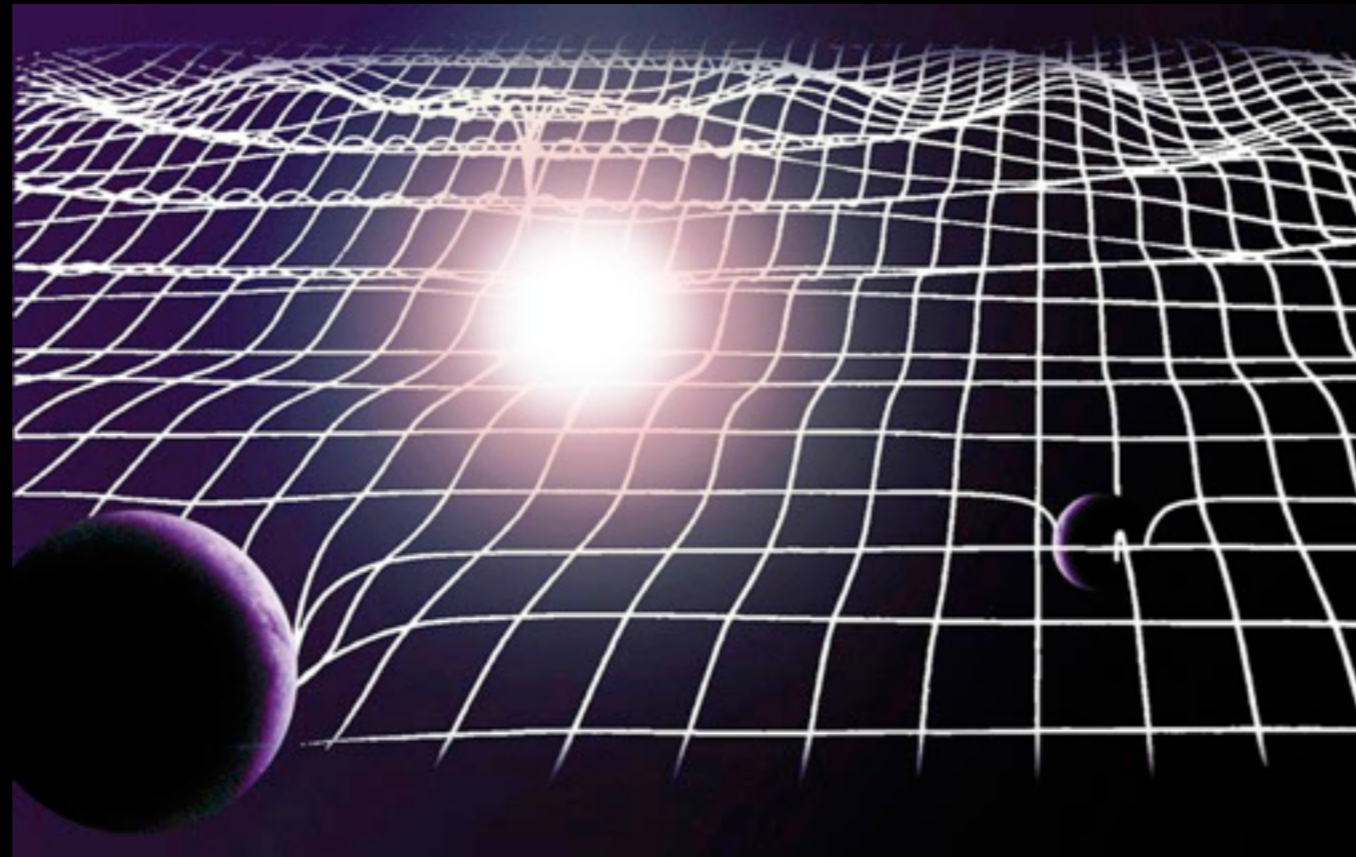
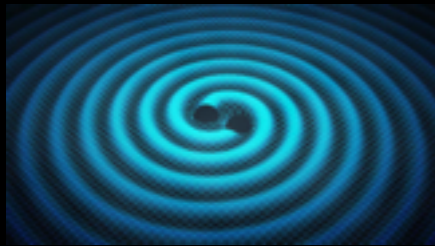


Gravitational Wave Astrophysics

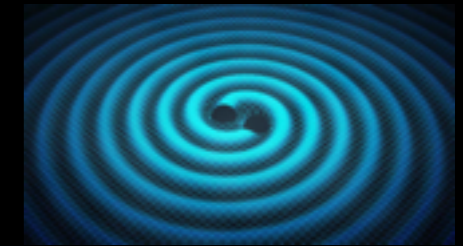
Johan Samsing
Princeton University



Part I



Gravitational Wave Properties



Frequency:

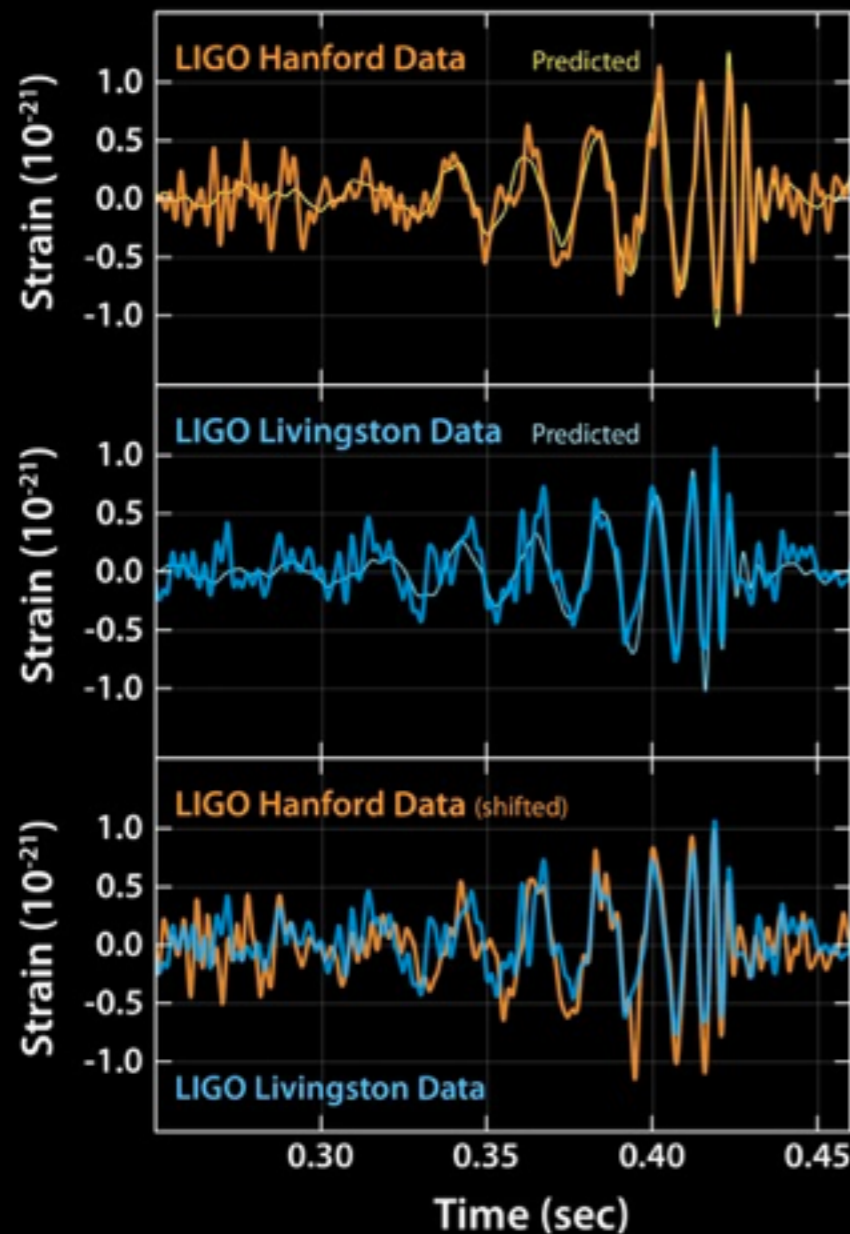
$$f_{GW} \sim 2f_{orb}$$

Strain:

$$h = 5 \times 10^{-22} \left(\frac{M}{2.8M_{\odot}} \right)^{2/3} \left(\frac{\mu}{0.7M_{\odot}} \right) \left(\frac{f}{100\text{Hz}} \right)^{2/3} \left(\frac{15\text{Mpc}}{r} \right)$$

Merger time:

$$t_{life} \propto a^4 m^{-3} (1 - e^2)^{7/2}$$



Objects:

NS Binary:

$$f_{GW(NS)} \sim 10^3 \text{ Hz}$$

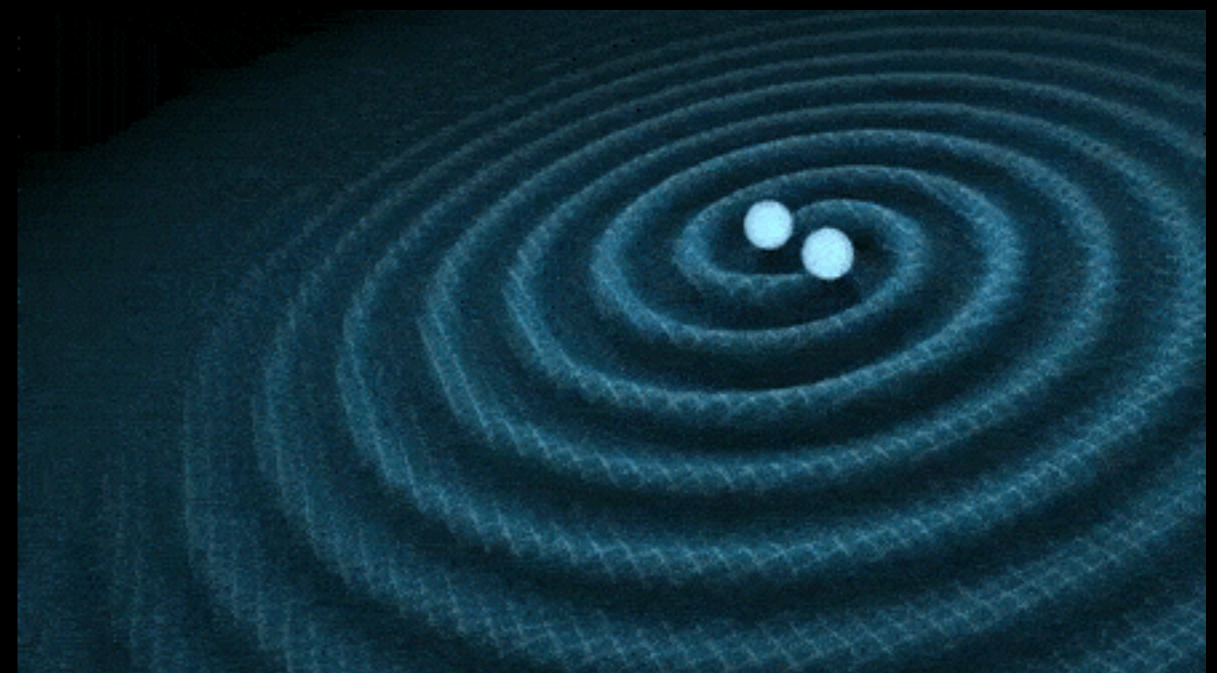
WD Binary:

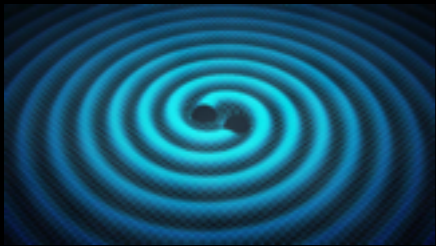
$$f_{GW(WD)} \sim 10^{-1} \text{ Hz}$$

Solar Binary:

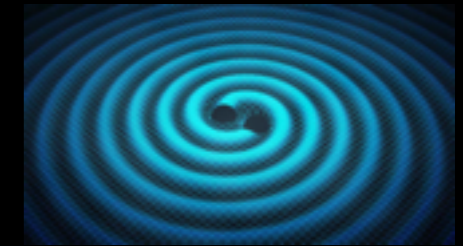
$$f_{GW(sun)} \sim 10^{-4} \text{ Hz}$$

In an expanding U: *redshift, (1+z)*





Gravitational Wave Properties



Frequency:

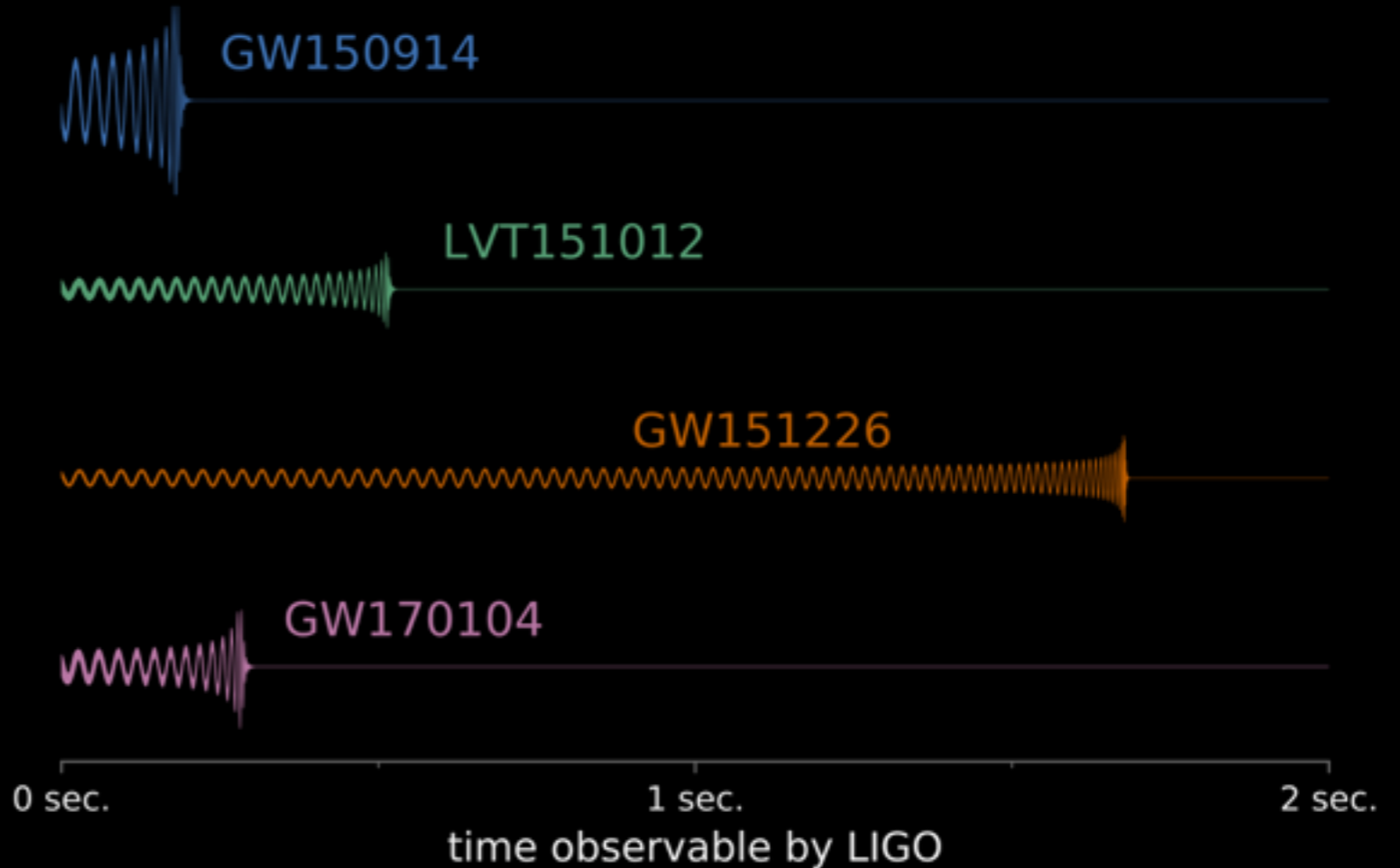
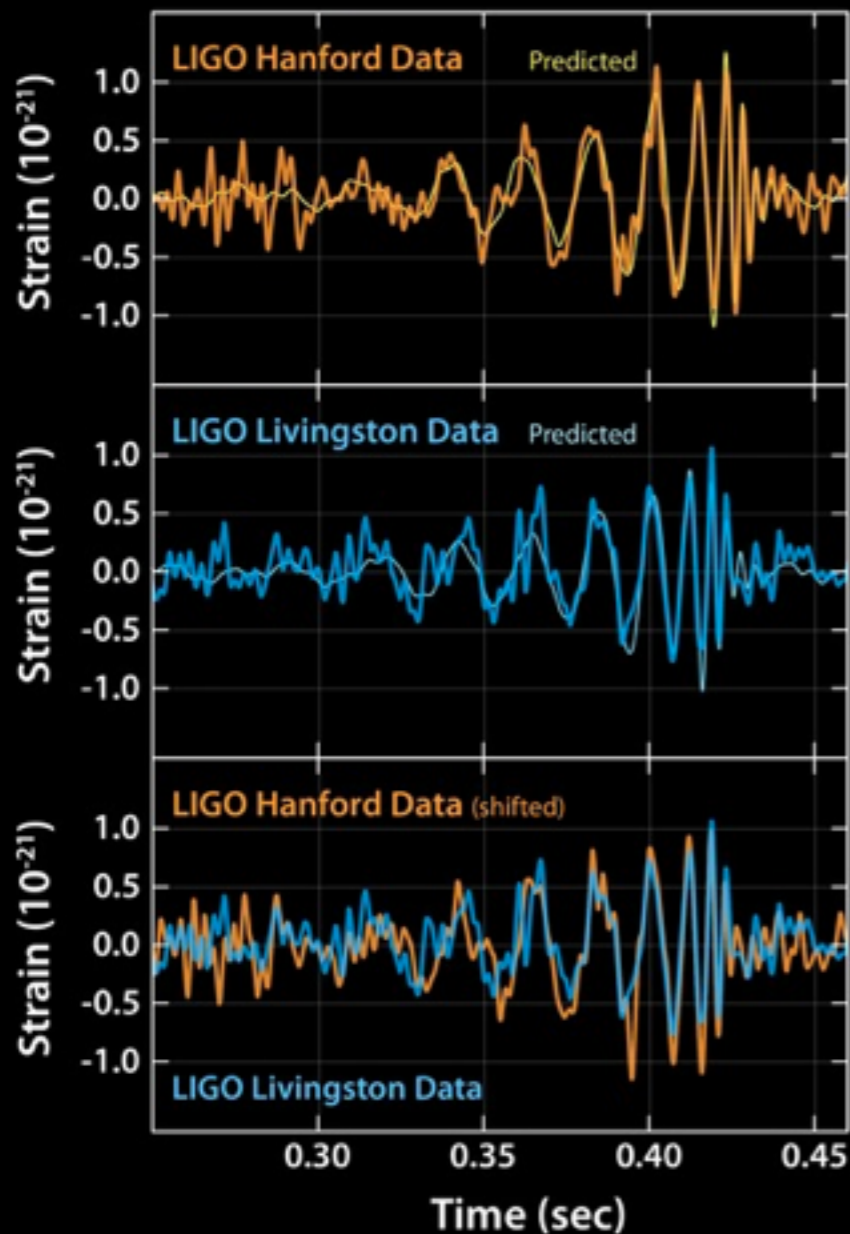
$$f_{GW} \sim 2f_{orb}$$

Strain:

$$h = 5 \times 10^{-22} \left(\frac{M}{2.8M_{\odot}} \right)^{2/3} \left(\frac{\mu}{0.7M_{\odot}} \right) \left(\frac{f}{100\text{Hz}} \right)^{2/3} \left(\frac{15\text{Mpc}}{r} \right)$$

Merger time:

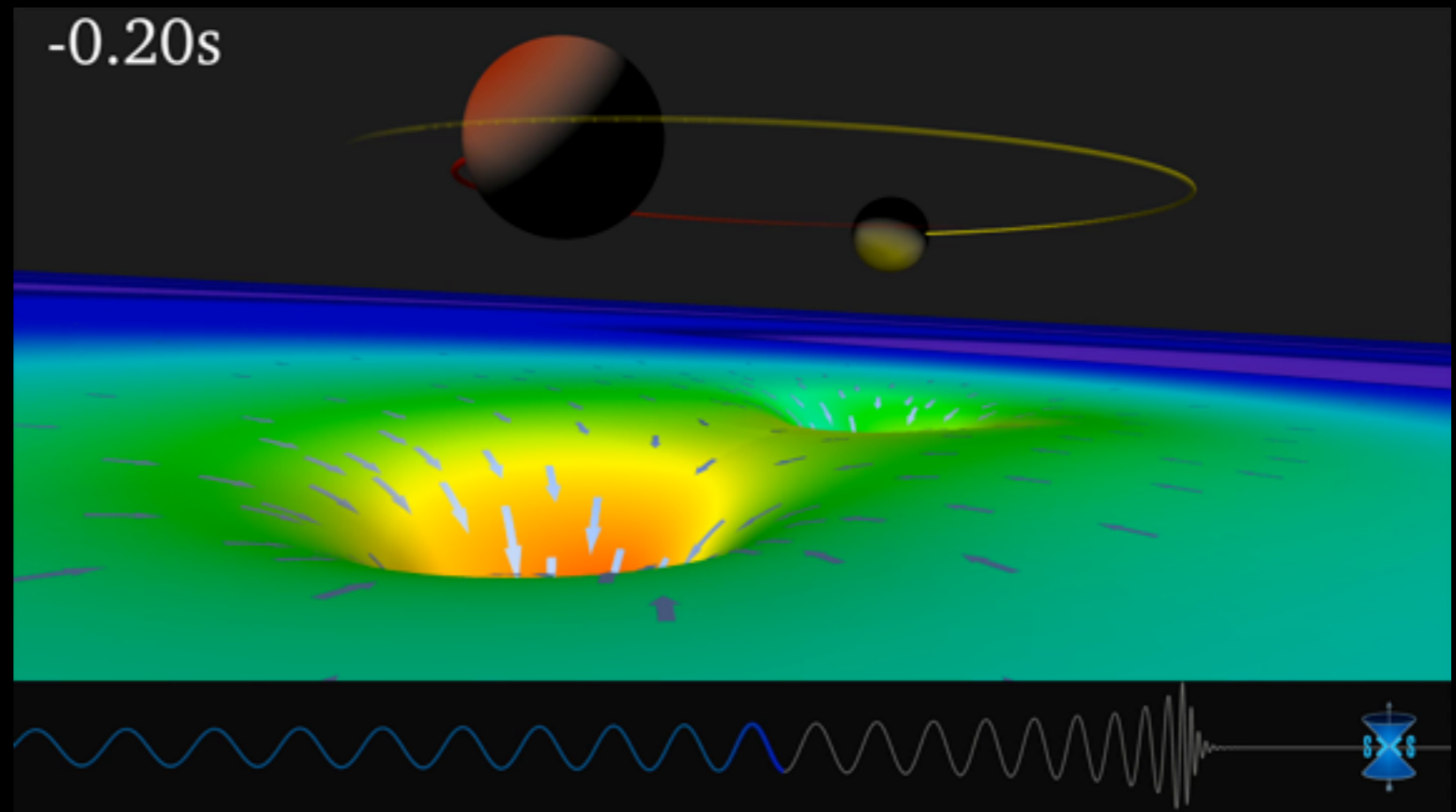
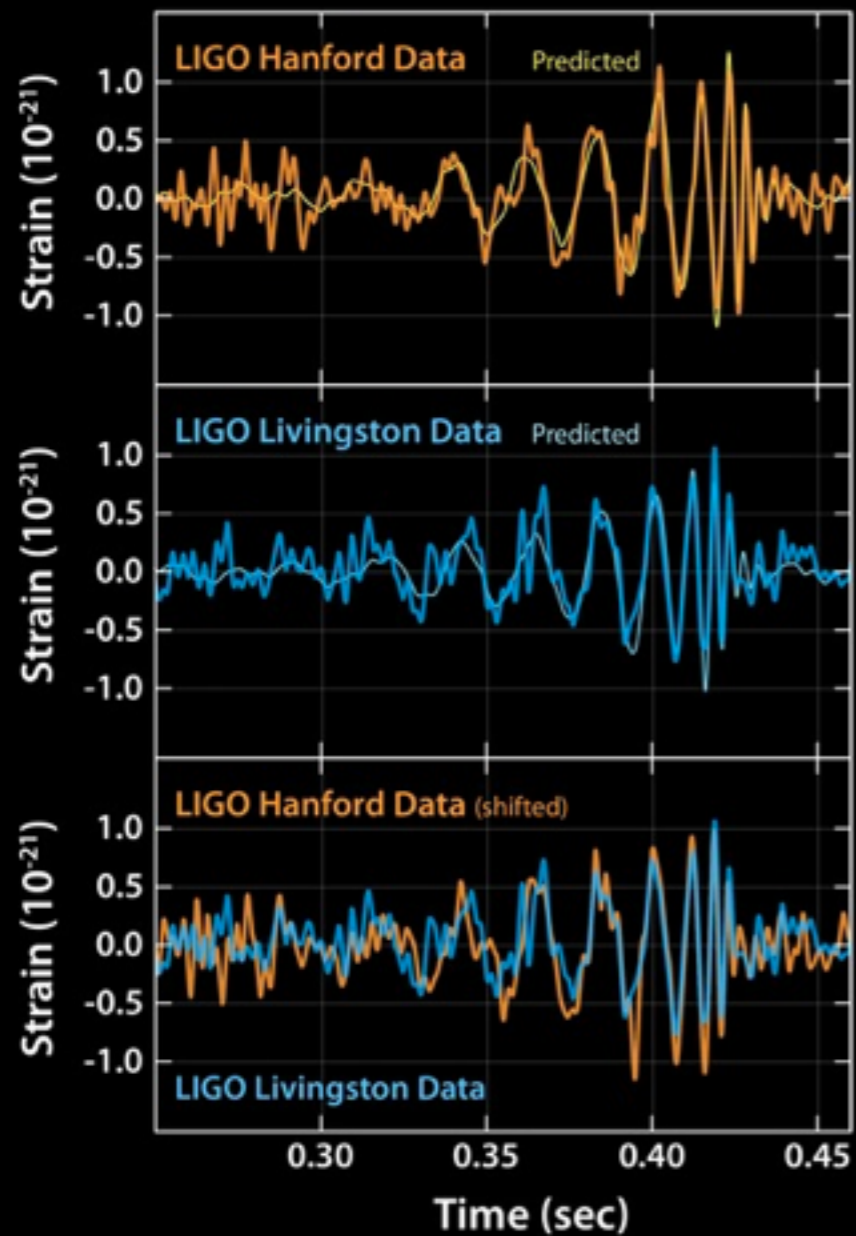
$$t_{life} \propto a^4 m^{-3} (1 - e^2)^{7/2}$$



Gravitational Waves have now been observed!

