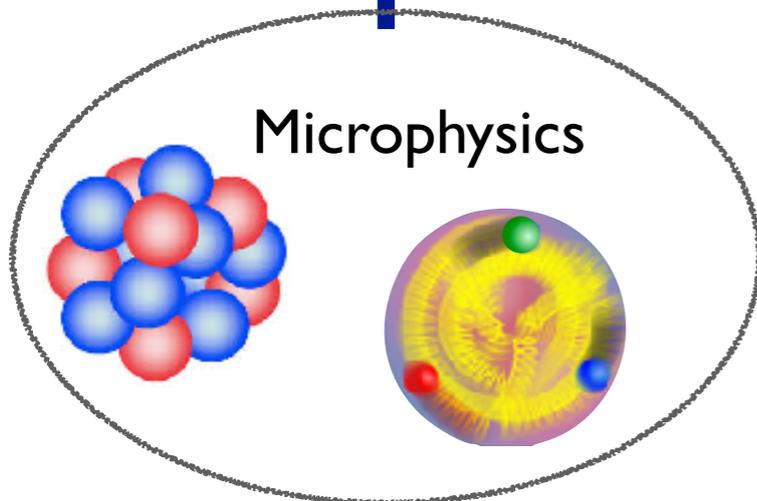
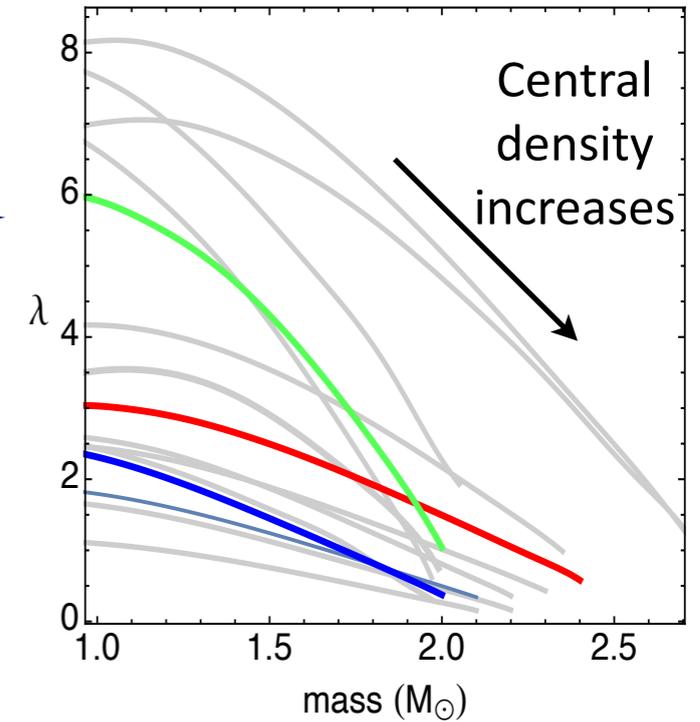
Einstein field equations & stress-energy conservation

Stationary perturbations to equilibrium
[TH 2008, many others]

or wave scattering

[Creci, TH,Steinhoff 2021]

Tidal deformability λ vs. mass

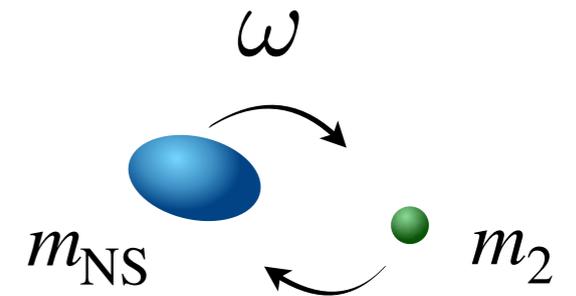


Main influence on GWs

- **Energy** goes into deforming the NS:

$$E \sim E_{\text{orbit}} + \frac{1}{4} Q \varepsilon$$

$$Q = -\lambda \varepsilon$$



$$M = m_{\text{NS}} + m_2$$

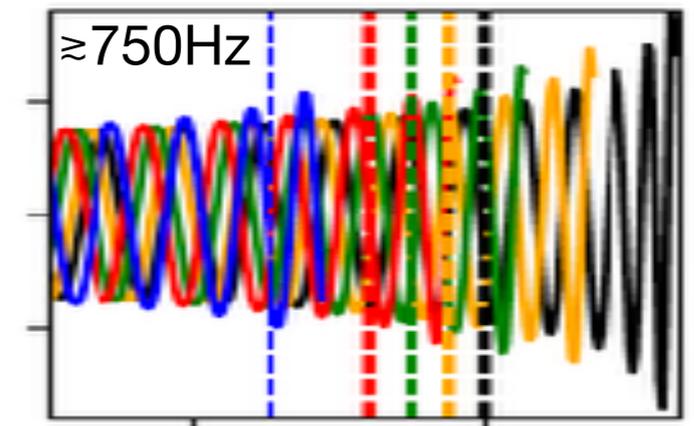
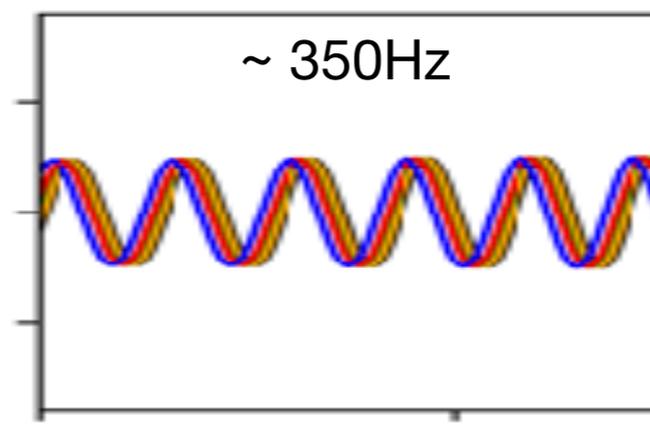
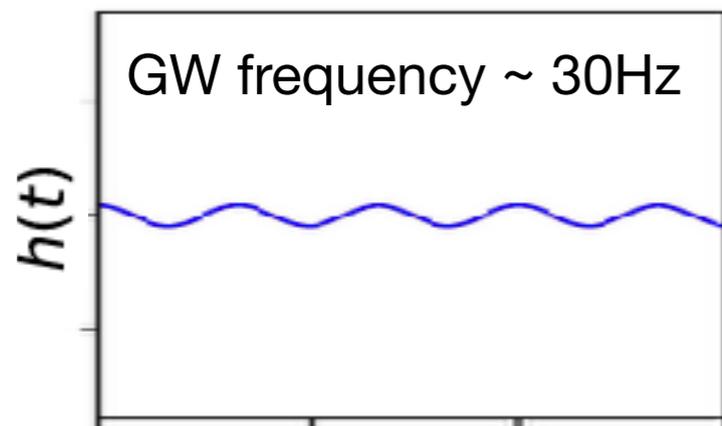
- moving multipoles contribute to **gravitational radiation**

$$\dot{E}_{\text{GW}} \sim \left[\frac{d^3}{dt^3} (Q_{\text{orbit}} + Q) \right]^2$$

