Revolutions in Our Understanding of the Laws of Nature

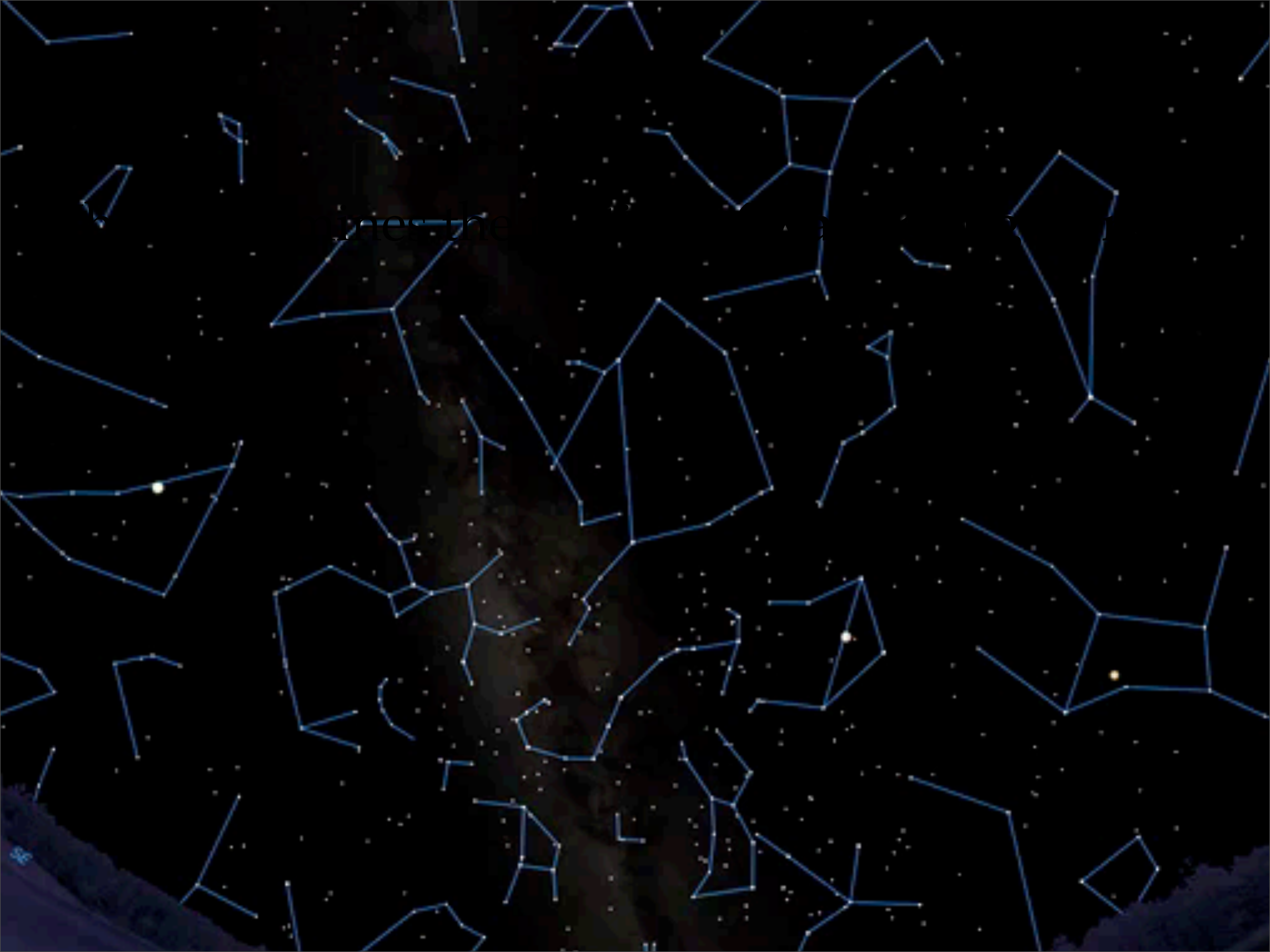
Jacob L. Bourjaily

The Niels Bohr International Academy

Part 1:
Classical Laws of Nature

Prelude to Understanding

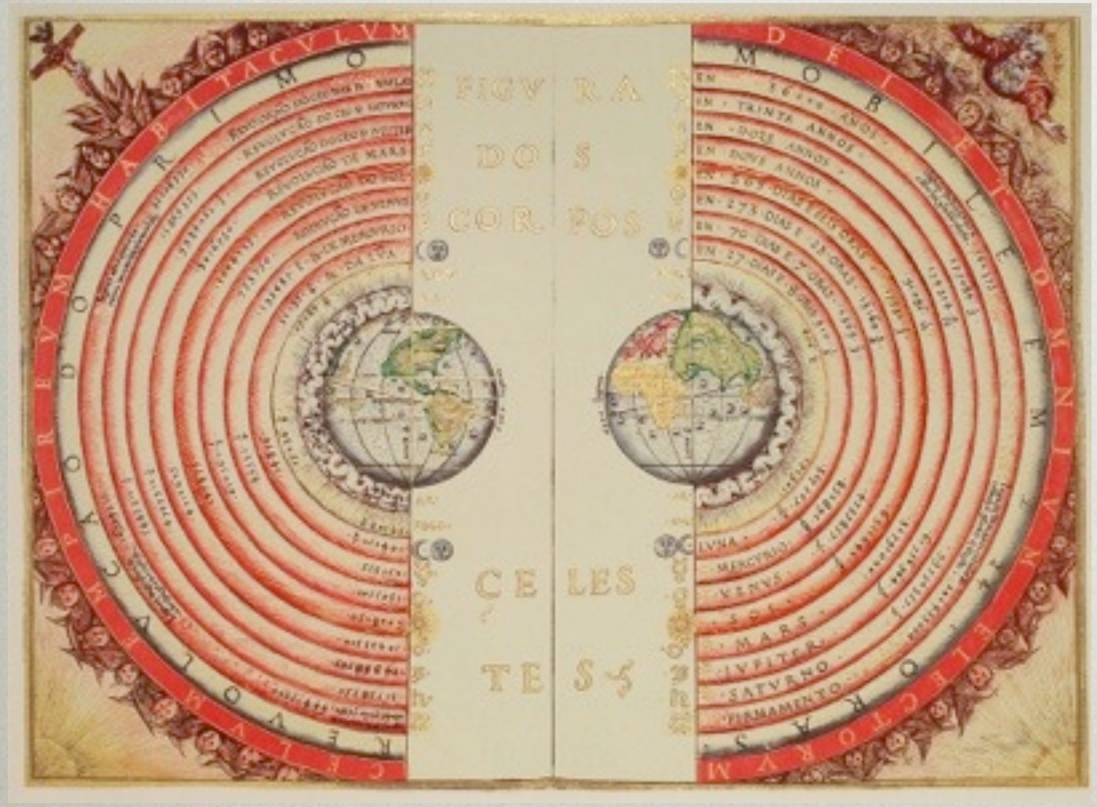
What determines the motion of **wandering stars**?



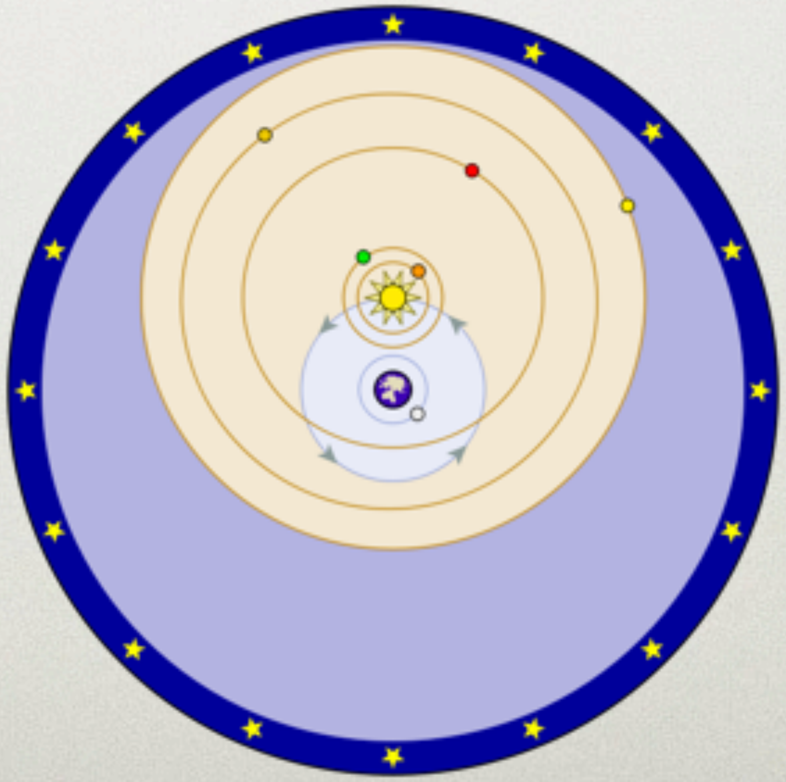
Prelude to Understanding

What determines the motion of **wandering stars**?

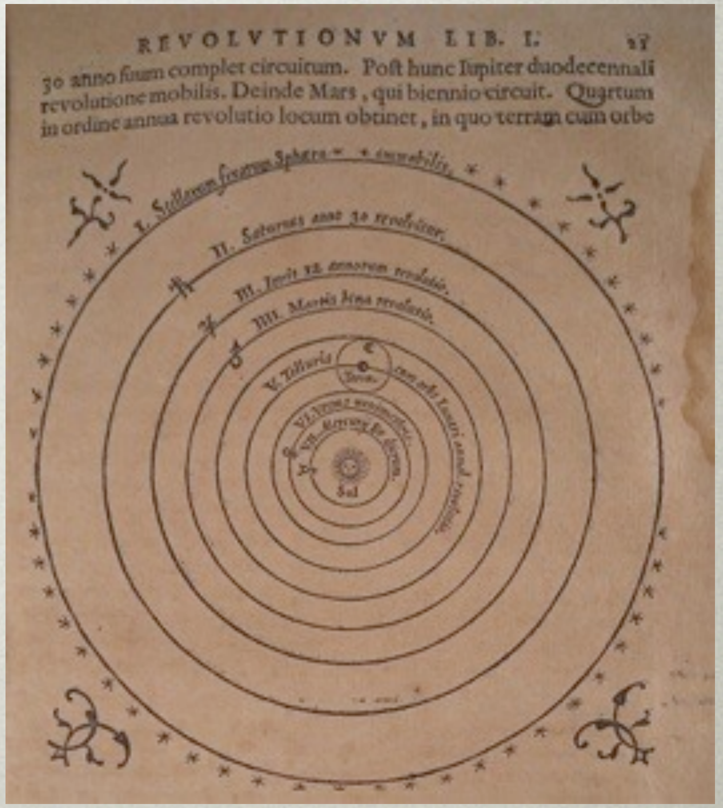
- motion seems orderly; many theories proposed:



Geocentric



Geo-Heliocentric

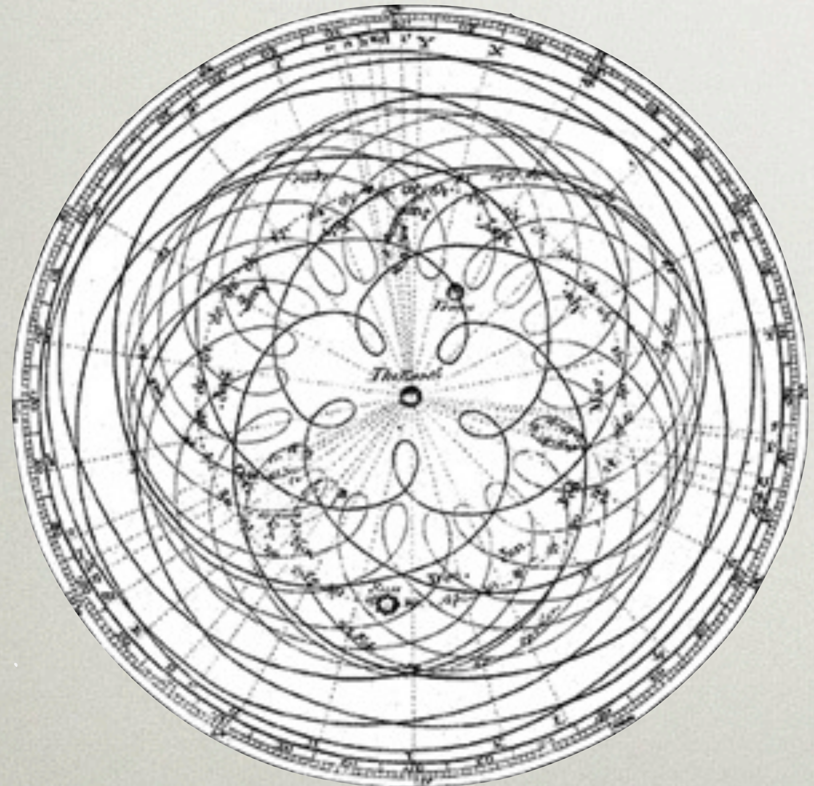


Heliocentric

Prelude to Understanding

What determines the motion of **wandering stars**?

- motion seems orderly;   many theories proposed:
  - the main difficulty with all these theories was that
    - to whatever extent they were ***predictive,***
      they were ***obviously wrong***



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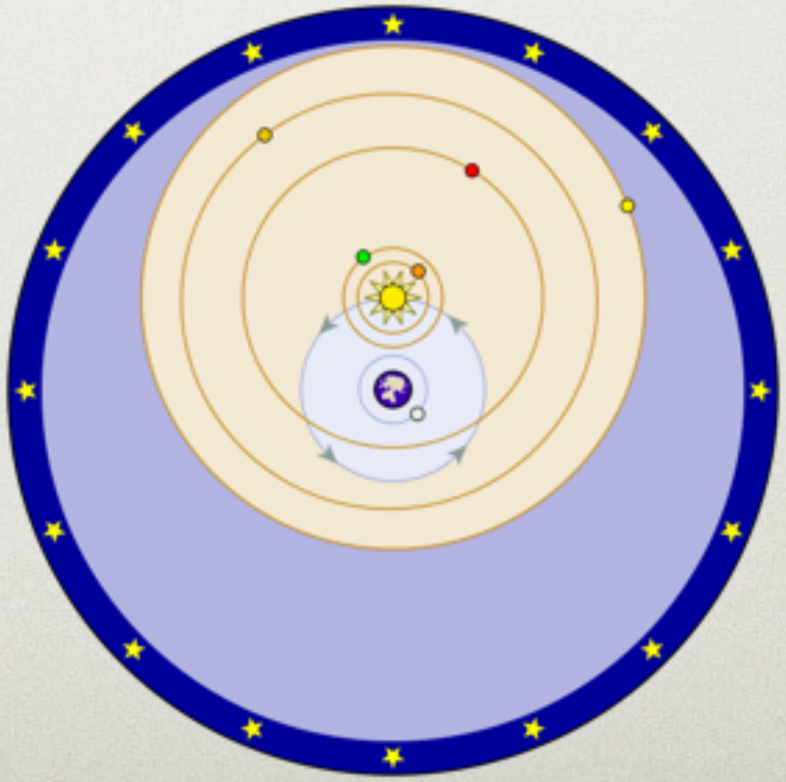

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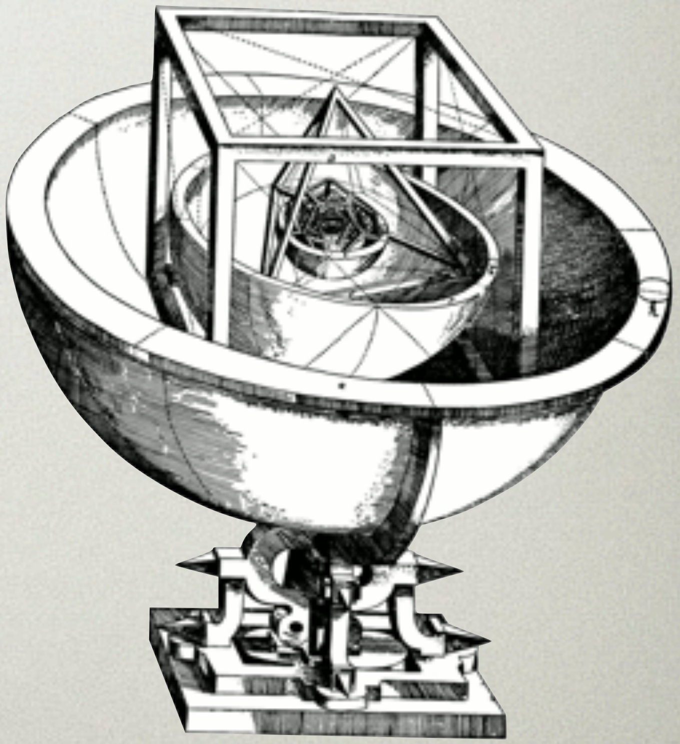
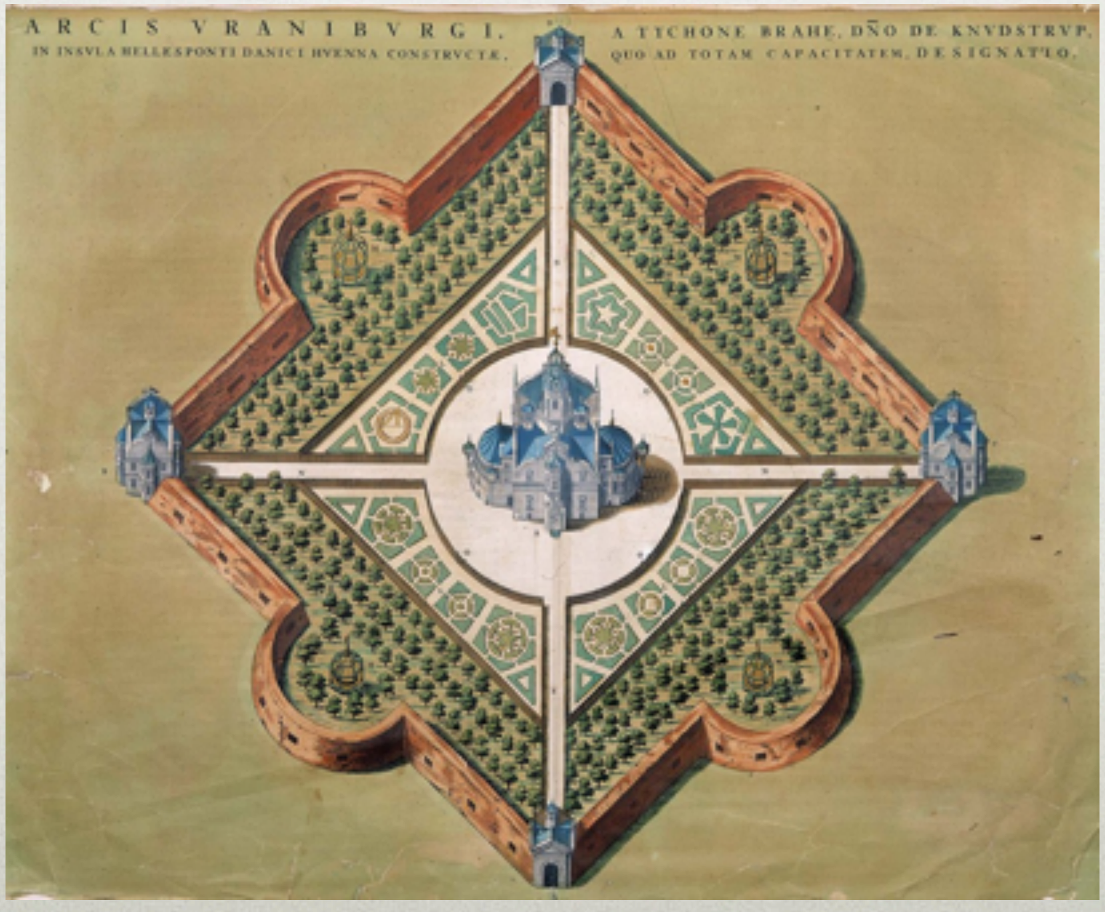


Heliocentric

Refinements through Observation

- **Tycho Brahe** suggested settling this debate by carefully observing how planets *actually* moved
    - he persuaded Emperor Rudolph II to build him an observatory, and **Johanes Kepler** to analyze the data