Real Signal:

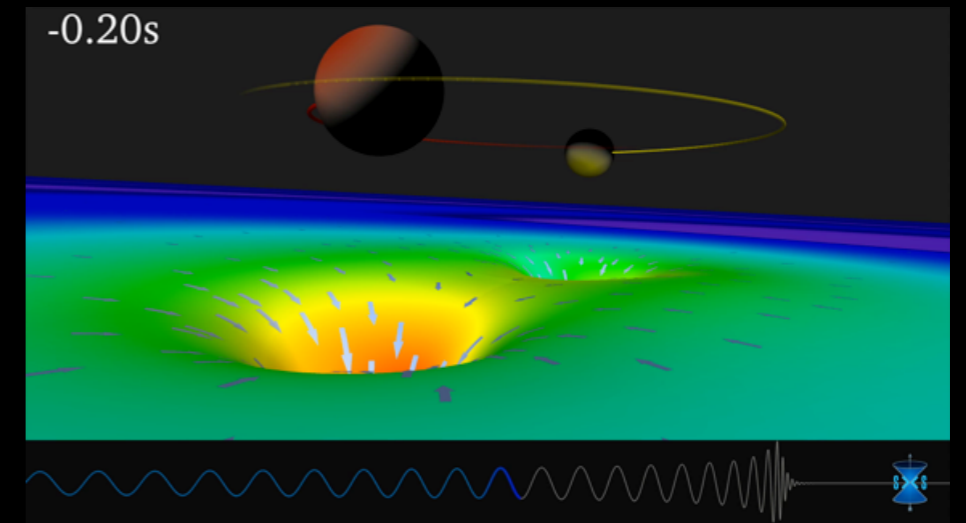
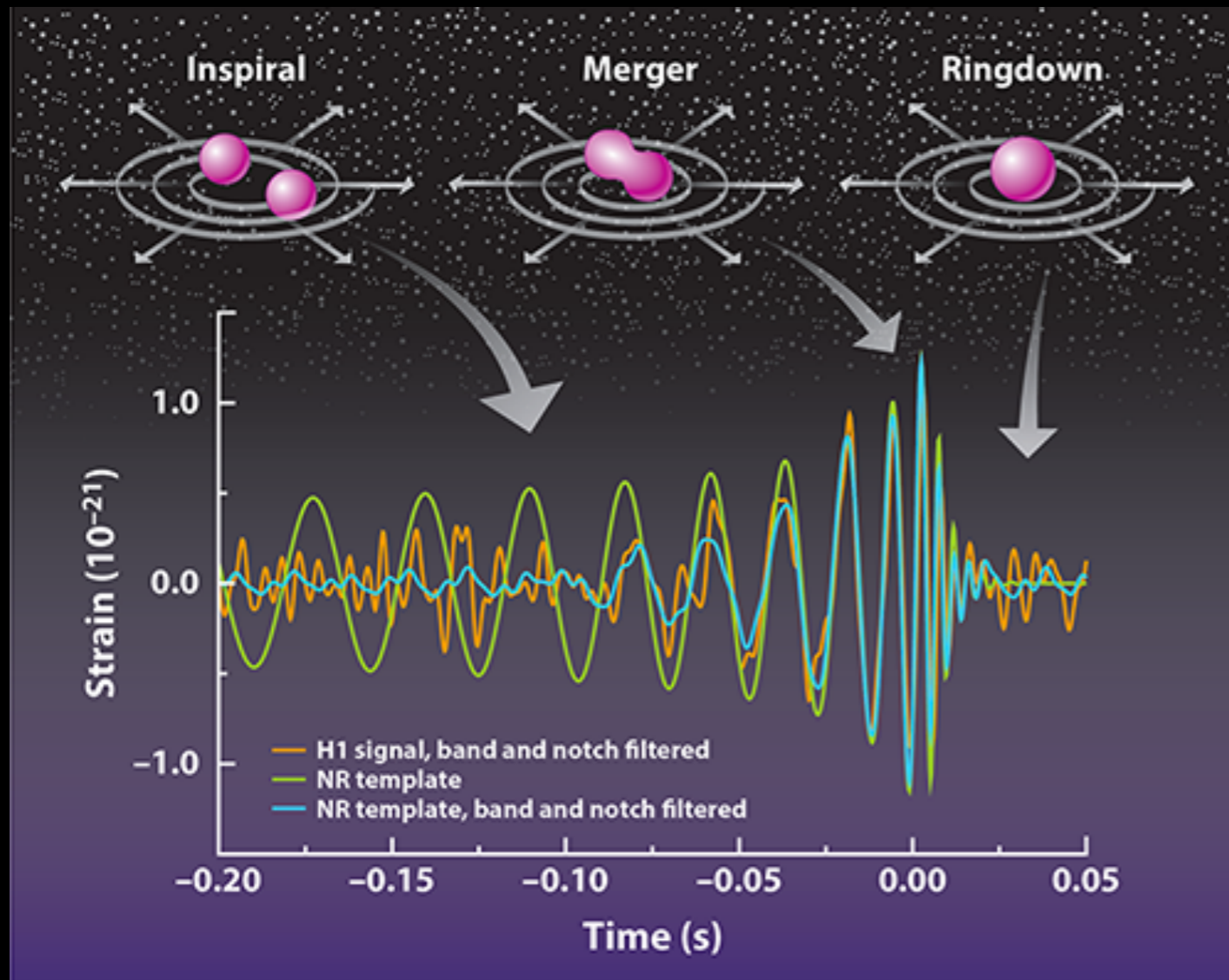
Animation:



Nobel Prize: 2017

Dynamics, Merger, and Simulation

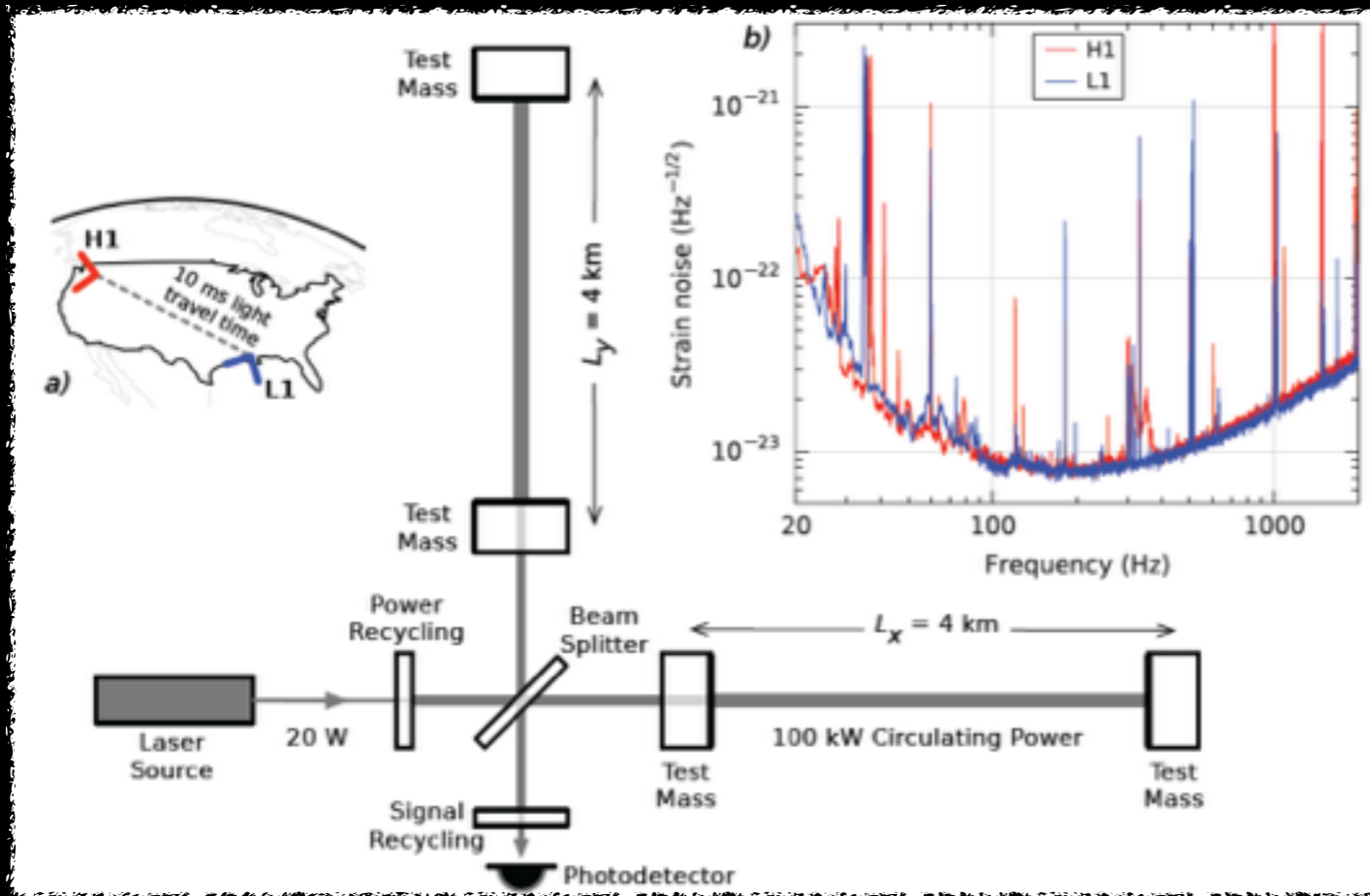
The three stages of merger:



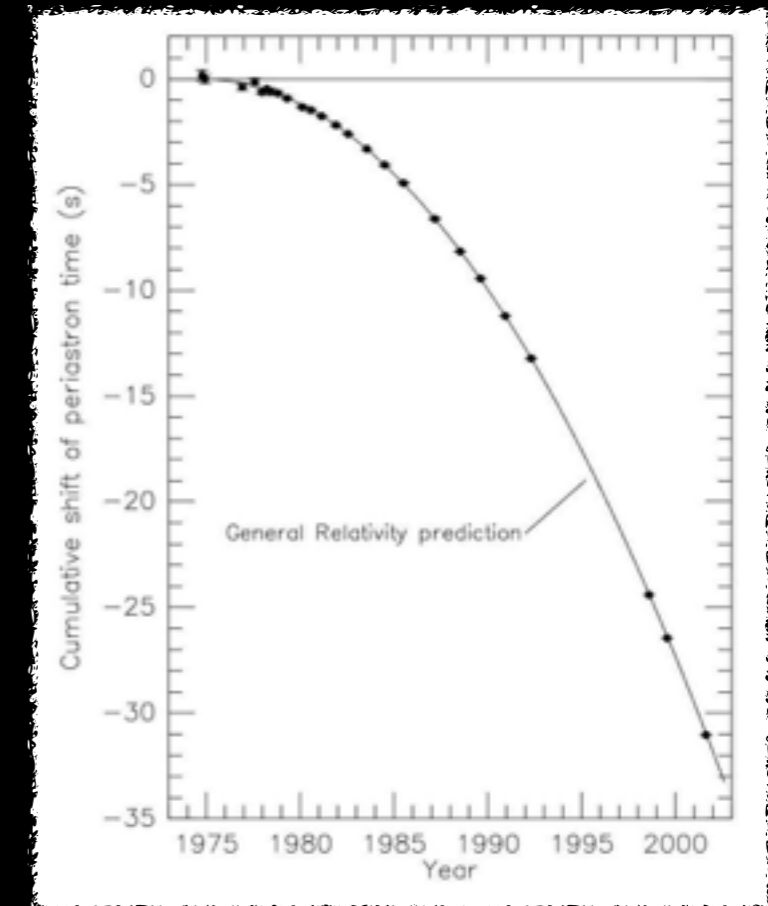
First GR sim.

How Gravitational Waves are Detected

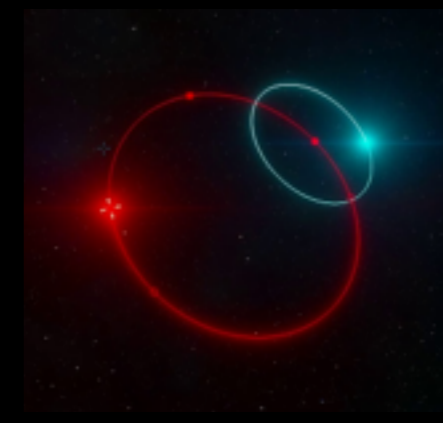
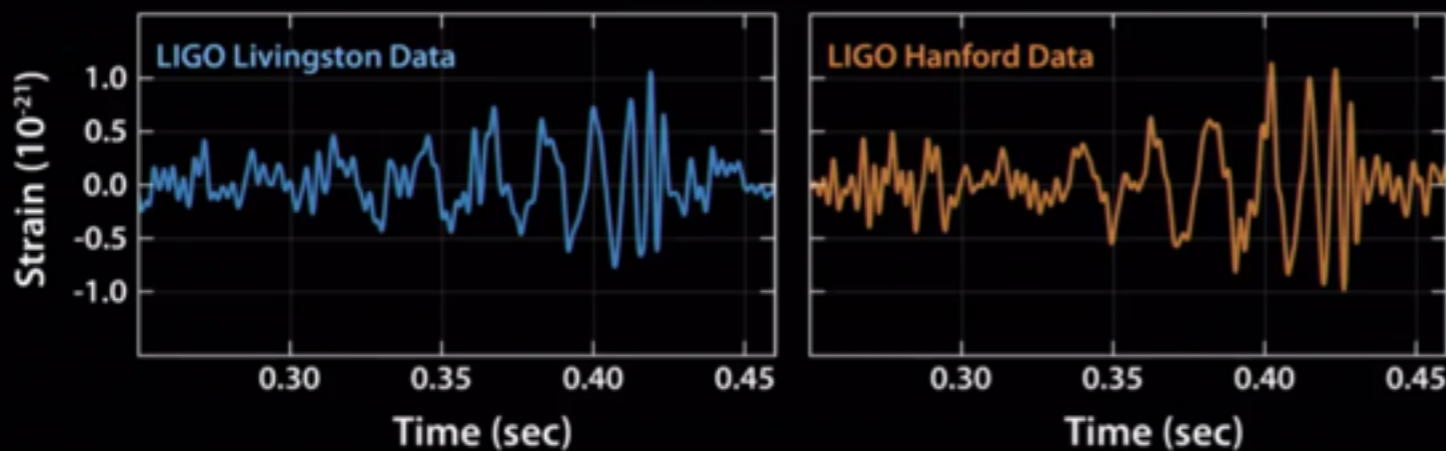
Direct:

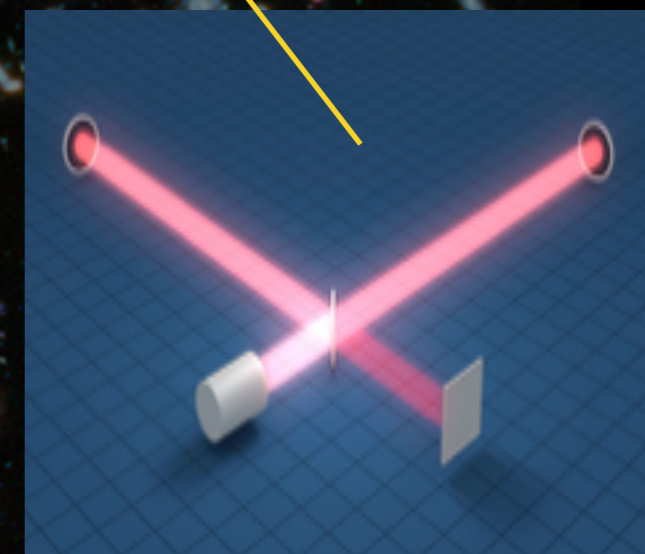
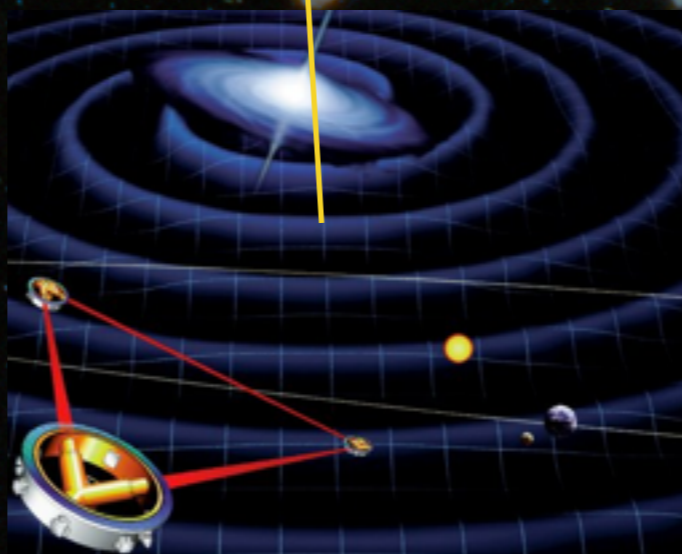
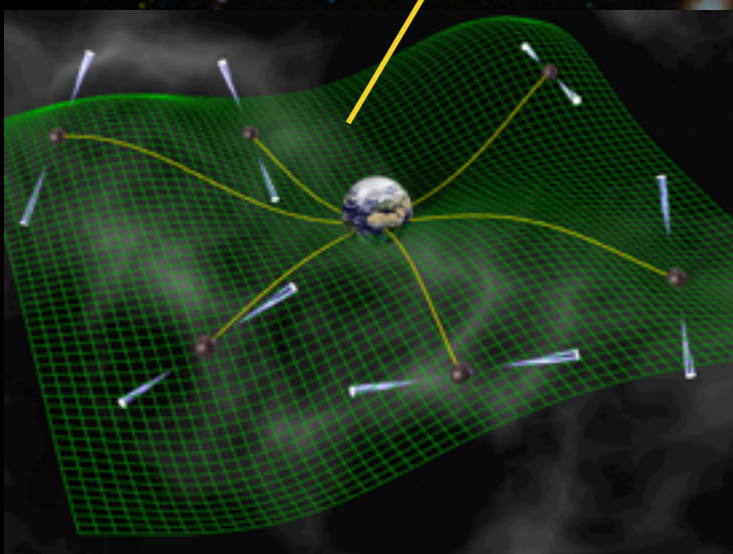
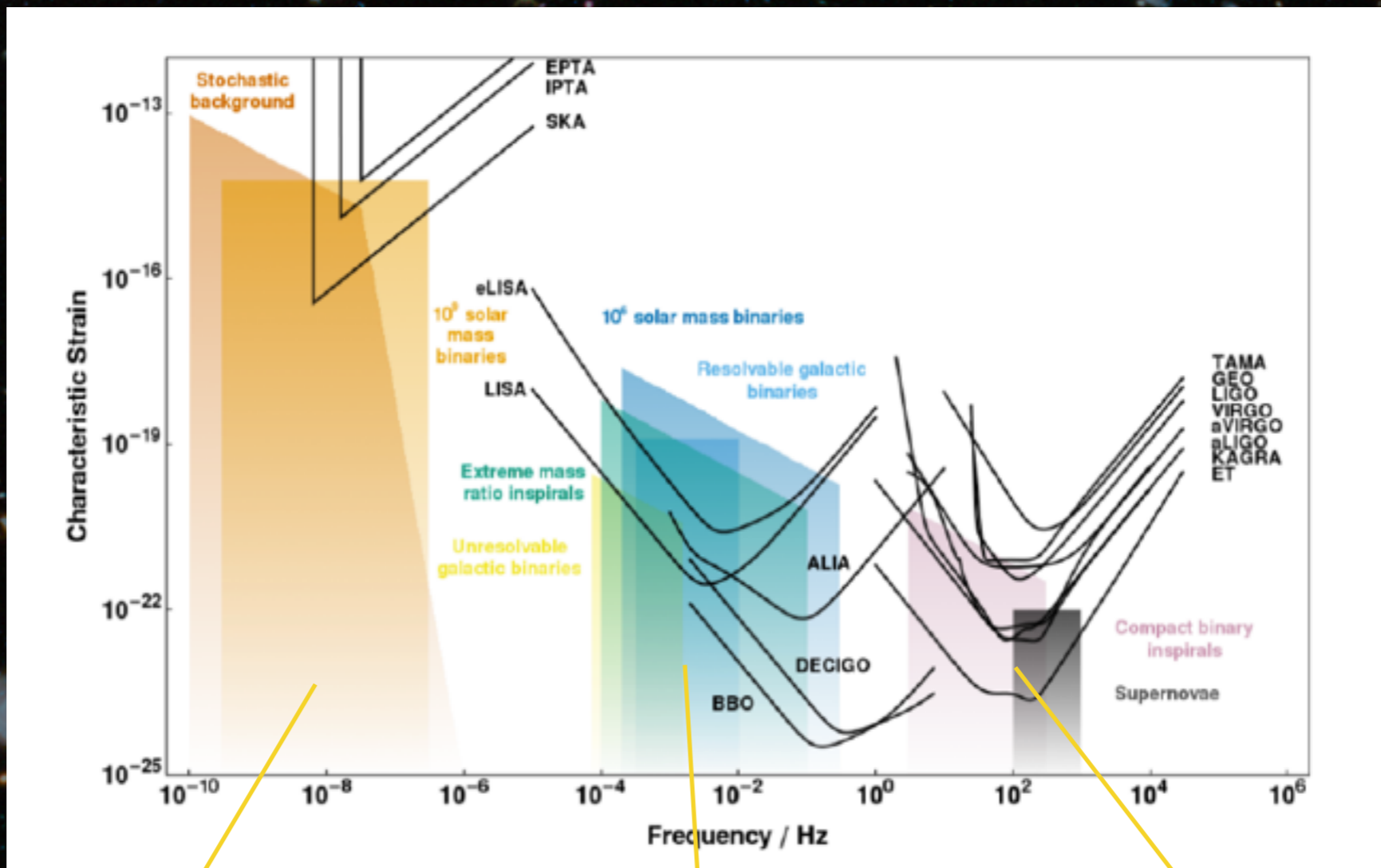


Indirect:



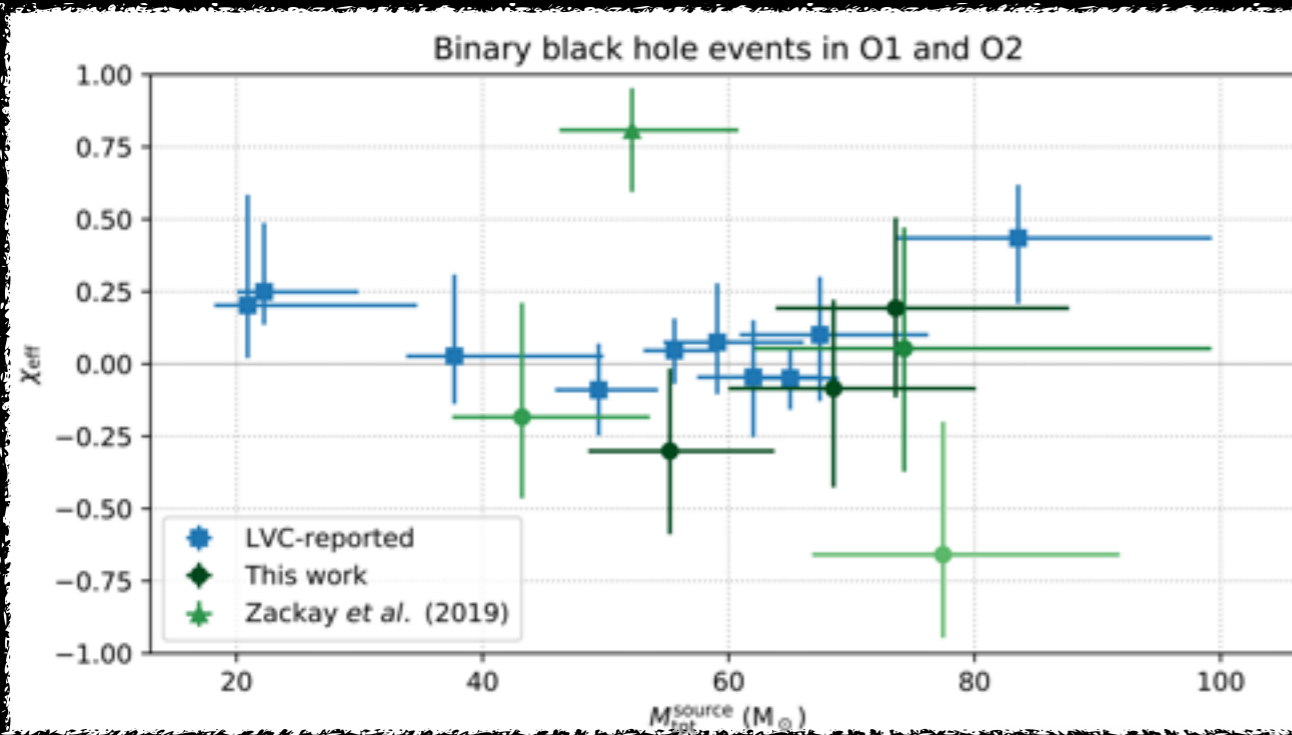
Hulse-Taylor Pulsar



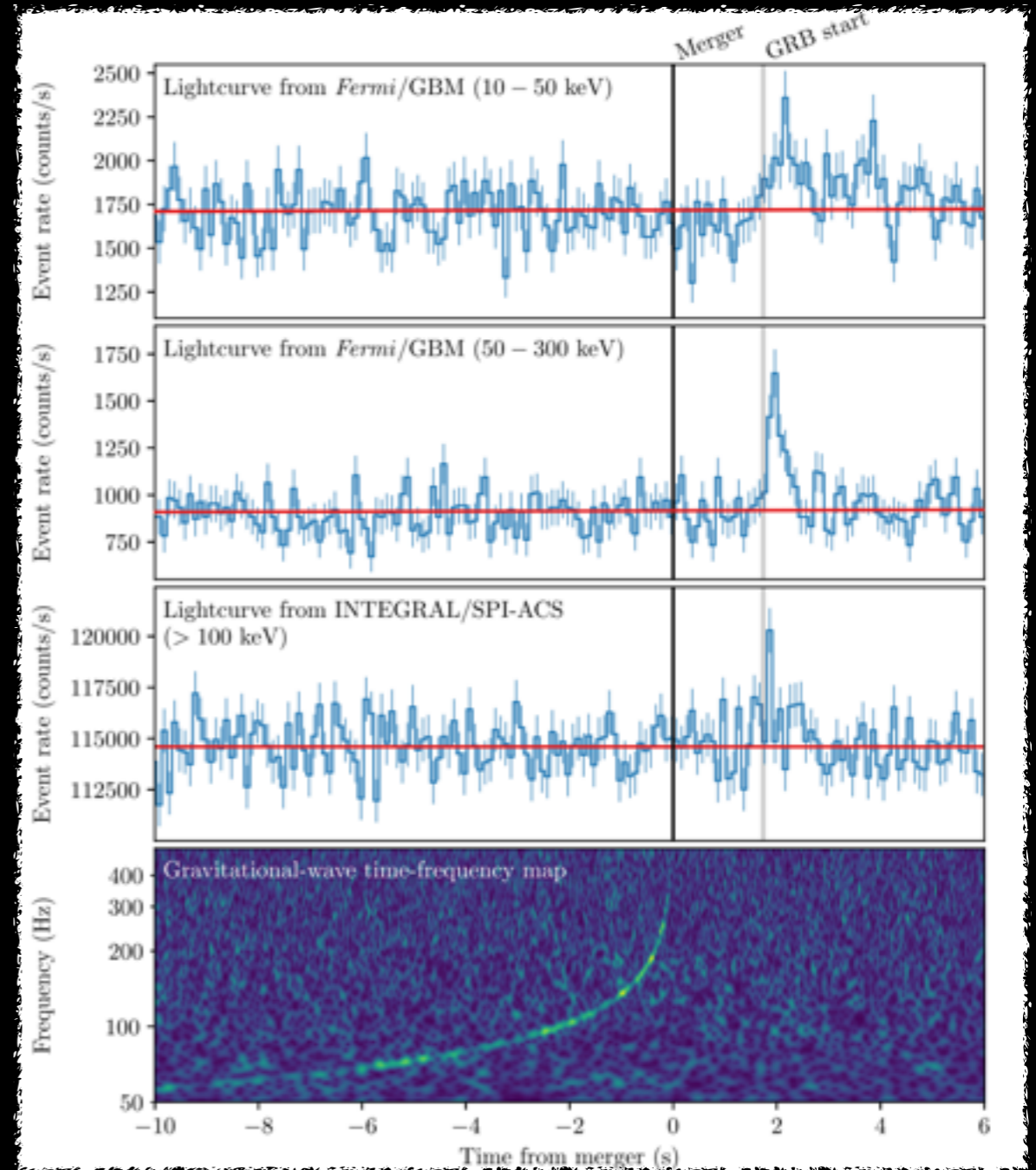


Current Status: Any Surprises yet?

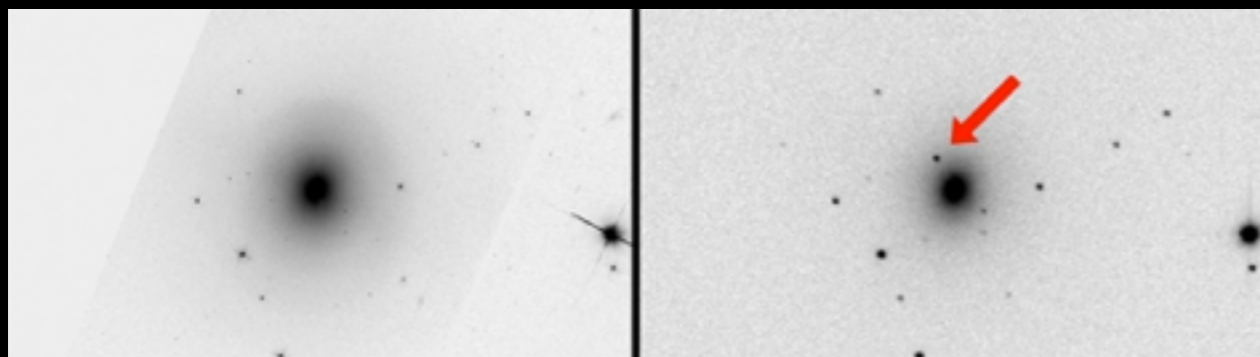
Black Holes:



Neutron Stars:



NS-NS optical signal:



What observations are we waiting for?

How do we know its Black Holes?

Solar Binary:

$$f_{GW}(sun) \sim 10^{-4} \text{ Hz}$$

WD Binary:

$$f_{GW}(WD) \sim 10^{-1} \text{ Hz}$$

NS/BH Binary:

$$f_{GW}(NS) \sim 10^3 \text{ Hz}$$

What can we learn from Gravitational Waves?

- What we measure:

- Masses
- Spins
- Eccentricity
- Dist./Pos./Redshift
- Acceleration

- What we can learn about:

- Gravity
- Space time: Extra Dim?
- Cosmology: Expansion
- Nuclear Physics
- Astrophysics

What can we learn from Gravitational Waves?

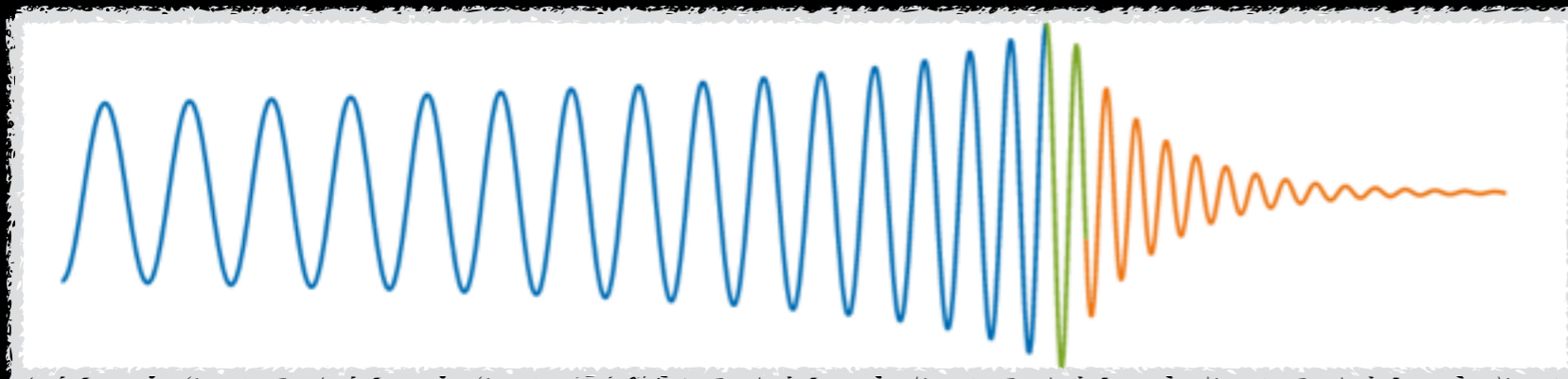
- What we measure:

- Masses
- Spins
- Eccentricity
- Dist./Pos./Redshift
- Acceleration

- What we can learn about:

- Gravity
- Space time: Extra Dim?
- Cosmology: Expansion
- Nuclear Physics
- Astrophysics

Generic Inspiral:



What can we learn from Gravitational Waves?