- approx. **GW phase evolution** from energy balance:

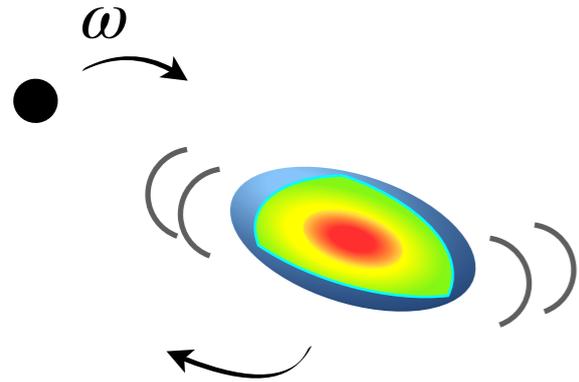
$$\Delta \phi_{\text{GW}}^{\text{tidal}} \sim \lambda \frac{(M\omega)^{10/3}}{M^5}$$

Examples
for different
EoSs
aligned at
30 Hz



Dashed lines: 1kHz

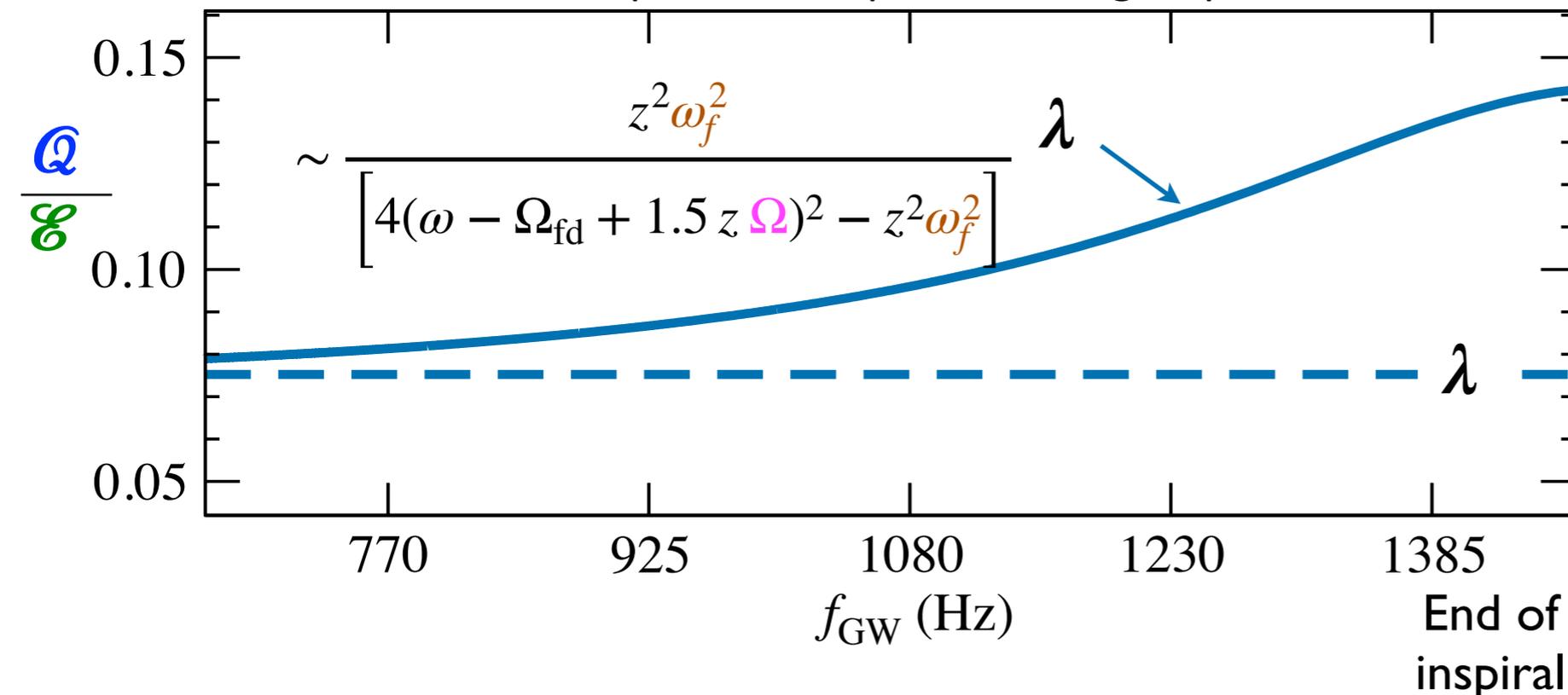
Taking into account more realistic physics



- Q corresponds (mainly) to the **f**undamental oscillation modes
- **f**-mode frequency: $\omega_f \sim \sqrt{G m / R^3}$ (internal-structure-dependent)
- **tidal forcing** frequency: $\sim 2\omega \sim 2\sqrt{G M / r^3}$ [circular orbits]

Enhanced tidal effects even if the resonance is not fully excited

Example tidal response during inspiral



During an inspiral:

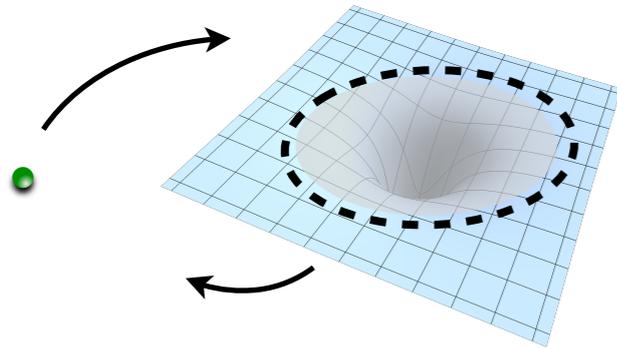
Response also impacted by:

- relativistic **redshift** z
- **frame dragging** Ω_{fd}
(from GR & companion's spin)
- **NS's spin** Ω

More realistic couplings of matter to orbital dynamics

Extended object viewed on the orbital scale:

- Central worldline + multipole moments
- Effective action for the binary dynamics:



gravitomagnetic tidal tensor

Matter contribution
to induced current
quadrupole

$$\mathcal{B}_{ij} = {}^*R_{0i0j}$$

Internal dynamics of the
multipoles (modes)

$$S \approx S_{pp} + \int d\tau \left[-\frac{z}{2} Q_{ij} \mathcal{E}_{ij} - \frac{z}{2} \dot{Q}_{ij}^{\mathcal{B}} \mathcal{B}_{ij} + L^{\text{Coriolis}} + L^{\text{FD}} + L^{\text{osc}} + \dots \right]$$

point-particle
part

multipoles interact with
companion's spacetime
curvature

Effects of NS's spin
on tidal response &
restoring force for
gravitomagnetic
modes

Q's angular momentum interacts
with orbital angular momentum
& companion's spin

More realistic couplings of matter to orbital dynamics

Multipoles behave as harmonic oscillators:

$$L^{\text{osc}} \approx \sum_n \frac{z}{4\lambda_{(n)} z^2 \omega_{(n)}^2} \frac{dQ_{ij}^{(n)}}{d\tau} \frac{dQ_{ij}^{(n)}}{d\tau} - \frac{z}{4\lambda_{(n)}} Q_{ij}^{(n)} Q_{ij}^{(n)} + \dots$$

$$+ \frac{3z}{32(\sigma_{\text{stat}} - \sigma_{\text{irrot}})} \frac{d\dot{Q}_{ij}^{\mathcal{B}}}{d\tau} \frac{d\dot{Q}_{ij}^{\mathcal{B}}}{d\tau} + \frac{2z\sigma_{\text{stat}}}{3} \mathcal{B}_{ij} \mathcal{B}_{ij} + \dots$$

Different quadrupolar modes contribute, dominated by fundamental modes + similarly for higher multipoles

subdominant but important for future GW detections

two different magnetic tidal deformabilities

[Landry, Poisson, Pani+, Damour, Nagar, ...]

$$S \approx S_{pp} + \int d\tau \left[-\frac{z}{2} Q_{ij} \mathcal{E}_{ij} - \frac{z}{2} \dot{Q}_{ij}^{\mathcal{B}} \mathcal{B}_{ij} + L^{\text{Coriolis}} + L^{\text{FD}} + L^{\text{osc}} + \dots \right]$$

point-particle part

multipoles interact with companion's spacetime curvature

Effects of NS's spin on tidal response & restoring force for gravitomagnetic modes

Q's angular momentum interacts with orbital angular momentum & companion's spin

[Steinhoff, TH +2016, 2020, 2021]

Many subtleties in the GR interplay of matter with gravity

- ▶ Nontrivial to define a ‘worldline skeleton’ [Dixon 1970]
- ▶ 1990s: does GR give rise to new couplings between internal degrees of freedom and orbital dynamics? E.g. seemingly numerical evidence for a ‘relativistic crushing force’ in NS binaries

VOLUME 75, NUMBER 23

PHYSICAL REVIEW LETTERS

4 DECEMBER 1995

Instabilities in Close Neutron Star Binaries

J. R. Wilson¹ and G. J. Mathews²

+ follow-up papers

... surprising evidence that GR effects may cause otherwise stable stars to individually collapse prior to merging

Sociological account: Kennefick 2000 ‘‘Star crushing: theoretical practice & the theoretician’s regress’’

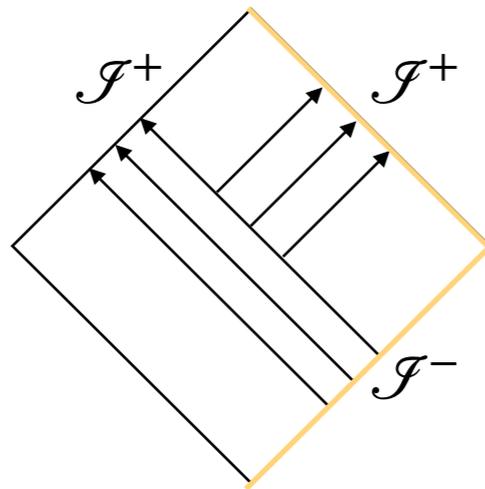
- ▶ rigorous analysis using **matched asymptotic expansions** of spacetimes showed that there are **no such new forces** under the assumptions for this case (c.f. also other numerical studies)

[Flanagan 1998: ‘‘GR coupling between orbital motion & internal degrees of freedom ..’’ [paper](#)]

- ▶ More recently: seemingly numerical findings of large tidal fields at higher post-Newtonian order, subtleties with gravitomagnetic tidal response, ambiguities in tidal deformability?, ...

Computing the tidal response from scattering

- Some concerns in the literature about potential ambiguities in tidal deformability
e.g. S. Gralla: On the Ambiguity in Relativistic Tidal Deformability, arXiv:1710.11096
- Advantages of scattering calculations:
 - **Identifications** at null infinity, using double-null coordinates (geometric meaning)
 - **invariant** scattering amplitudes



Parameters in effective action in flat space matched to asymptotics of relativistic perturbations

- work with **in- and outgoing waves** instead of stationary perturbations

Computing the tidal response from scattering

- Response determined by the **in-** and **outgoing wave amplitudes** through:

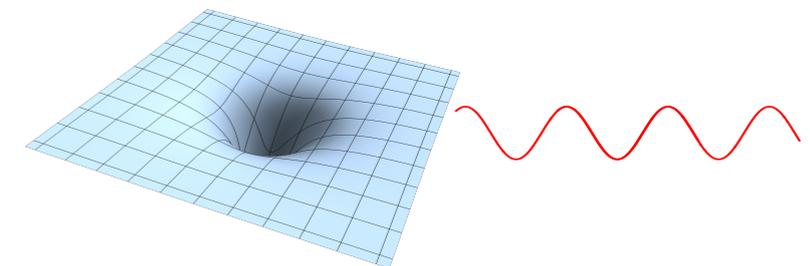
$$\lambda_\ell(\omega) = i \Xi_\ell \left[1 - 2 \left(1 + \frac{C_\ell^{\text{in}}}{C_\ell^{\text{out}}} e^{i\frac{\pi}{2}(D-1)} \right)^{-1} \right] \quad D=\text{spatial dimensions}$$