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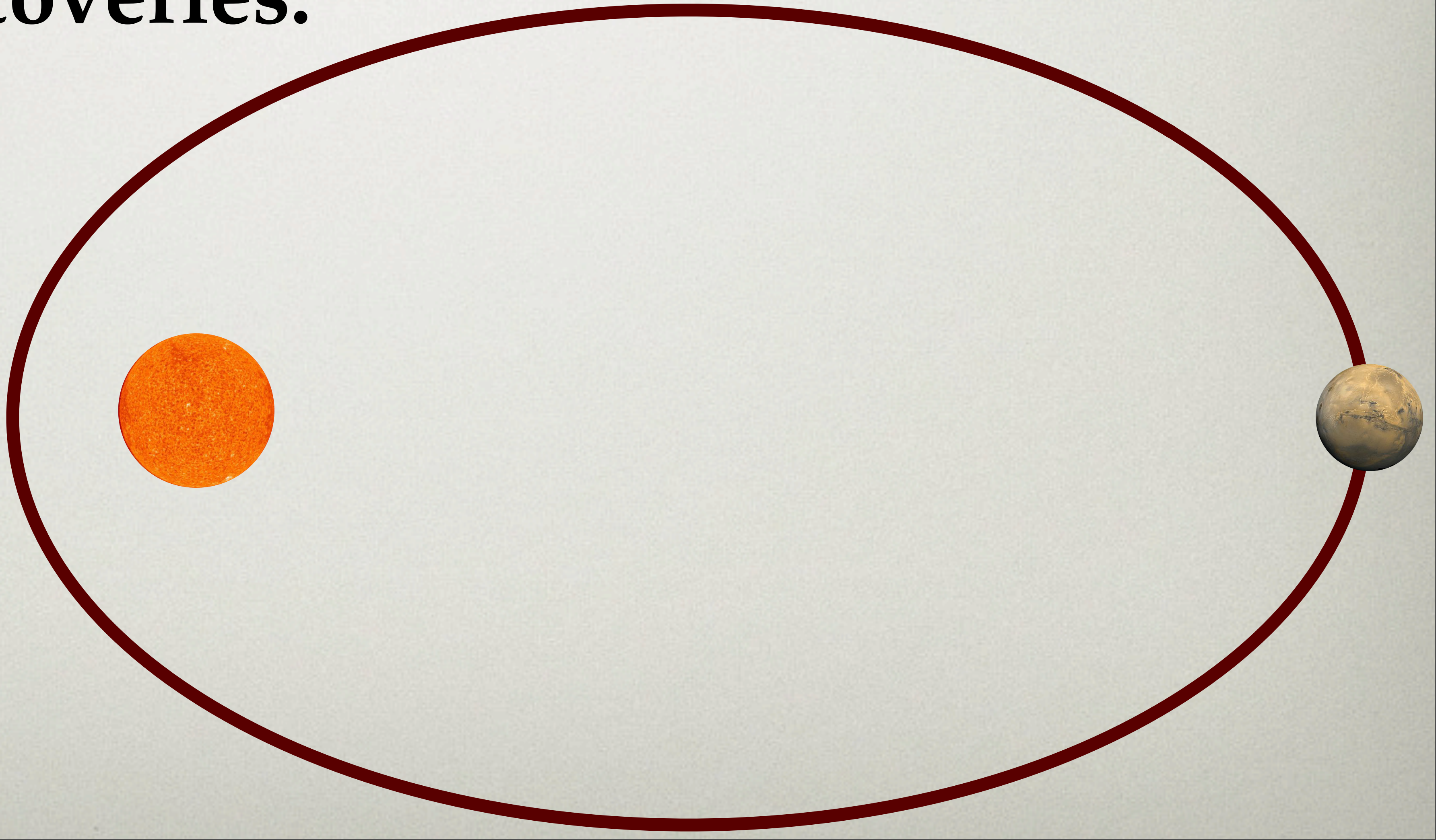
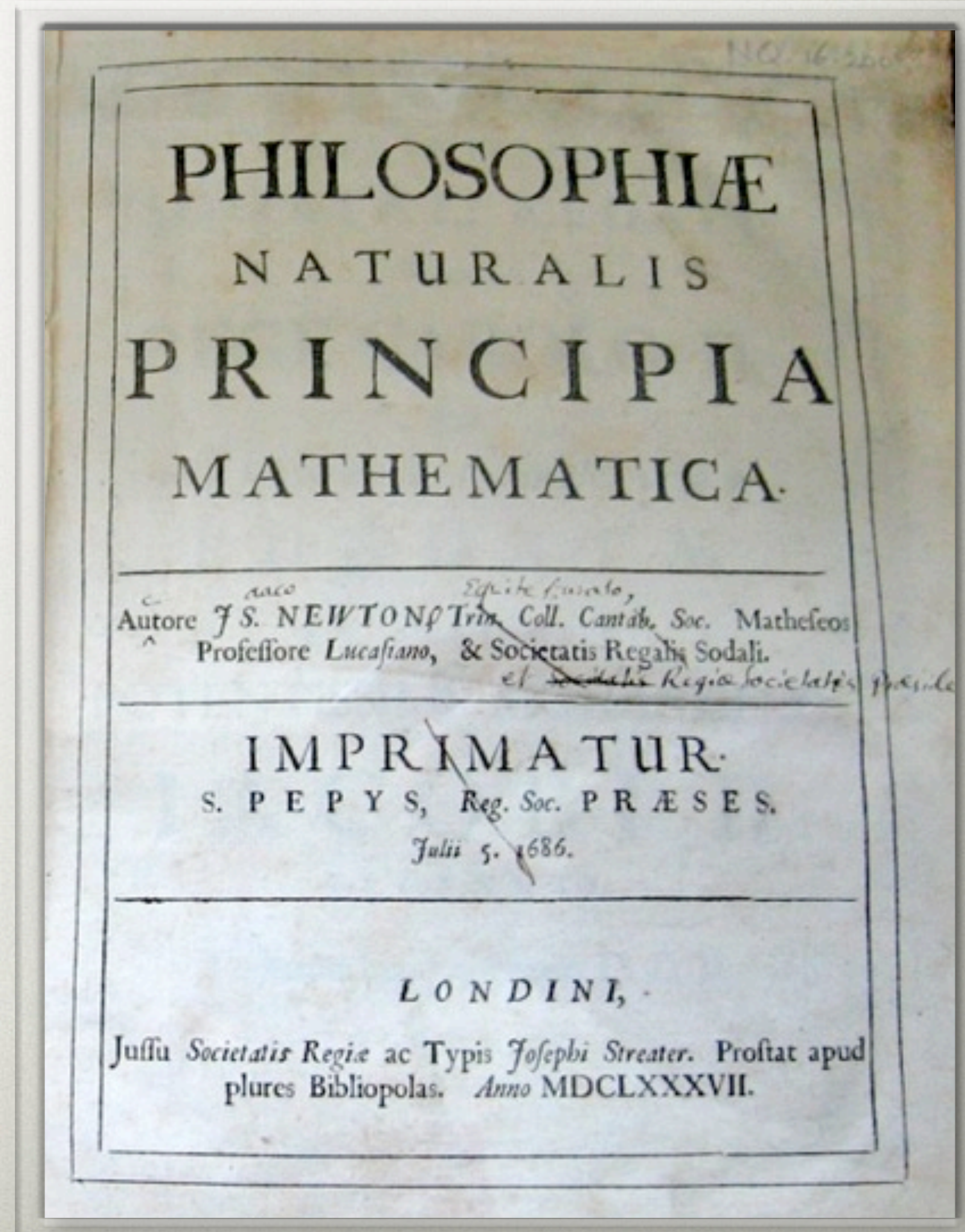
Two shocking discoveries:

- planetary orbits are not **circular** but ***elliptical***
- the sun is not at the **center**, but at a **focus**



Newton's System of the World

- In 1687, Newton puts forth his *System of the World* giving his *Laws of Motion*:



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore JS. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

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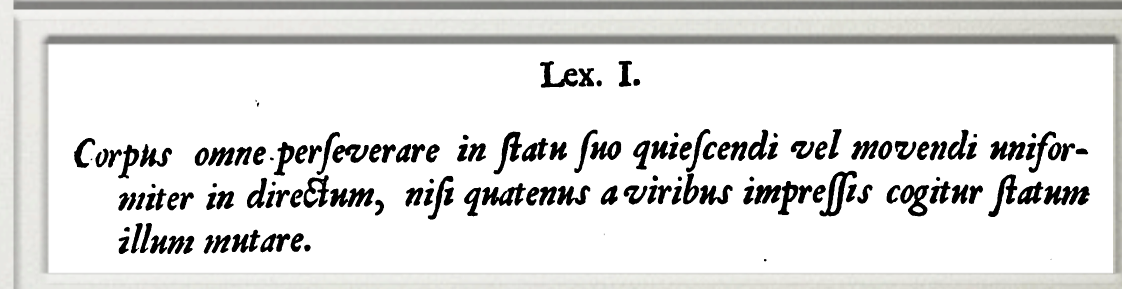
    - *inertia* (*Galilean relativity*)

AXIOMATA
SIVE
LEGES MOTUS



Lex. I.

Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

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Lex. II.

Mutationem motus proportionalem eſſe vi motrici impreſſæ, & fieri ſecundum lineam rectam qua vis illa imprimitur.

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$$F = ma$$

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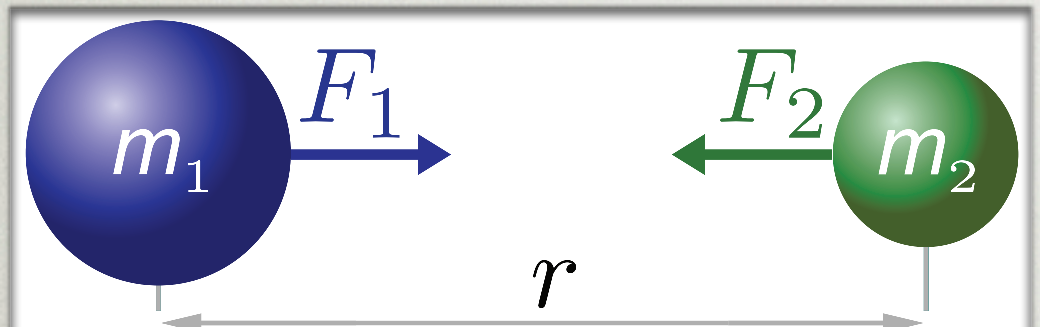
    - *Law of Gravitation*



$$F_1 = F_2 = G\,\frac{m_1\,m_2}{r^2}$$

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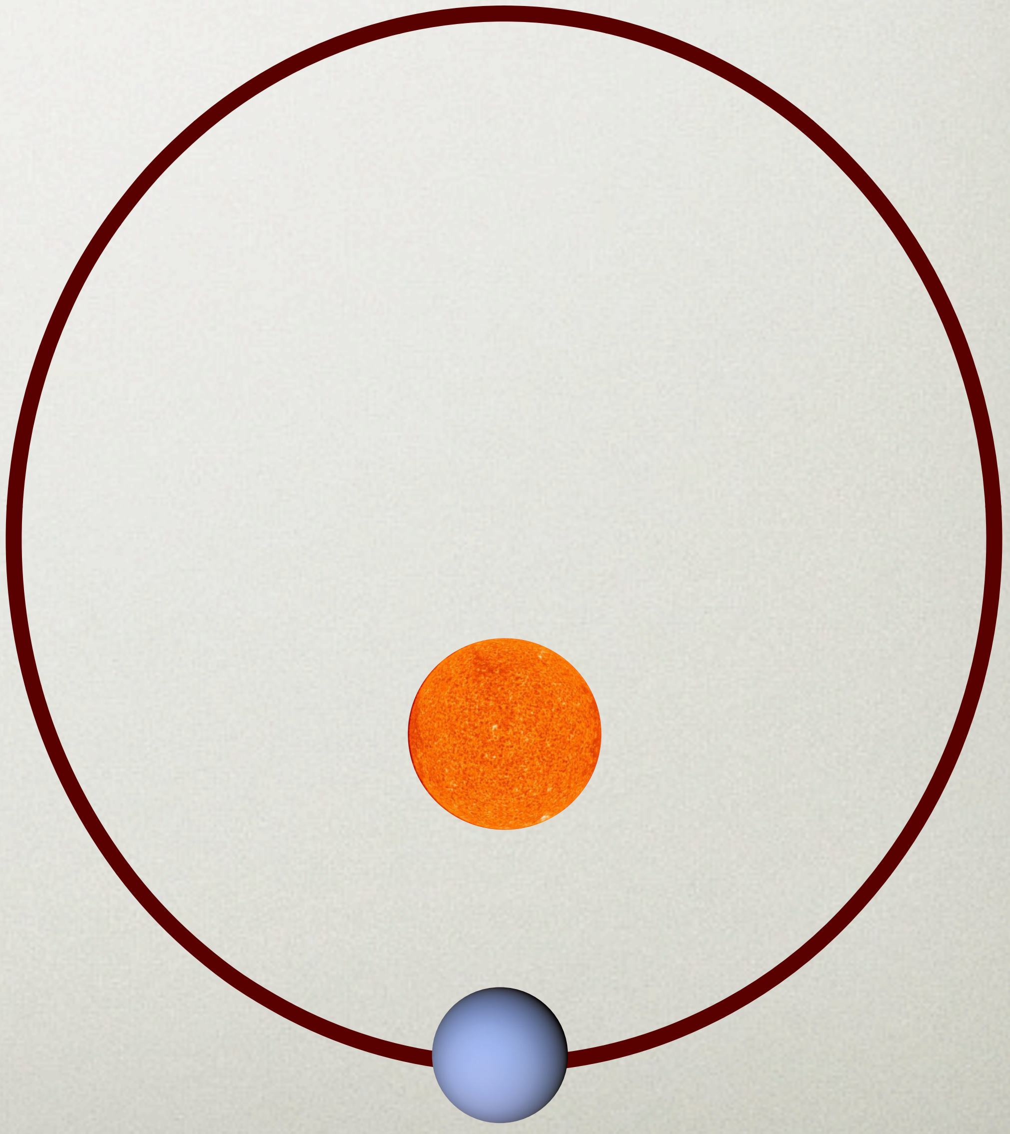

Confrontations with Experiment I

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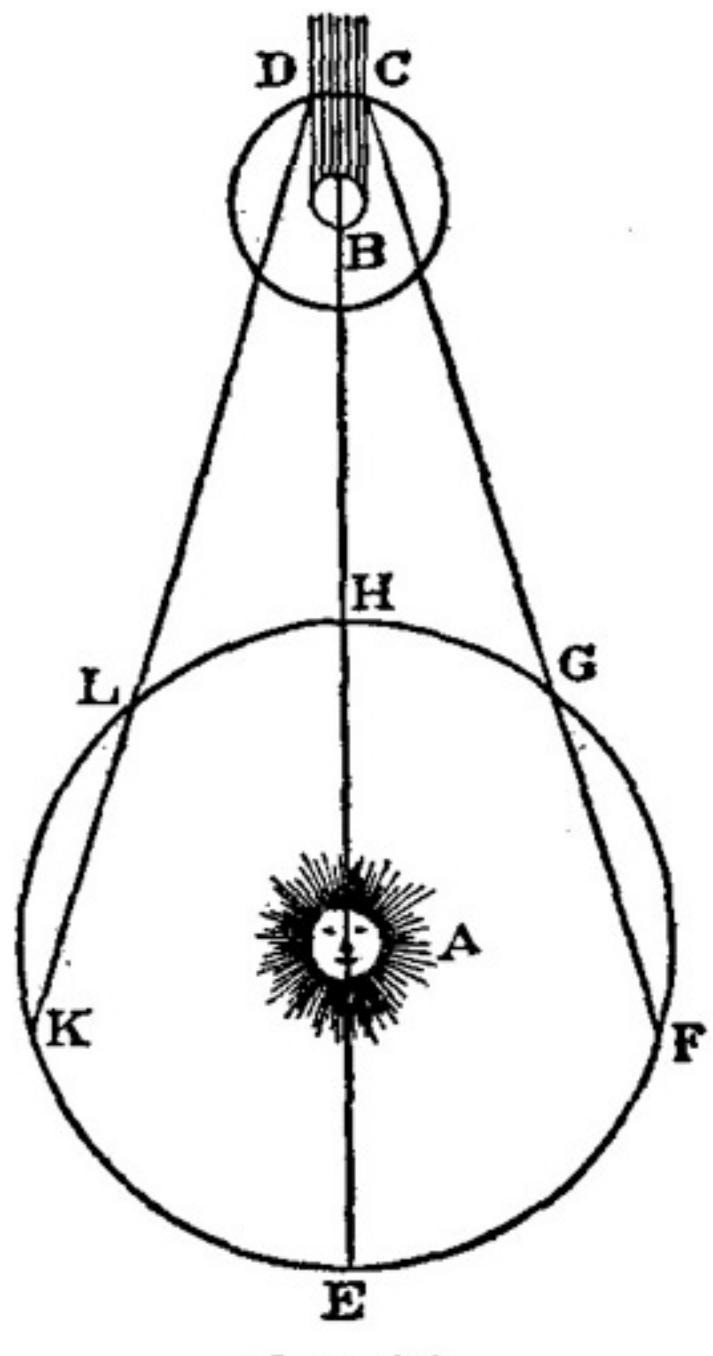
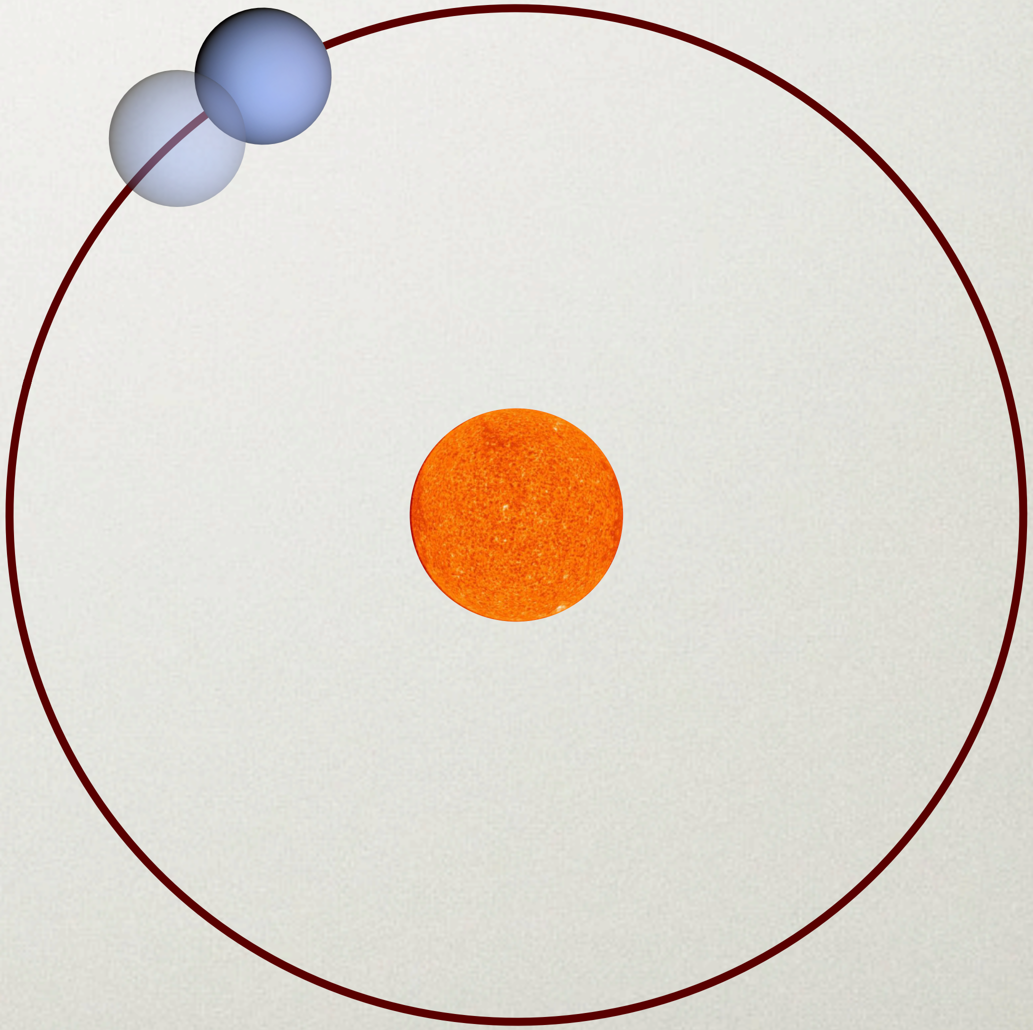


Fig. 70.