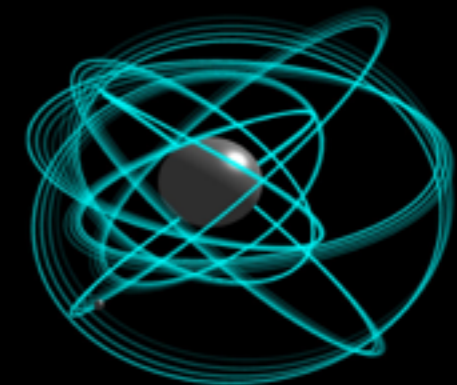
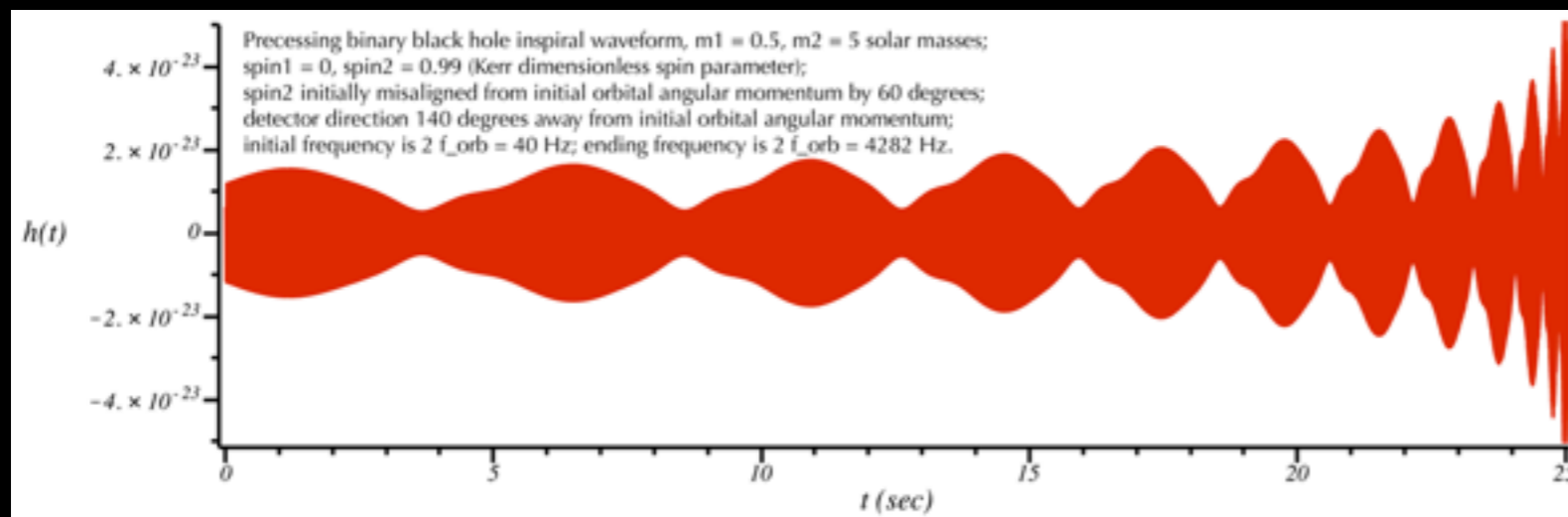
- What we measure:

- **Masses**
- **Spins**
- Eccentricity
- Dist./Pos./Redshift
- Acceleration

- What we can learn about:

- Gravity
- Space time: Extra Dim?
- Cosmology: Expansion
- Nuclear Physics
- Astrophysics

Highly Spinning:



What can we learn from Gravitational Waves?

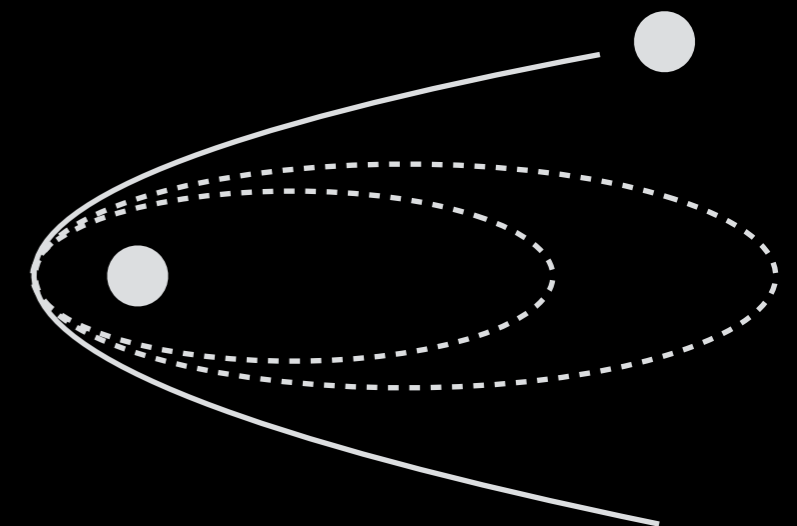
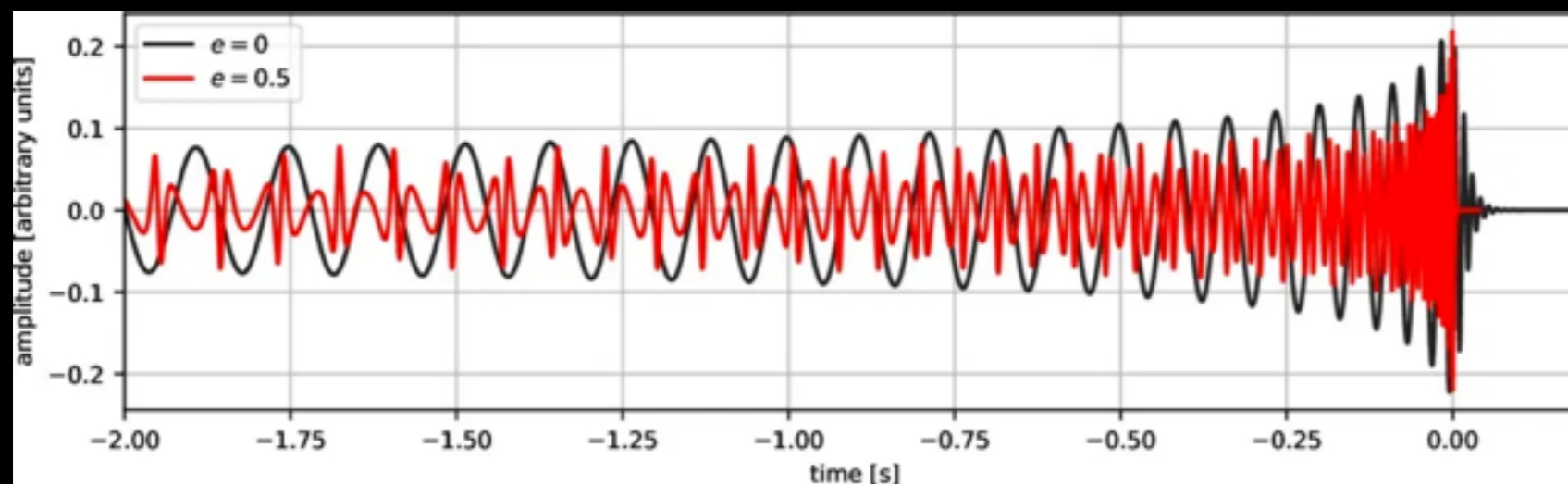
- What we measure:

- Masses
- Spins
- **Eccentricity**
- Dist./Pos./Redshift
- Acceleration

- What we can learn about:

- Gravity
- Space time: Extra Dim?
- Cosmology: Expansion
- Nuclear Physics
- Astrophysics

Highly Eccentric:



What can we learn from Gravitational Waves?

- What we measure:

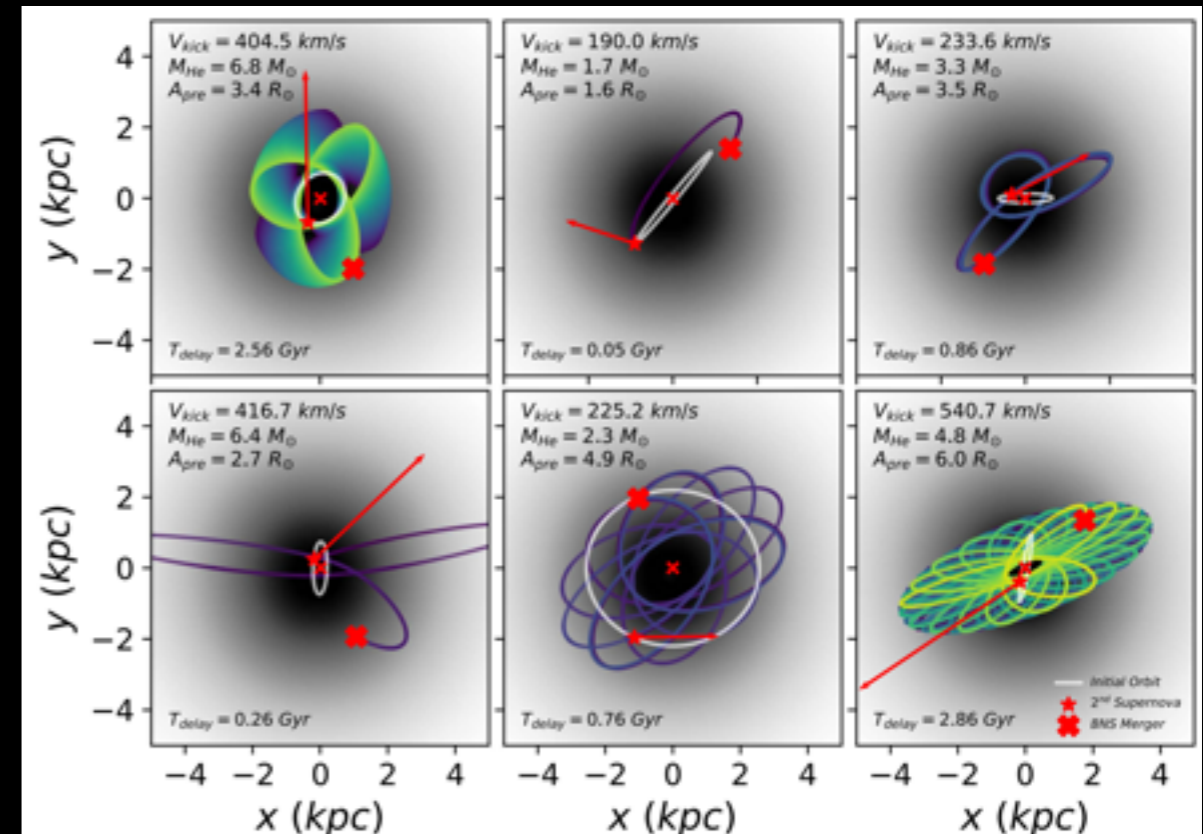
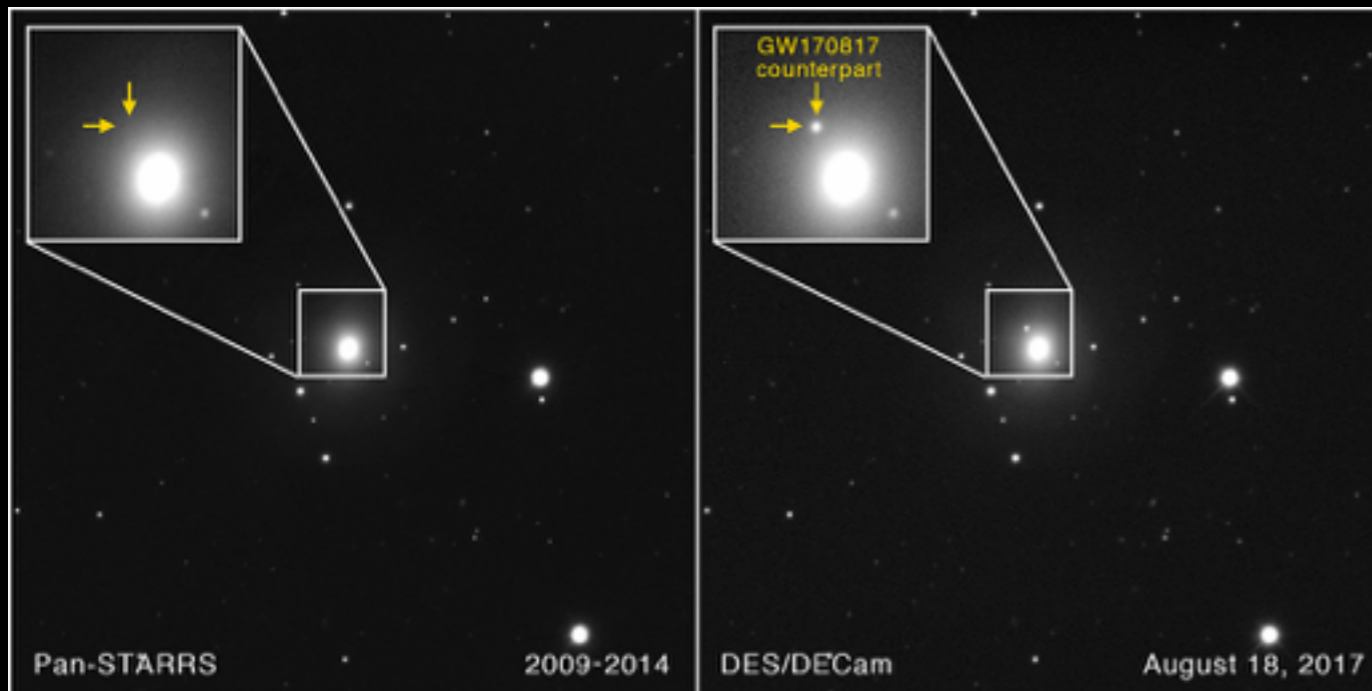
- Masses
- Spins
- Eccentricity
- **Dist./Pos./Redshift**
- Acceleration

- What we can learn about:

- Gravity
- Space time: Extra Dim?
- Cosmology: Expansion
- Nuclear Physics
- Astrophysics

Observation:

Simulation:



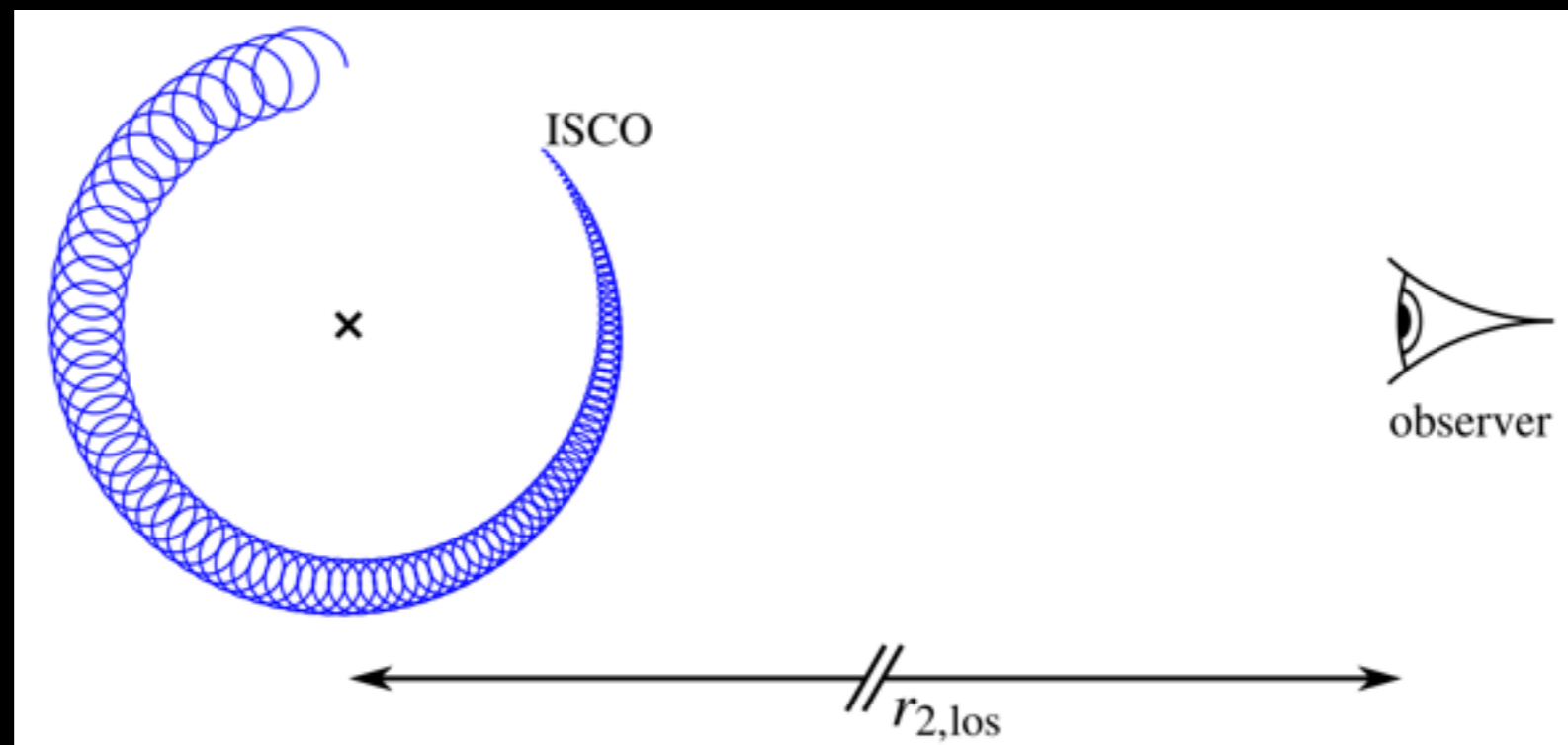
What can we learn from Gravitational Waves?

- What we measure:

- Masses
- Spins
- Eccentricity
- Dist./Pos./Redshift
- **Acceleration**

- What we can learn about:

- Gravity
- Space time: Extra Dim?
- Cosmology: Expansion
- Nuclear Physics
- Astrophysics



What can we learn from Gravitational Waves?

- What we measure:

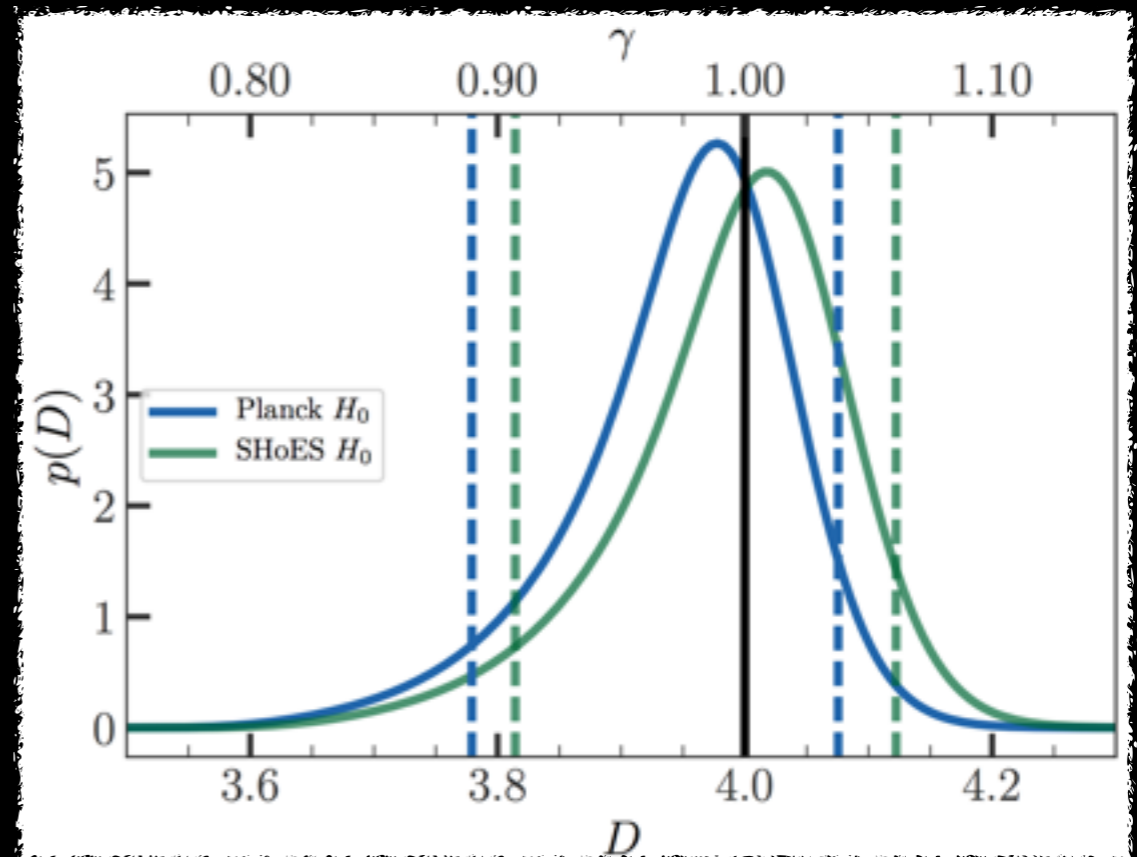
- Masses
- Spins
- Eccentricity
- Dist./Pos./Redshift
- Acceleration

Constraining the number of space-time dimensions:

$$h \propto \frac{1}{d_L^\gamma}$$

- What we can learn about:

- **Gravity (modify waveform)**
- **Space time: Extra Dim?**
- Cosmology: Expansion
- Nuclear Physics
- Astrophysics



What can we learn from Gravitational Waves?

- What we measure:

- Masses
- Spins
- Eccentricity
- Dist./Pos./Redshift
- Acceleration

- What we can learn about:

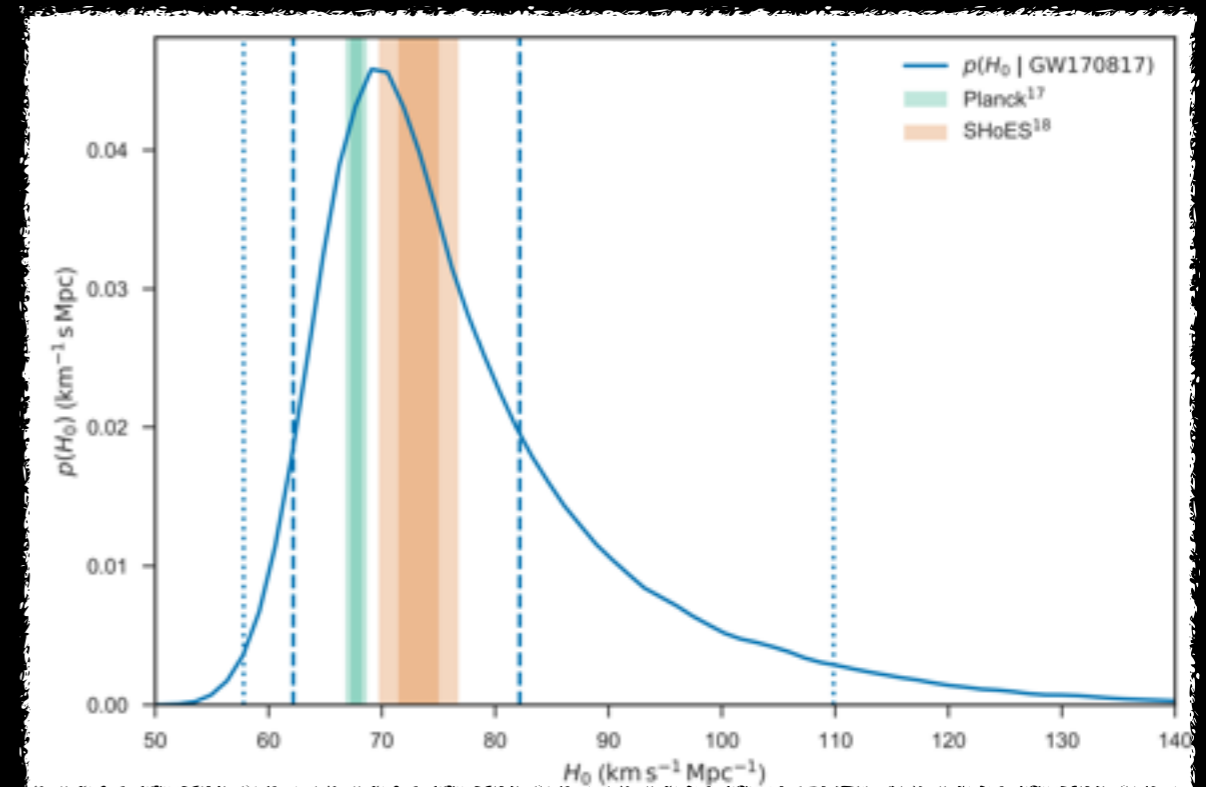
- Gravity
- Space time: Extra Dim?
- **Cosmology: Expansion**
- Nuclear Physics
- Astrophysics

Hubble's law:

$$v_H = H_0 d$$

The cosmological params.
can also be measured!

$$D_L(z) = \frac{c(1+z)}{H_0 \sqrt{\Omega_K}} \sinh \left[\sqrt{\Omega_K} \int_0^z \frac{H_0}{H(z')} dz' \right]$$



What can we learn from Gravitational Waves?

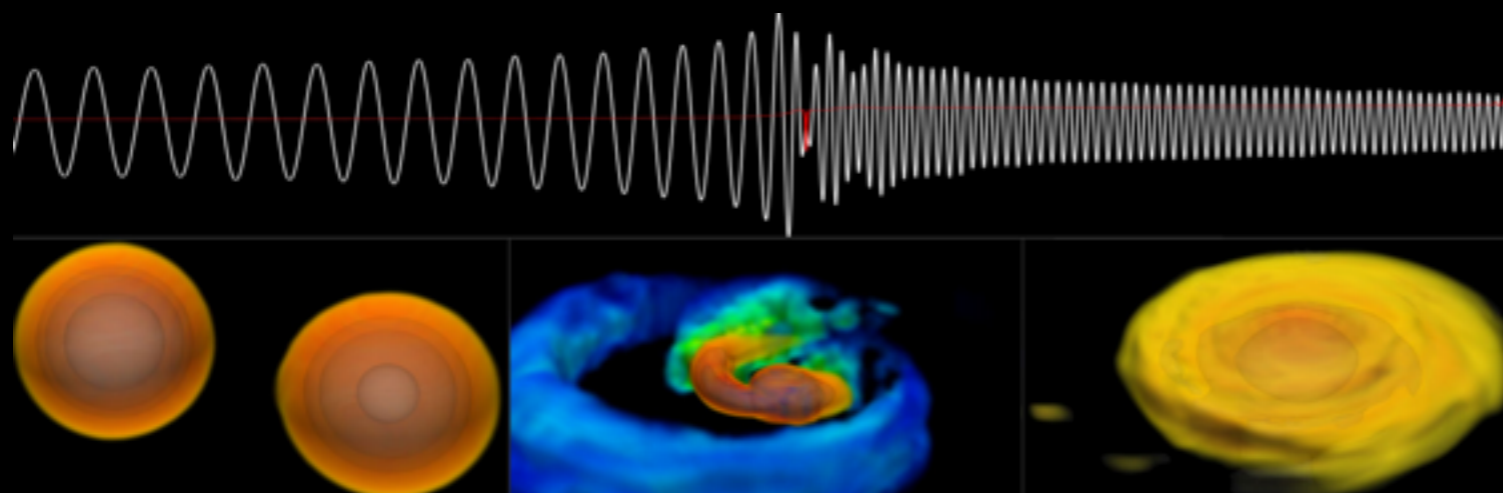
- What we measure:

- Masses
- Spins
- Eccentricity
- Dist./Pos./Redshift
- Acceleration

- What we can learn about:

- Gravity
- Space time: Extra Dim?
- Cosmology: Expansion
- **Nuclear Physics**
- Astrophysics

Tidal coupling and disruption:



Jet/Enrichment:



What can we learn from Gravitational Waves?

- What we measure:

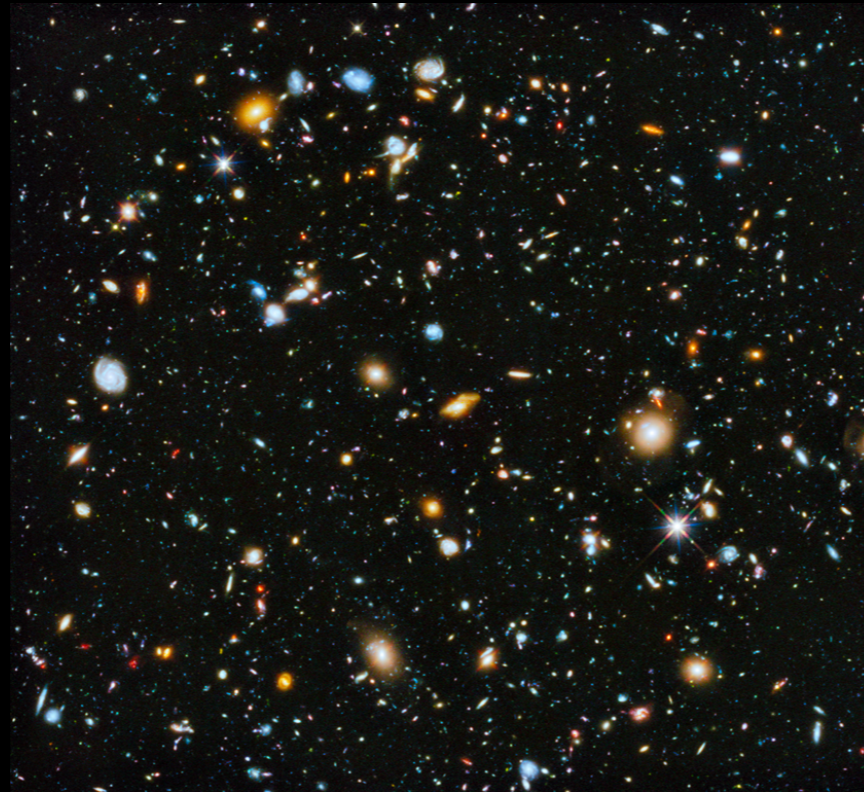
- Masses
- Spins
- Eccentricity
- Dist./Pos./Redshift
- Acceleration

- What we can learn about:

- Gravity
- Space time: Extra Dim?
- Cosmology: Expansion
- **Nuclear Physics**
- **Astrophysics**

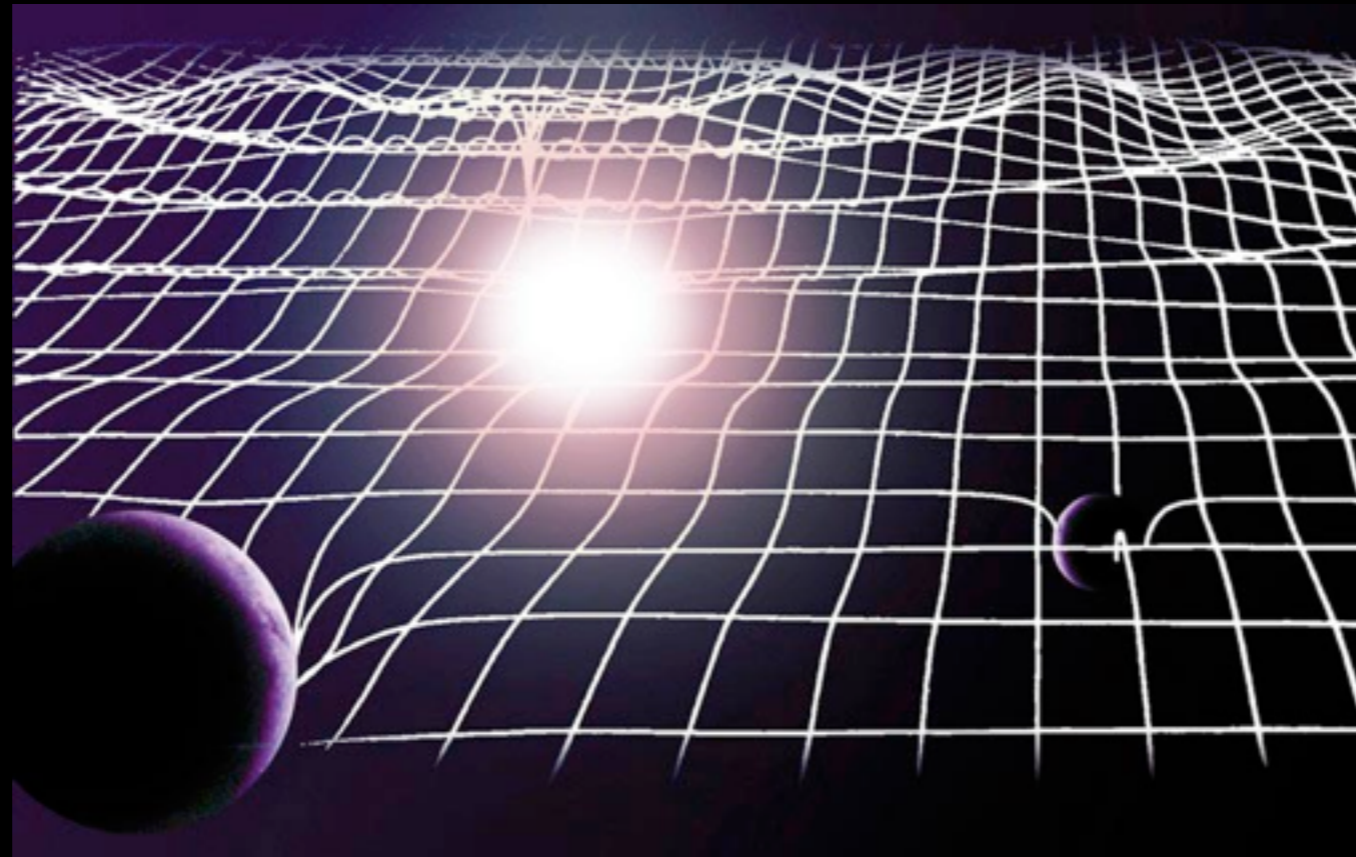
Astrophysics: How do Black Holes Form and Merge?