Matching reveals:

$$\frac{C_\ell^{\text{in}}}{C_\ell^{\text{out}}}\Big|_{\text{Minkowski}} = \frac{A_{\text{in}}^\infty}{A_{\text{out}}^\infty}\Big|_{\text{Schwarzschild}} \quad \Xi_\ell = -\frac{4\pi^{(D-2)/2}}{2^\ell} \left(\frac{2}{\omega}\right)^{(D-2)+2\ell} \Gamma\left(\frac{D-2}{2} + \ell + 1\right)$$



Calculated from relativistic perturbation analysis in the full theory



Final result for the scalar case

- Substituting details: information contained in the response function is as expected:
 - e.g. in $D = 3$ spatial dimensions, in the limit $M\omega \ll 1$

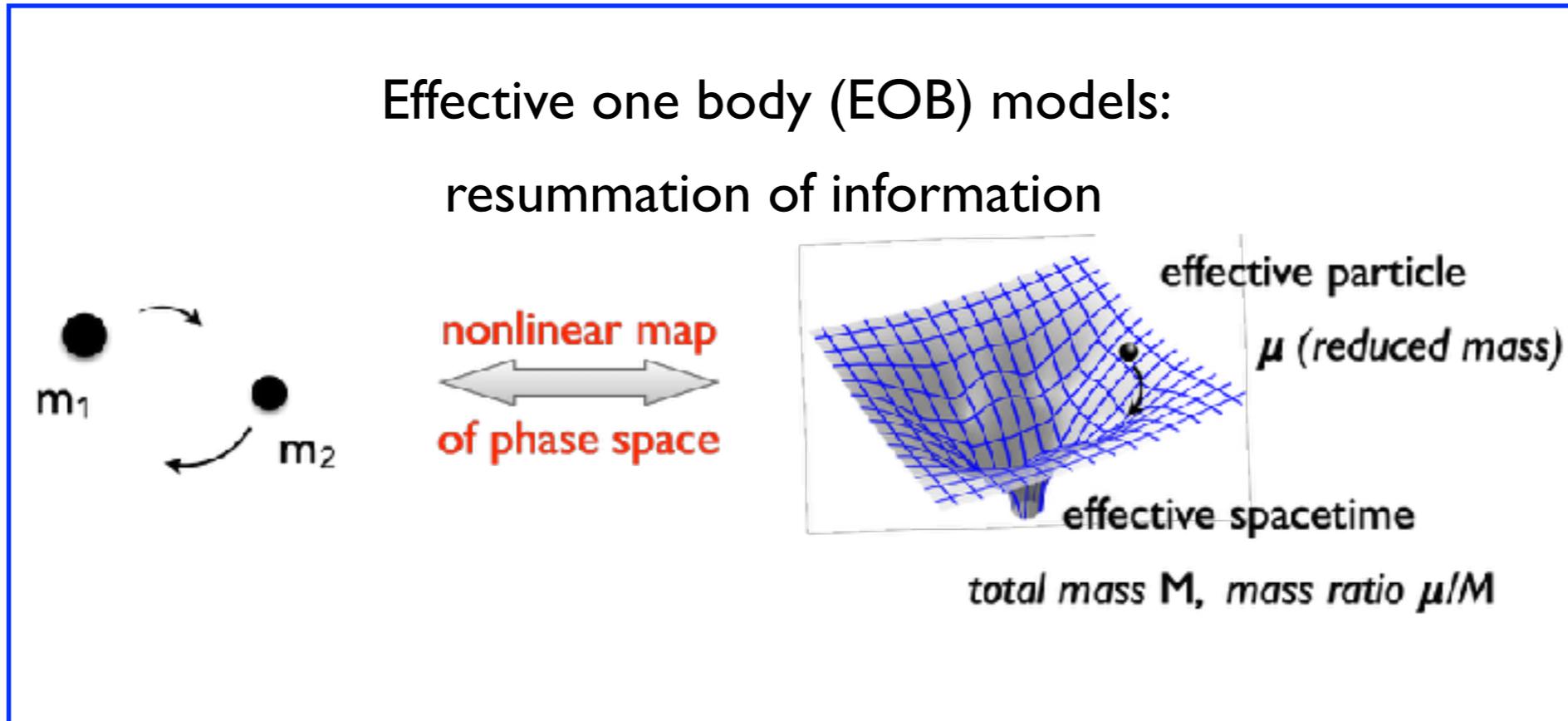
$$\lambda_{\ell=0}(\omega) = - \underbrace{4r_{\text{horizon}}^2}_{\text{Black hole absorption cross section (scalar waves)}} \pi i \omega$$

Black hole **absorption** cross section (scalar waves)