Confrontations with Experiment II

- Allowing for light's delay, observations were in pefect agreement for nearly 200 years of further data
  - until Uranus slowed down around 1820

Urbain LeVerrier, 1845



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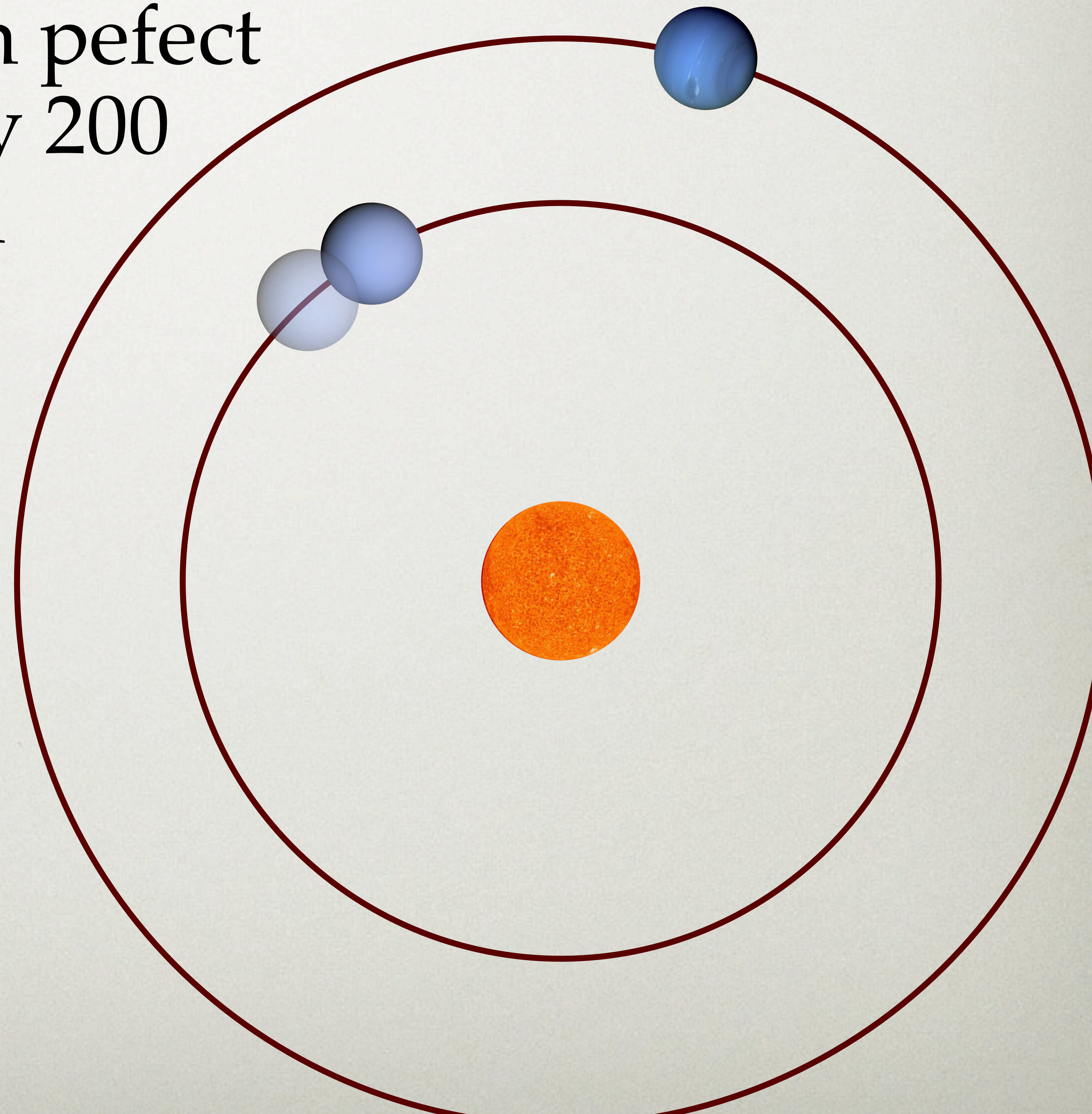
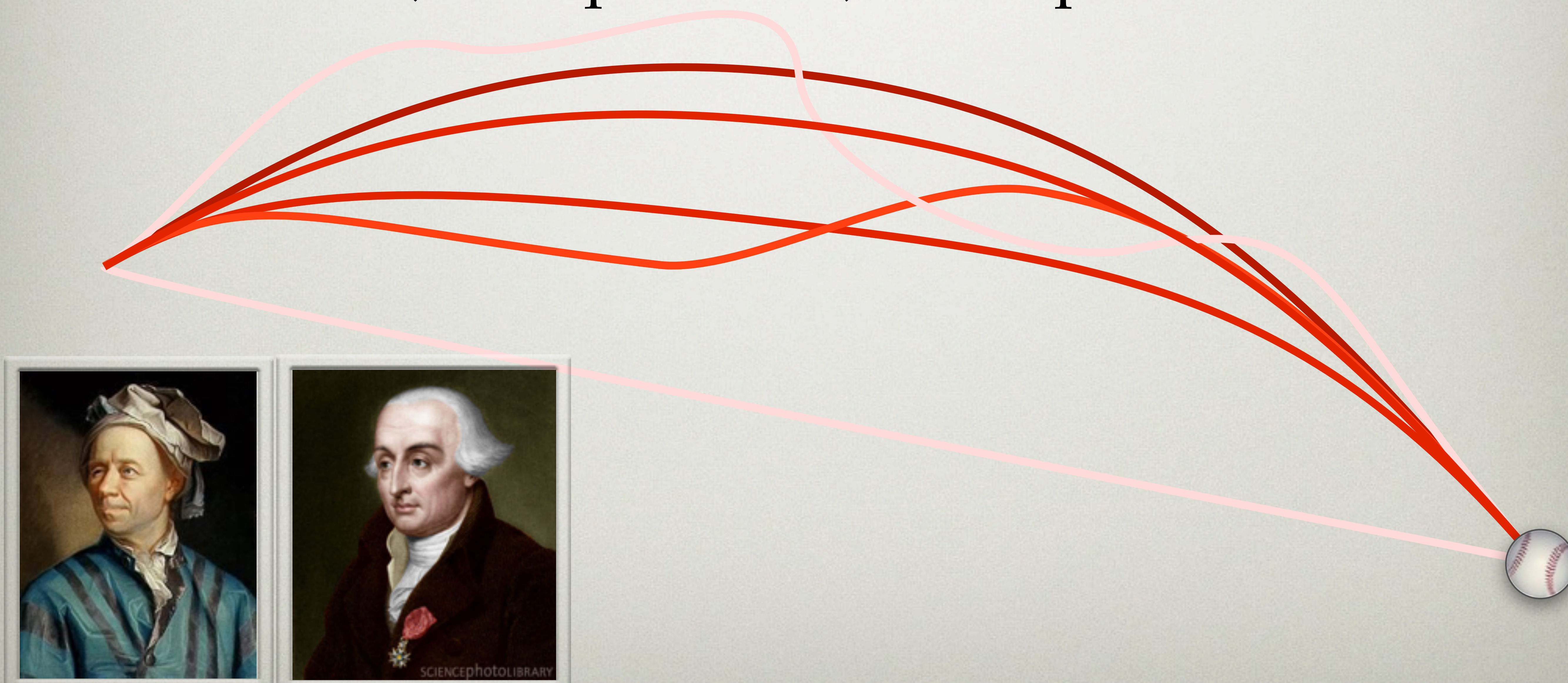


Radical Reformulation of the Rules

- We can understand motion as arising through a succession of forces acting at each instant of time
  - while intuitive, often quite mathematically challenging

Radical Reformulation of the Rules

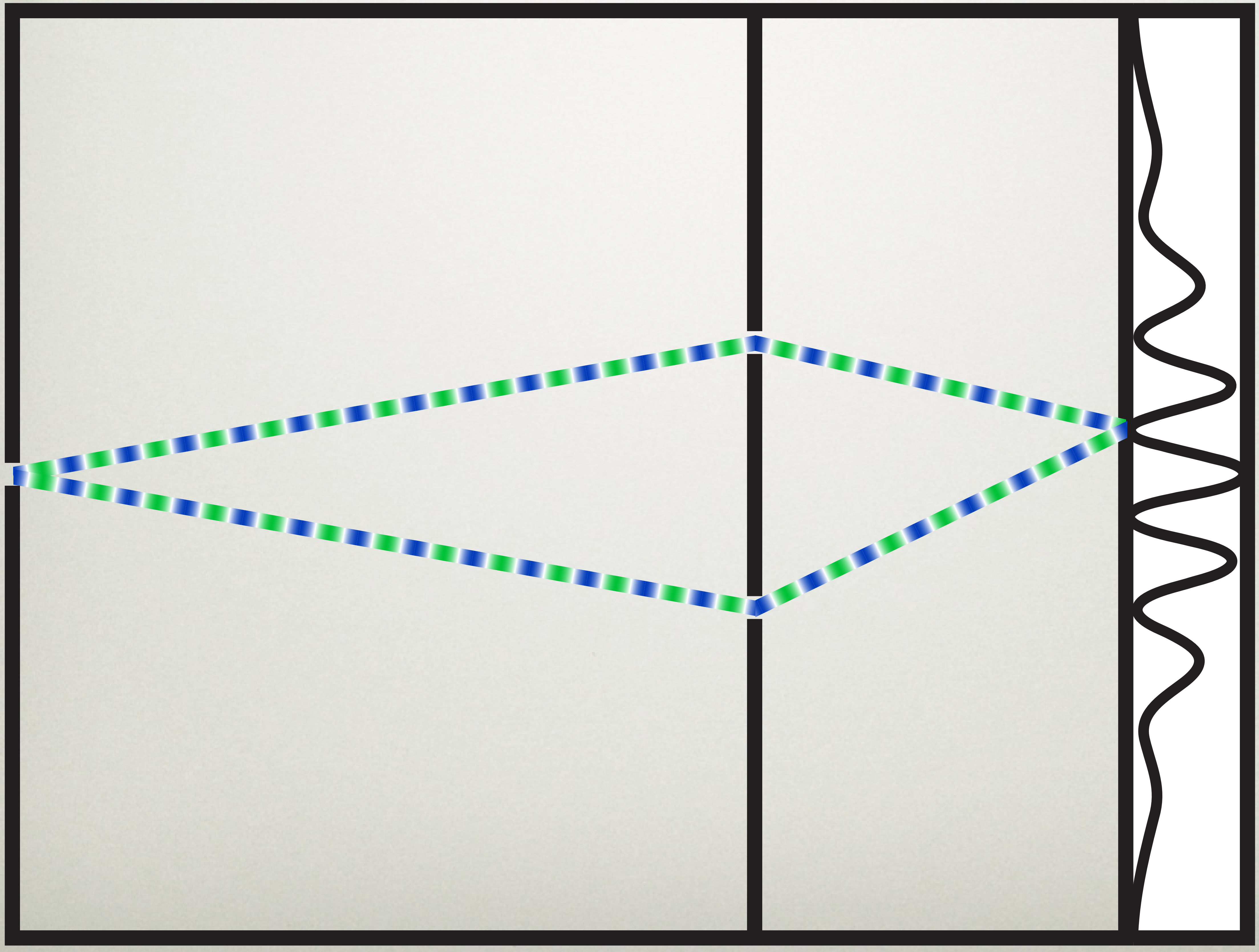
- We can understand motion as arising through a succession of forces acting at each instant of time
- Within a century after Newton, a new, radically different (but equivalent) description was found



Part 2:
Quantum Field Theory

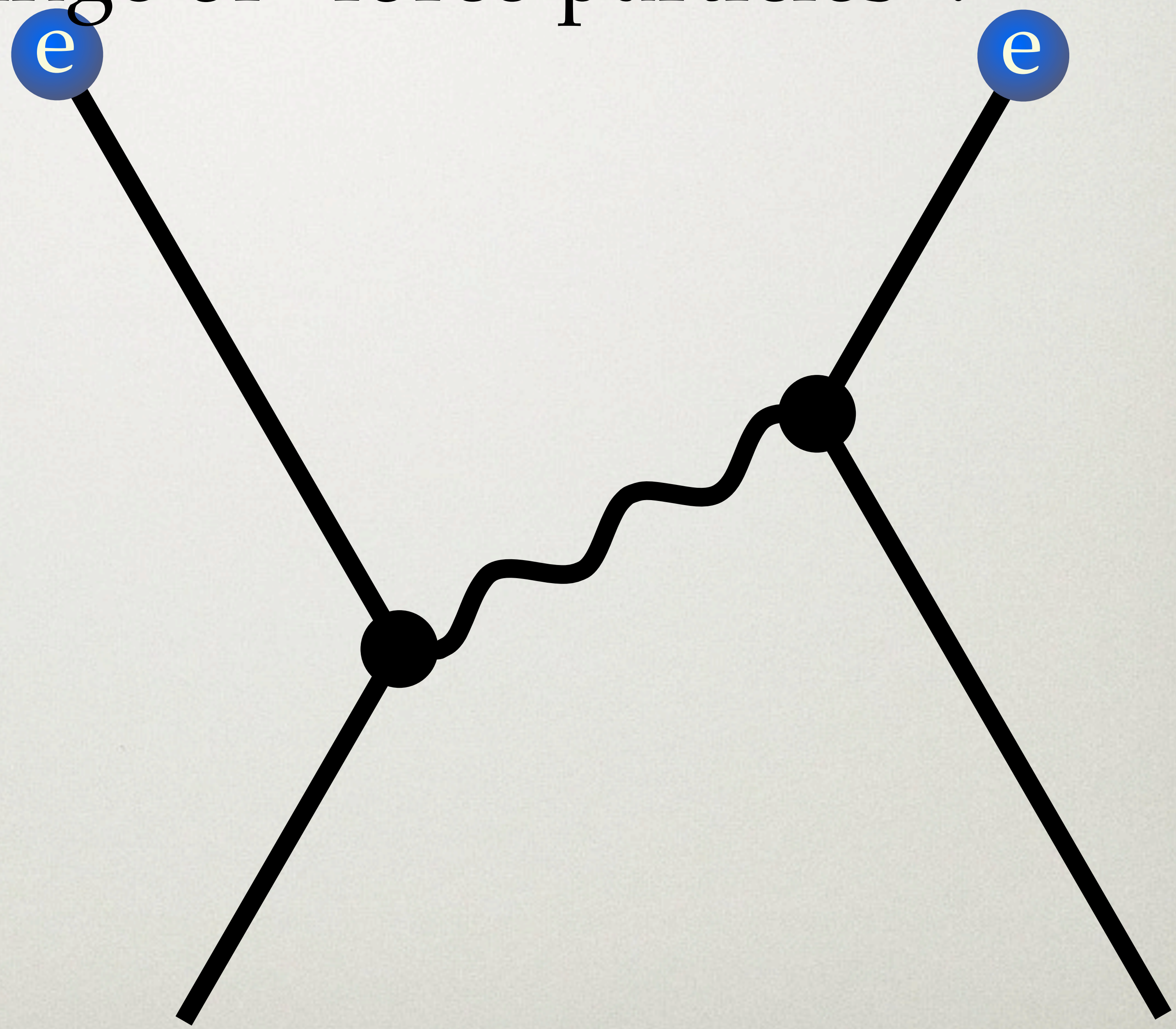
Quantum Particles & Probability

Quantum Particles & Probability



Feynman's Formulation of Force

- Forces (both classical and quantum) arise from the exchange of "force particles":



Summing Over Histories

- Shining light on an electron "rotates" it by a certain amount, called the *gyromagnetic ratio*, g_e

$$g_e^{\text{theory}} = 2 \qquad [1928]$$

$$g_e^{\text{expt}} \approx 2$$

The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

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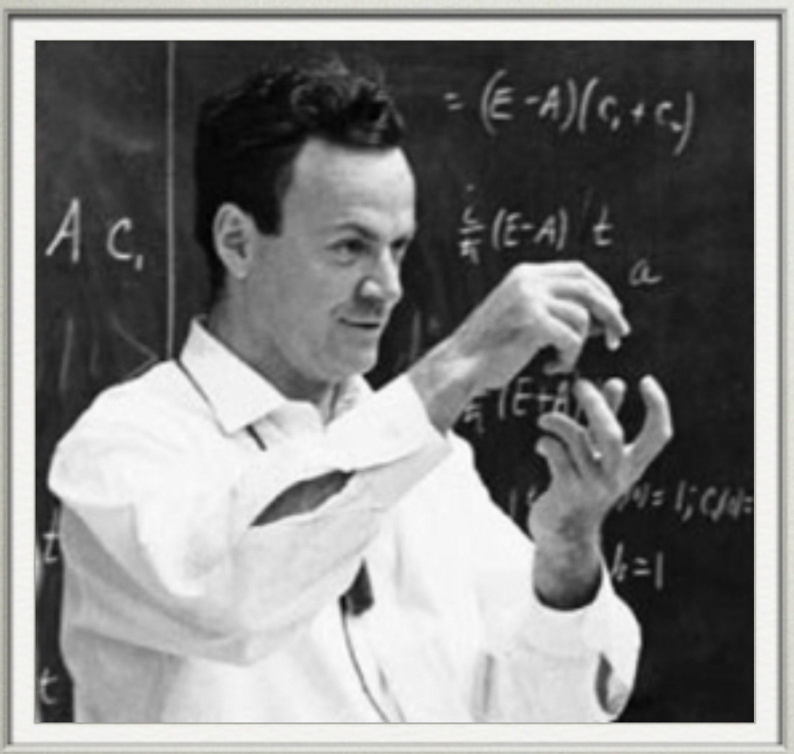

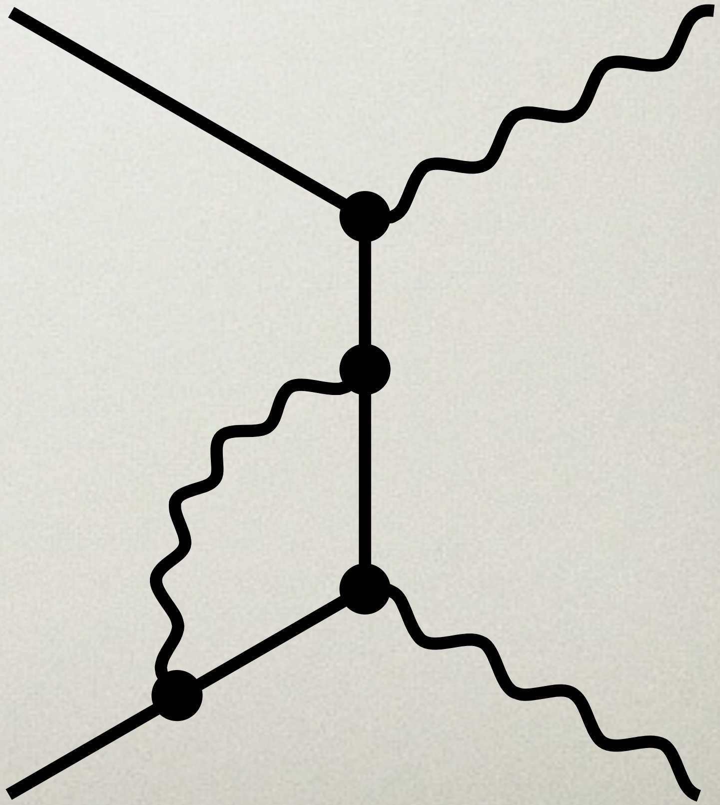

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$$g_e^{\text{theory}} = 2 + \frac{\alpha}{\pi} + \ldots \qquad [1947]$$

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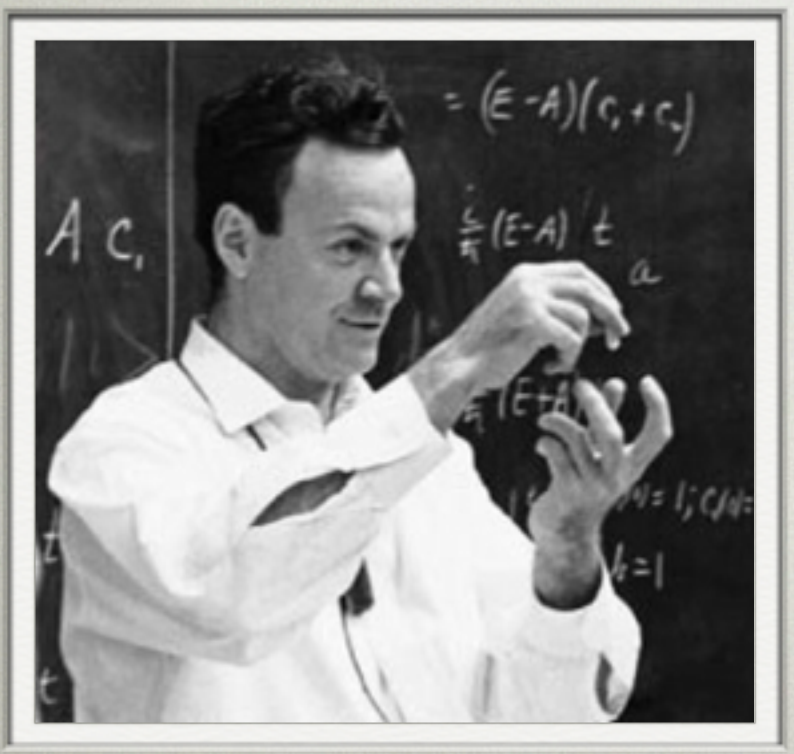