- *Primordial BHs*
- *Galactic Nuclei*
- *Supernovae*
- *Isolated BSE*
- *3,4-body BSE*
- *AGN Disk*
- *NS Mountains*
- *... ?*



Gravitational Wave Astrophysics

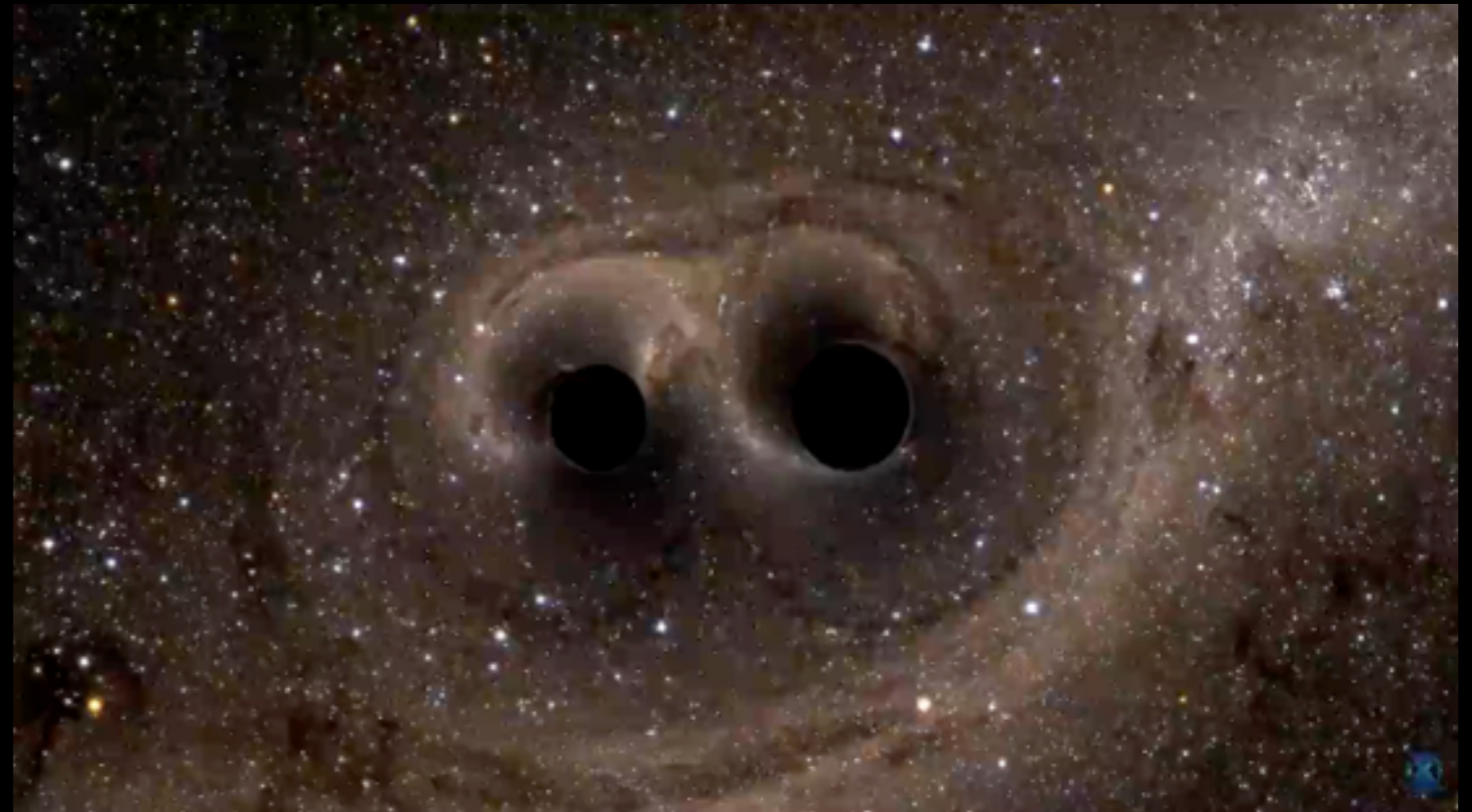
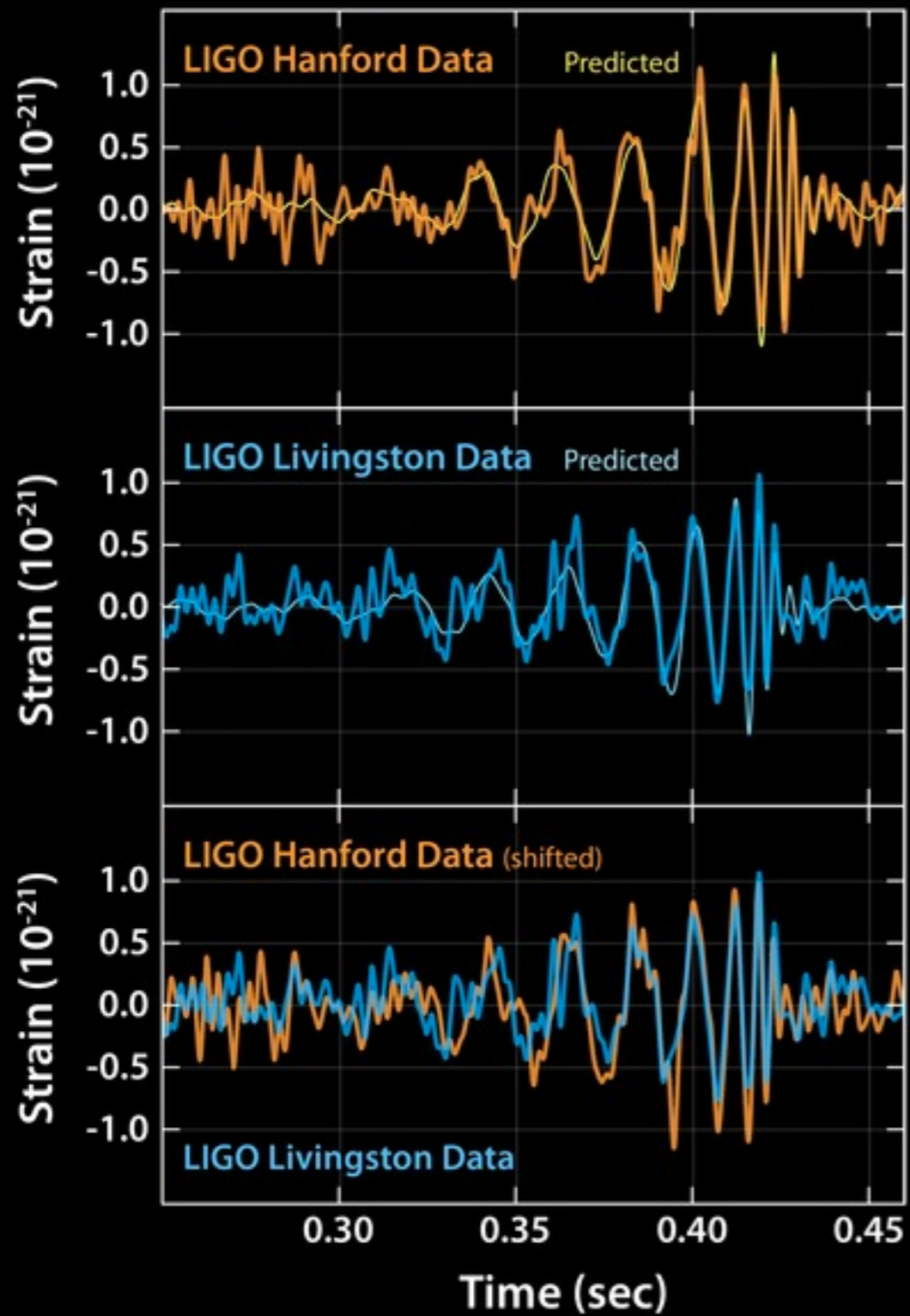
Johan Samsing
Princeton University



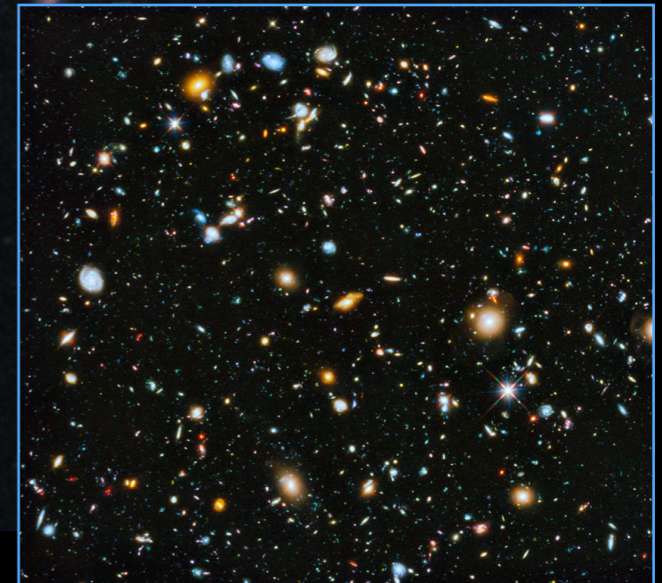
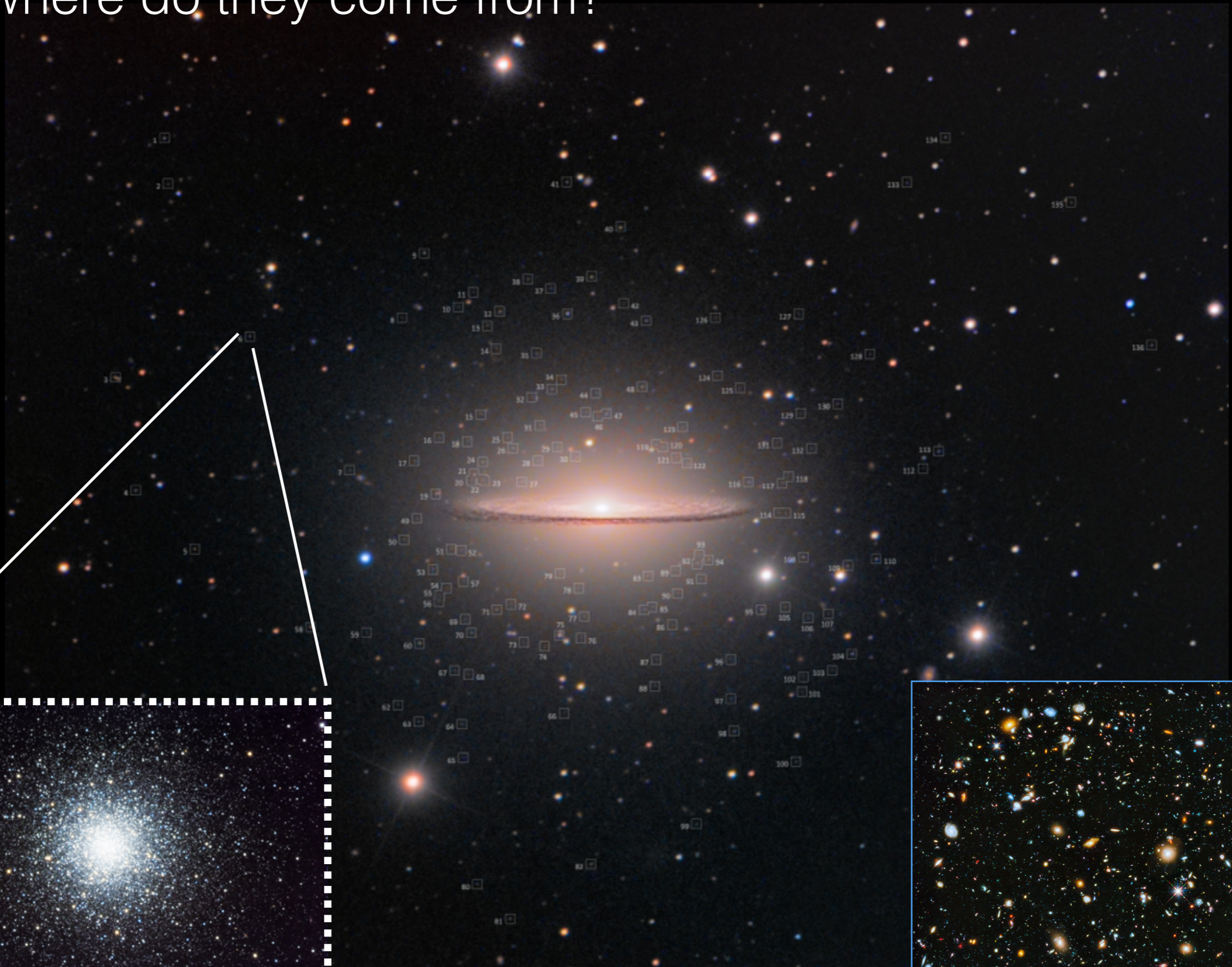
Part II

Observation:

Simulation:

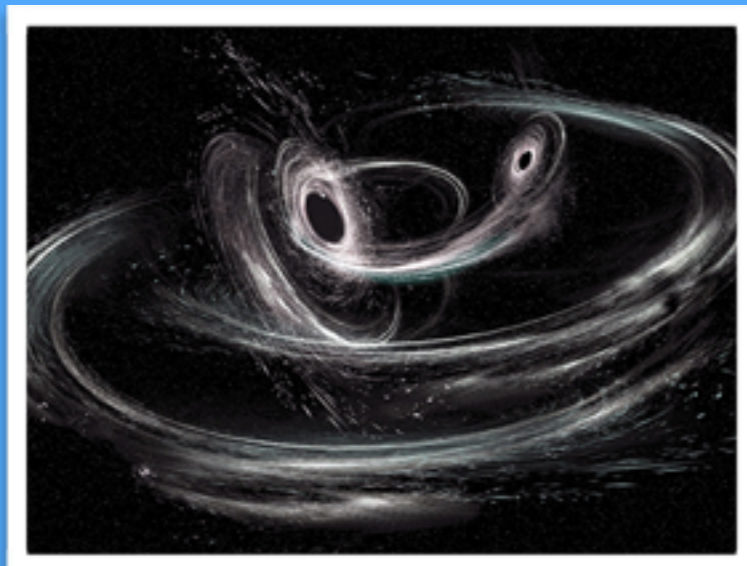


But where do they come from?

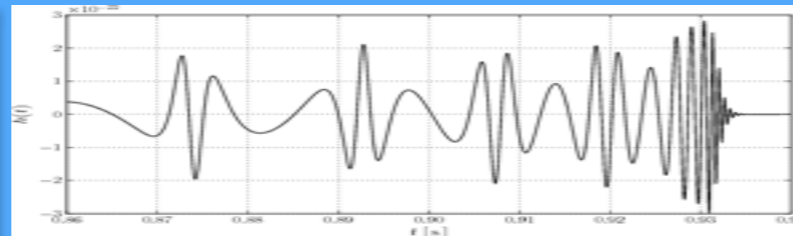


What is the origin of BBH mergers?

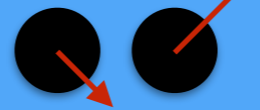
Clusters



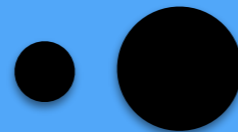
parameters



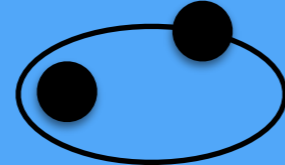
Spin



Mass



Eccentricity



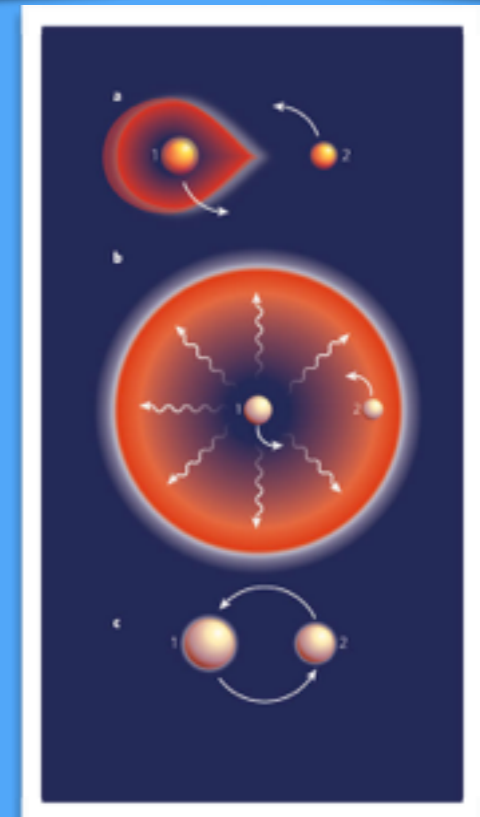
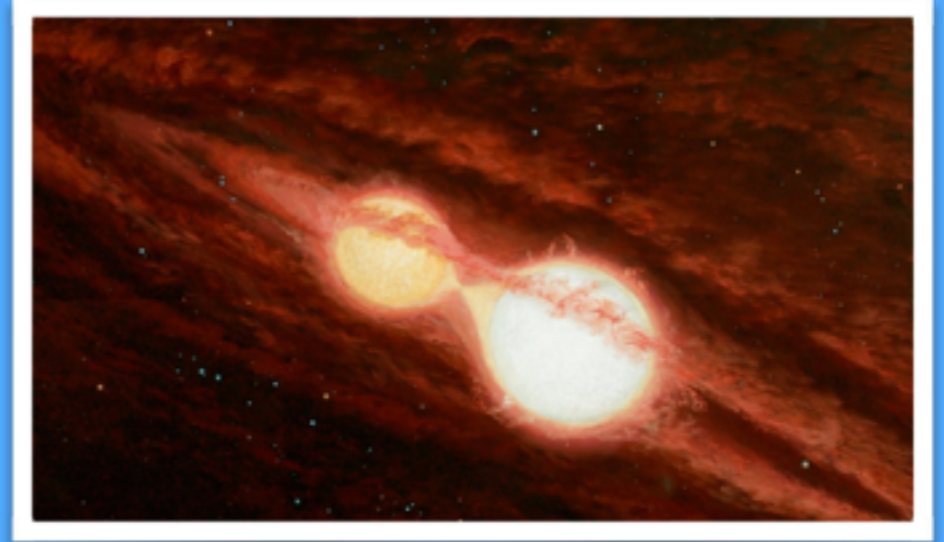
Rates



Location

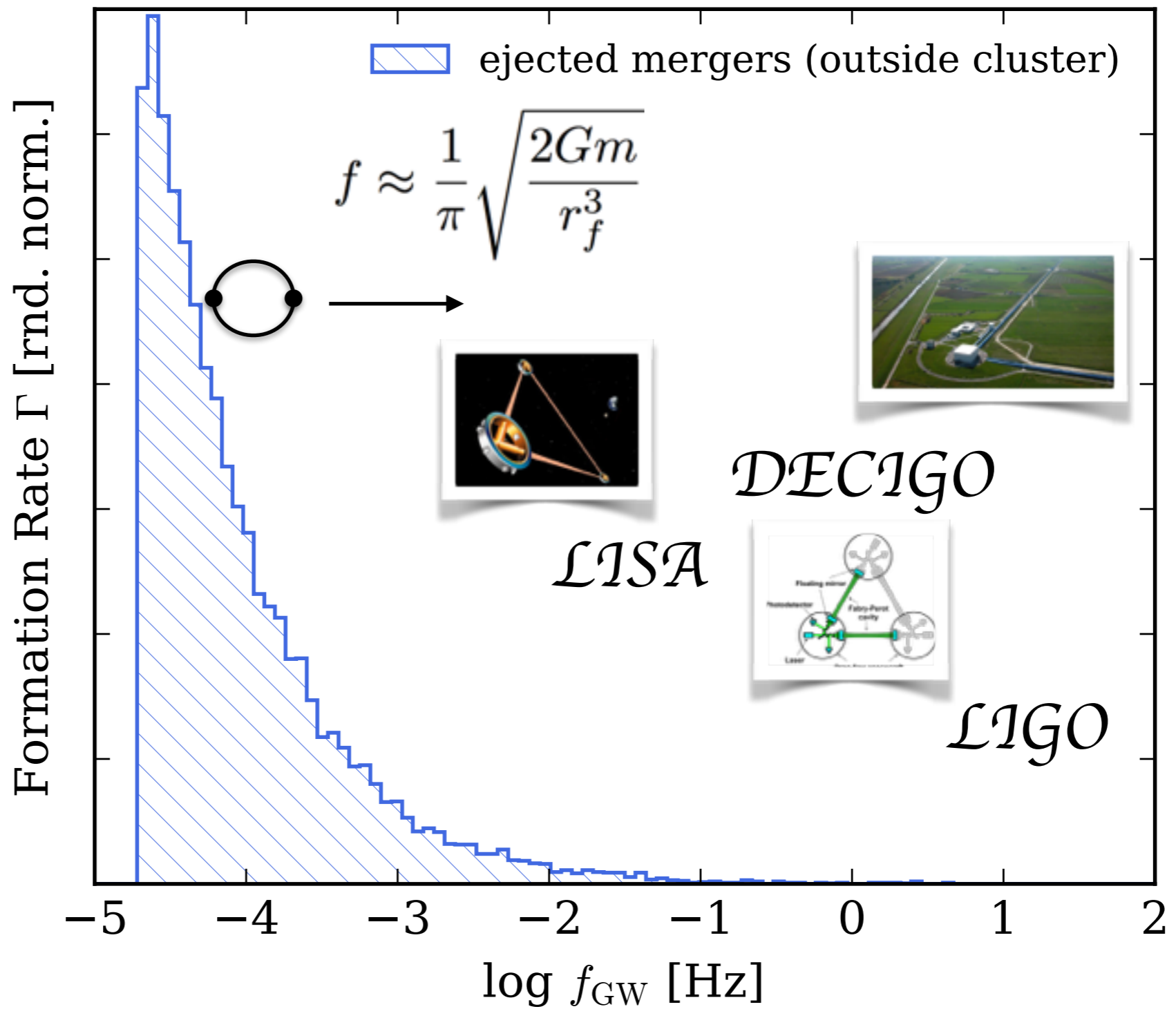


Field



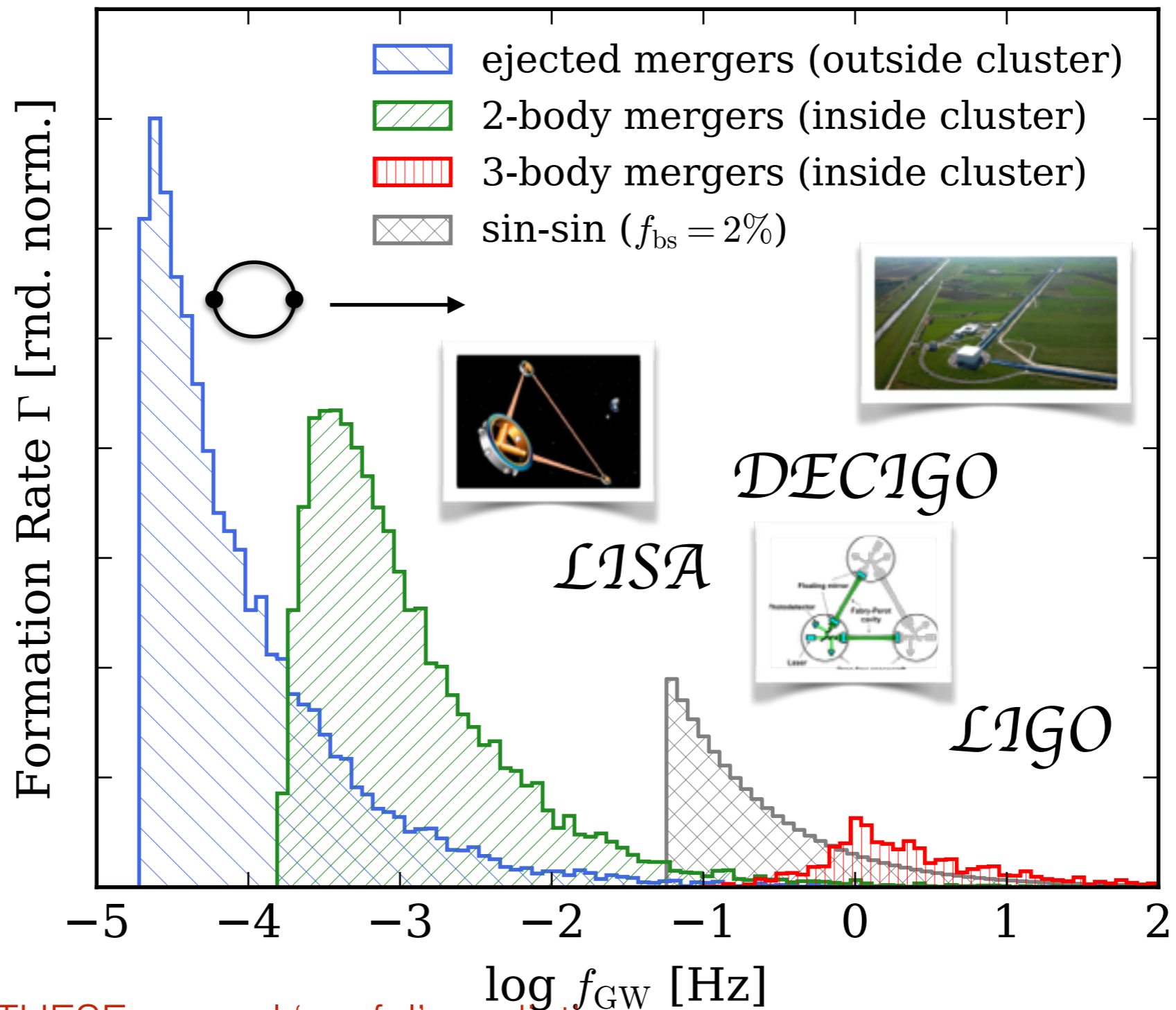
Old Newtonian Studies

< 2017

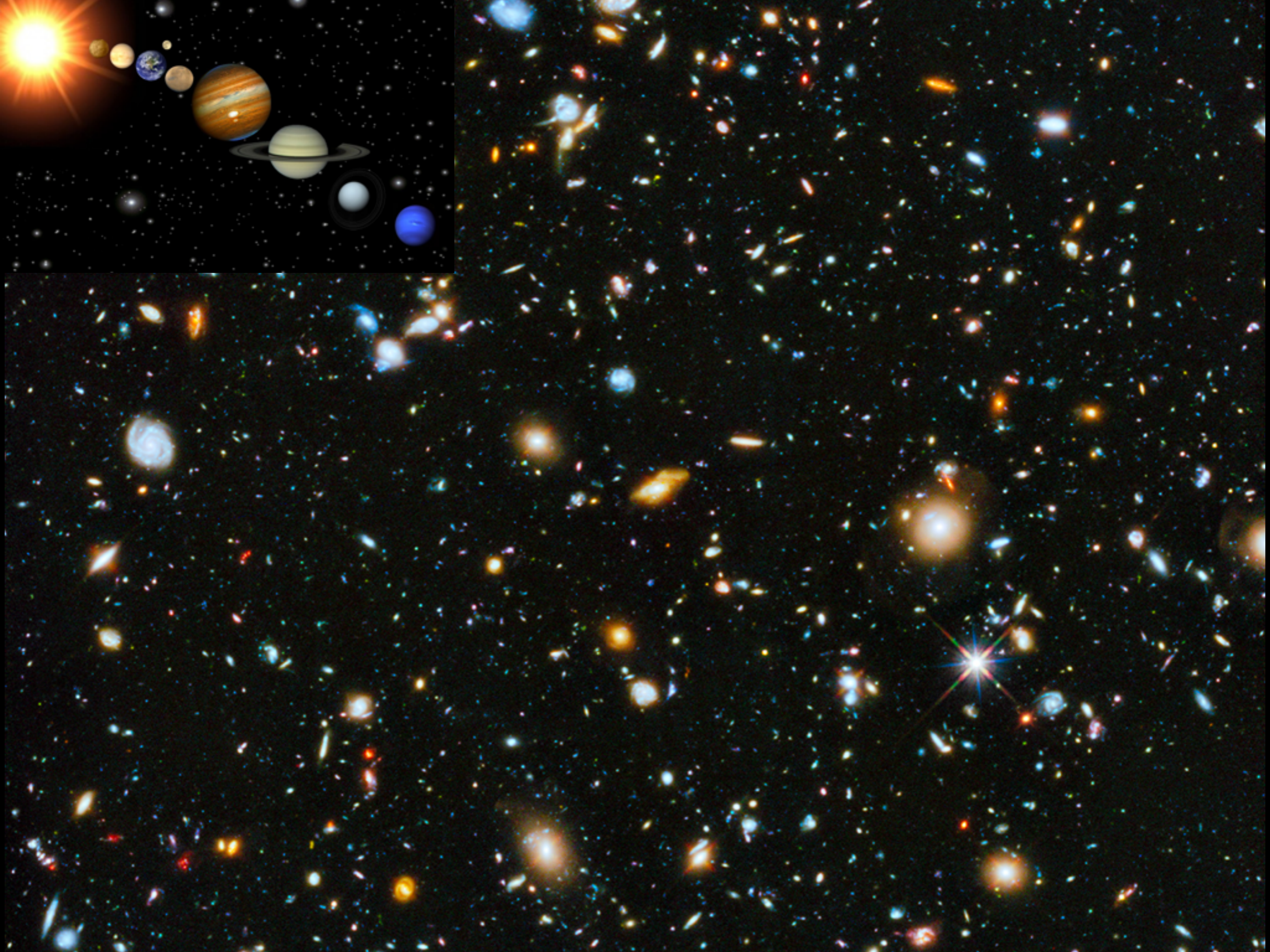
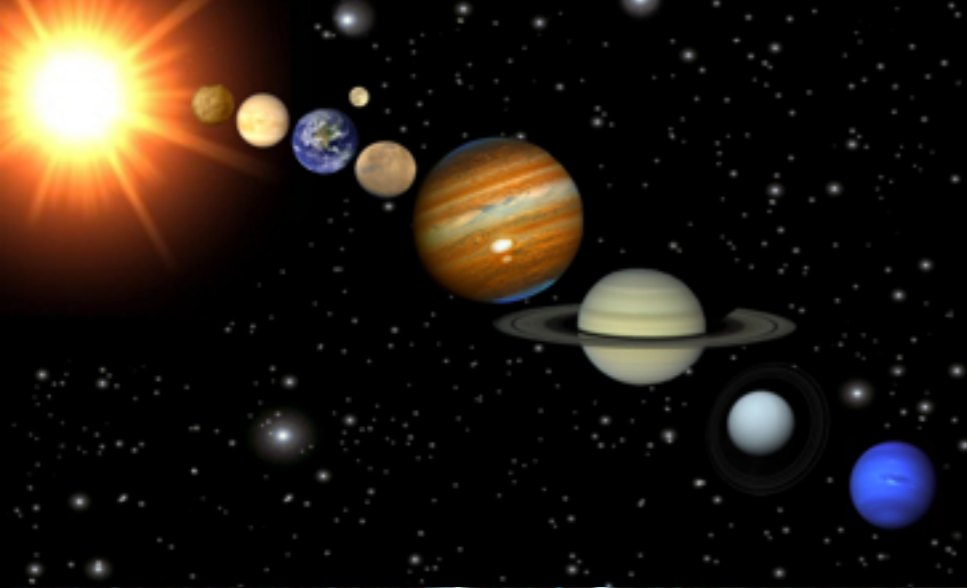