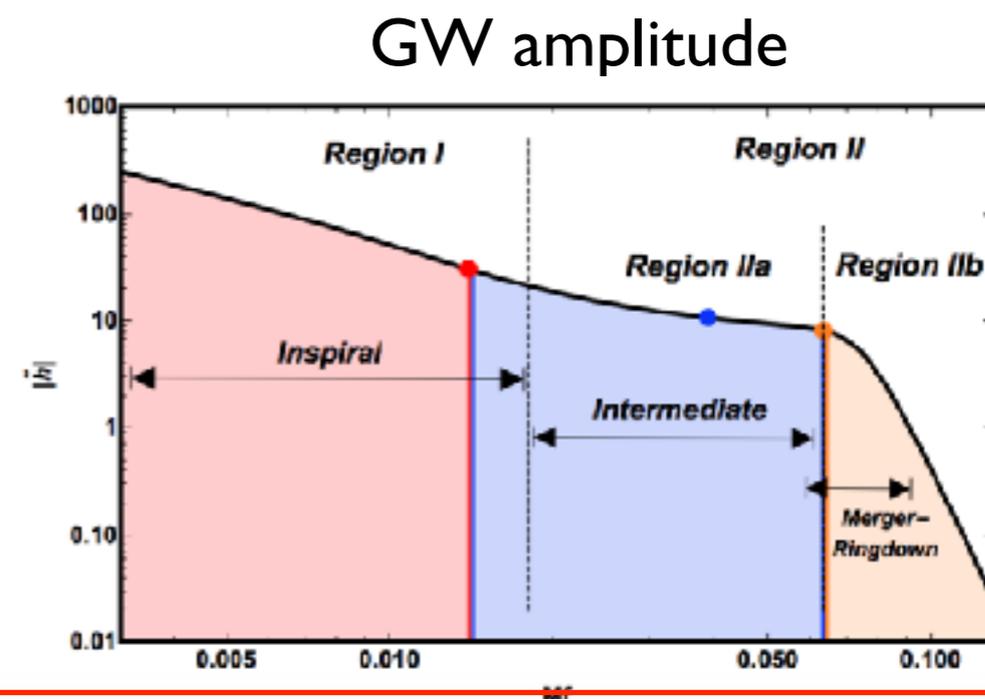
- **Real** part of the response is zero: **tidal Love number** vanishes
- Similar to frequency-dependent response in optics \leftrightarrow material's refractive index
 - **Imaginary part: absorption**
 - Real part: refraction, phase shift compared to incident beam

Finite size effects included in models for data analysis

Matter effects on top of full black hole baseline models



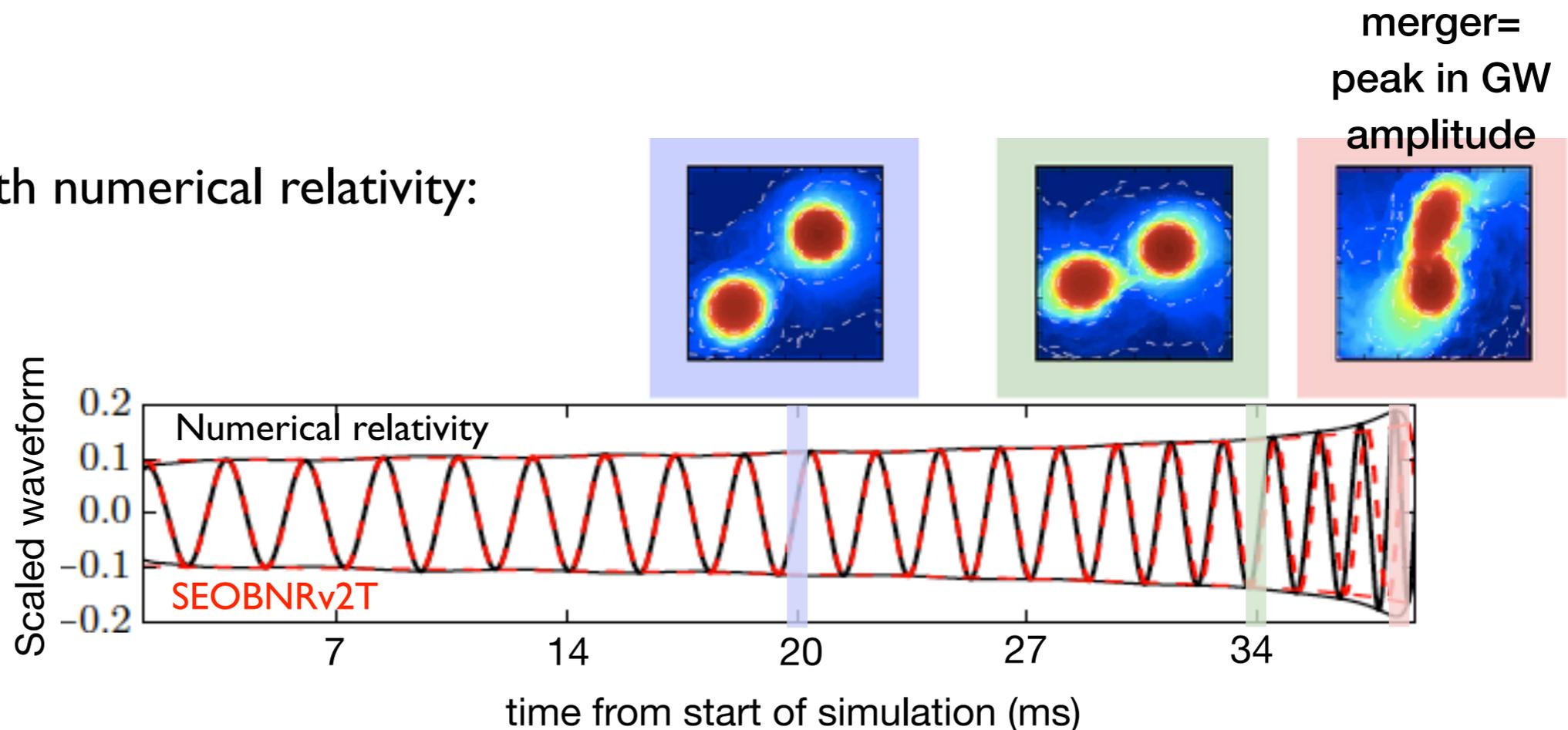
Phenomenological (Phenom)
models:
closed-form expressions for
frequency-domain GWs



Matter effects in models for data analysis

- For inspirals: variety of physics & assumptions, e.g. **some but not all of the models**
 - Rely on quasi-universal relations used to reduce matter parameters to λ_1, λ_2
 - Are calibrated to numerical relativity
 - Include some dynamical tidal effects
 - ...