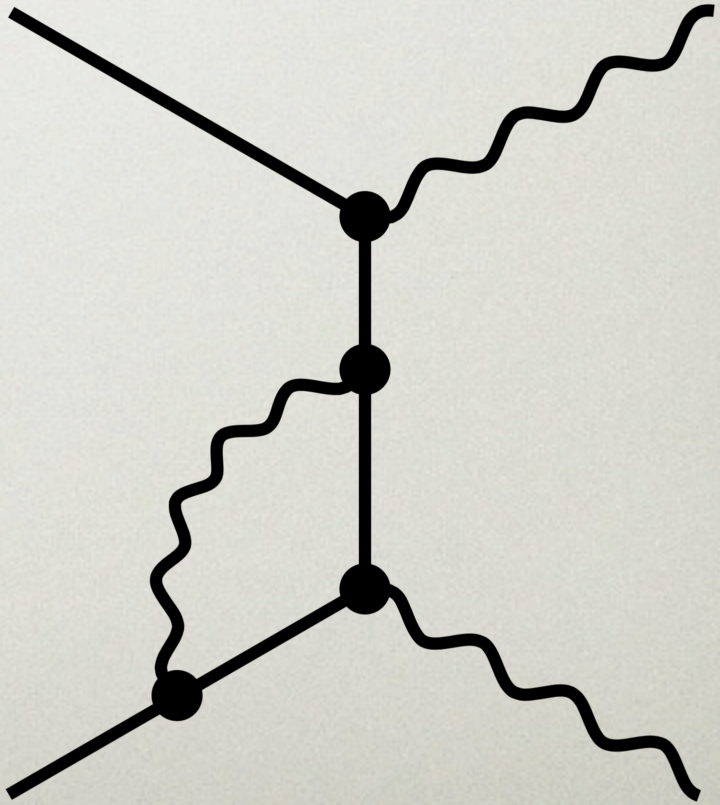

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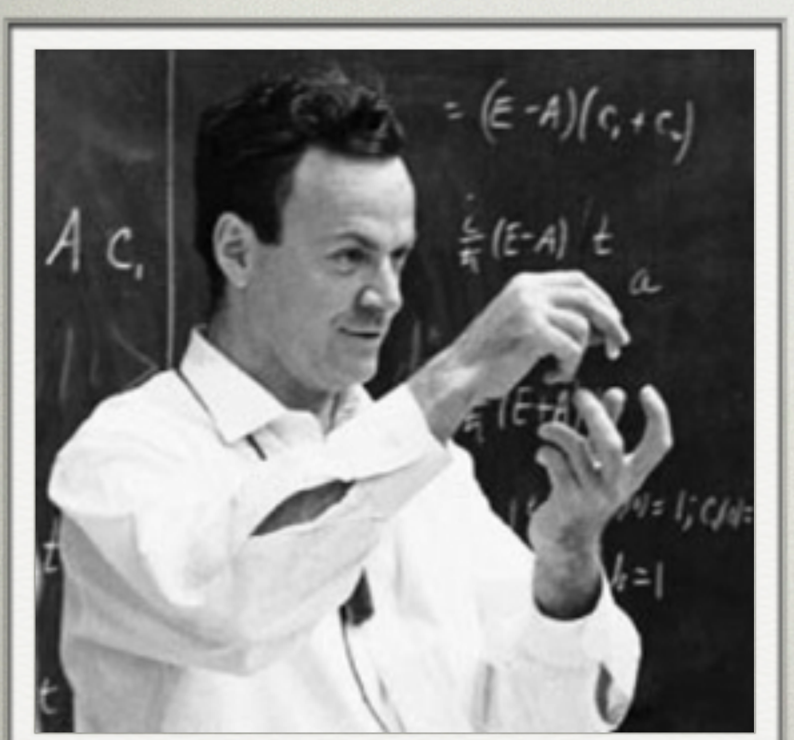

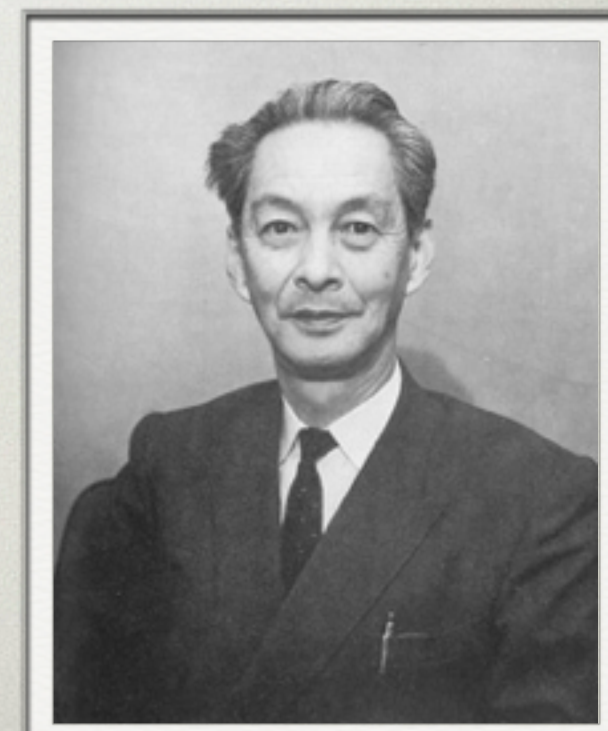

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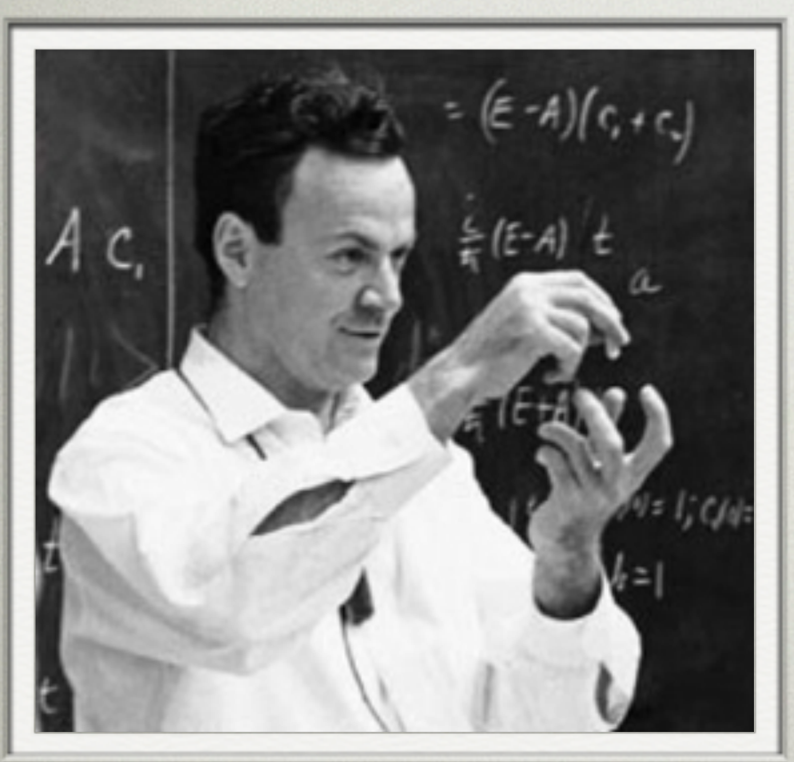

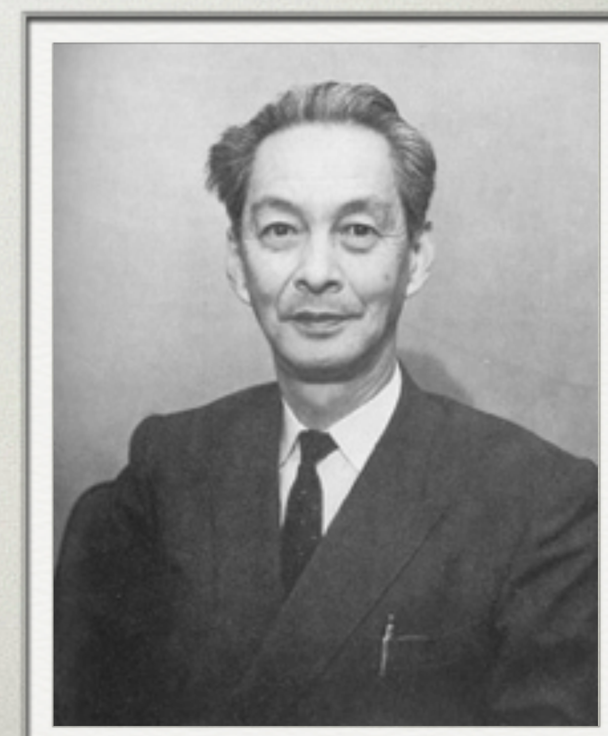

Summing Over Histories

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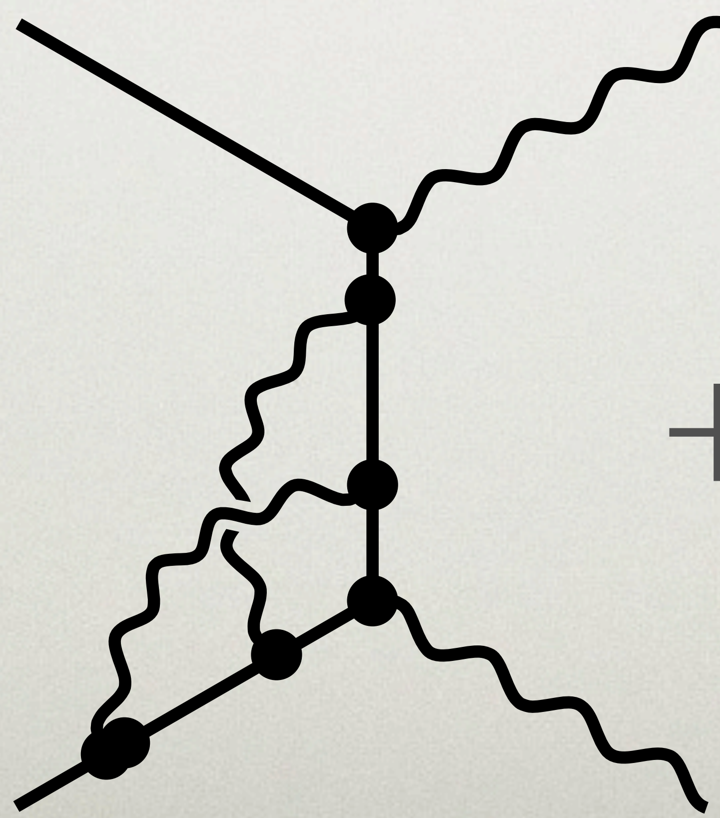
$$g_e^{\text{theory}} = 2.0023193\ldots \qquad [1957]$$

$$g_e^{\text{expt}} = 2.00231931\ldots \qquad [1972]$$



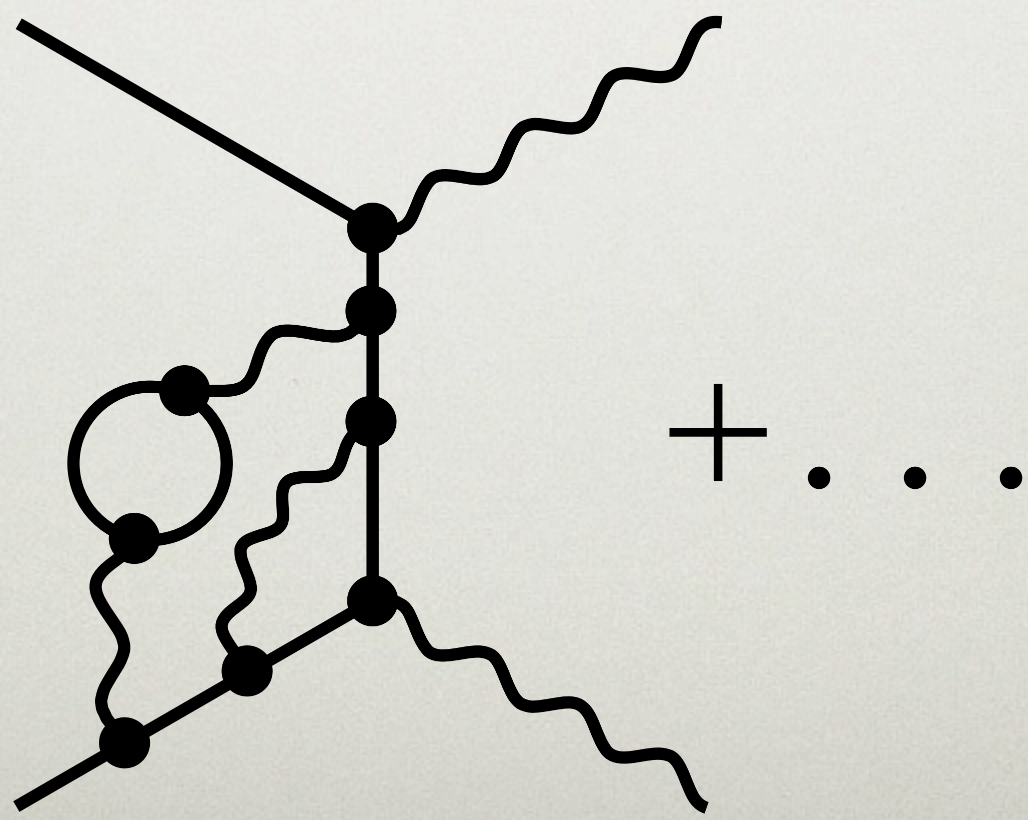
$+\ldots$

Summing Over Histories

- Shining light on an electron "rotates" it by a certain amount, called the *gyromagnetic ratio*, g_e

$$g_e^{\text{theory}} = 2.0023193044\ldots \qquad [1990]$$

$$g_e^{\text{expt}} = 2.00231931\ldots \qquad [1972]$$



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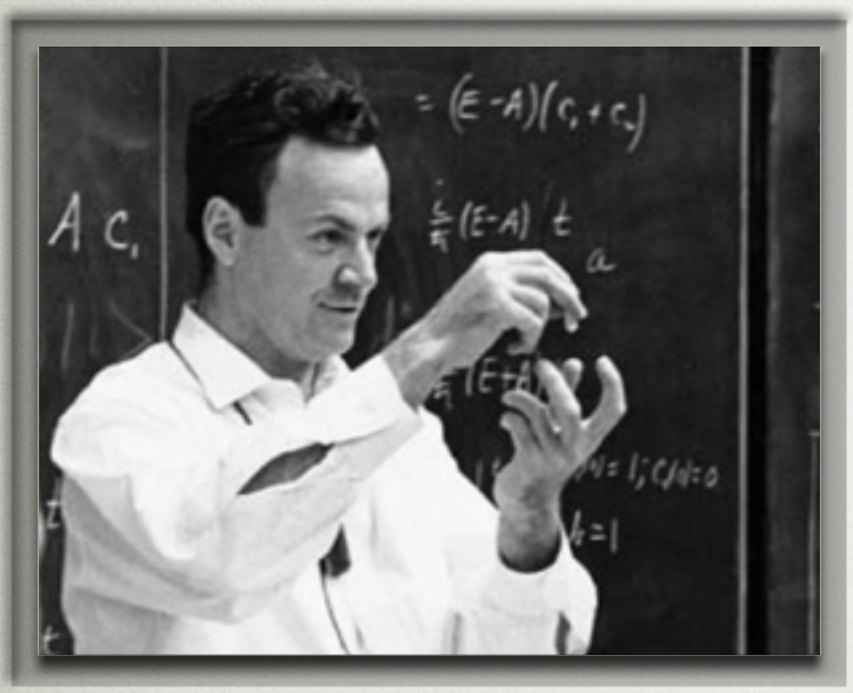
$$g_e^{\text{theory}} = 2.00231930435801\ldots \qquad [2012]$$

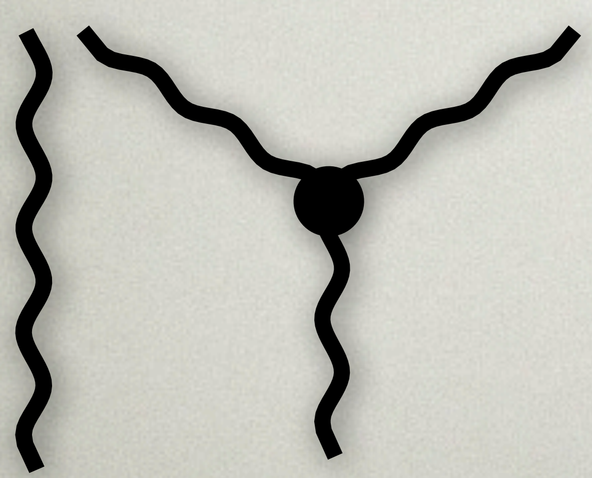
$$g_e^{\text{expt}} = 2.002319304361\ldots \qquad [2011]$$

Modern Laws of Nature: QFT

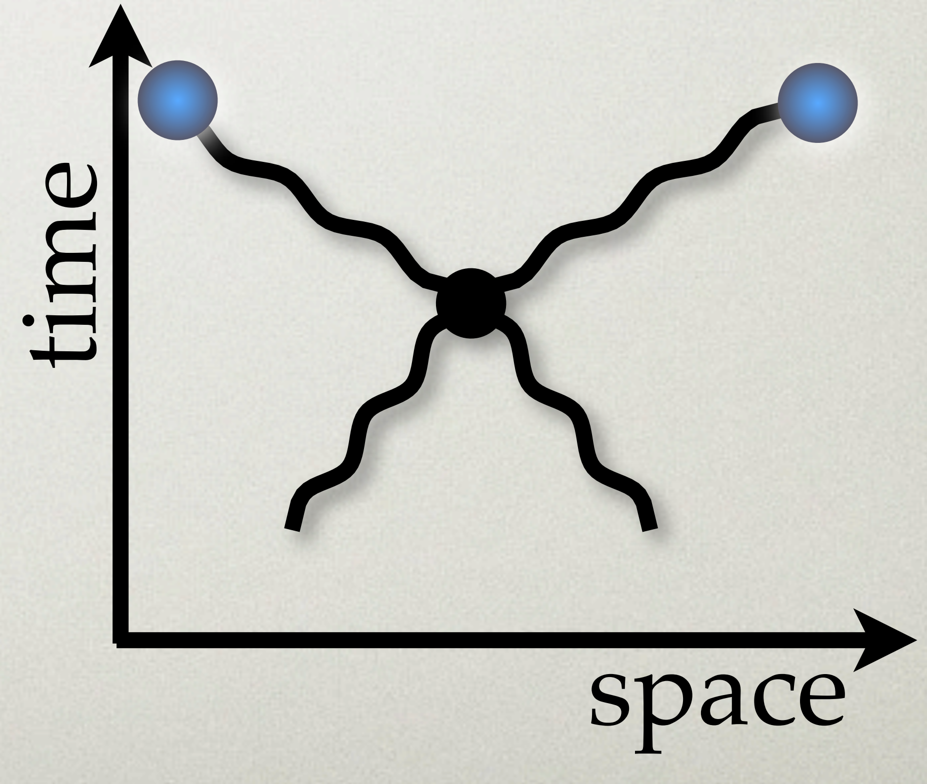


- Quantum field theories are specified by a list of **elementary** processes

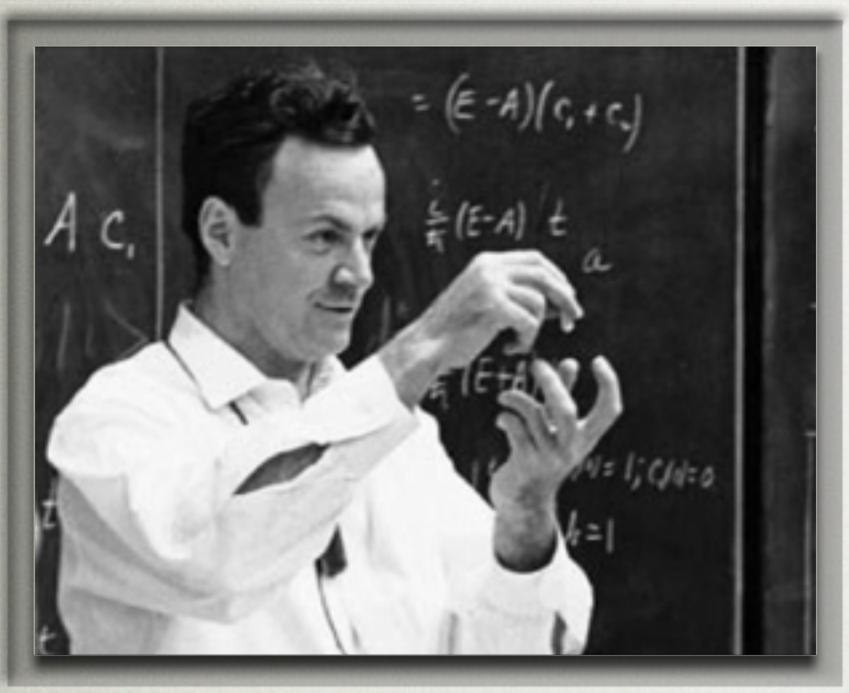
Quantum Chromodynamics



Feynman Diagrams



Modern Laws of Nature: QFT



- Quantum field theories are specified by a list of **elementary** processes
- The predicted distribution of outcomes for any experiment can then be calculated by summing over all possible "histories"