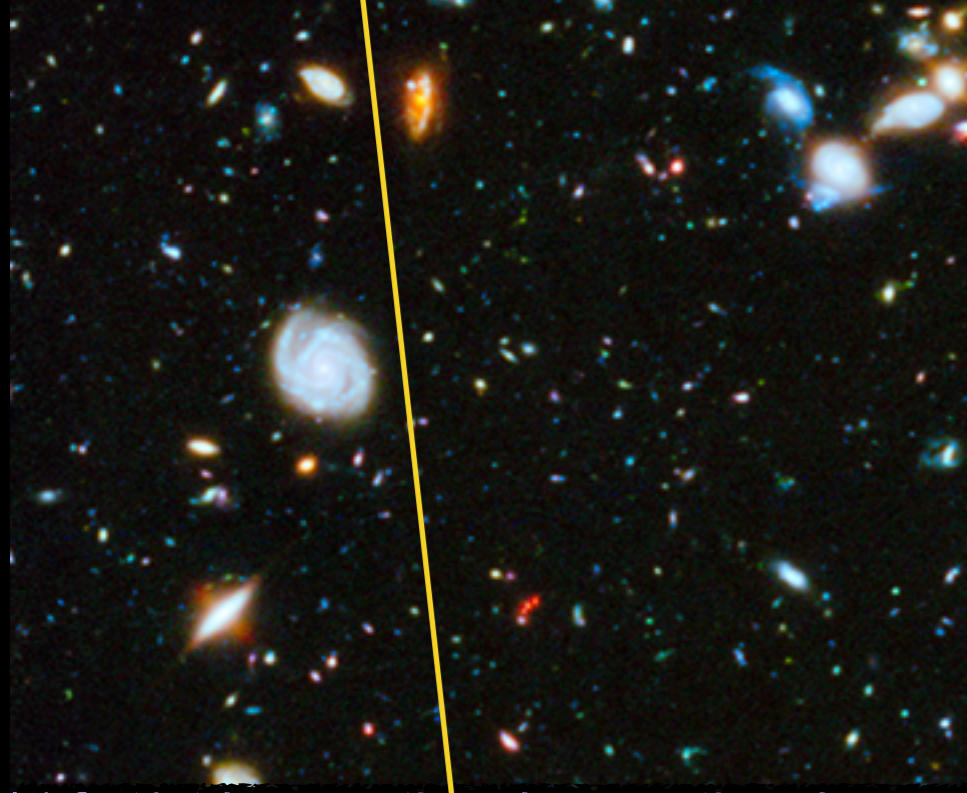
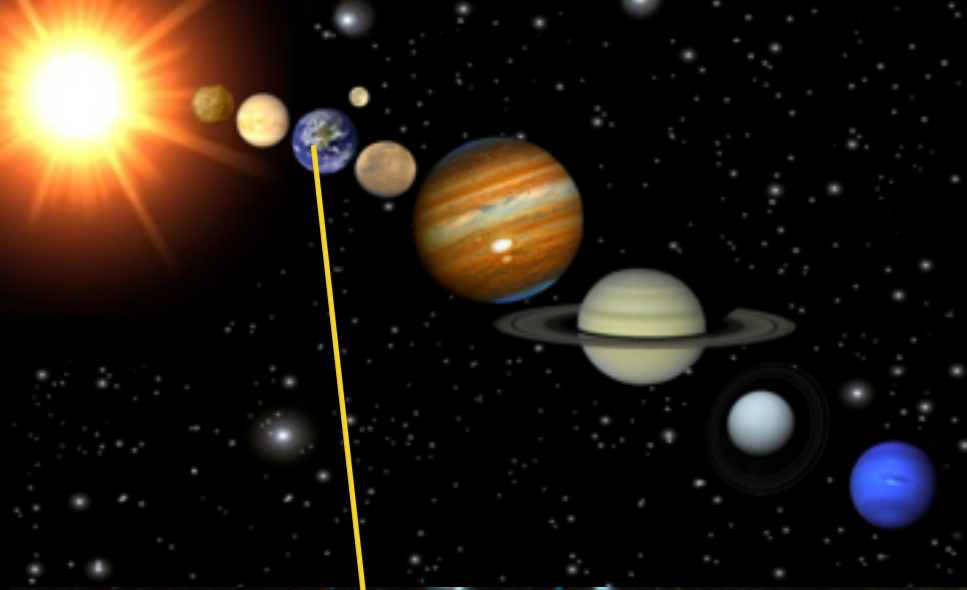
New Post-Newtonian Studies

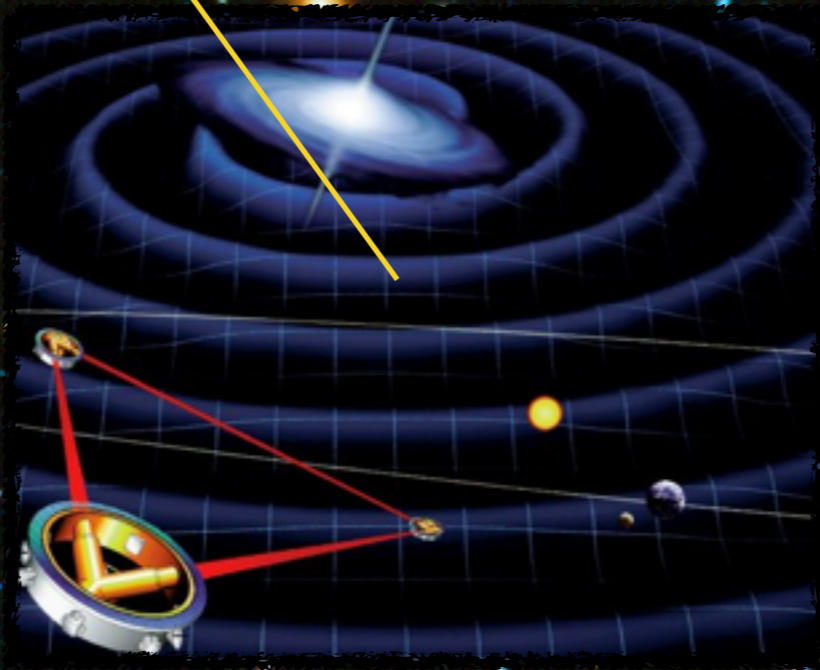
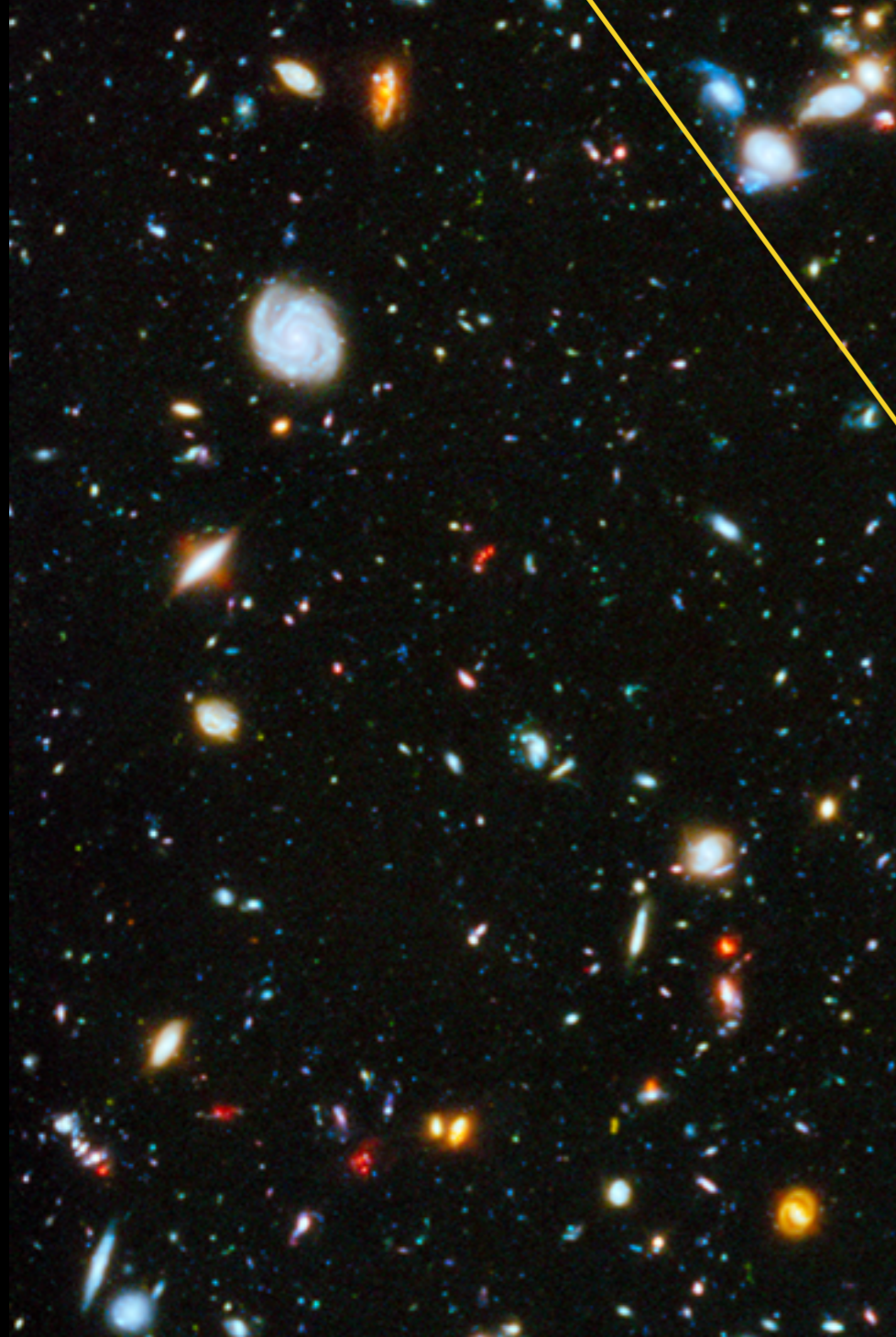
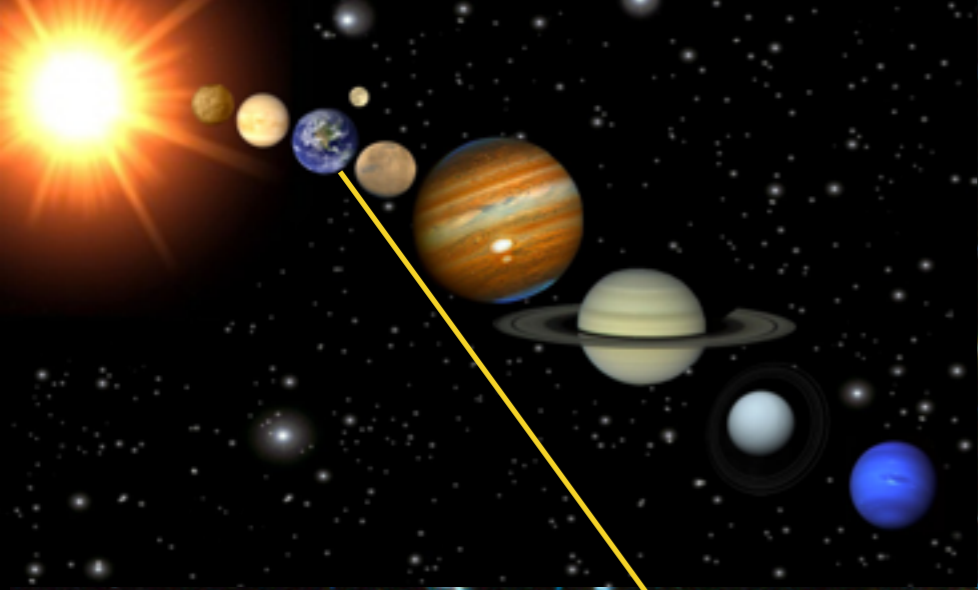
> 2017

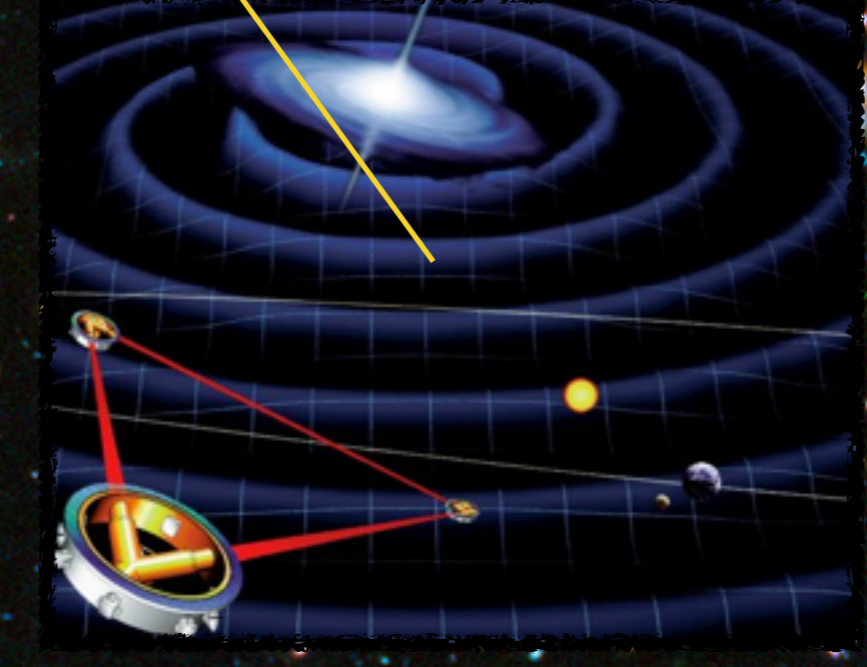
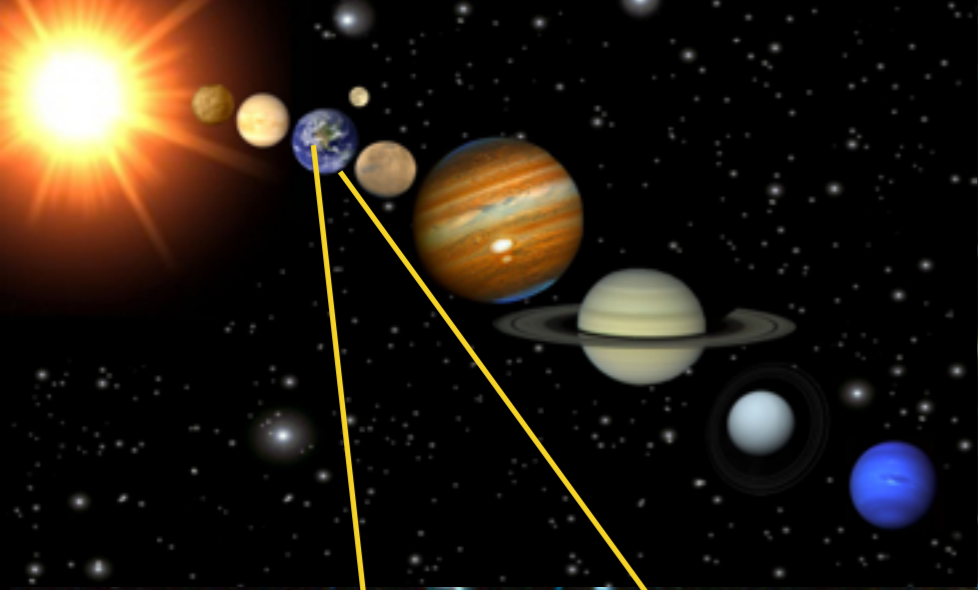


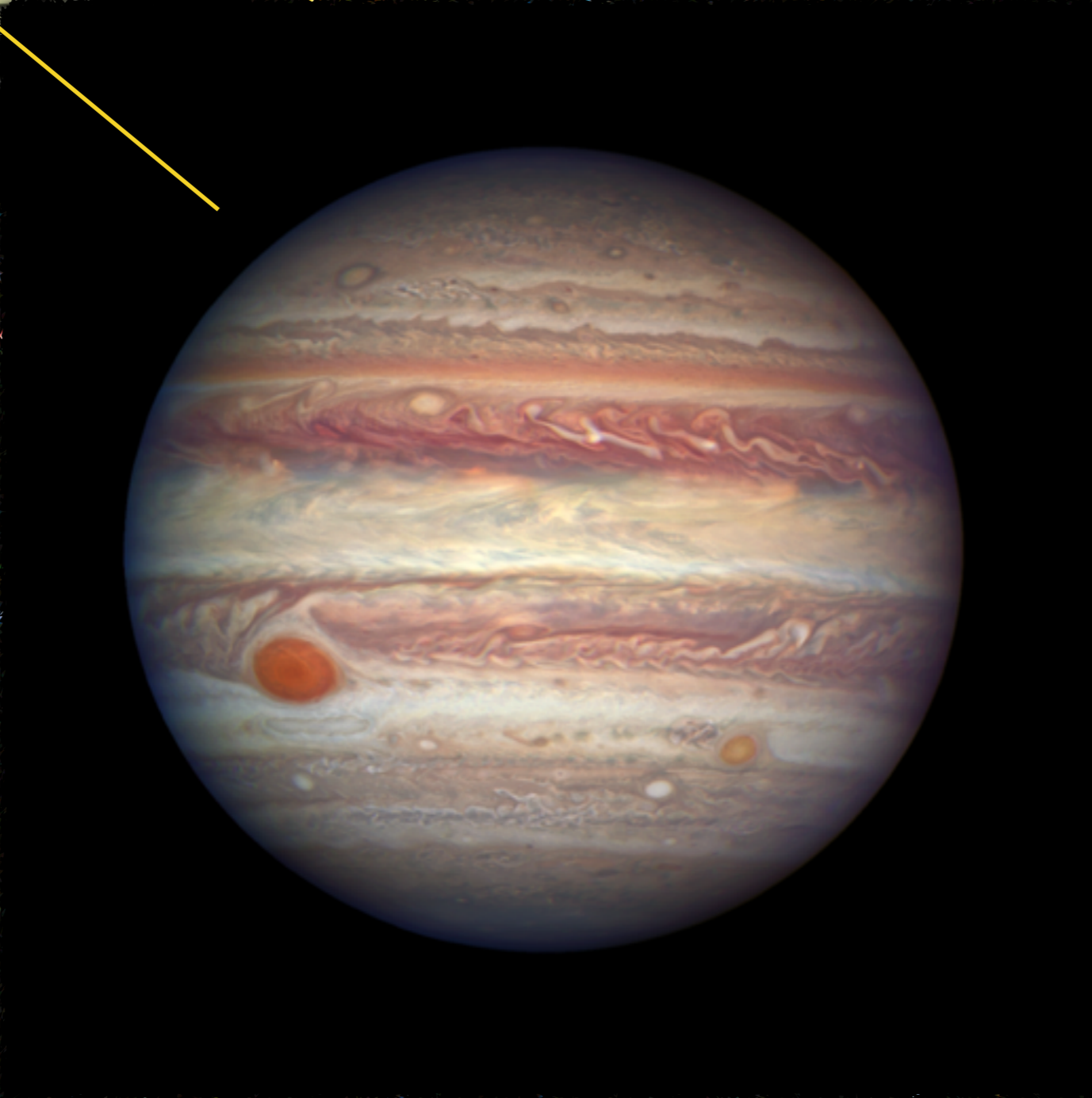
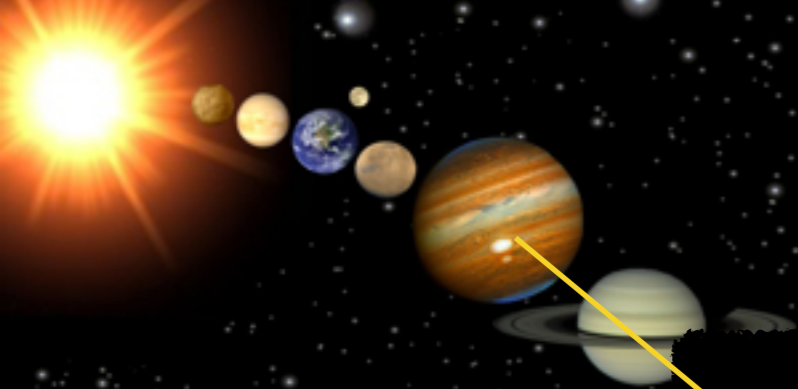
THESE are real 'useful' predictions...

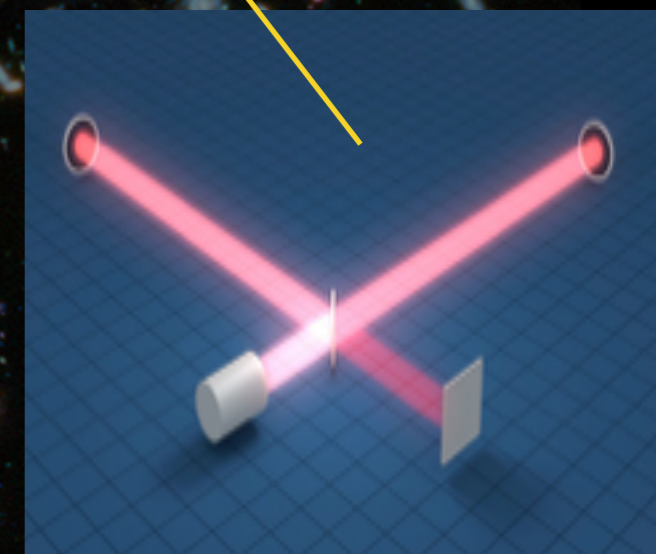
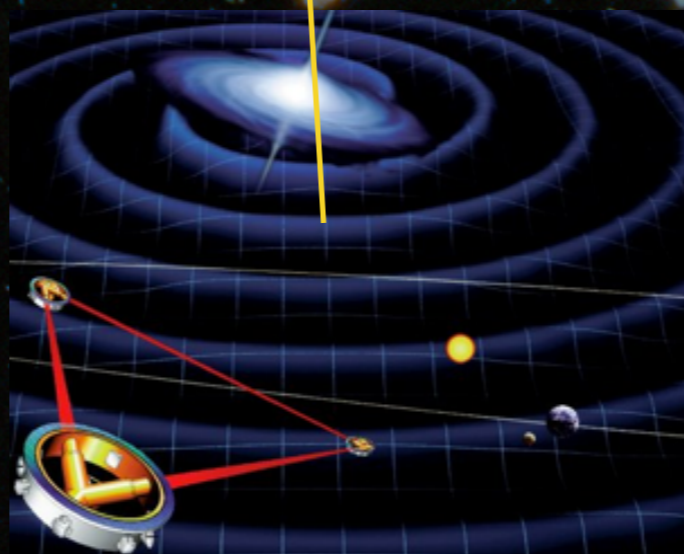
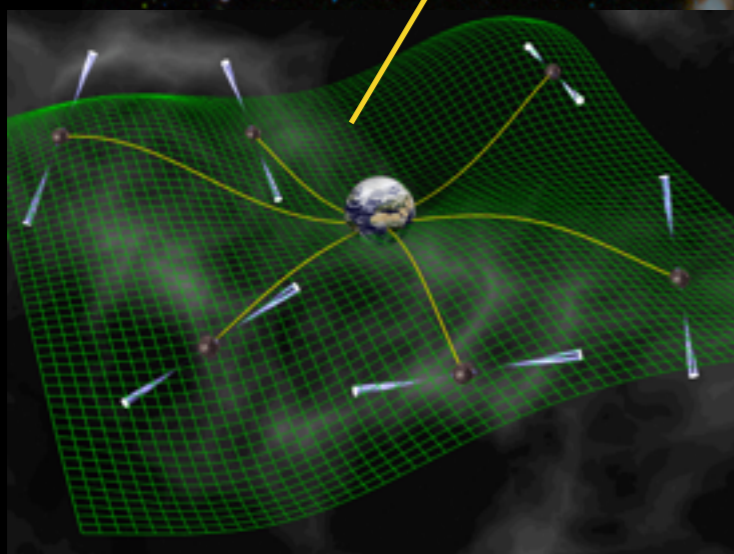
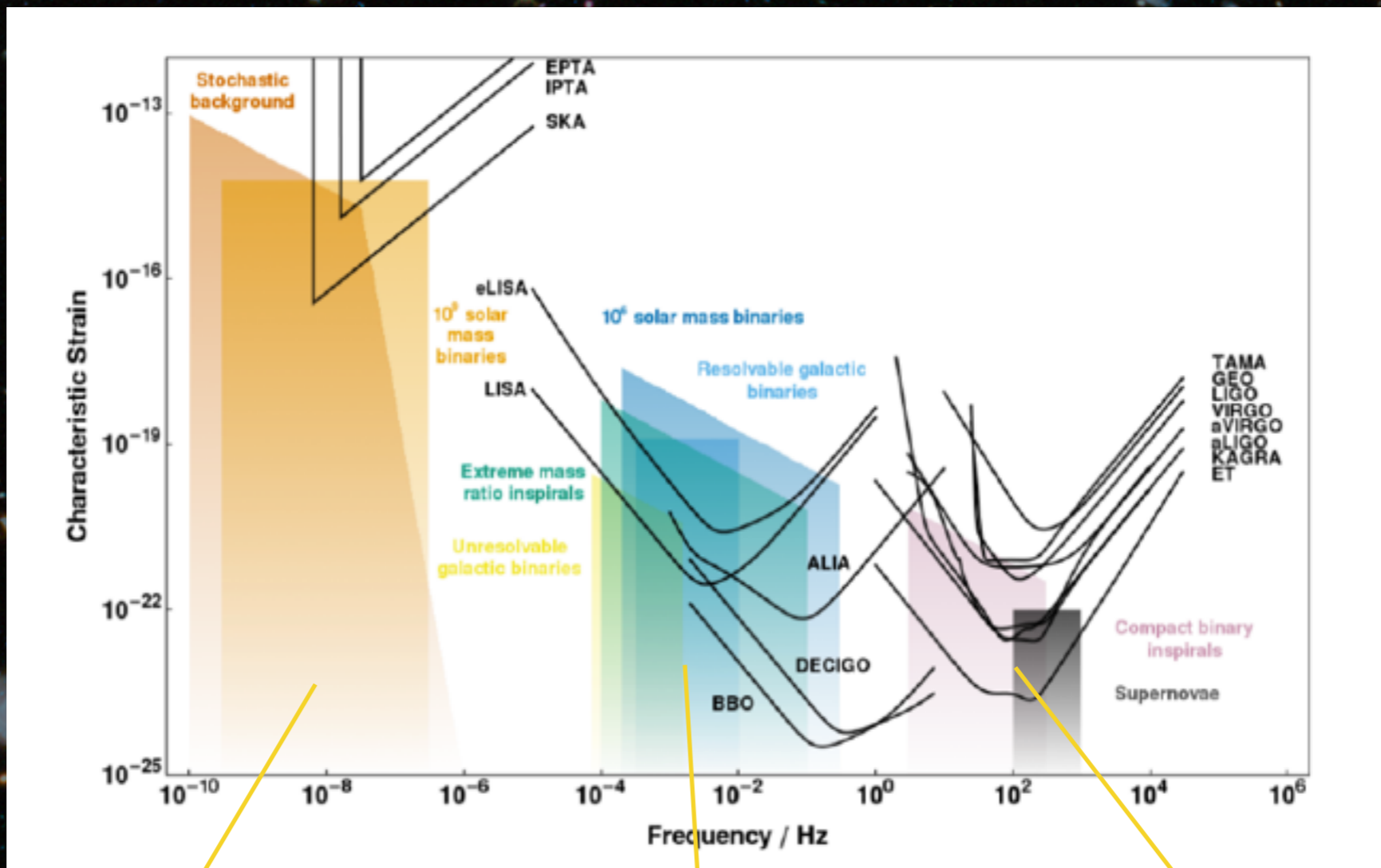