Comparison with numerical relativity:



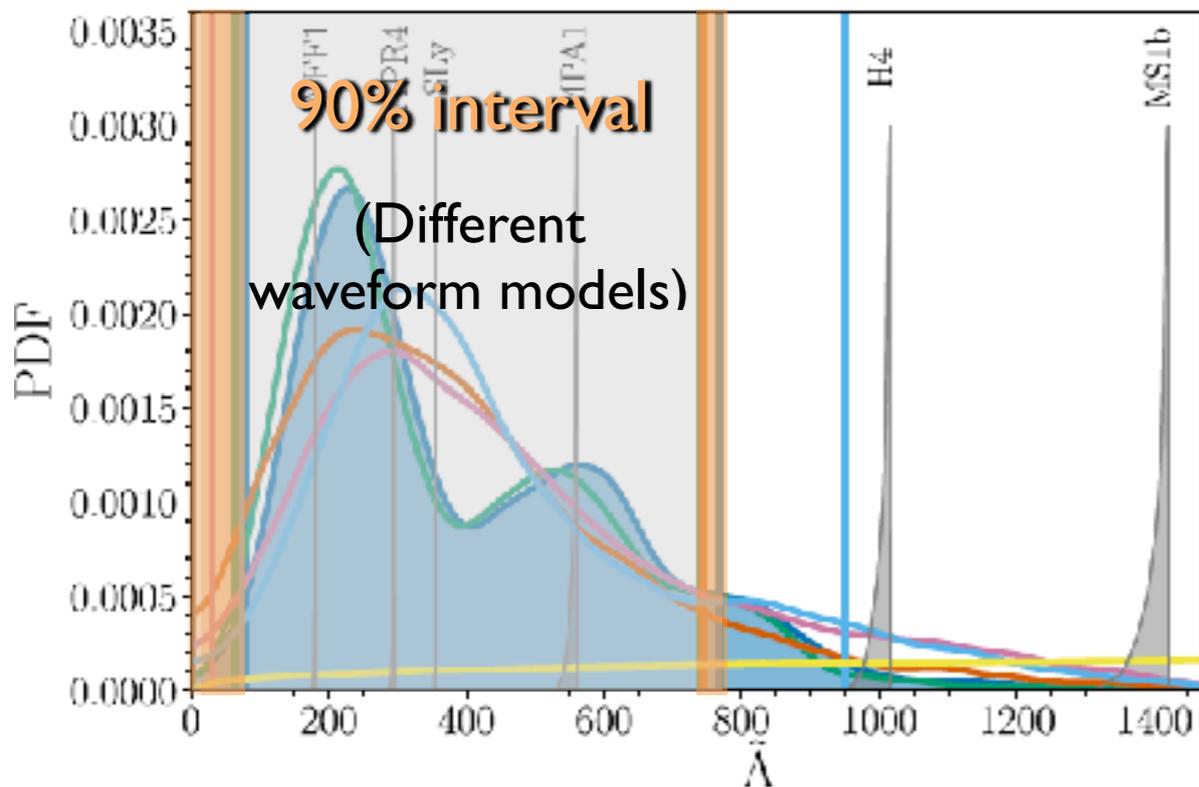
Measurements/constraints on tidal deformability

Results with **low spin priors** (Dimensionless spins < 0.05)

GW170817

Total mass $\sim 2.8 M_{\text{sun}}$

Distance $\sim 40 \text{Mpc}$



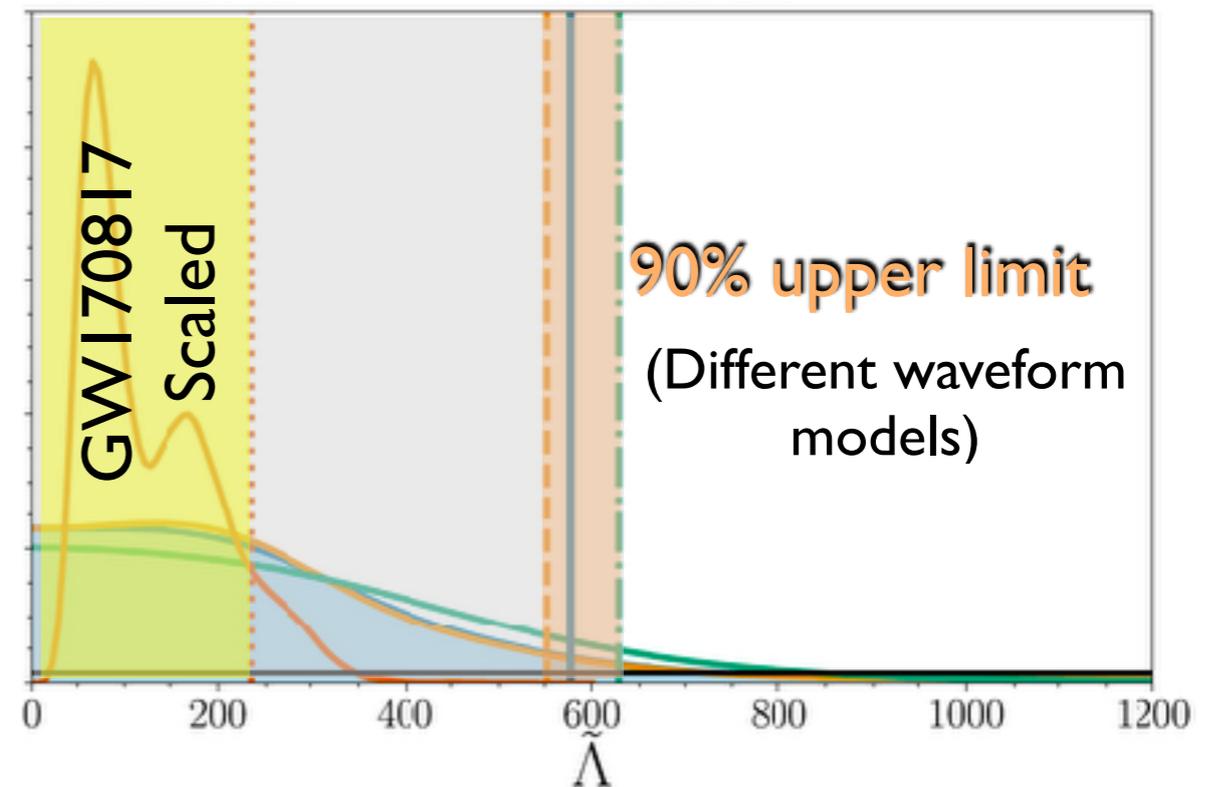
LVC GWTC-1

[updated results compared to initial papers,
e.g. detector calibration, ...]