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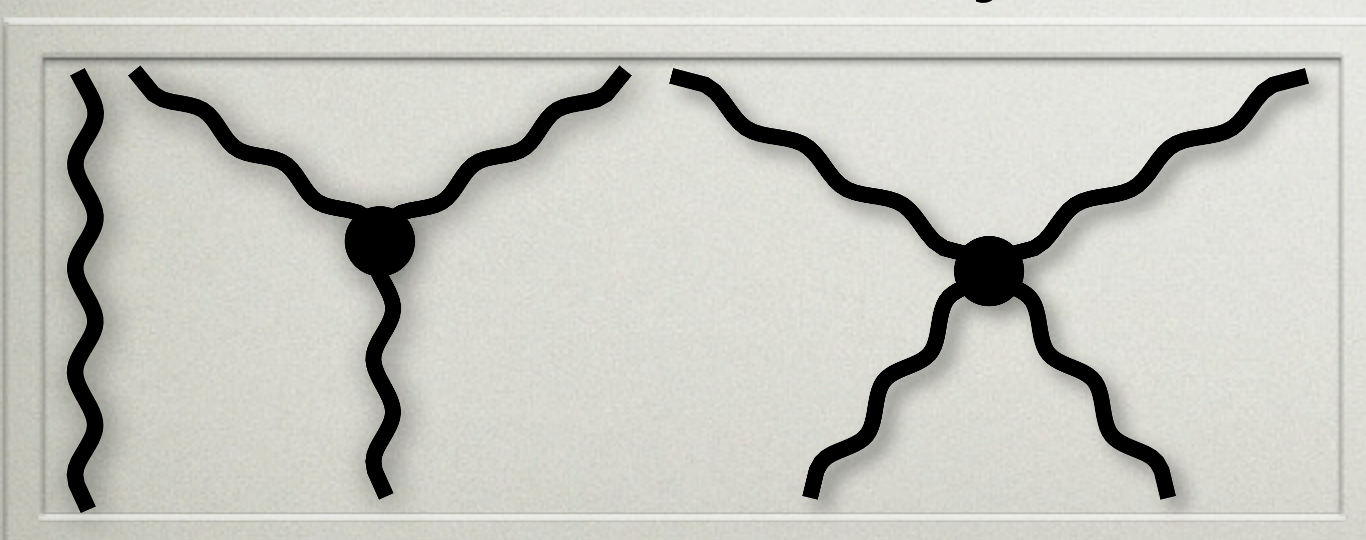
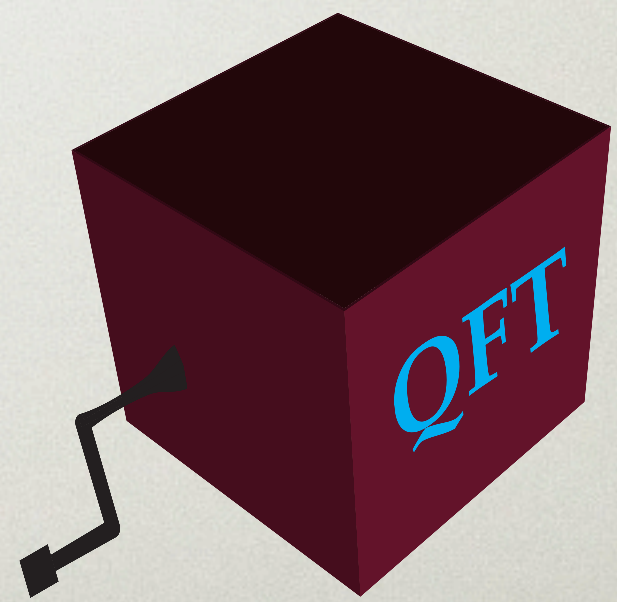
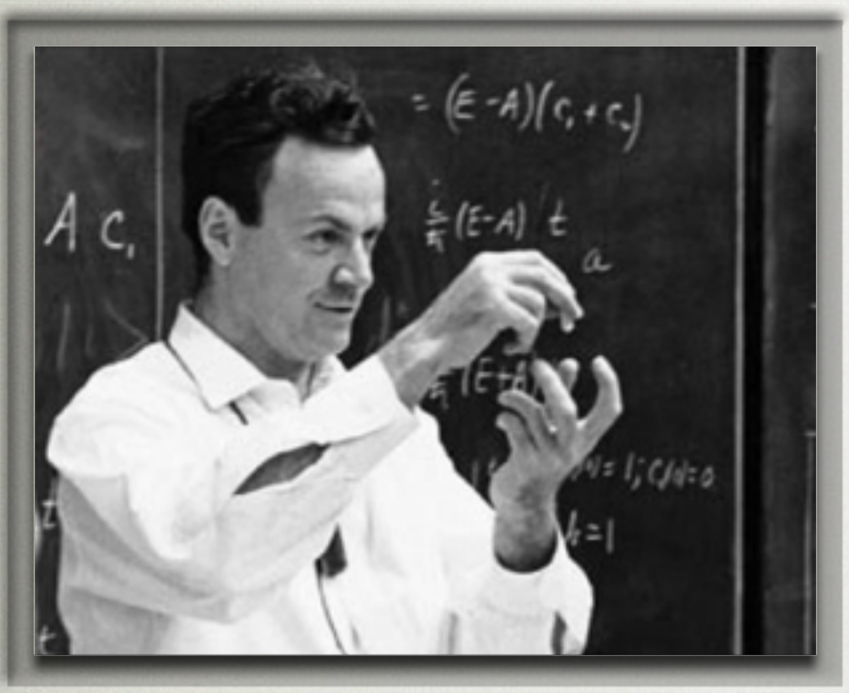




Modern Laws of Nature: QFT



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Exempli Gratia: collide two gluons, and find four coming out

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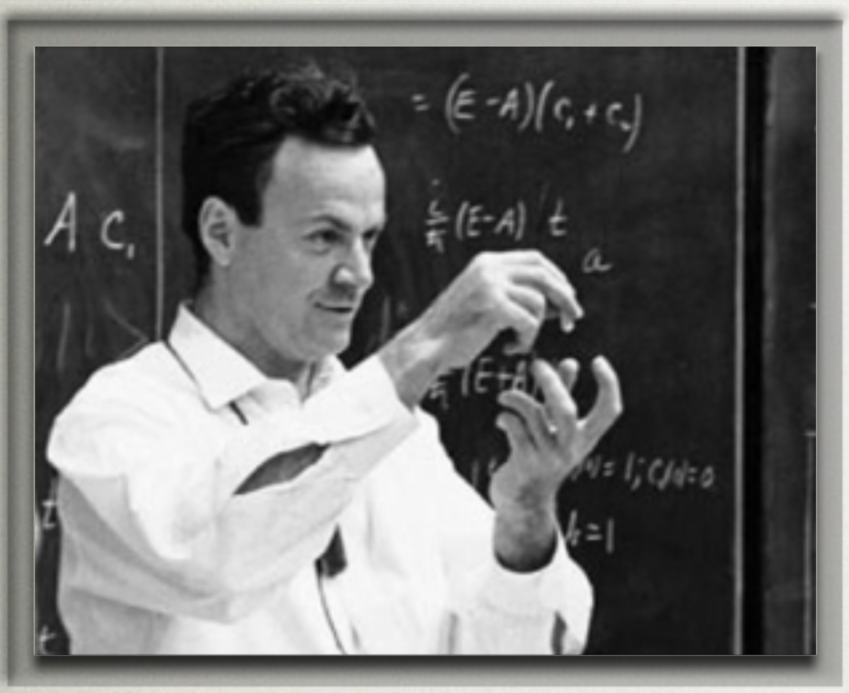

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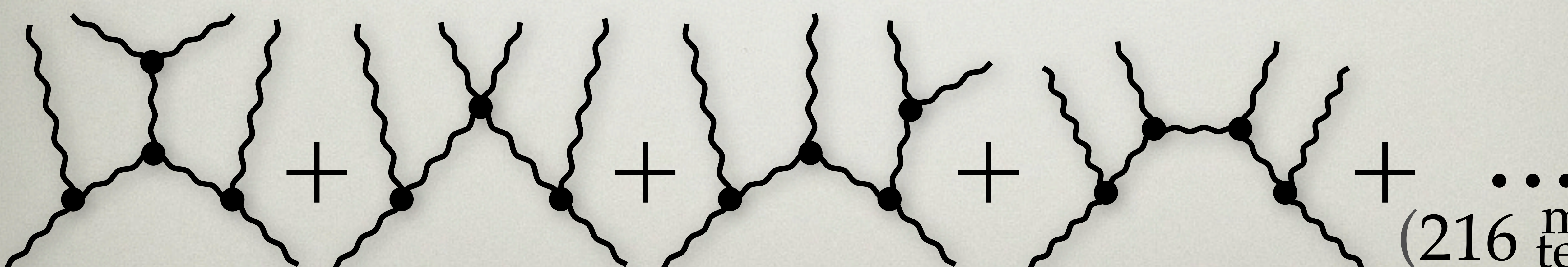
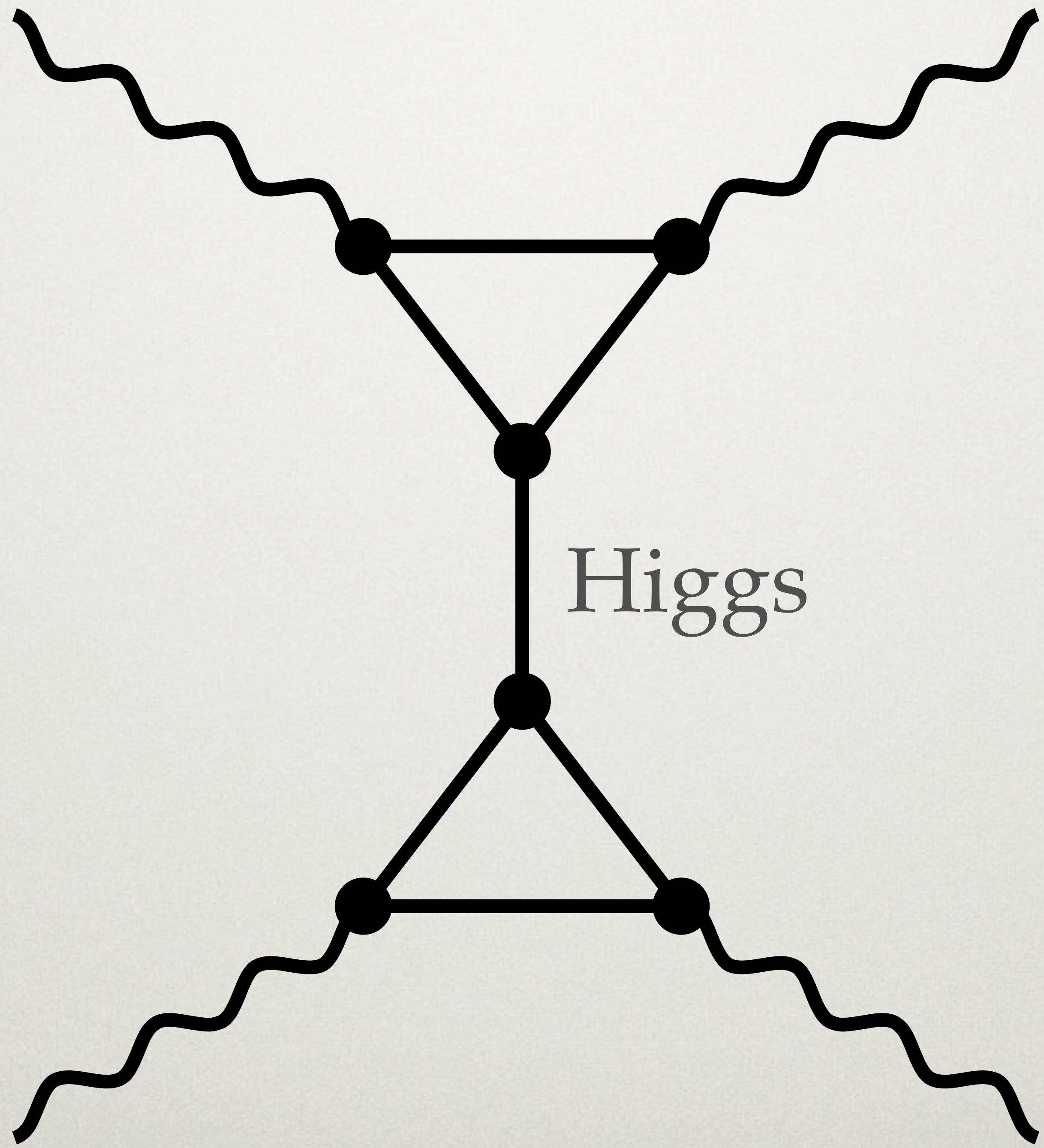


(216 more terms)

Discovery of the Higgs Boson



Challenges and Triumphs of QFT

- The amplitude for (2 gluons) ⟶ (4 gluons) was computed by **Parke and Taylor** in 1985
  - the calculation required many clever tricks, and one of the most powerful supercomputers in the world (at the time)
- The final formula: **8 pages**

THE CROSS SECTION FOR FOUR-GLUON PRODUCTION BY GLUON–GLUON FUSION

Stephen J. PARKE and T.R. TAYLOR

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA

Received 13 September 1985

The cross section for two-gluon to four-gluon scattering is given in a form suitable for fast numerical calculations.

gluons. The cross section for the scattering of two gluons with momenta p_1, p_2 into four gluons with momenta p_3, p_4, p_5, p_6 is obtained from eq. (5) by setting $I = 2$ and replacing the momenta p_3, p_4, p_5, p_6 by $-p_3, -p_4, -p_5, -p_6$.

As the result of the computation of two hundred and forty Feynman diagrams, we obtain

$$A_{\binom{0}{2}}(p_1, p_2, p_3, p_4, p_5, p_6) = (\mathcal{D}^\dagger, \mathcal{D}^\dagger_\rho, \mathcal{D}^\dagger_\sigma, \mathcal{D}^\dagger_\tau)_{\binom{0}{2}} \cdot \begin{pmatrix} K & K_\rho & K_\sigma & K_\tau \\ K_\rho & K & K_\tau & K_\sigma \\ K_\sigma & K_\tau & K & K_\rho \\ K_\tau & K_\sigma & K_\rho & K \end{pmatrix} \cdot \begin{pmatrix} \mathcal{D} \\ \mathcal{D}_\rho \\ \mathcal{D}_\sigma \\ \mathcal{D}_\tau \end{pmatrix}_{\binom{0}{2}}, \quad (6)$$

where $\mathcal{D}$, $\mathcal{D}_\rho$, $\mathcal{D}_\sigma$ and $\mathcal{D}_\tau$ are 11-component complex vector functions of the momenta p_1, p_2, p_3, p_4, p_5 and p_6, and K, K_ρ, K_σ and K_τ are constant 11×11 symmetric matrices. The vectors $\mathcal{D}_\rho$, $\mathcal{D}_\sigma$ and $\mathcal{D}_\tau$ are obtained from the vector $\mathcal{D}$ by the permutations $(p_2 \leftrightarrow p_3)$, $(p_5 \leftrightarrow p_6)$ and $(p_2 \leftrightarrow p_3, p_5 \leftrightarrow p_6)$, respectively, of the momentum variables in $\mathcal{D}$. The individual components of the vector $\mathcal{D}$ represent the sums of all contributions proportional to the appropriately chosen eleven basis color factors. The matrices K, which are the suitable sums over the color indices of products of the color bases, contain two independent structures, proportional to $N^4(N^2-1)$ and $N^2(N^2-1)$, respectively (N is the number of colors, $N = 3$ for QCD):

$$K = \tfrac{1}{8}g^8 N^4(N^2-1)K^{(4)} + \tfrac{1}{2}g^8 N^2(N^2-1)K^{(2)}. \quad (7)$$

Here g denotes the gauge coupling constant. The matrices $K^{(4)}$ and $K^{(2)}$ are given in table 1. The vector $\mathcal{D}$ is related to the thirty-three diagrams $D^{G}(I = 1-33)$ for two-gluon to four-scalar scattering, eleven diagrams $D^{F}(I = 1-11)$ for two-fermion to four-scalar scattering and sixteen diagrams $D^{S}(I = 1-16)$ for two-scalar to four-scalar scattering, in the following way:

$$\mathcal{D}_0 = \frac{2s_{14}}{\sqrt{|s_{15}s_{45}s_{16}s_{46}|s_{23}s_{56}}}\{t_{123}^2 C^{G} \cdot D_0^{G} - 4s_{14}t_{123}E(p_5+p_6, p_6)C^{F} \cdot D_0^{F} - 2s_{14}G(p_5+p_6, p_5+p_6)C^{S} \cdot D_0^{S}\},$$

$$\mathcal{D}_2 = \frac{s_{56}}{s_{23}} C^{G} \cdot D_2^{G}, \quad (8)$$

where the constant matrices $C^{G}(11\times 33)$, $C^{F}(11\times 11)$ and $C^{S}(11\times 16)$ are given in table 2. The Lorentz invariants s_{ij} and t_{ijk} are defined as $s_{ij} = (p_i + p_j)^2$, $t_{ijk} = (p_i + p_j + p_k)^2$ and the complex functions E and G are given by

$$E(p_i, p_j) = \tfrac{1}{2}\{(p_1 p_4)(p_i p_j) - (p_1 p_i)(p_j p_4) - (p_1 p_j)(p_i p_4) + i\varepsilon_{\mu\nu\rho\lambda} p_1^\mu p_i^\nu p_j^\rho p_4^\lambda\}/(p_1 p_4),$$

$$G(p_i, p_j) = E(p_i, p_5)E(p_j, p_6), \quad (9)$$

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  - the calculation required many clever tricks, and one of the most powerful supercomputers in the world (at the time)
- The final formula: **8 pages**

of our calculation, the most powerful test does not rely on the gauge symmetry, but on the appropriate permutation symmetries. The function $A_0(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under arbitrary permutations of the momenta (p_1, p_2, p_3) and separately, (p_4, p_5, p_6), whereas the function $A_2(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under the permutations of (p_1, p_2, p_3, p_4) and separately, (p_5, p_6). This test is extremely powerful, because the required permutation symmetries are hidden in our supersymmetry relations, eqs. (1) and (3), and in the structure of amplitudes involving different species of particles. Another, very important test relies on the absence of the double poles of the form $(s_{ij})^{-2}$ in the cross section, as required by general arguments based on the helicity conservation. Further, in the leading $(s_{ij})^{-1}$ pole approximation, the answer should reduce to the two goes to three cross section [3, 4], convoluted with the appropriate Altarelli-Parisi probabilities [5]. Our result has succesfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

We thank Keith Ellis, Chris Quigg and especially Estia Eichten for many useful discussions and encouragement during the course of this work. We acknowledge the hospitality of Aspen Center for Physics, where this work was being completed in a pleasant, strung-out atmosphere.