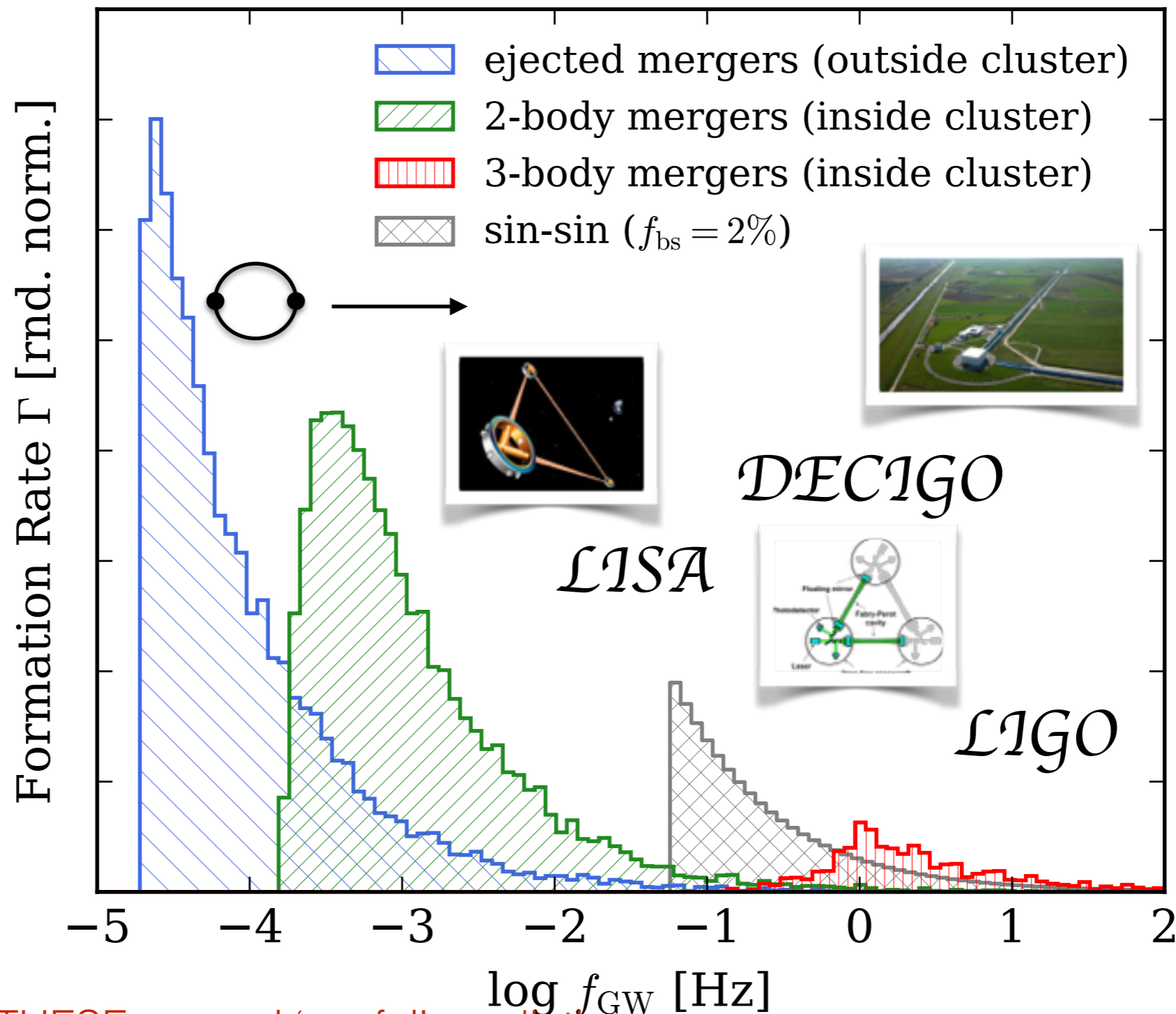






New Post-Newtonian Studies

> 2017



THESE are real 'useful' predictions...

Background

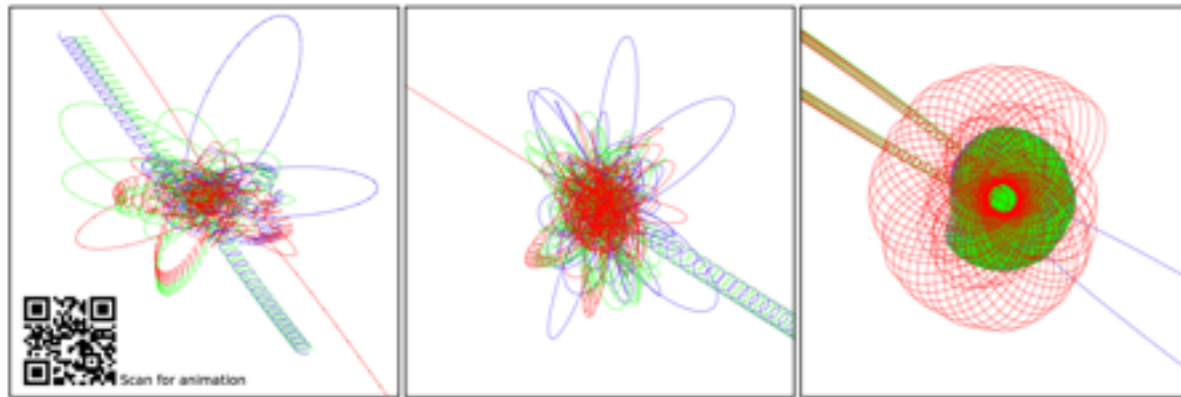
THE FORMATION OF ECCENTRIC COMPACT BINARY INSPIRALS AND THE ROLE OF GRAVITATIONAL WAVE EMISSION IN BINARY-SINGLE STELLAR ENCOUNTERS

JOHAN SAMSING¹, MORGAN MACLEOD², ENRICO RAMIREZ-RUIZ²

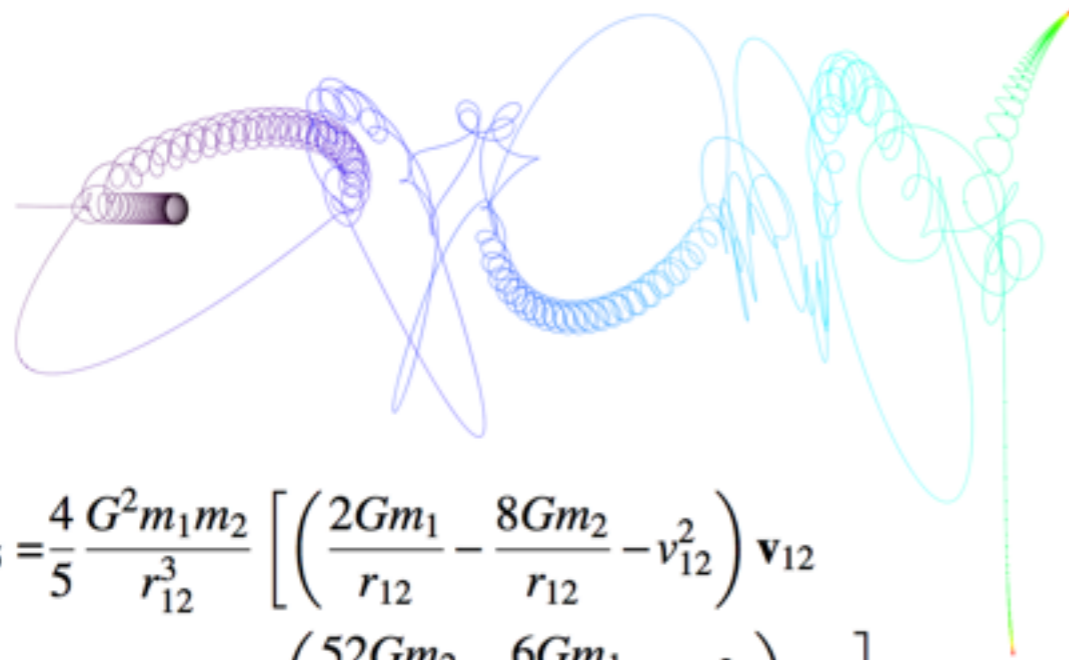
Draft version October 29, 2018

ABSTRACT

The inspiral and merger of eccentric binaries leads to gravitational waveforms distinct from those generated by circularly merging binaries. Dynamical environments can assemble binaries with high eccentricity and peak frequencies within the *LIGO* band. In this paper, we study binary-single stellar scatterings occurring in dense



eccentric black hole mergers forming in globular clusters
Samsing, 18.



$$\mathbf{a}_5 = \frac{4}{5} \frac{G^2 m_1 m_2}{r_{12}^3} \left[\left(\frac{2Gm_1}{r_{12}} - \frac{8Gm_2}{r_{12}} - v_{12}^2 \right) \mathbf{v}_{12} + (\hat{\mathbf{r}}_{12} \cdot \mathbf{v}_{12}) \left(\frac{52Gm_2}{3r_{12}} - \frac{6Gm_1}{r_{12}} + 3v_{12}^2 \right) \hat{\mathbf{r}}_{12} \right]$$

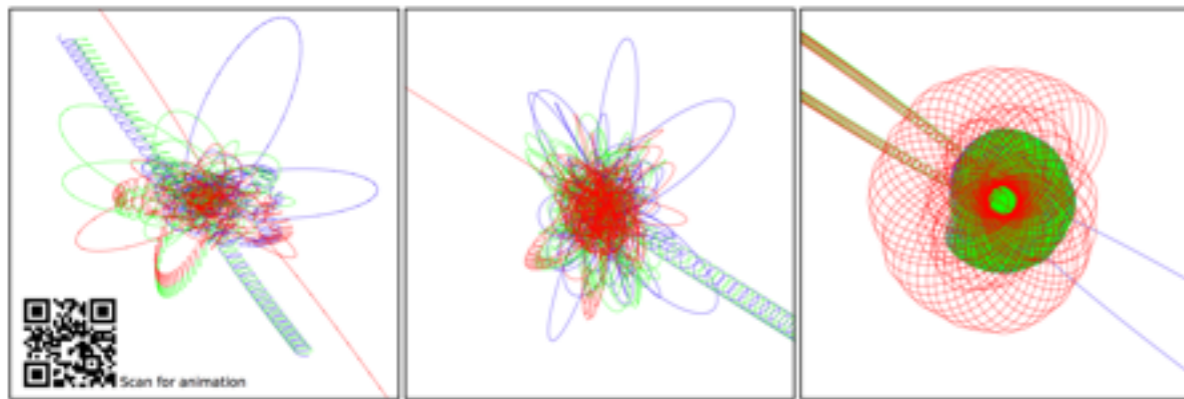
Background

THE FORMATION OF ECCENTRIC COMPACT BINARY INSPIRALS AND THE ROLE OF GRAVITATIONAL WAVE EMISSION IN BINARY-SINGLE STELLAR ENCOUNTERS

JOHAN SAMSING¹, MORGAN MACLEOD², ENRICO RAMIREZ-RUIZ²
Draft version October 29, 2018

ABSTRACT

The inspiral and merger of eccentric binaries leads to gravitational waveforms distinct from those generated by circularly merging binaries. Dynamical environments can assemble binaries with high eccentricity and peak frequencies within the *LIGO* band. In this paper, we study binary-single stellar scatterings occurring in dense

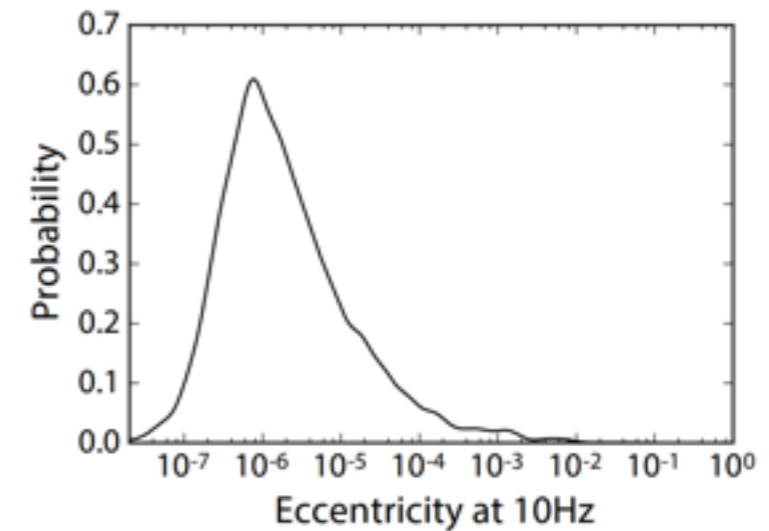


Binary Black Hole Mergers from Globular Clusters: Masses, Merger Rates, and the Impact of Stellar Evolution

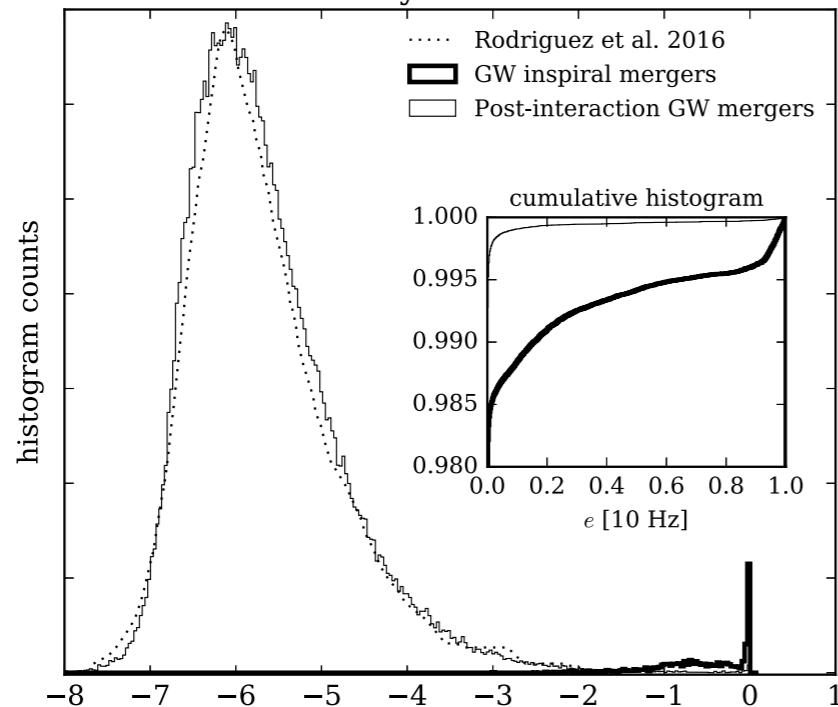
Carl L. Rodriguez,¹ Sourav Chatterjee,¹ and Frederic A. Rasio¹

¹*Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA) and Dept. of Physics and Astronomy, Northwestern University, 2145 Sheridan Rd, Evanston, IL 60208, USA*
 (Dated: March 25, 2016)

The recent discovery of GW150914, the binary black hole merger detected by Advanced LIGO,



BBH Eccentricity Distribution at 10Hz



(Samsing, Ramirez-Ruiz, 17) $\log e [10 \text{ Hz}]$



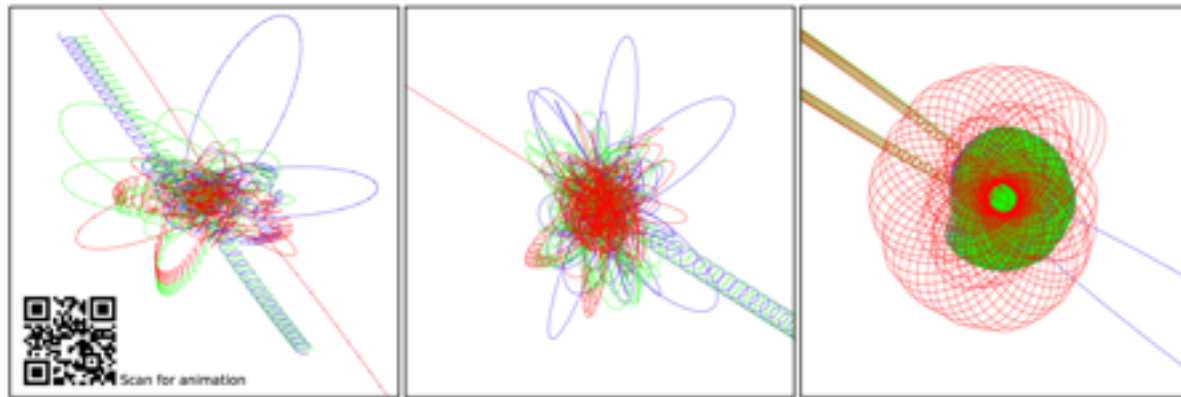
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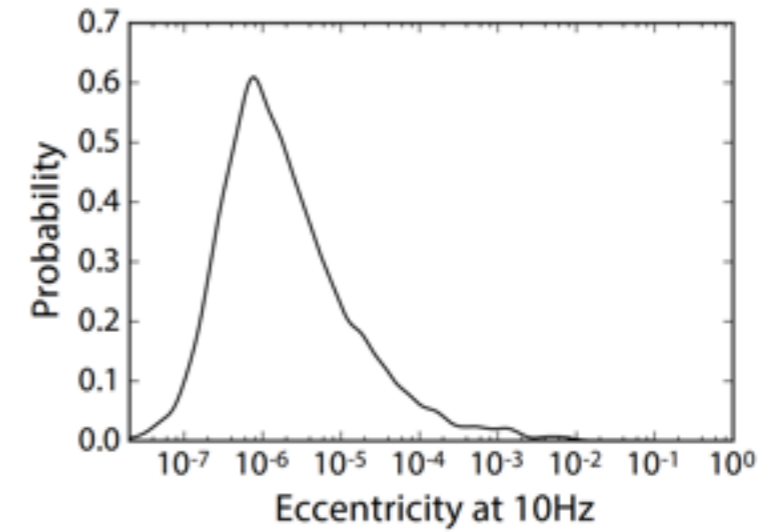


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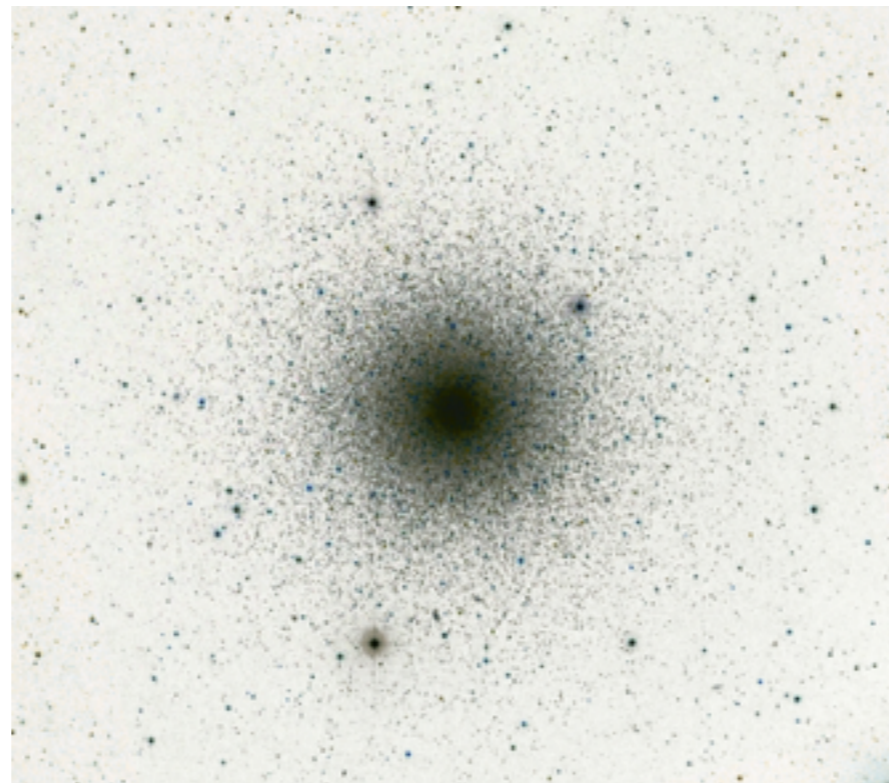
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Why is it so difficult?



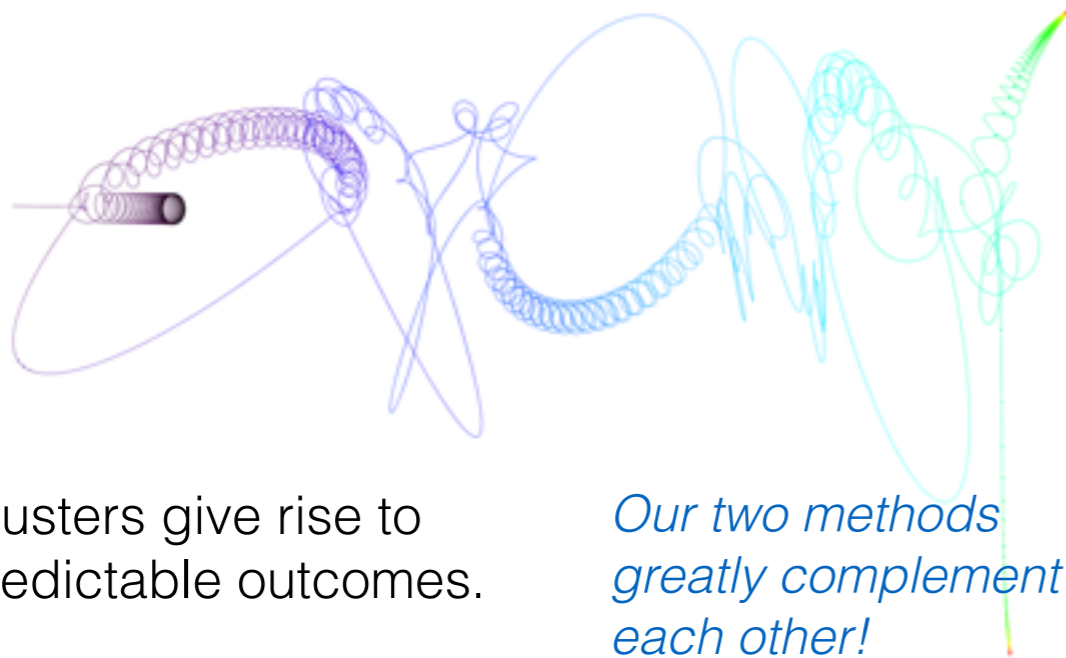
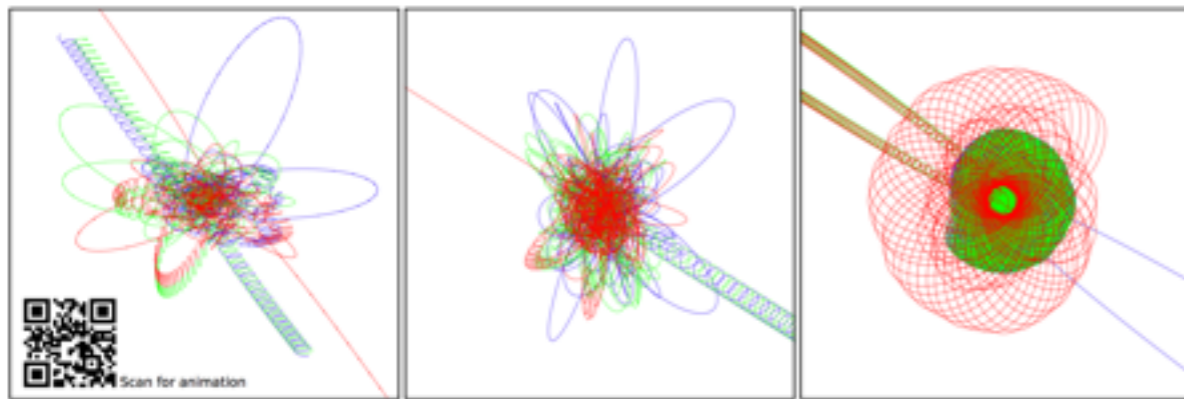
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Clusters give rise to predictable outcomes.

Our two methods greatly complement each other!

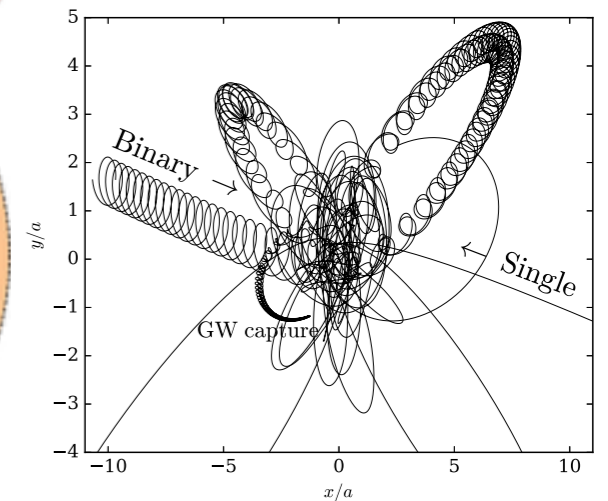
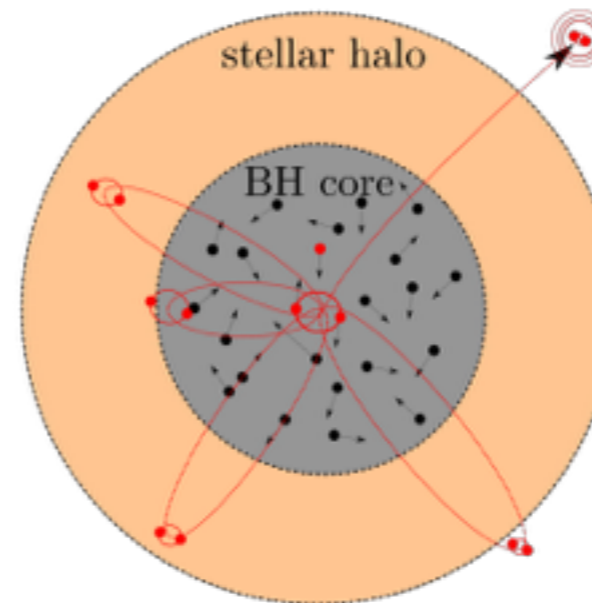
Pen and paper can reach percent precision!

Eccentric Black Hole Mergers Forming in Globular Clusters

Johan Samsing*

Department of Astrophysical Sciences, Princeton University,
 Peyton Hall, 4 Ivy Lane, Princeton, NJ 08544, USA.