GW190425

Total mass $\sim 3.4 M_{\text{sun}}$

Distance $\sim 160 \text{Mpc}$



LVC ApJ Lett. 892 L3 (2020)

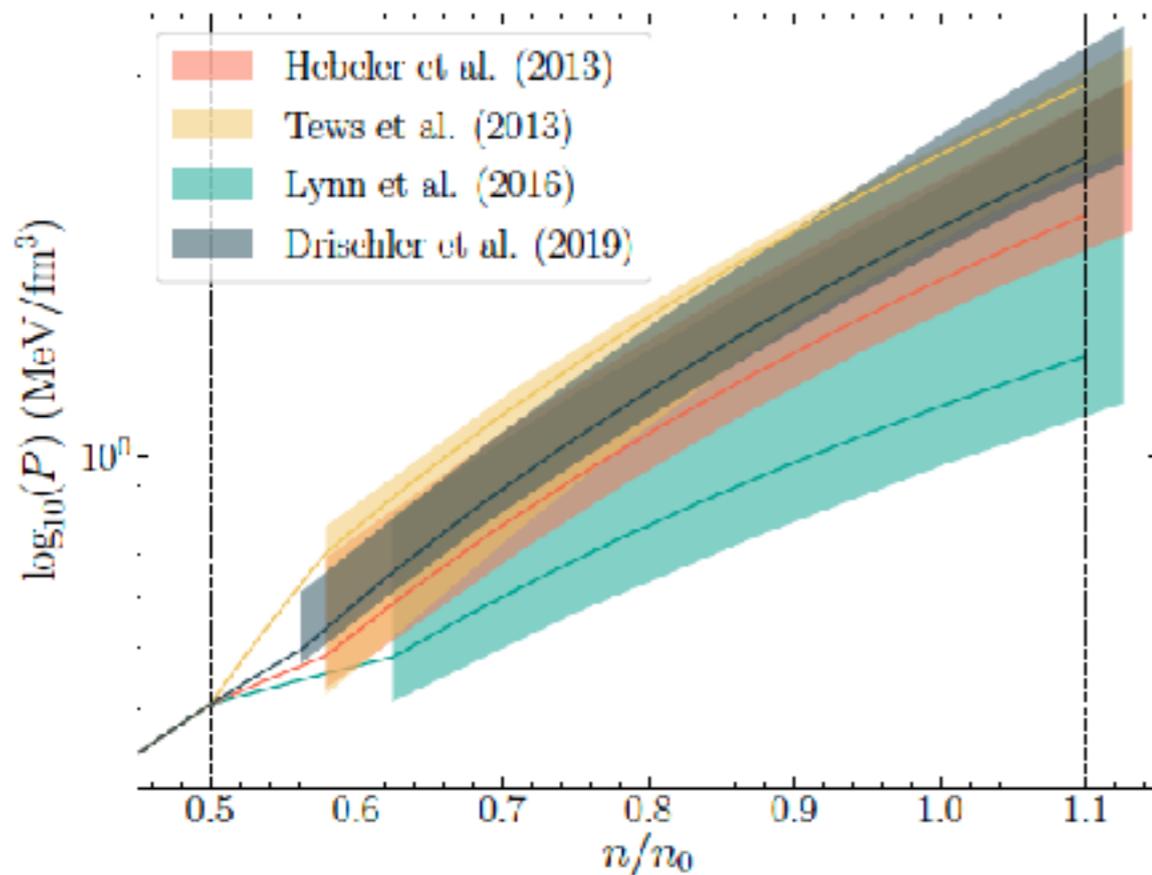
- ▶ for two NSs: GWs most sensitive to the combination (similar to chirp mass):

$$\tilde{\Lambda} = \frac{13c^{10}}{16 G^5 M^5} \left[\left(1 + 12 \frac{m_2}{m_1} \right) \lambda_1 + \left(1 + 12 \frac{m_1}{m_2} \right) \lambda_2 \right]$$

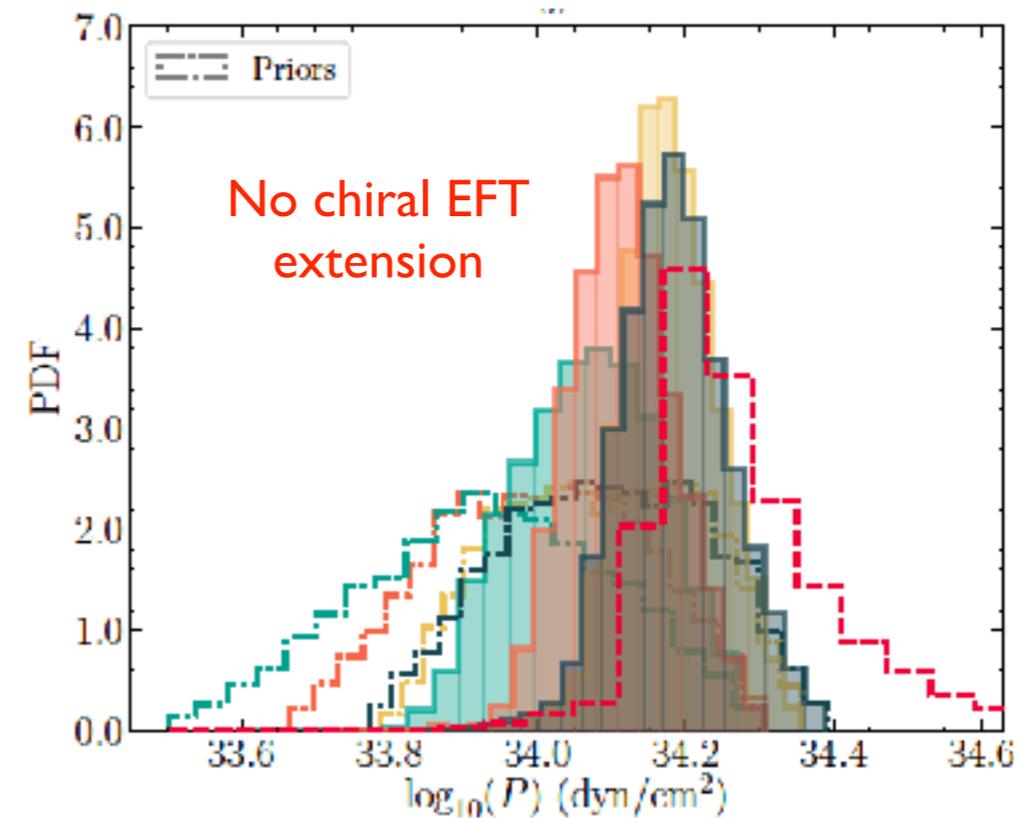
Example implications for subatomic physics