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F.A. Berends, R. Kleiss, P. de Causmaecker, R. Gastmans and T.T. Wu, Phys. Lett. 103B (1981) 1…
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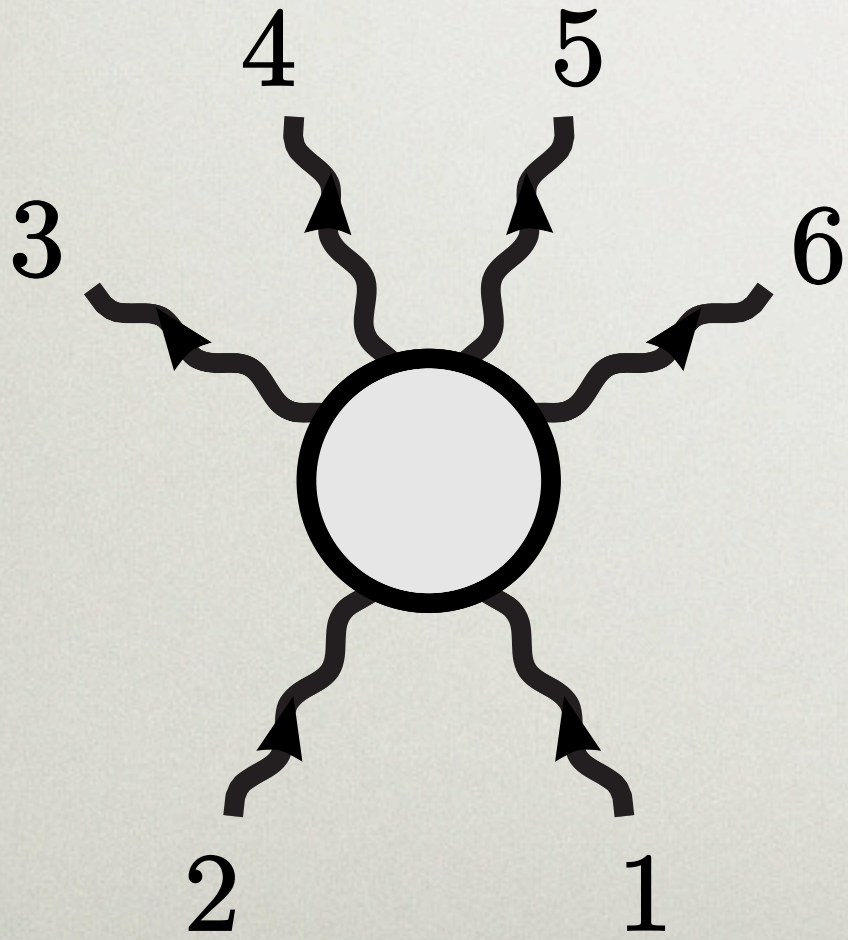
Shocking Simplicity of Amplitudes

- Six months later, they **guessed** a simplified form of their earlier answer—a formula which now bears their names



$$= \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle\langle 6\,1\rangle}$$

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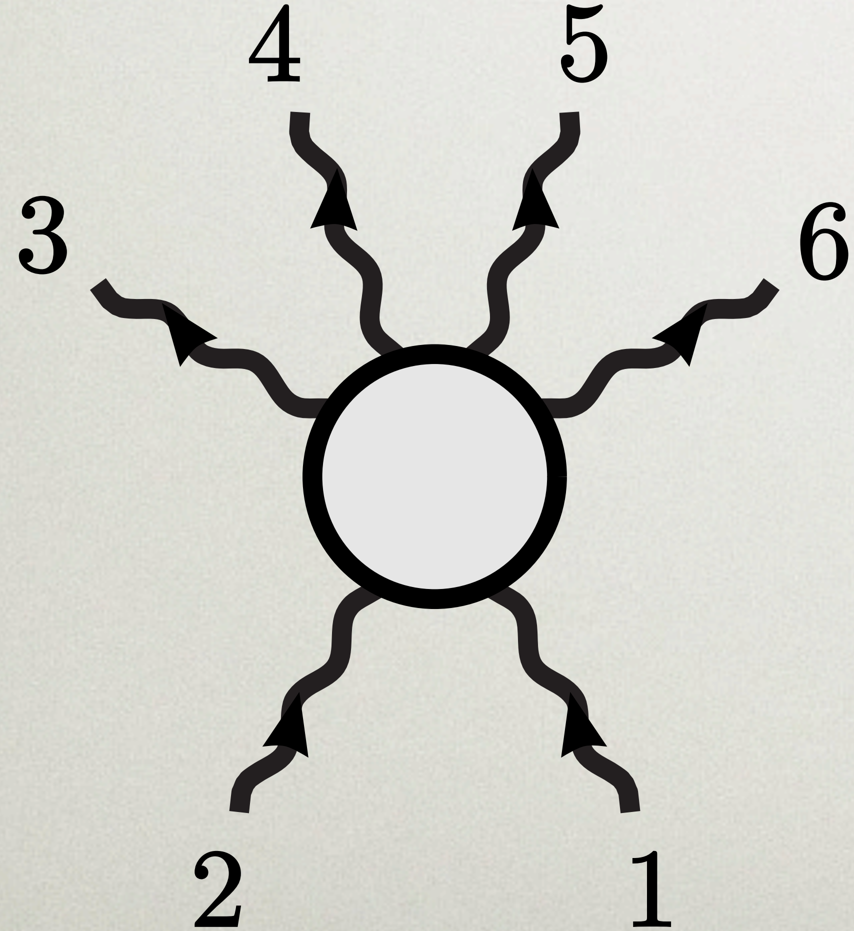

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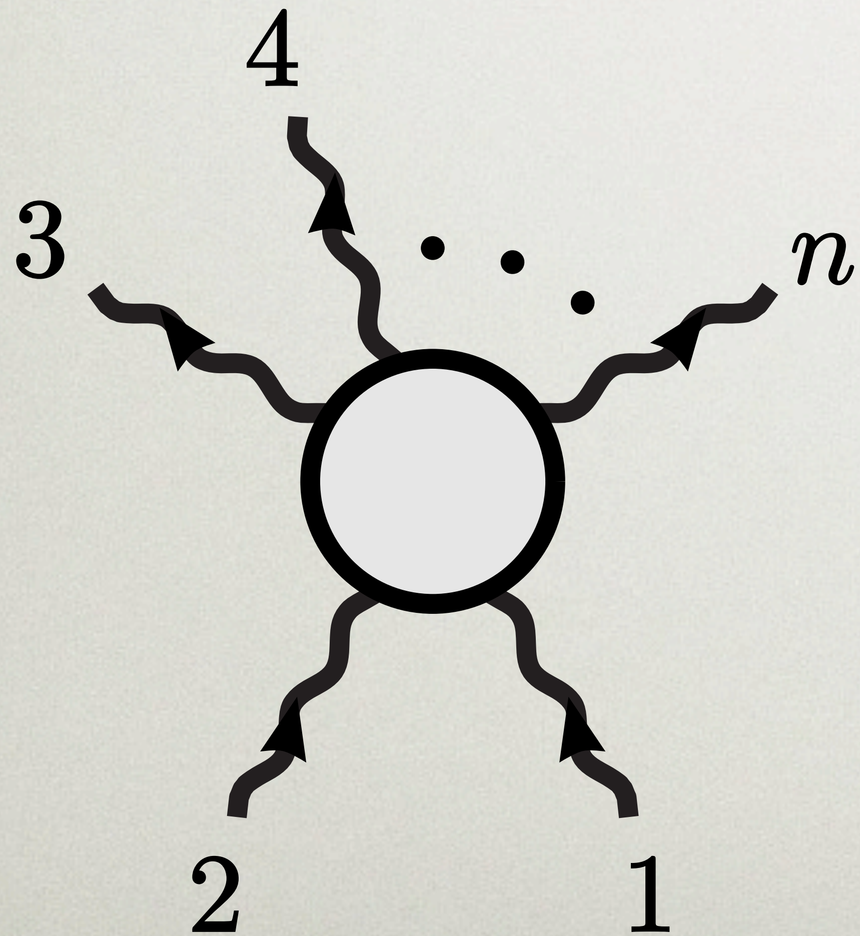
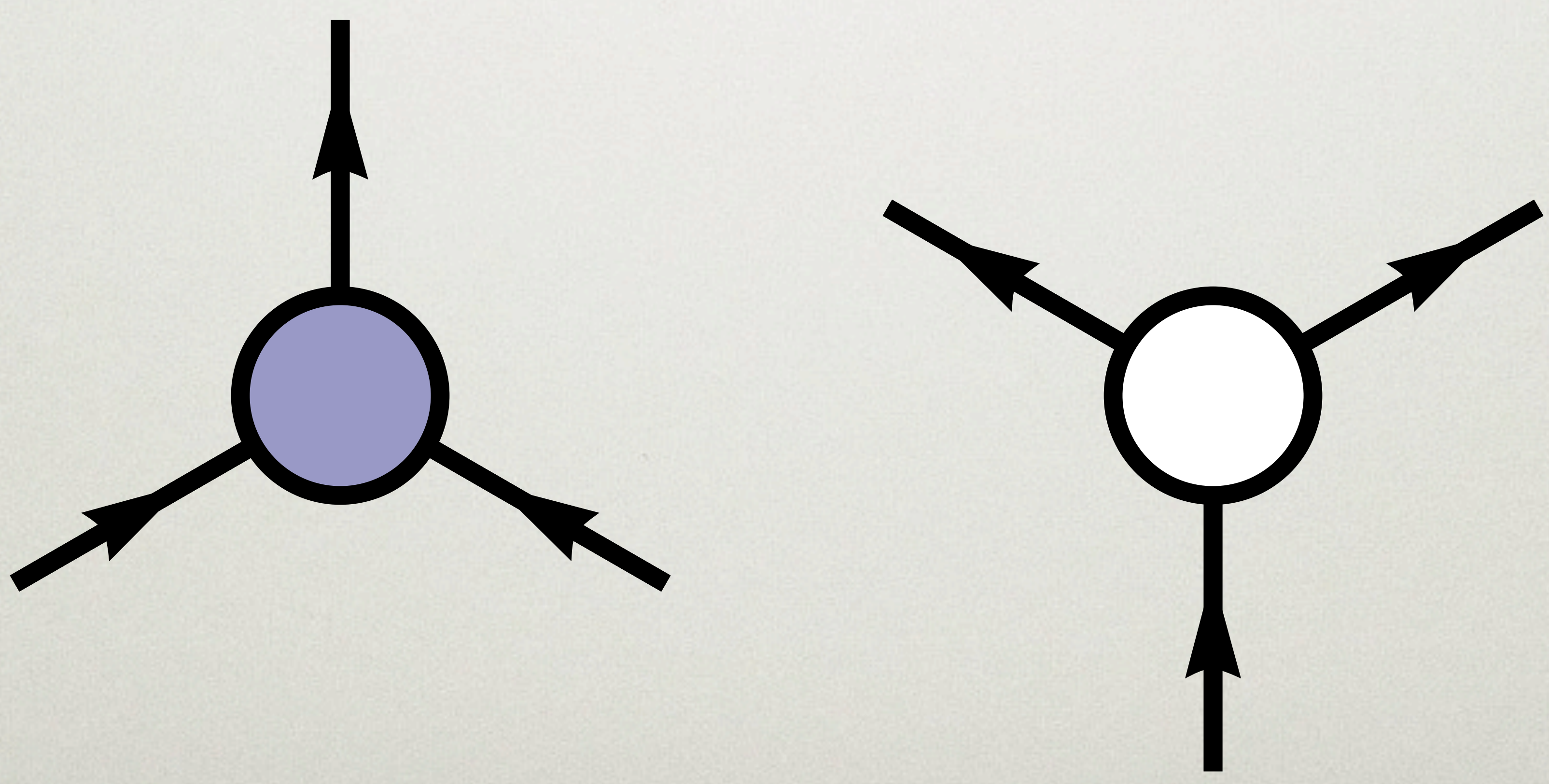


$$= \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\ \cdots\ \langle n\,1\rangle}$$

A Radical Reformulation of QFT

In 2004 Britto, Cachazo, and Feng proposed that amplitudes be determined recursively (to leading order) in terms of two basic processes, without **ever** introducing *imagined* 'histories':



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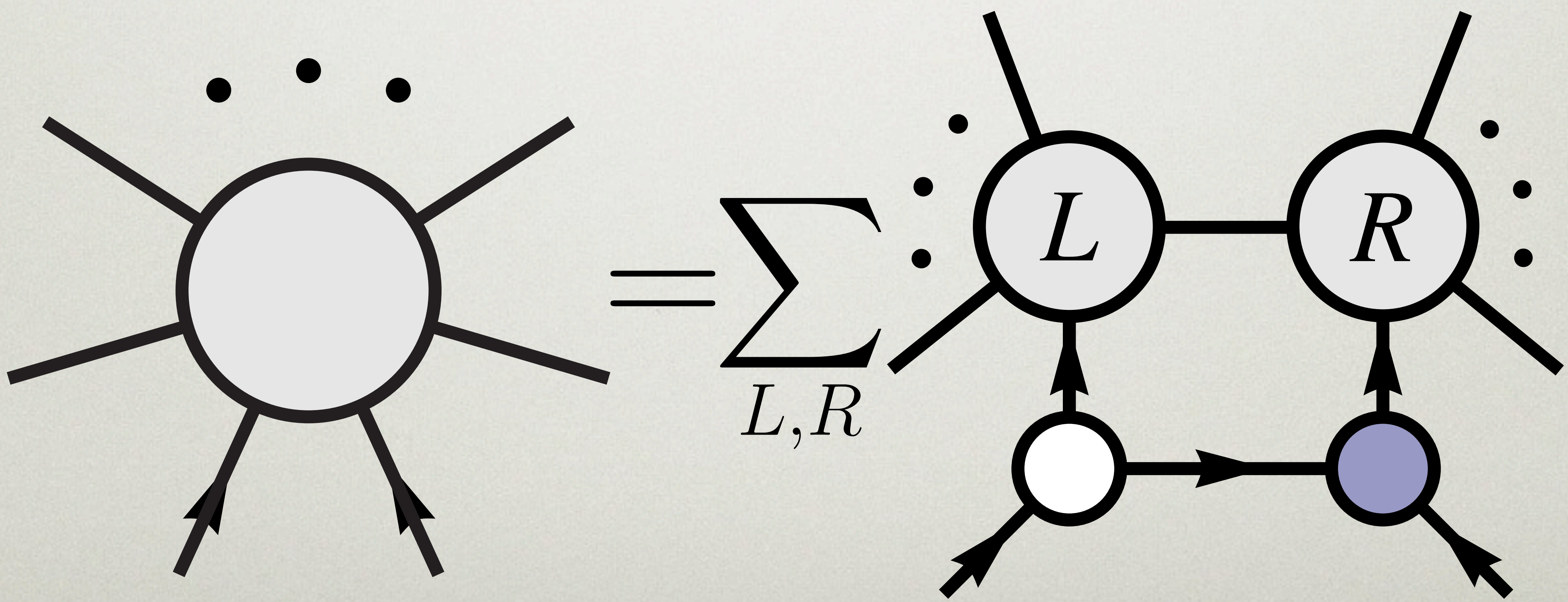

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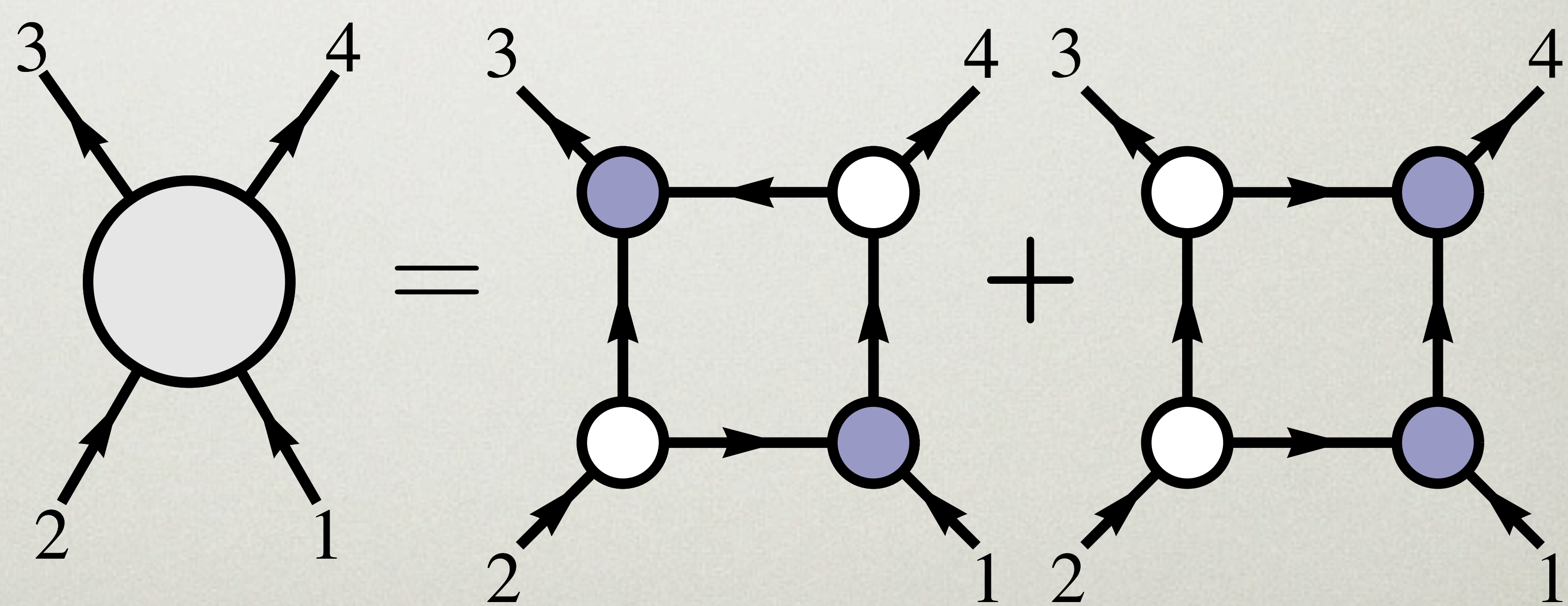

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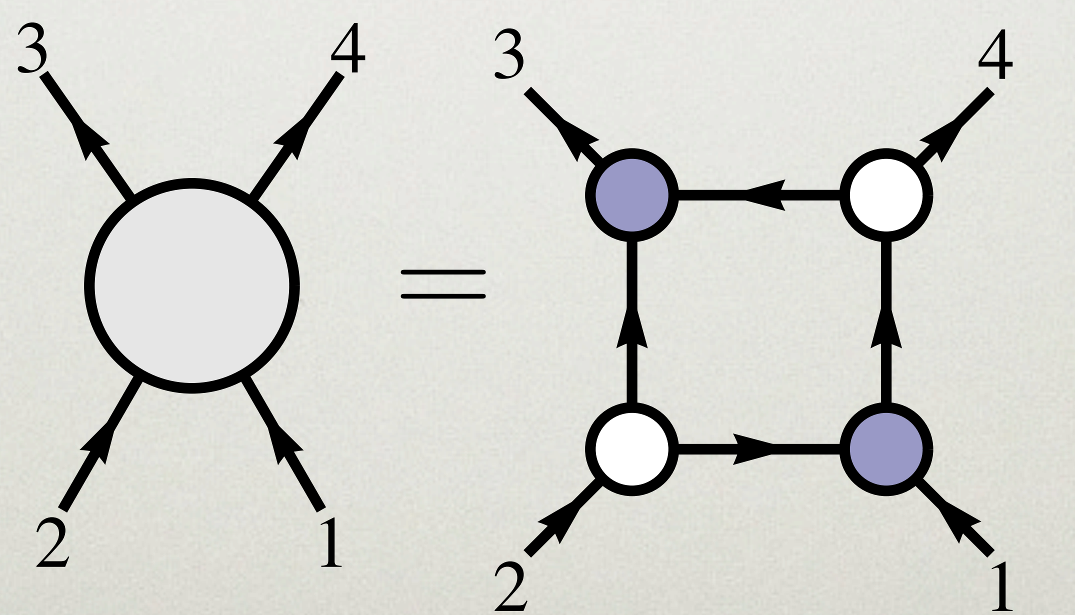

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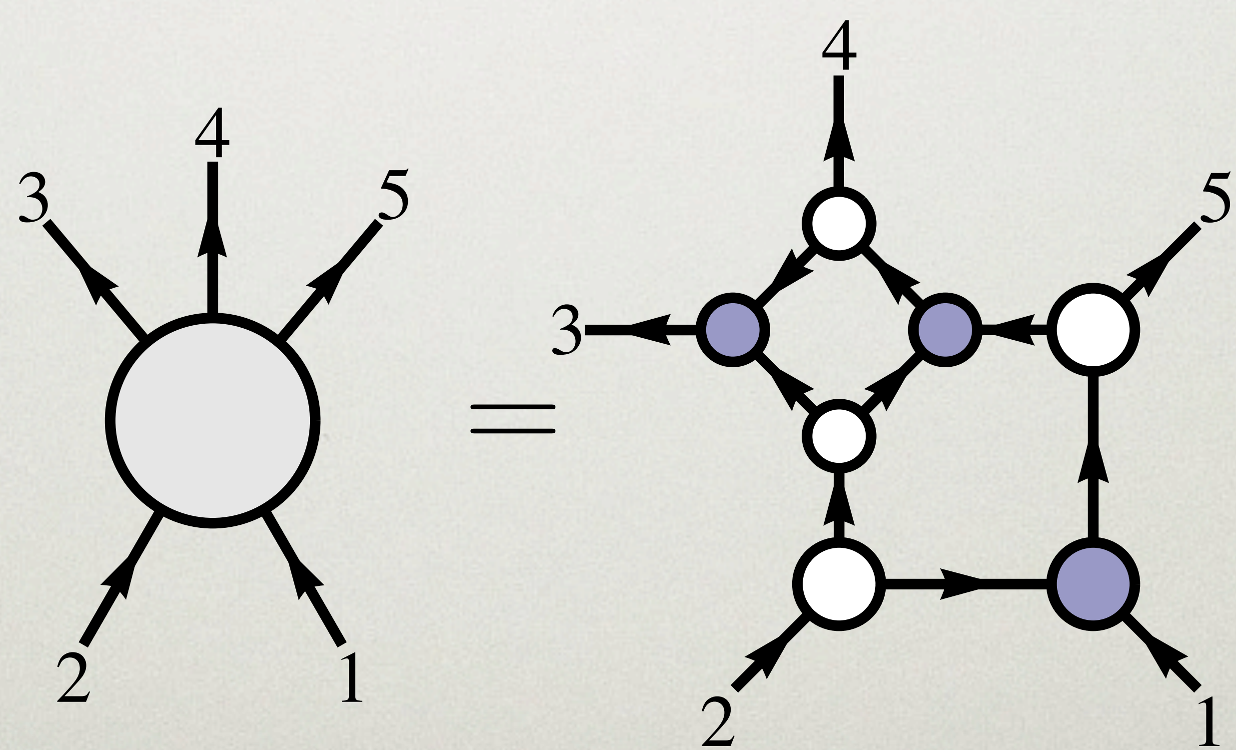