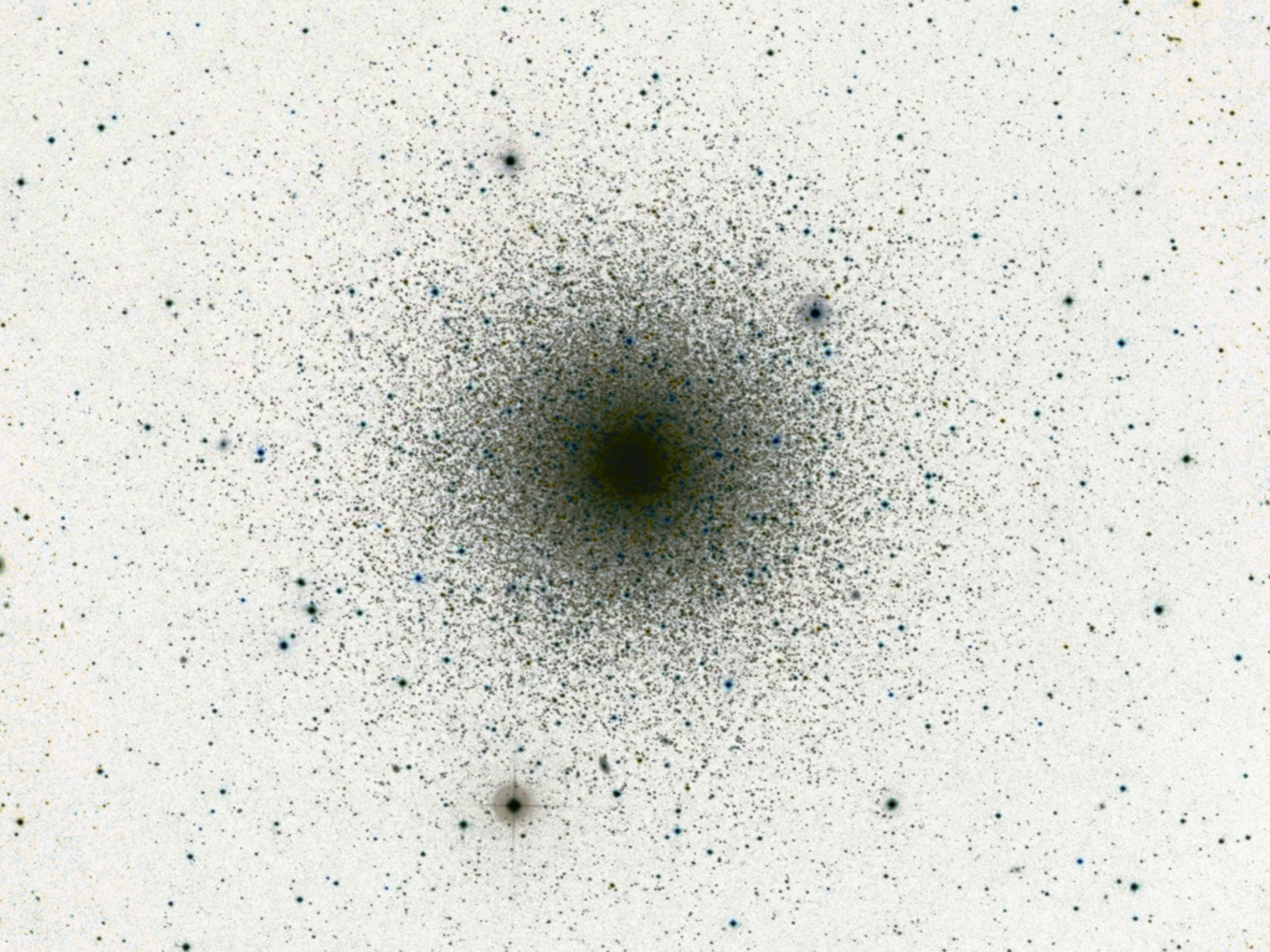
We derive the probability for a newly formed binary black hole (BBH) to undergo an eccentric gravitational wave (GW) merger during binary-single interactions inside a stellar cluster. By integrating over the hardening interactions such a BBH must undergo before ejection, we find that the observable rate of BBH mergers with eccentricity > 0.1 at 10 Hz relative to the rate of circular mergers can be as high as $\sim 5\%$ for a typical globular cluster (GC). This further suggests that BBH mergers forming through GW captures in binary-single interactions, eccentric or not, are likely to constitute $\sim 10\%$ of the total BBH merger rate from GCs. Such GW capture mergers can only

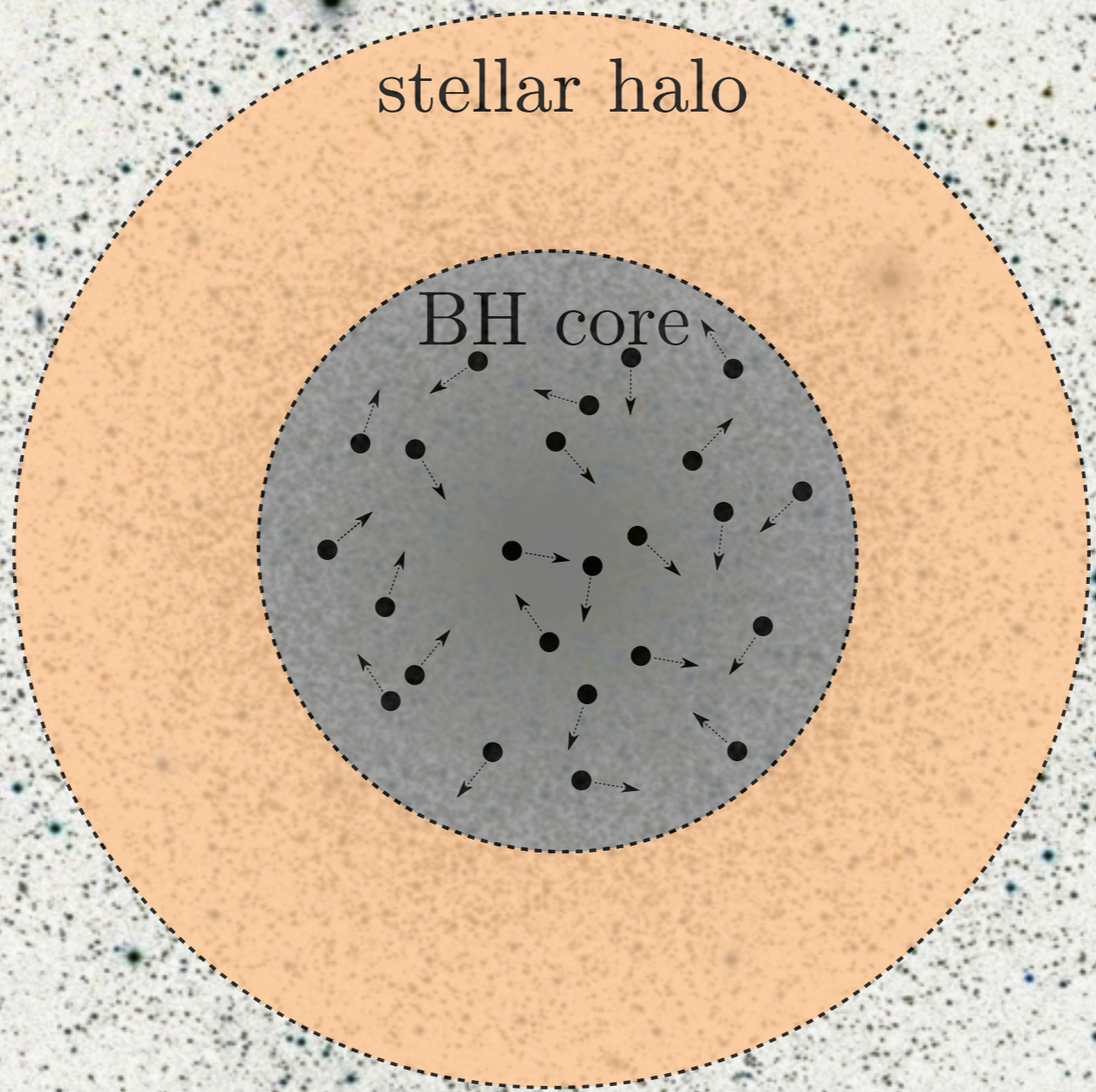


Post-Newtonian Dynamics in Dense Star Clusters: Formation, Masses, and Merger Rates of Highly-Eccentric Black Hole Binaries

Carl L. Rodriguez,¹ Pau Amaro-Seoane,² Sourav Chatterjee,³ Kyle Kremer,⁴
 Frederic A. Rasio,⁴ Johan Samsing,⁵ Claire S. Ye,⁴ and Michael Zevin⁴

Using state-of-the-art dynamical simulations of globular clusters, including radiation reaction during black hole encounters and a cosmological model of star cluster formation, we create a realistic population of dynamically-formed binary black hole mergers across cosmic space and time. We show that in the local universe, 10% of these binaries form as the result of gravitational-wave emission between unbound black holes during chaotic resonant encounters, with roughly half of those events having eccentricities detectable by current ground-based gravitational-wave detectors. The mergers

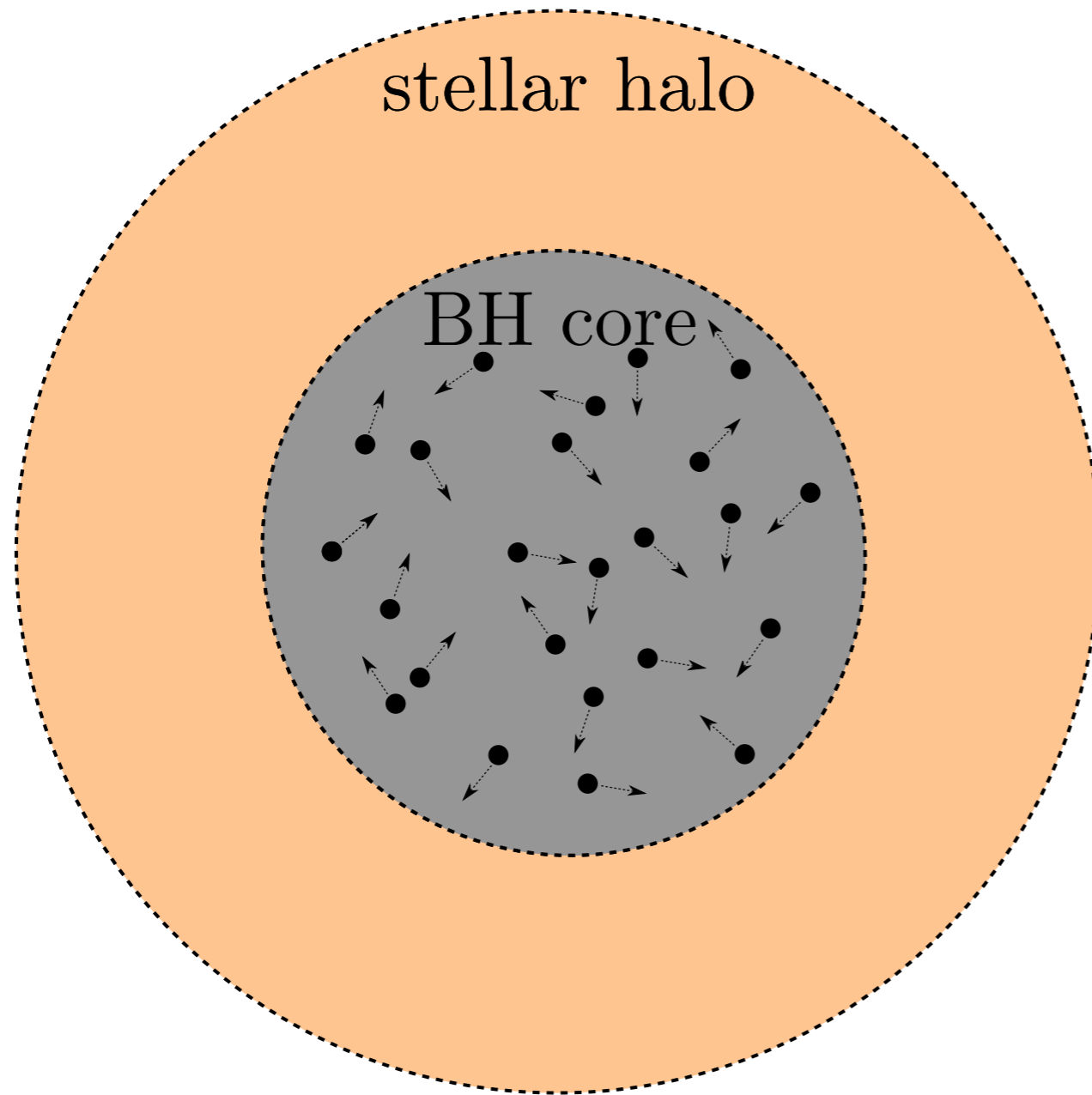




stellar halo

BH core

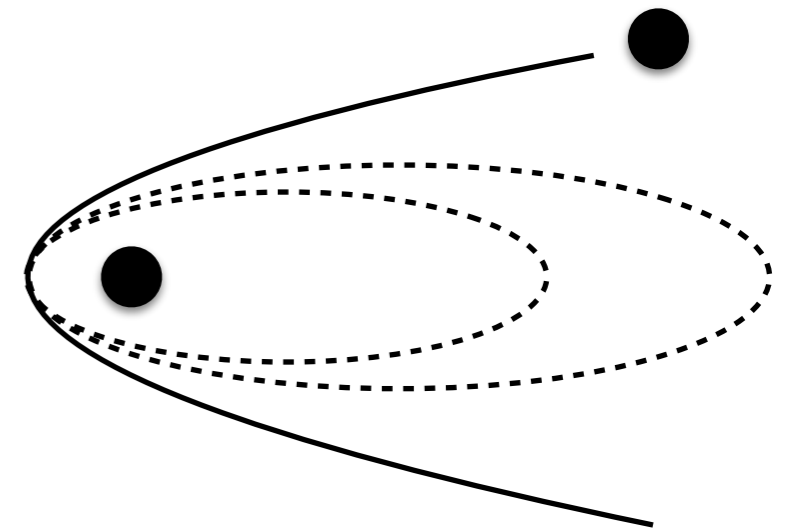
MODEL:



How do BBHs form and merge?

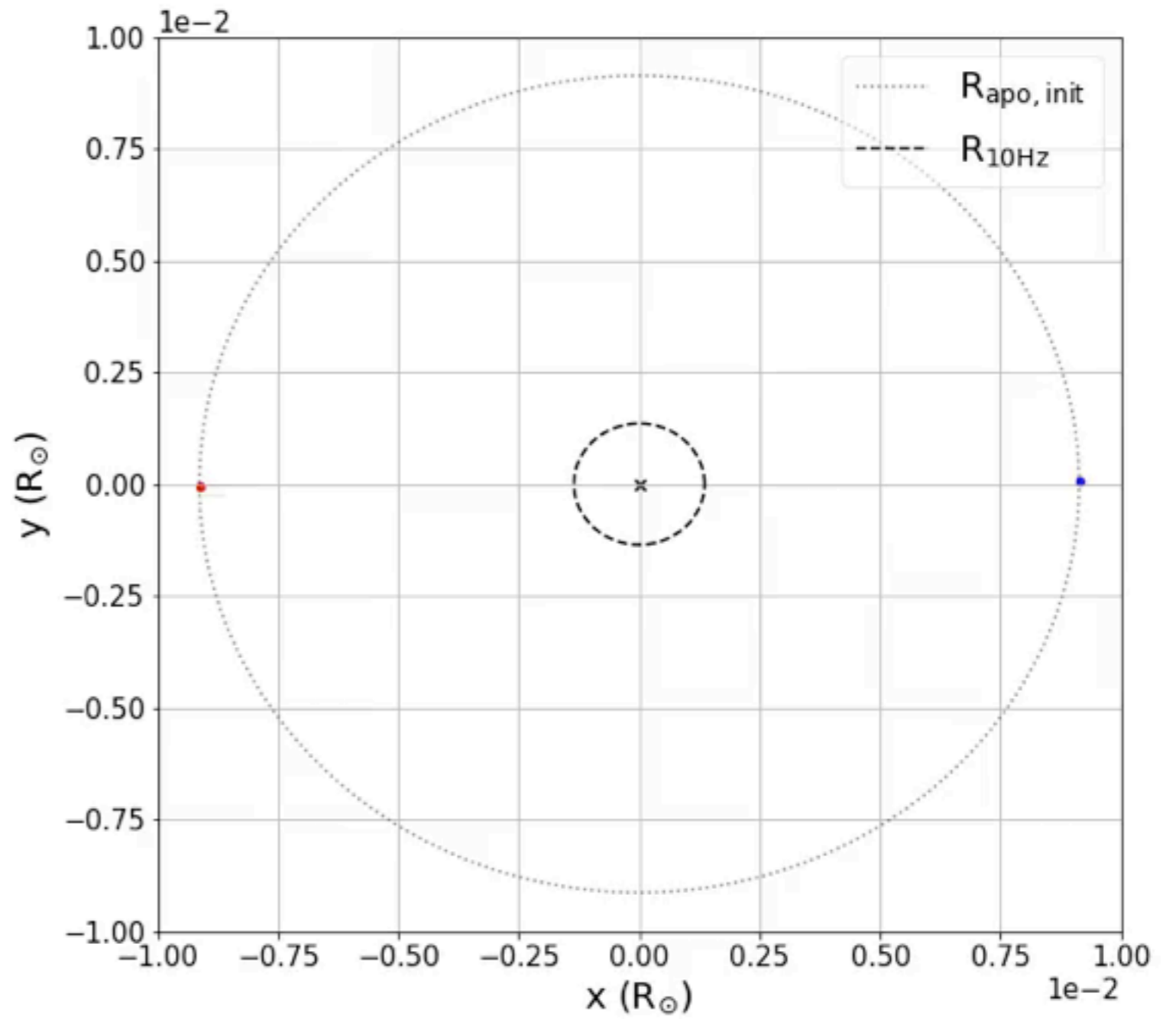
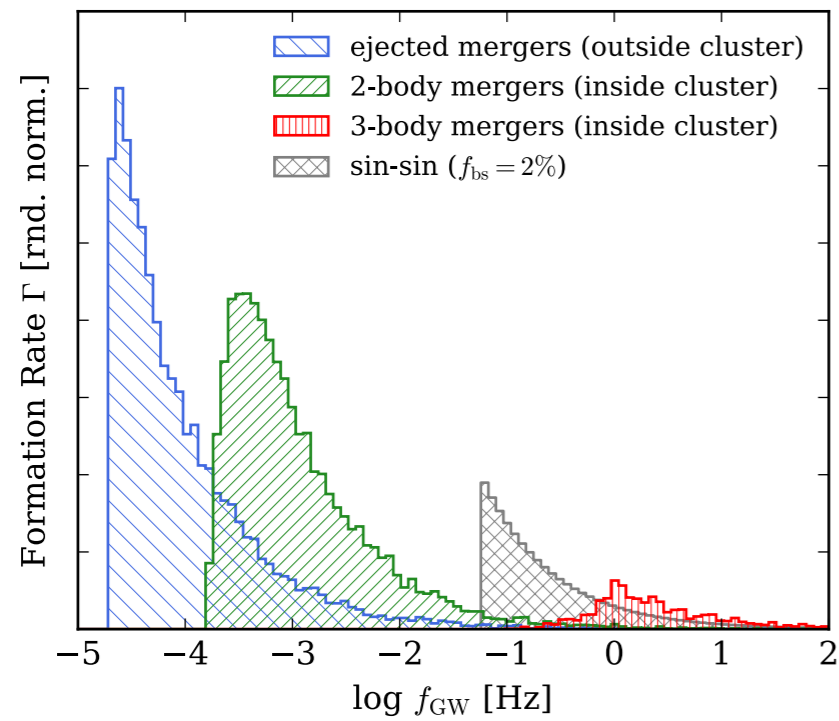
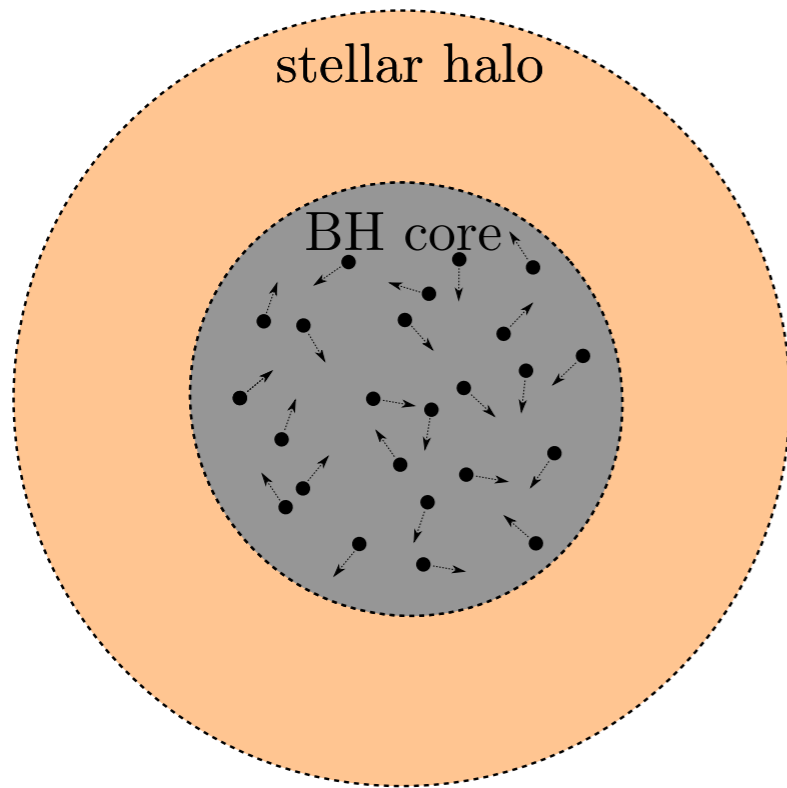
$$f \approx \frac{1}{\pi} \sqrt{\frac{2Gm}{r_f^3}}$$

$$r_f \approx \left(\frac{2Gm}{f^2 \pi^2} \right)^{1/3}$$



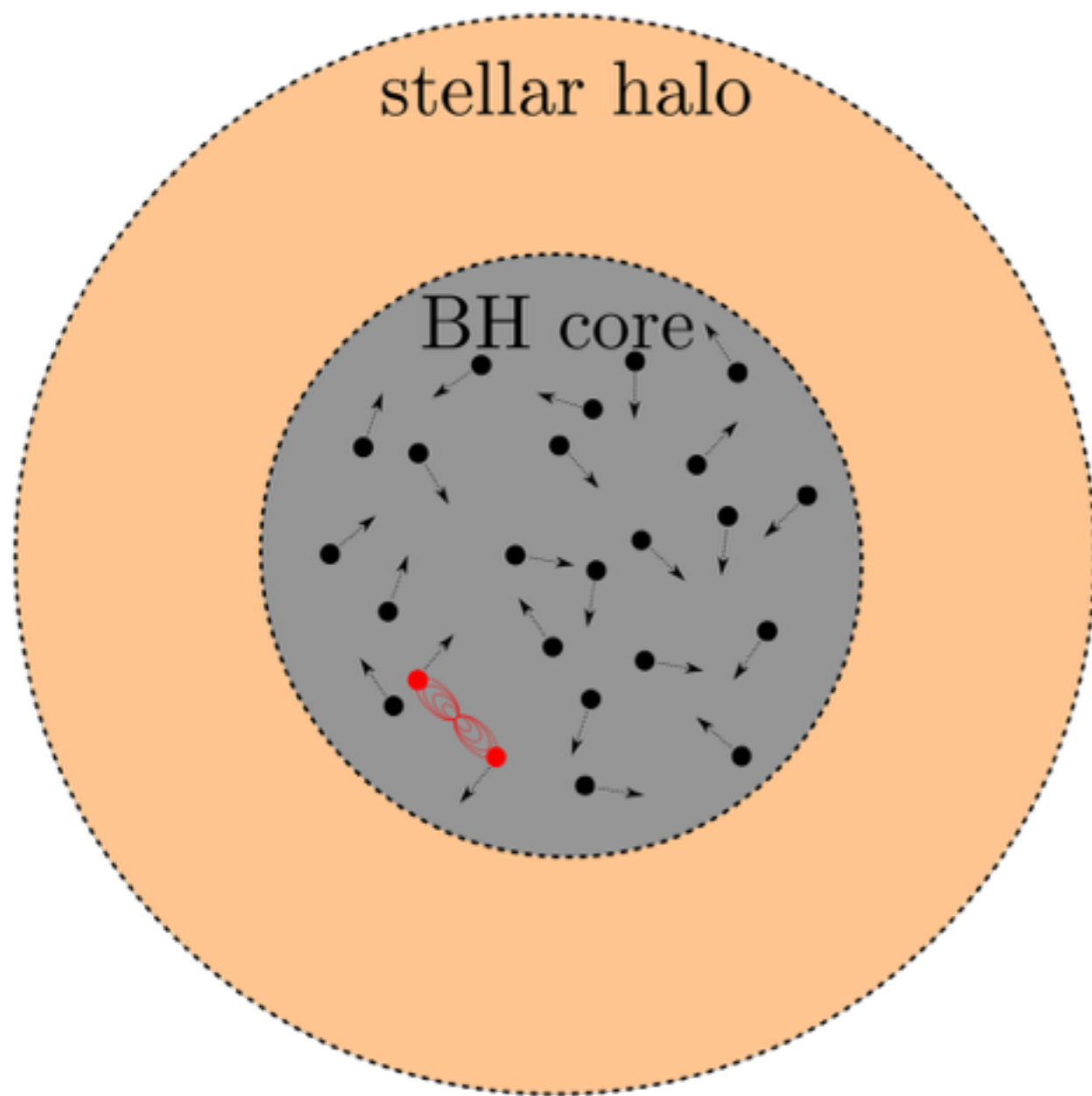
What is the peak freq. dist?
Correlate this with e!

MODEL:

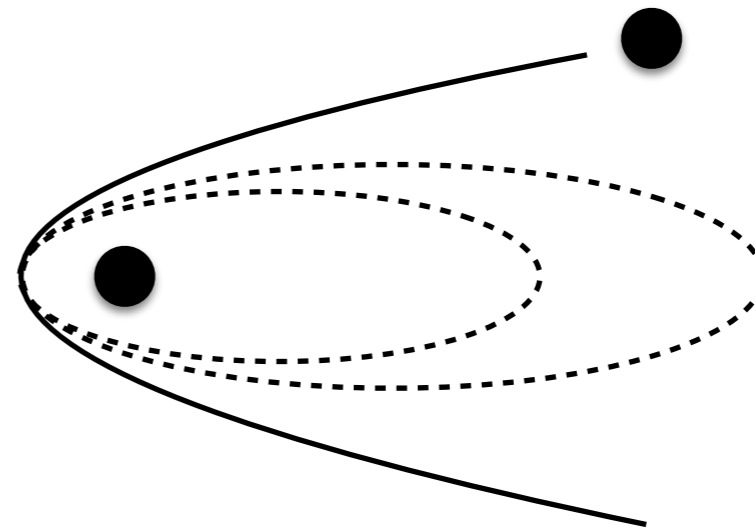


$$f \approx \frac{1}{\pi} \sqrt{\frac{2Gm}{r_f^3}}$$

Merger Type: Single-Single



Capture:

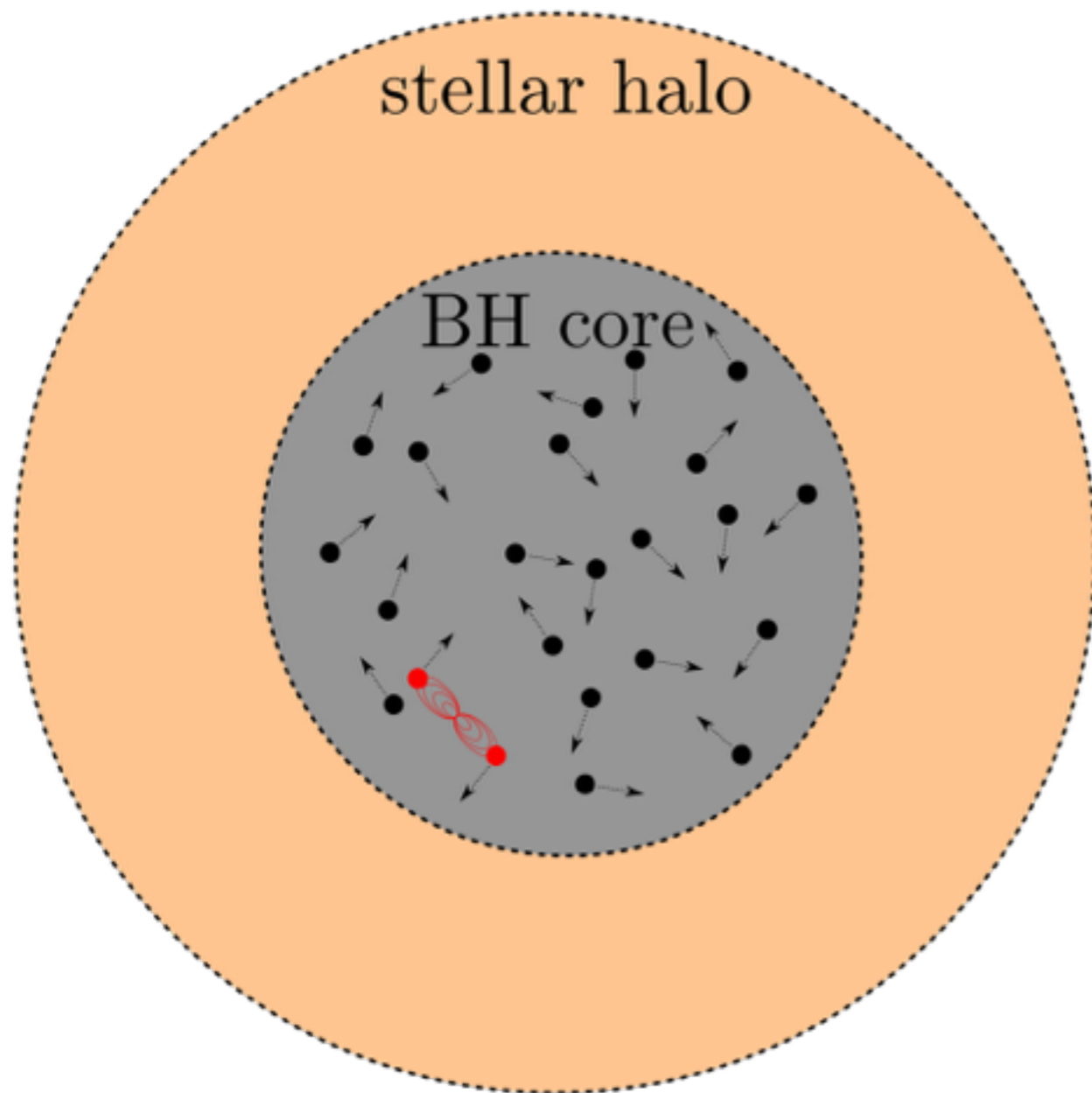


$$\Delta E_p \approx (85\pi/12)G^{7/2}c^{-5}m^{9/2}r_p^{-7/2}$$

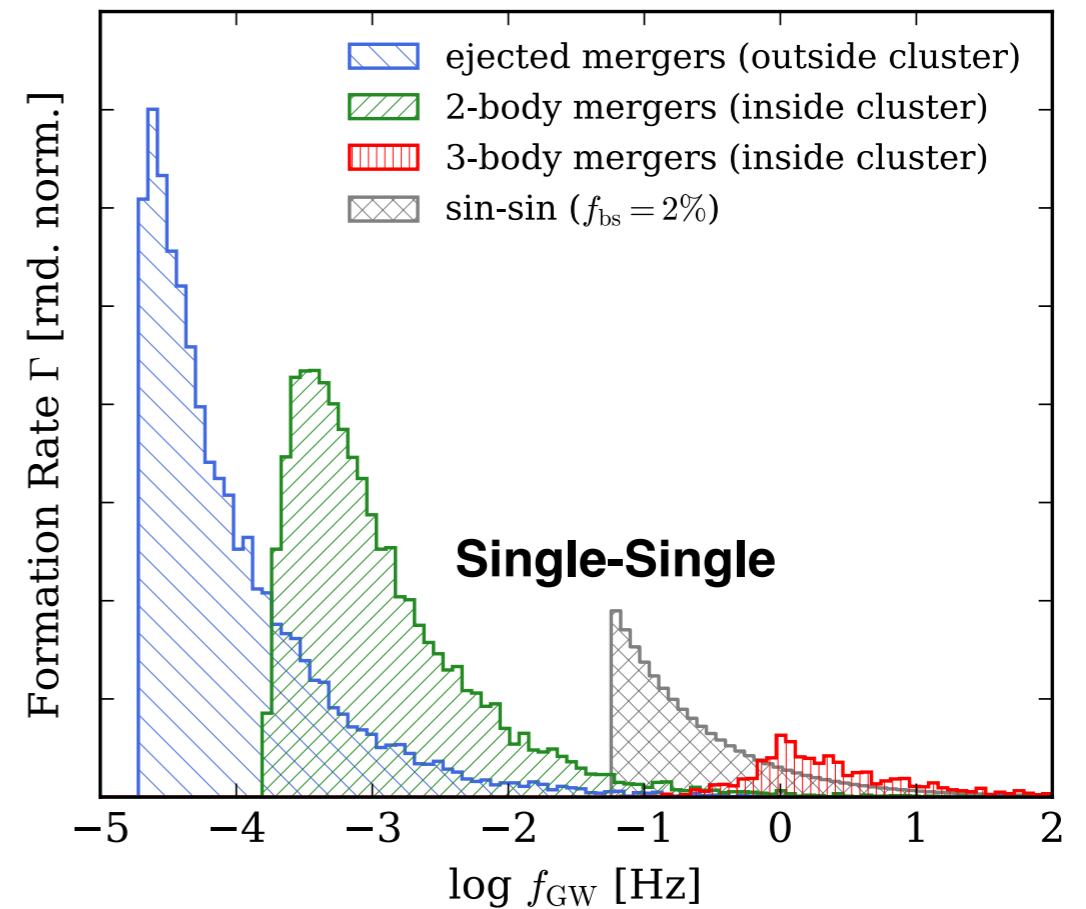
$$E_{ss} \approx \mu v^2/2$$

$$\mathcal{R}_{ss} = \left(\frac{85\pi}{24\sqrt{2}}\right)^{2/7} \times \mathcal{R}_m \left(\frac{c^2}{v^2}\right)^{2/7}$$