- Joint constraints with other observations (kilonova, x-ray, radio) + subatomic physics
- E.g. can start to inform chiral effective field theory extensions (nuclear multi-body interactions, symmetry energy ...):

Different nuclear physics models

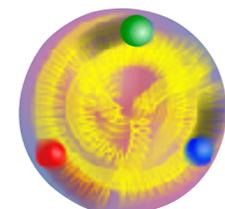
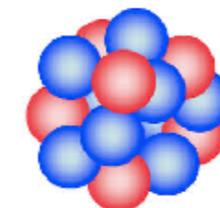


pressure at 1.5 nuclear saturation density
(different extensions)



Geert Raaijmakers+ 2021 arXiv:2105.06981

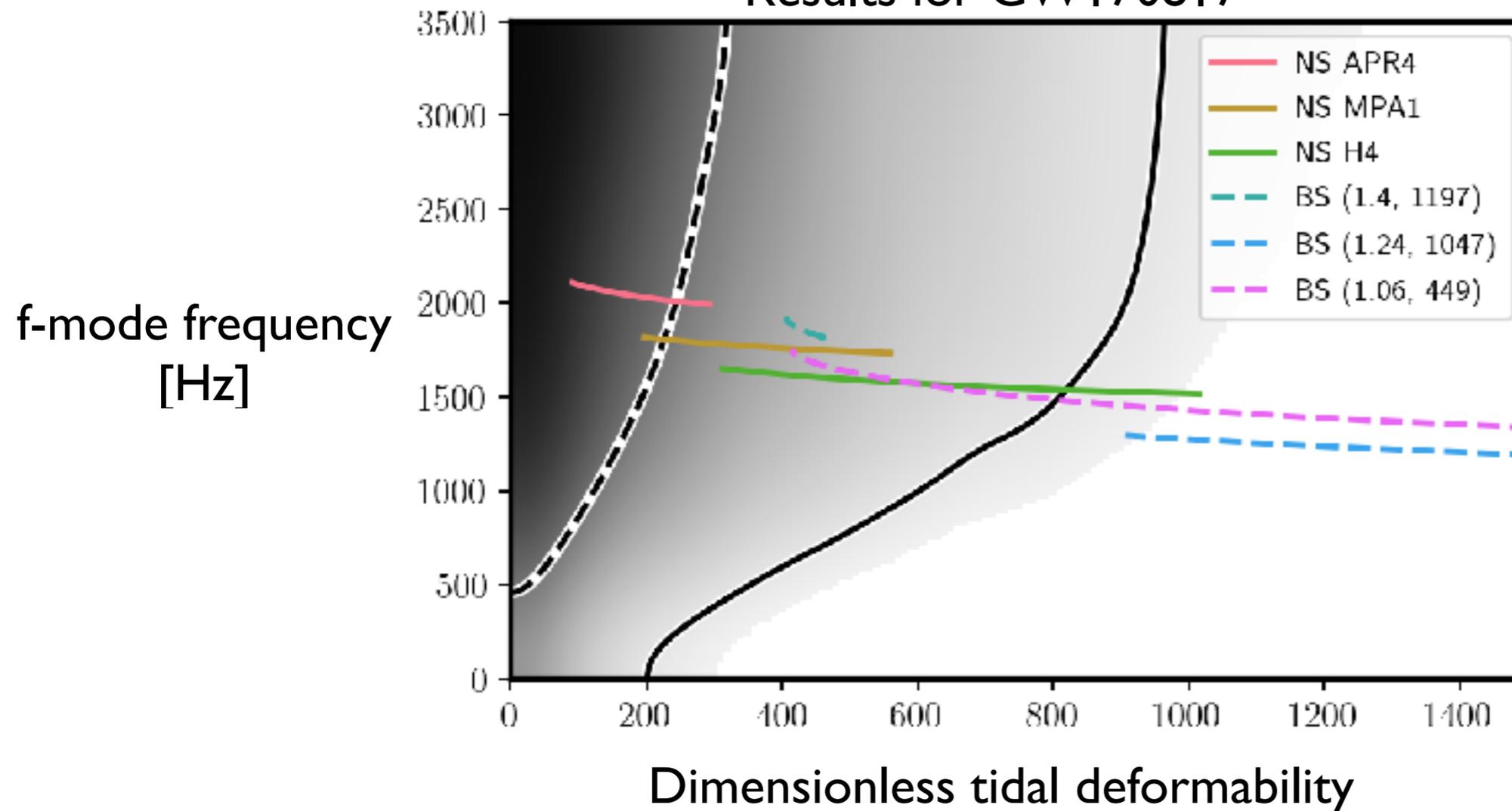
many different groups have studied all kinds of different aspects



Proof of principle: GW constraints on f -mode frequency

- Measuring both λ and ω_f
- quadrupole & octupole for each star: 8 matter parameters, expect deterioration in measurements
- more efficient approximate frequency-domain model [Schmidt, TH 2019]

Results for GW170817



Pratten, Schmidt, TH 2020

Near-term future prospects

next observing run O4: LIGO/Virgo near/reaching design sensitivity



Further upgrades scheduled



- More accurate measurements of nearby sources
- greater number & diversity of events

Plans for next-generation detectors moving ahead (~2035)

European
vision

Einstein Telescope

L=10 km triangle

- Prototype being built in Maastricht

- 10 times better sensitivity than LIGO/Virgo, wider frequency range
 - $O(100\ 000)$ binary merger detections per year
 - High precision studies of nearby sources

US vision

Cosmic
Explorer

L=40 km

A few examples of remaining theoretical challenges

Need high-accuracy and efficient waveforms over wide parameter space

- more matter effects & relativistic corrections
- arbitrary spins
- eccentricity, ...
- role of beyond zero-temperature, equilibrium matter?
-
- degeneracies (e.g. modified gravity, dark matter/BSM physics), ...
-



Conclusions

- GWs are new **probes of NS physics**: clean gravitational channel of information
- Exciting near- & longer-term prospects with **larger & more precise datasets**
- Simultaneous **advances in modeling** are **essential** to fully realize the **science potential**, reduce biases in measurements and interpretation



- Significant **progress** on understanding, modeling relevant phenomena but **much work remains**
- Synergy of theoretical approaches important (diverse analytical + numerical)
- Interdisciplinary cooperation needed on **connections** and fundamental **inputs**