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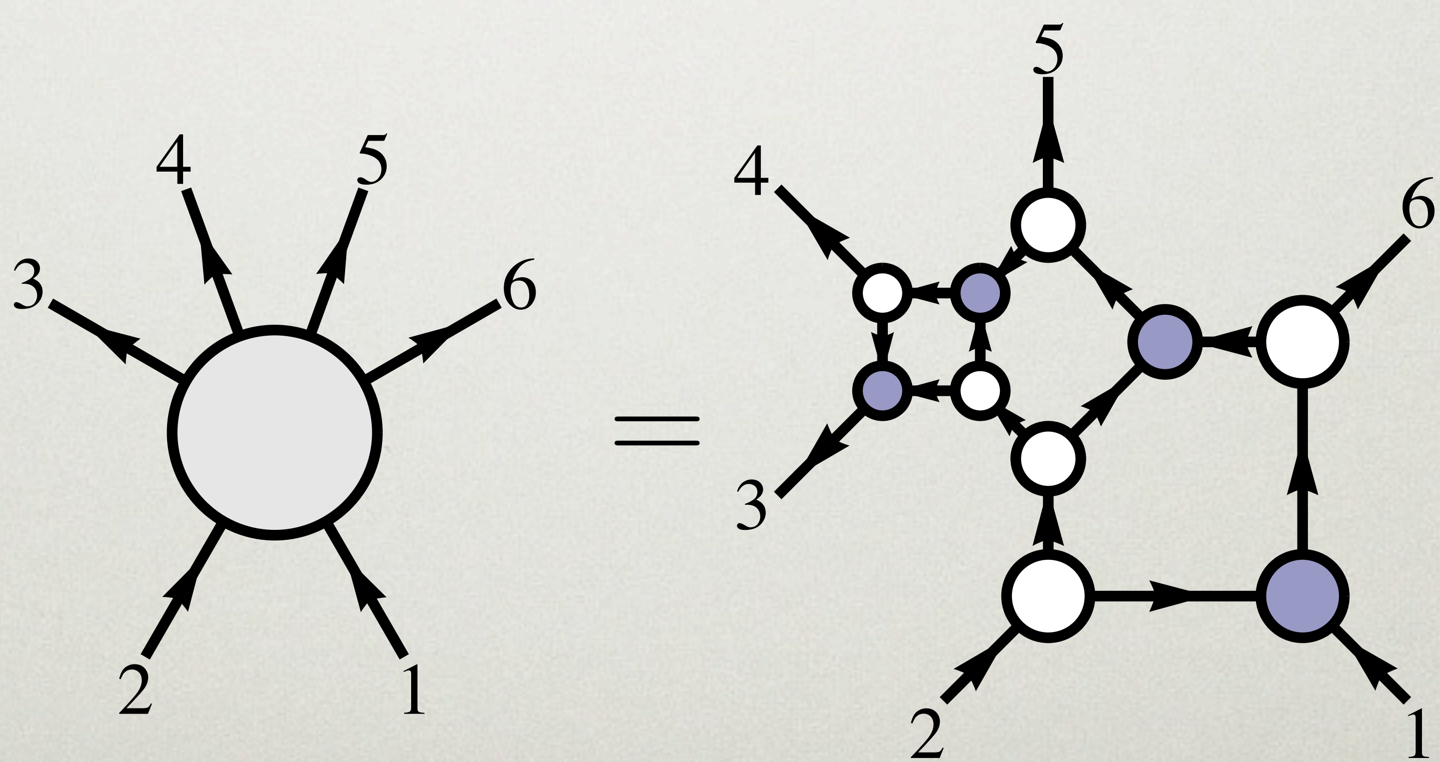

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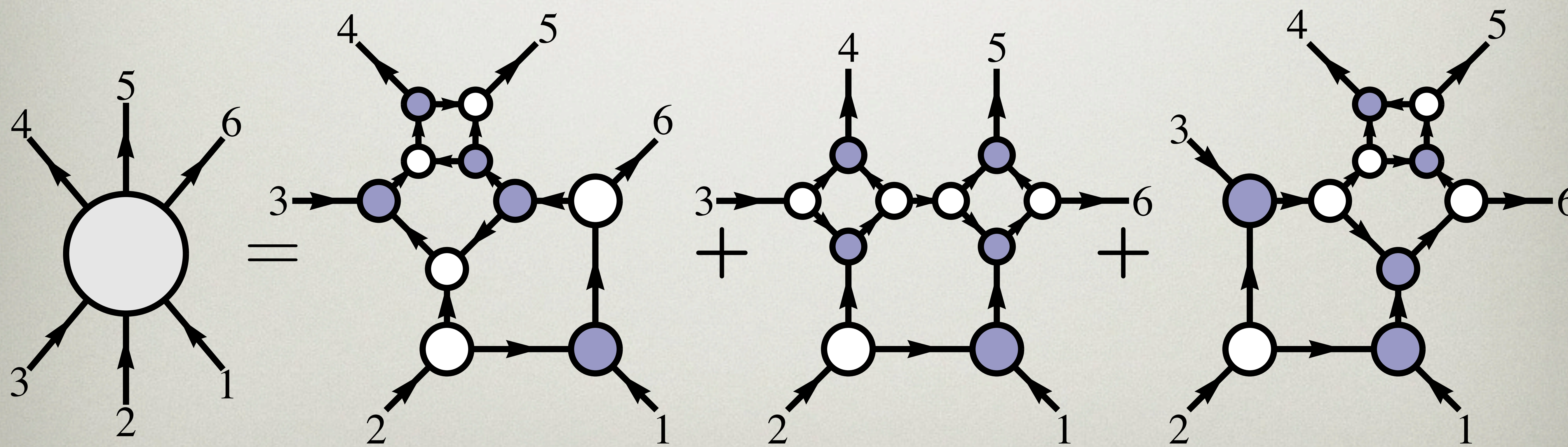

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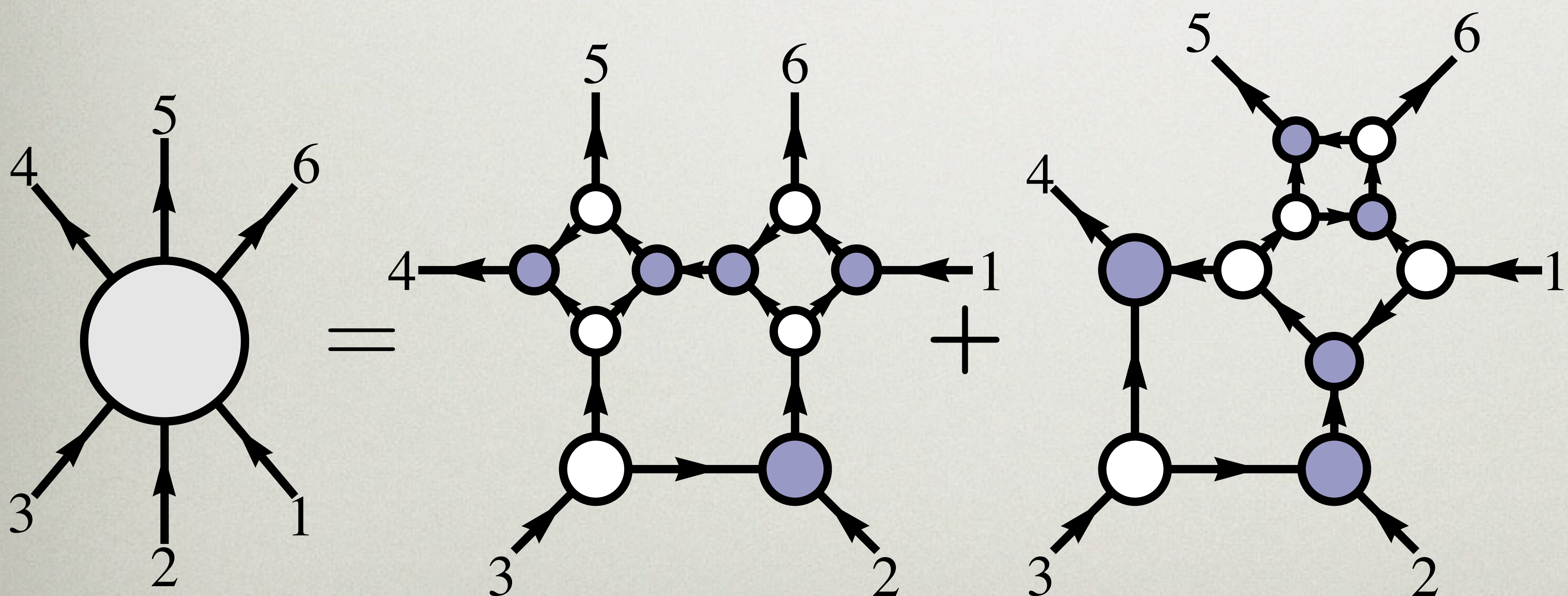

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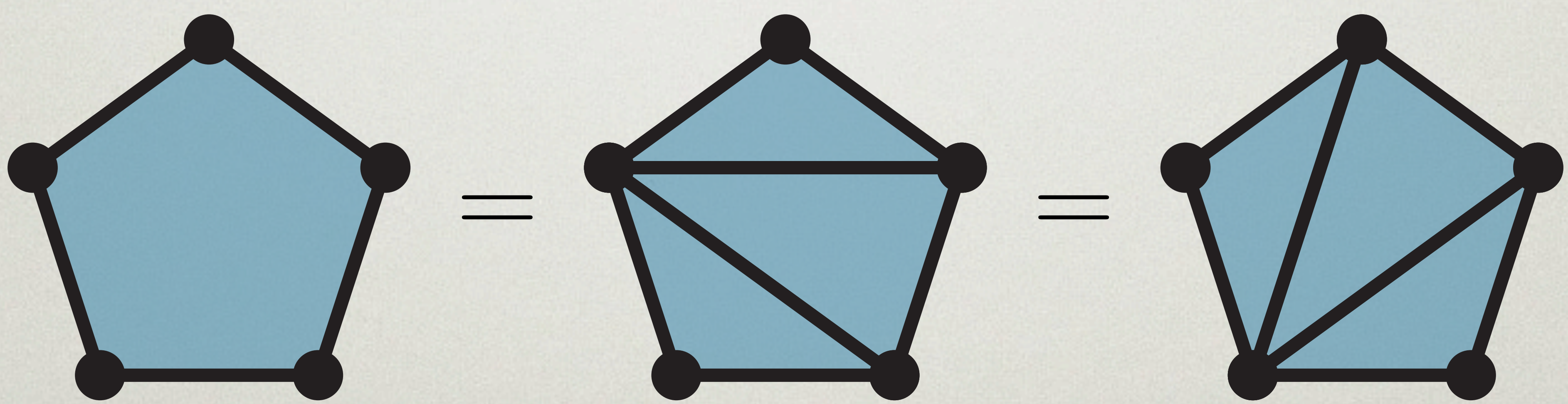

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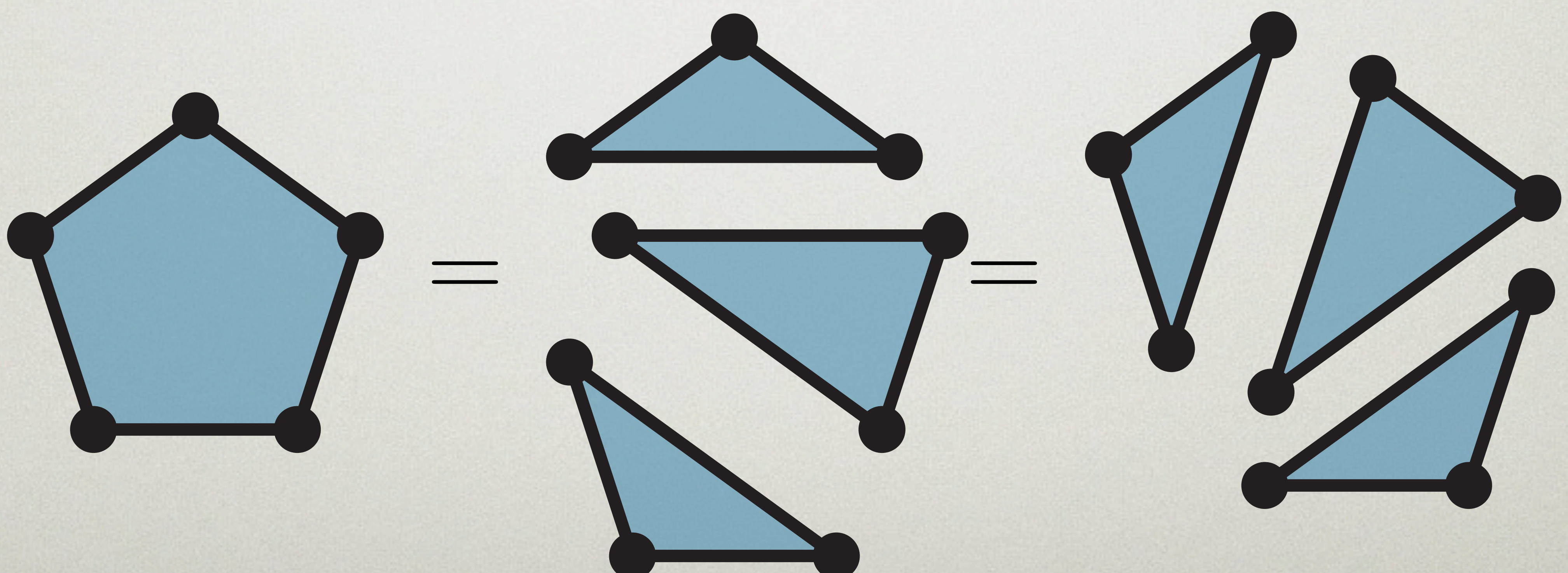
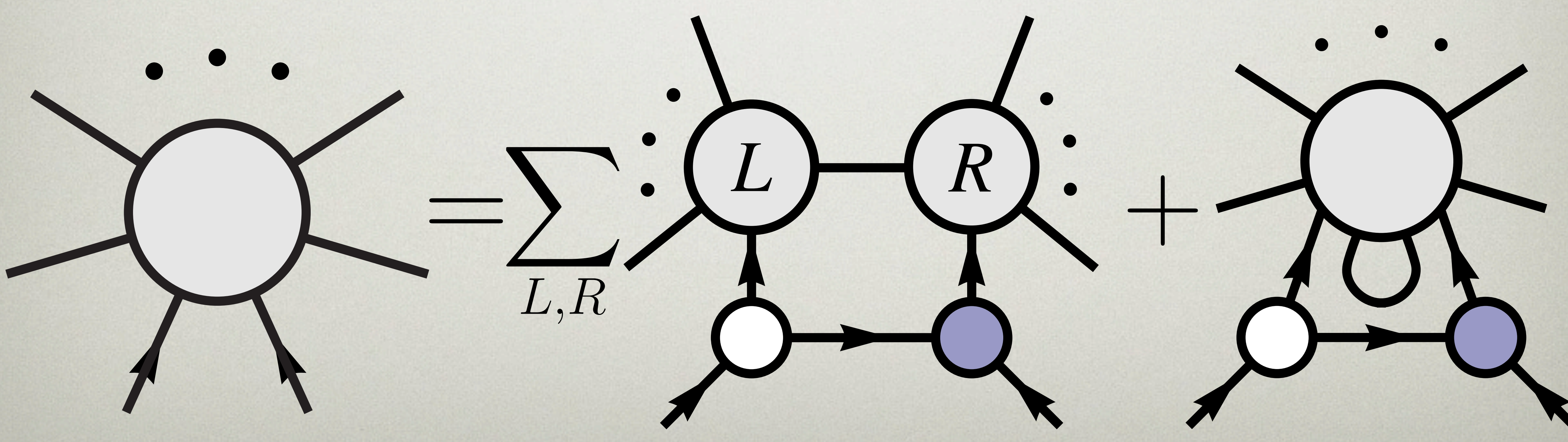


Completing the Revolution

Remarkable advances in recent years:

- the recursion relations for computing amplitudes were found to extend to **all** orders of complexity!



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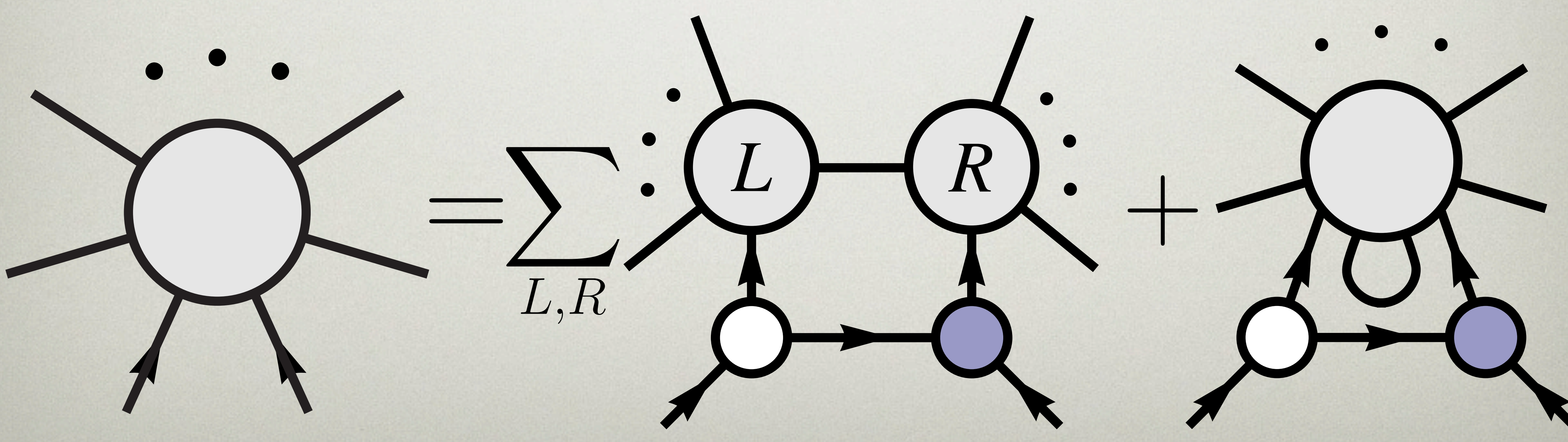

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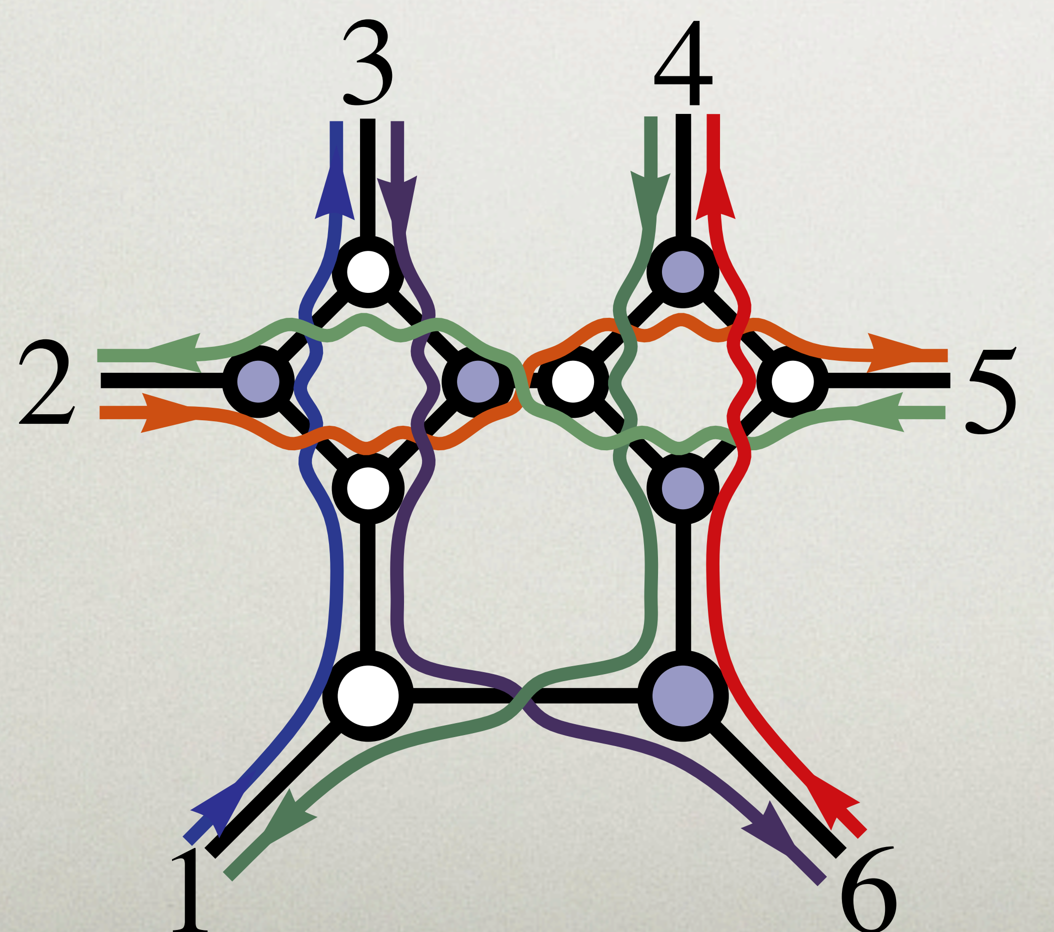

1 → 3
2 → 5
3 → 6
4 → 1
5 → 2
6 → 4

Completing the Revolution

Remarkable advances in recent years:

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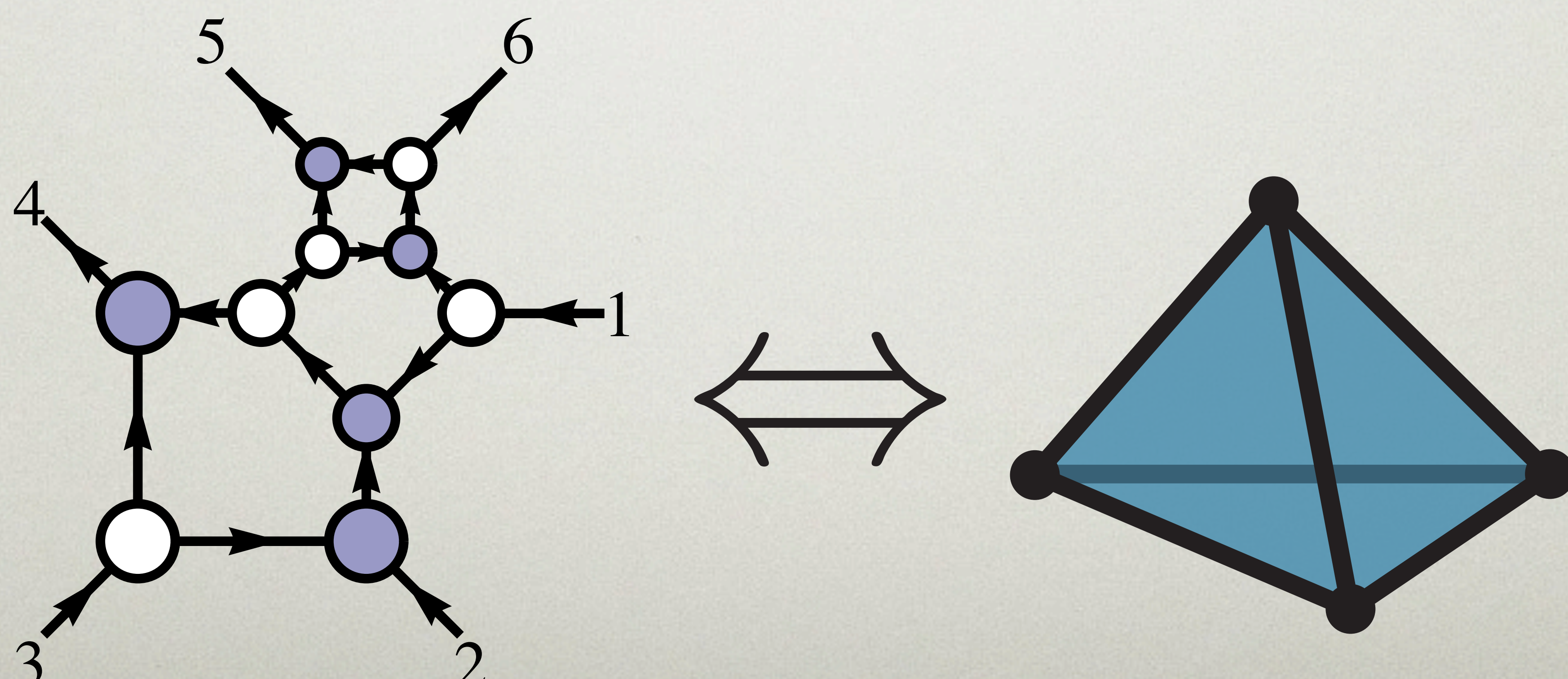
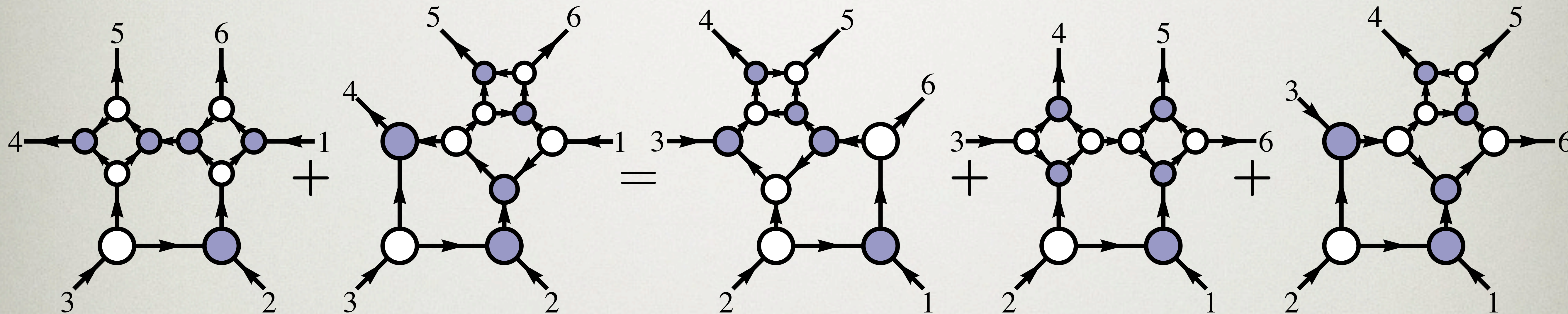
- each term can be characterized combinatorially
- each term represents the volume of a polytope



The Geometry of Amplitudes

Amplitudes can now be understood geometrically!



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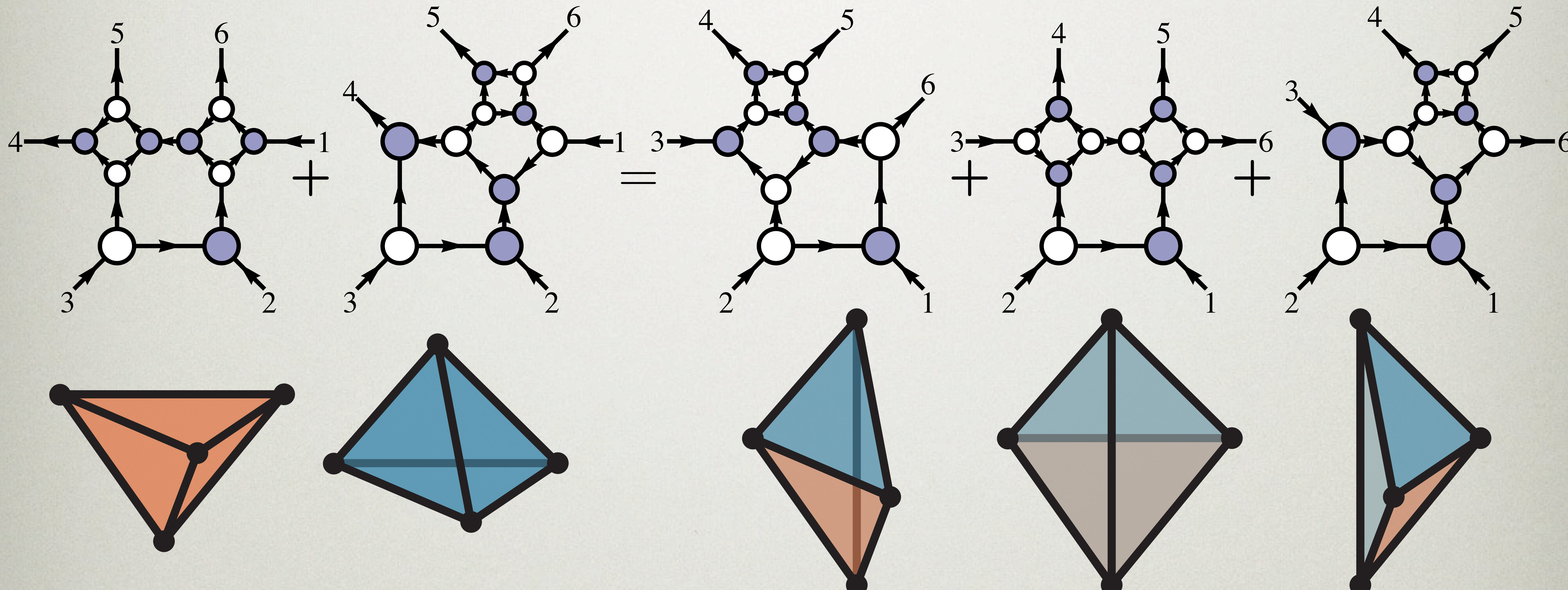

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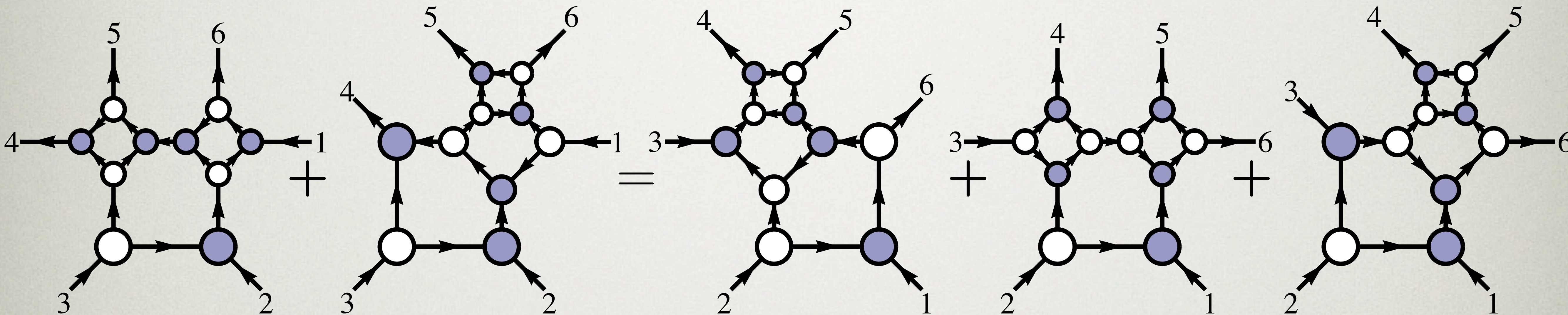

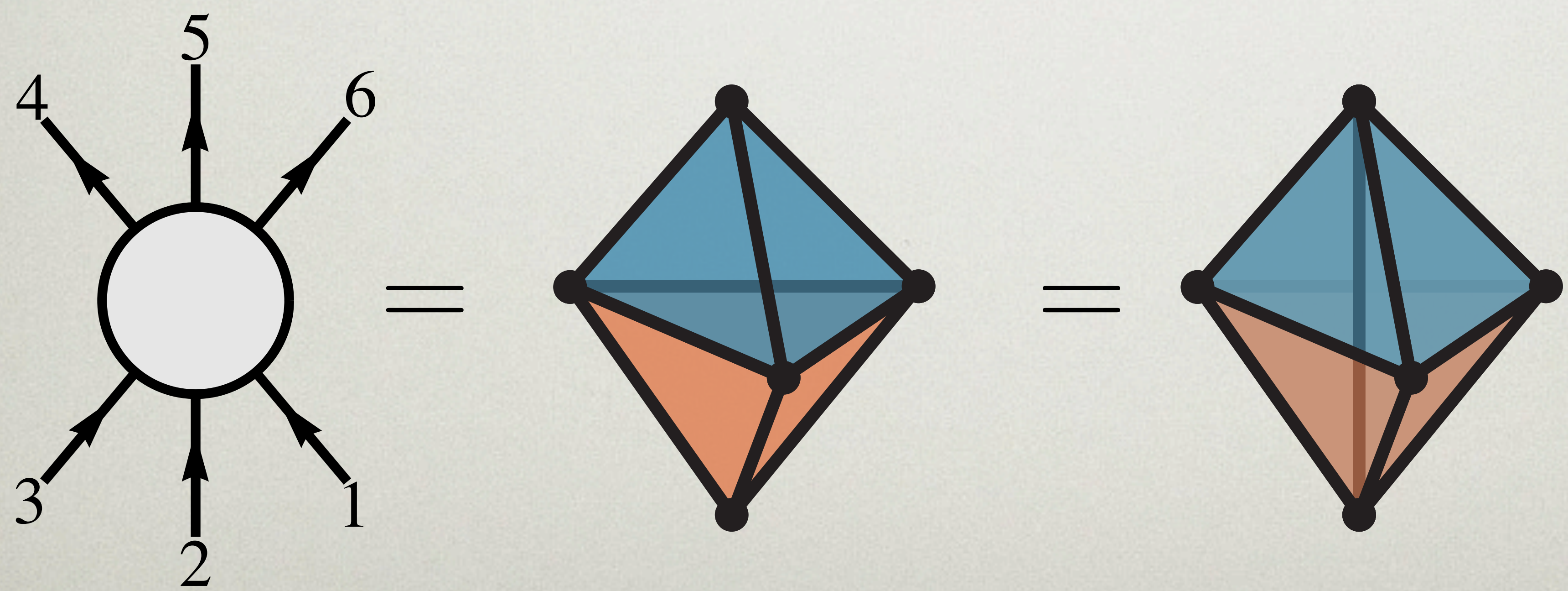

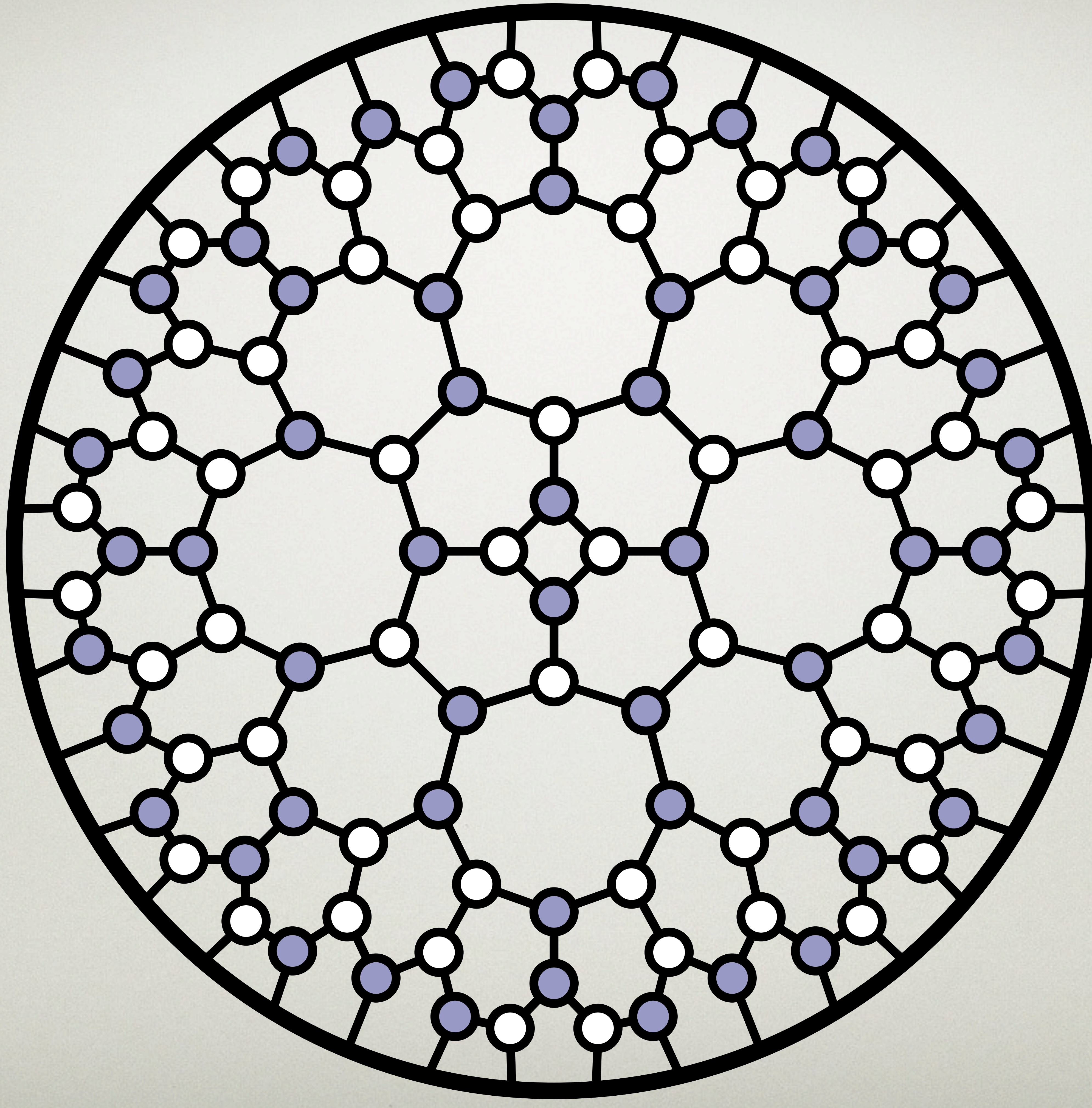

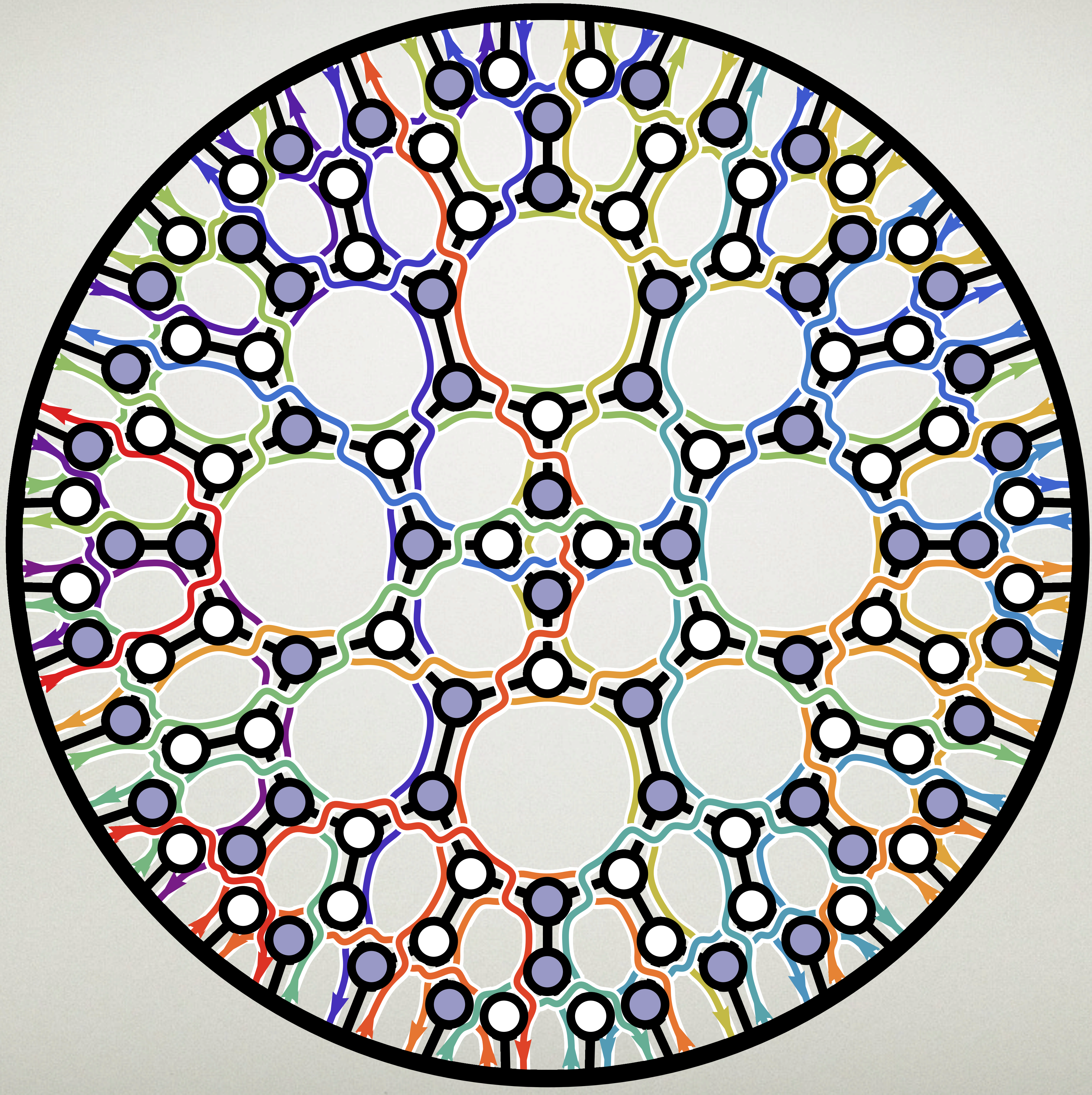

Thank You!

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Particle Fever

in this room

on Monday, 12 January @ 17:00