Merger Type: Single-Single



S-S captures do not only operate in Galactic Nuclei!



$$\mathcal{R}_{ss} = \left(\frac{85\pi}{24\sqrt{2}} \right)^{2/7} \times \mathcal{R}_m \left(\frac{c^2}{v^2} \right)^{2/7} \quad f \approx \frac{1}{\pi} \sqrt{\frac{2Gm}{r_f^3}}$$

Distribution depends on:

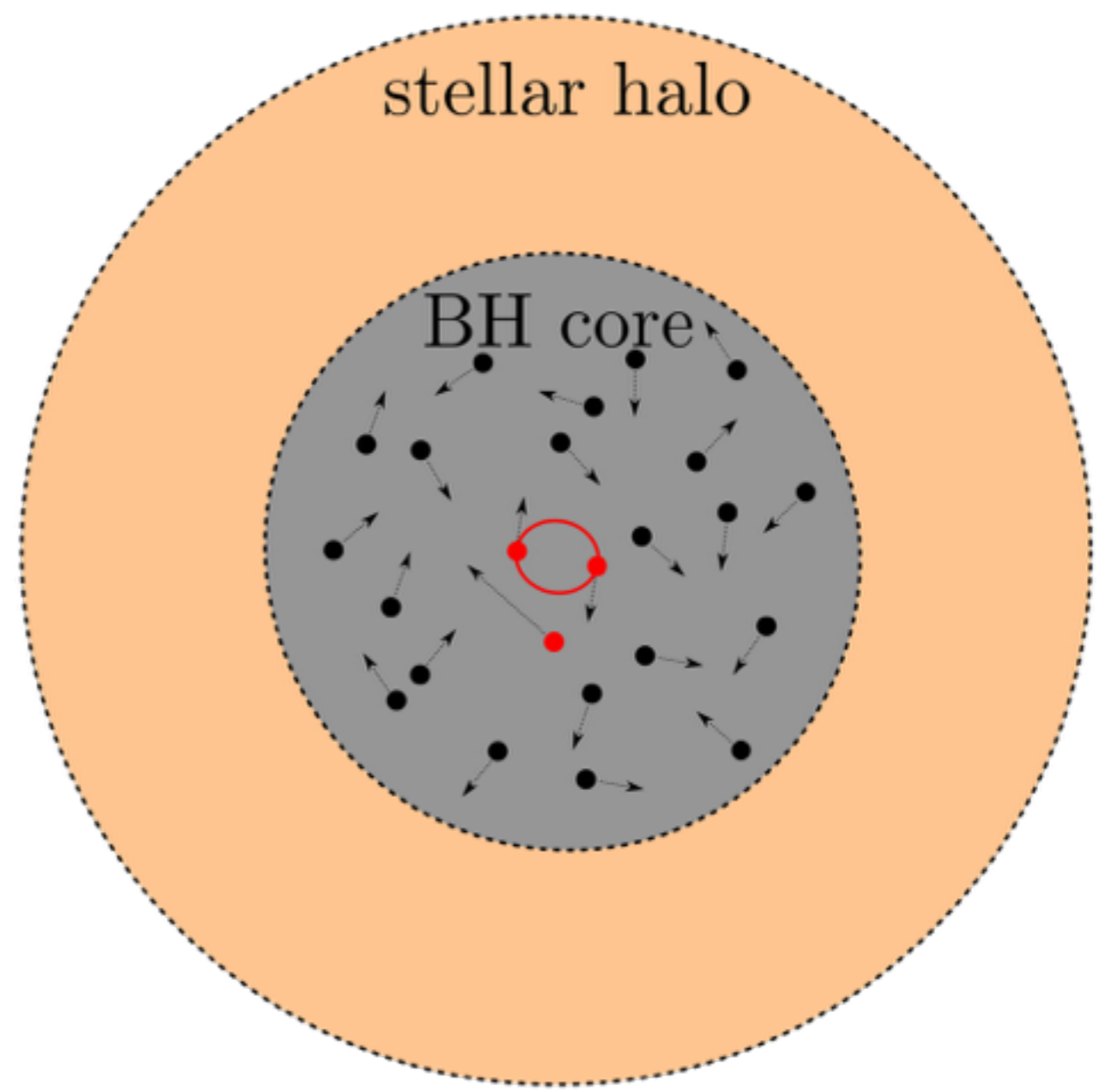
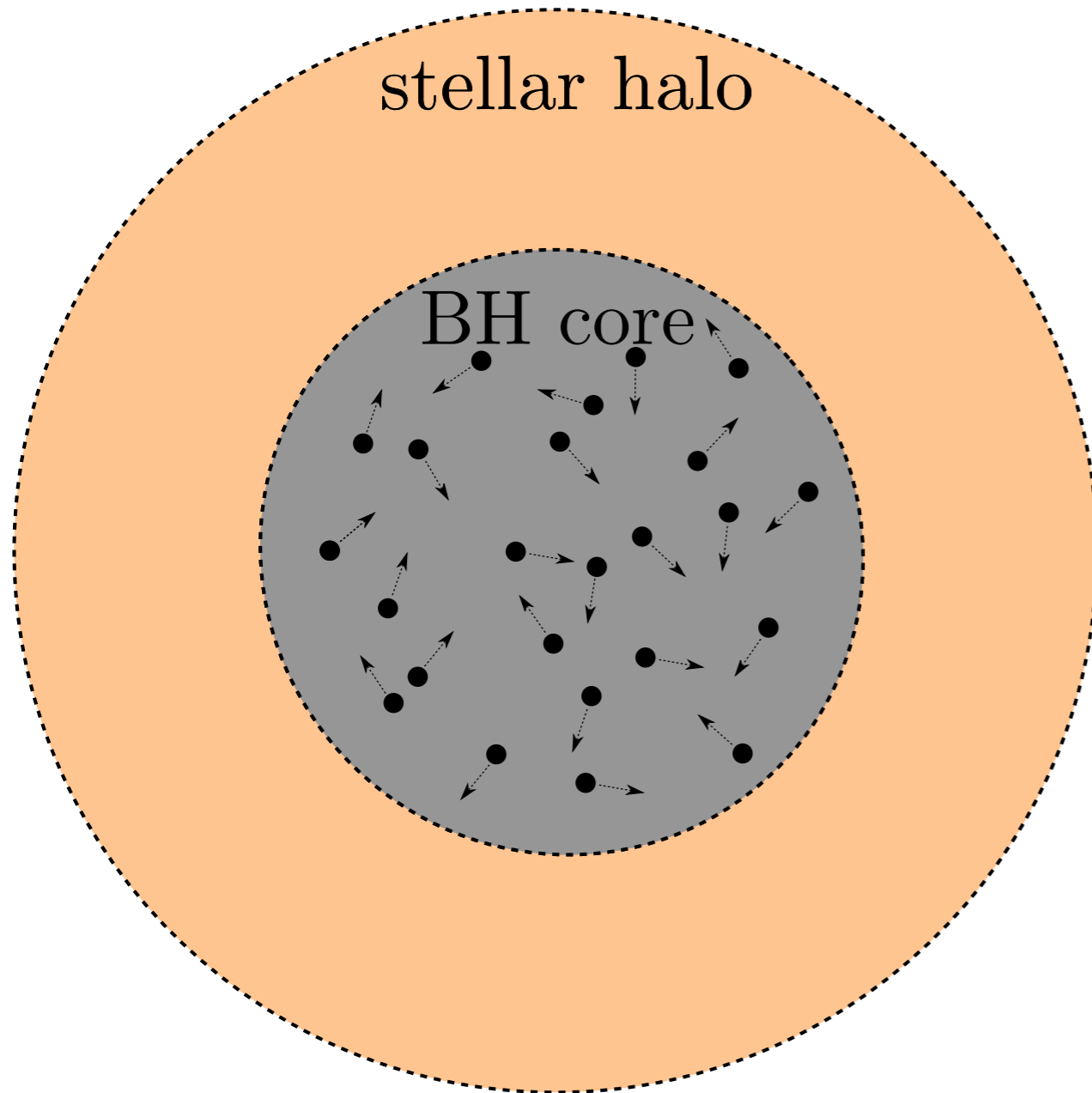
BH distribution (Plummer/Uniform..)

Binary Fraction

see our new paper:

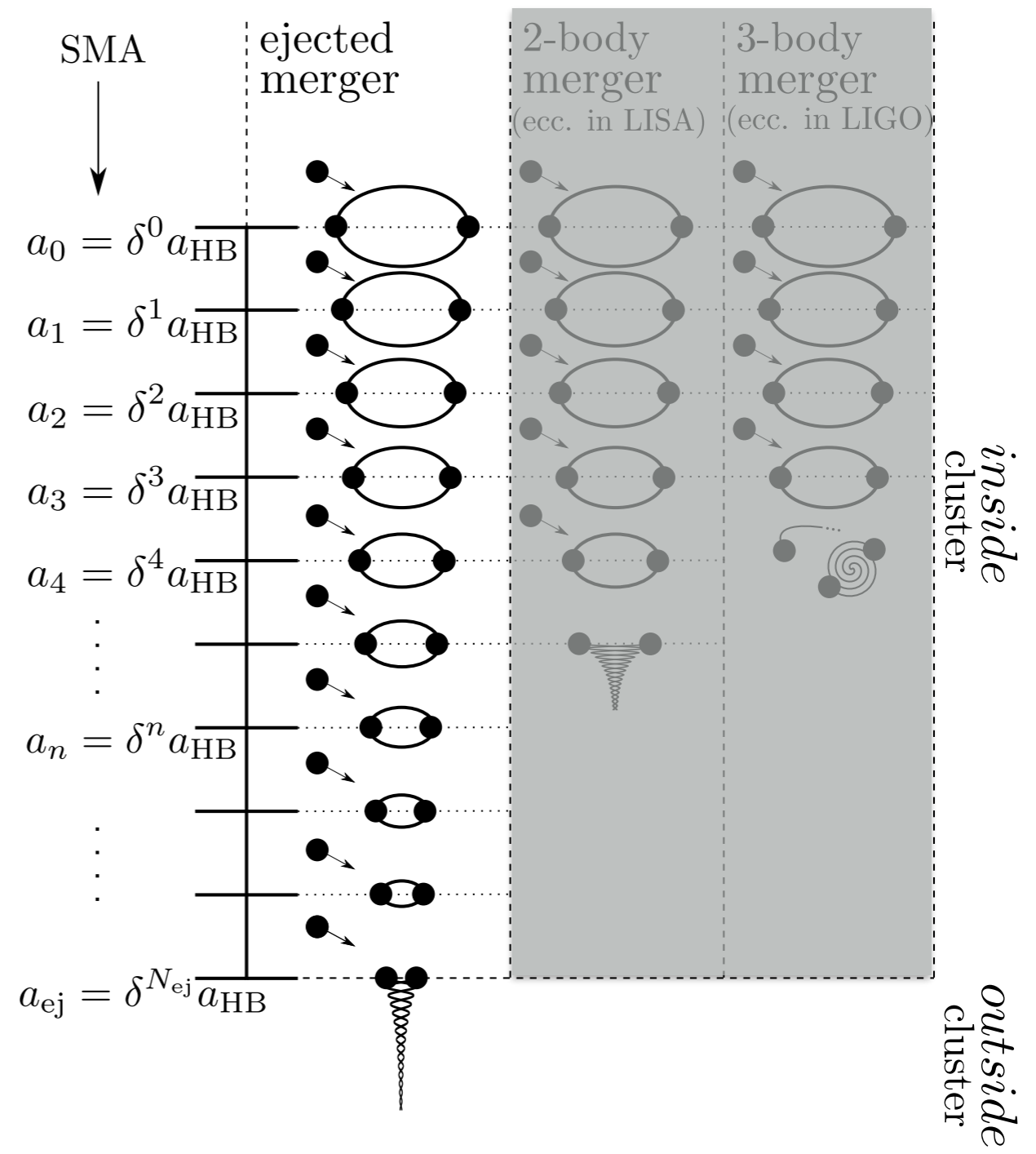
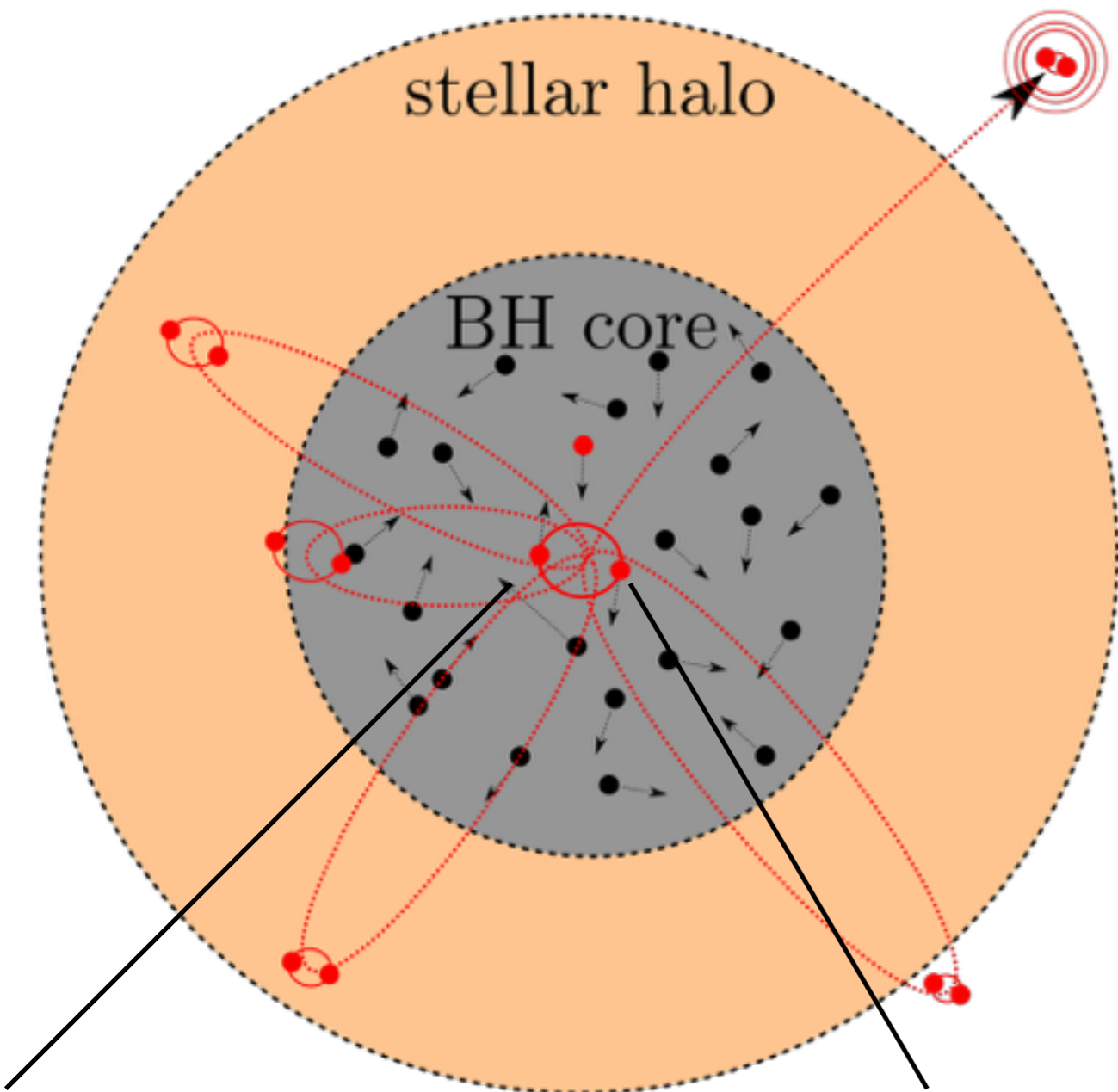
'GW captures of single BHs in GCs'
(Samsing, et al. 2019)

Few-body BBH mergers: Formation of a BBH

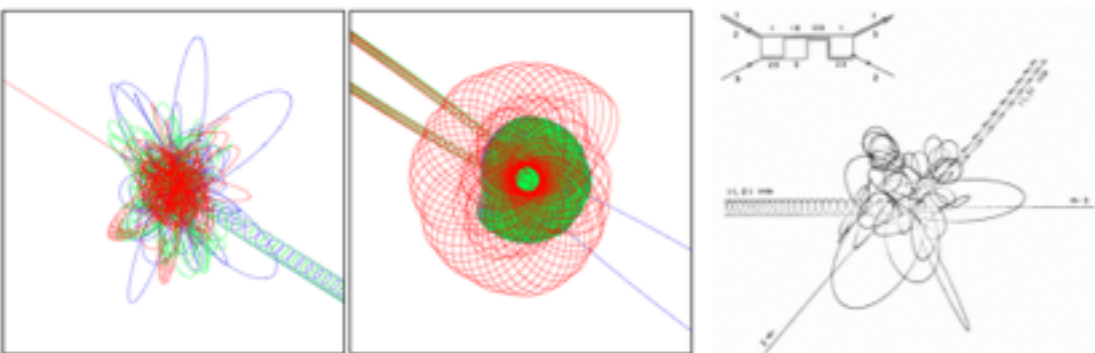


$$a_{\text{HB}} = \frac{3}{2} \frac{Gm_{\text{BH}}}{v_{\text{dis}}^2}$$

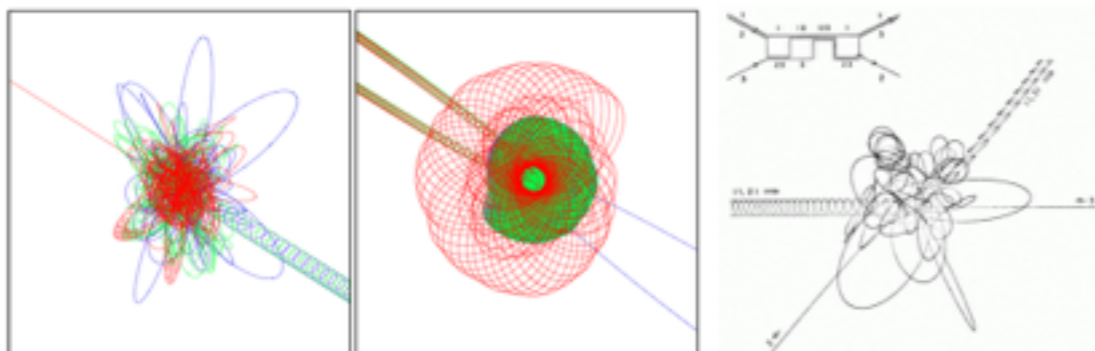
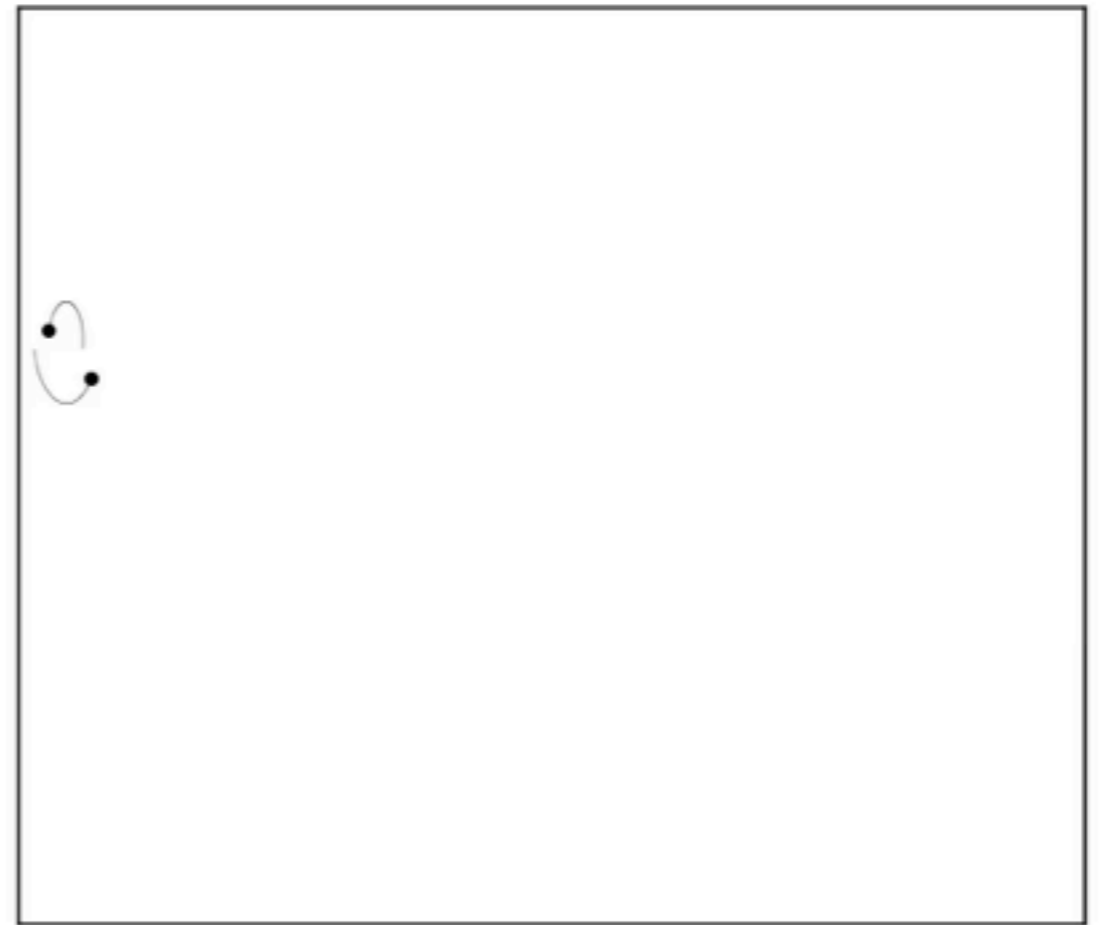
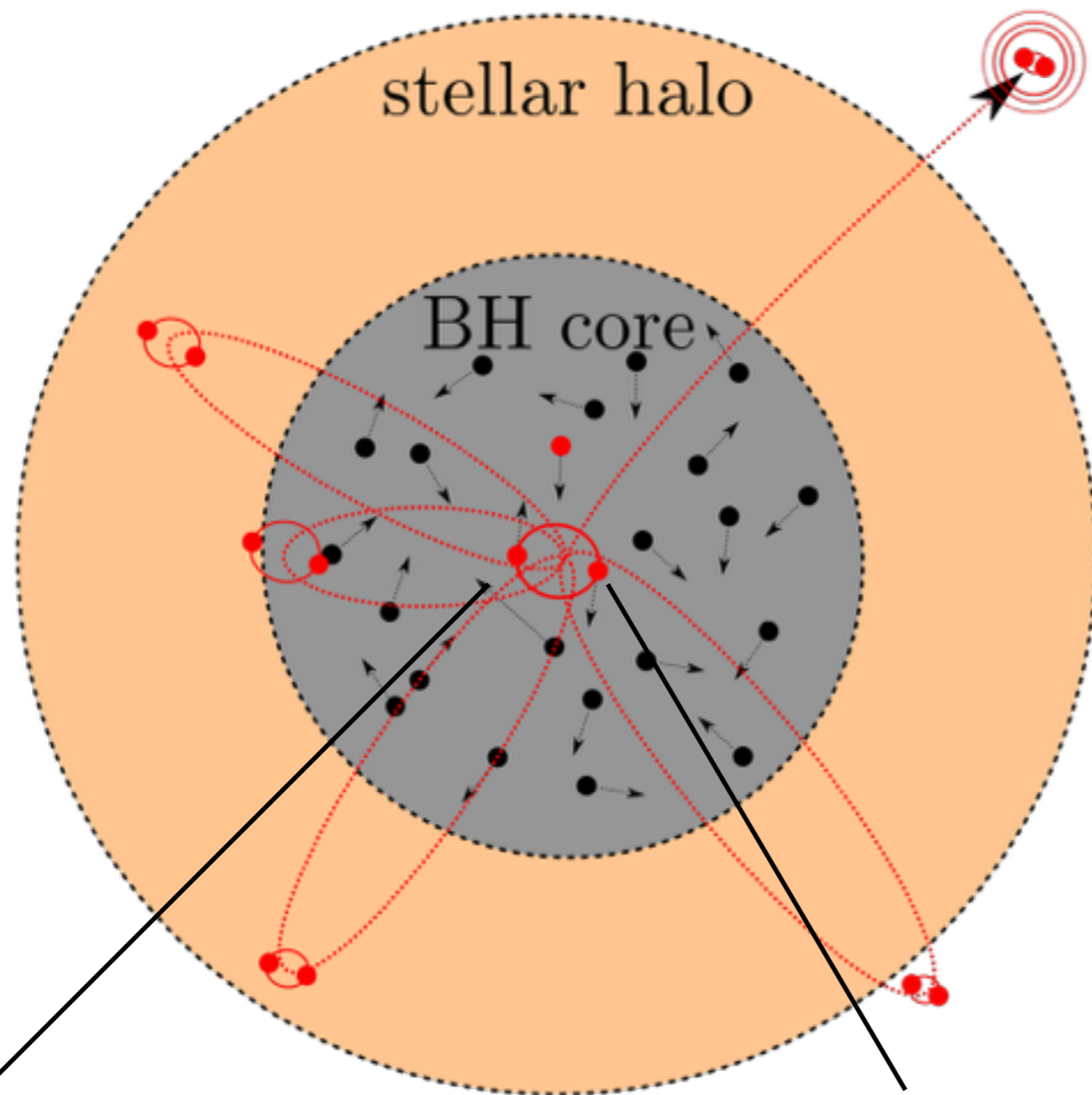
Merger Type: Ejected Merger



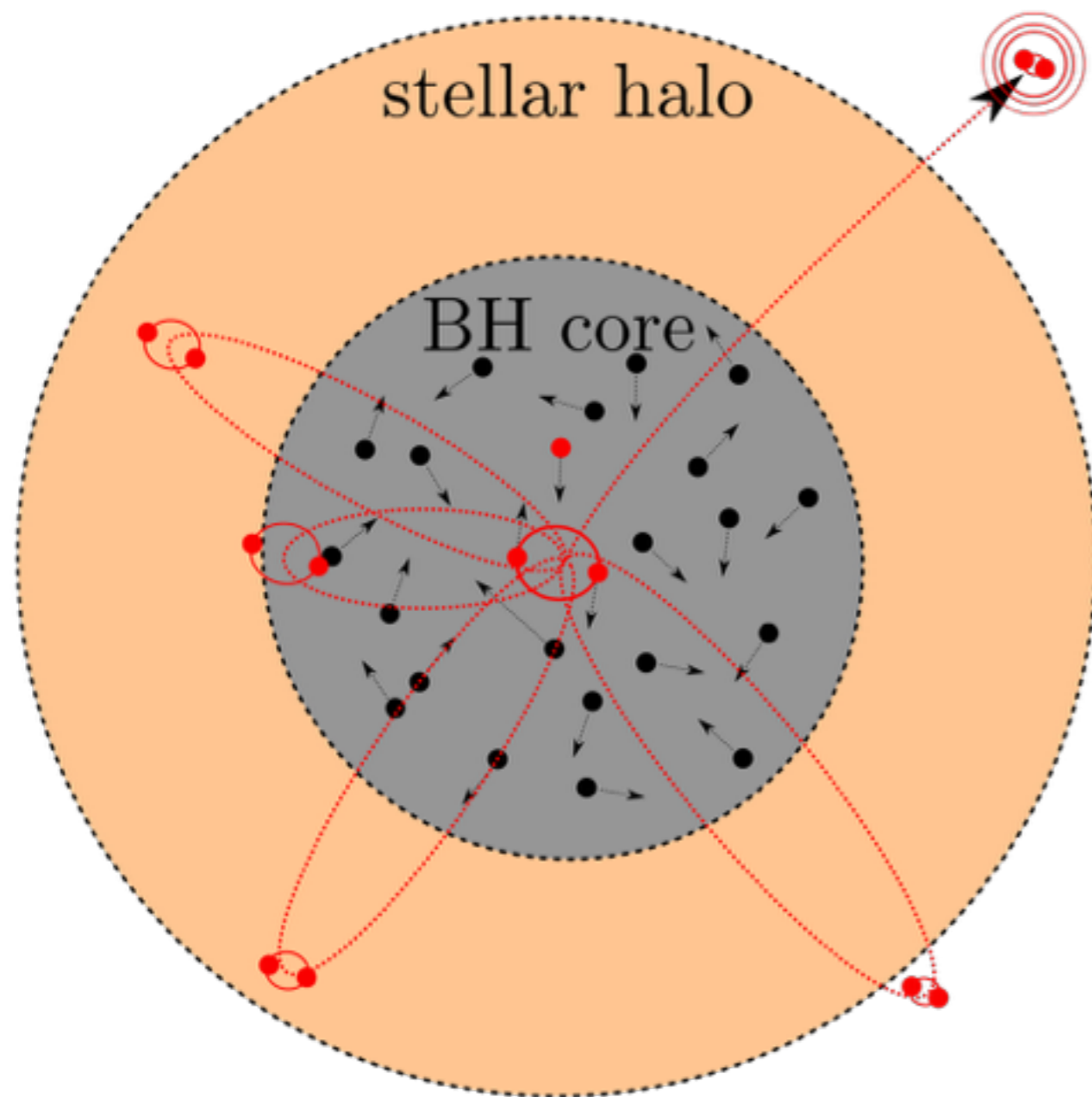
Samsing, D'Orazio. 18.



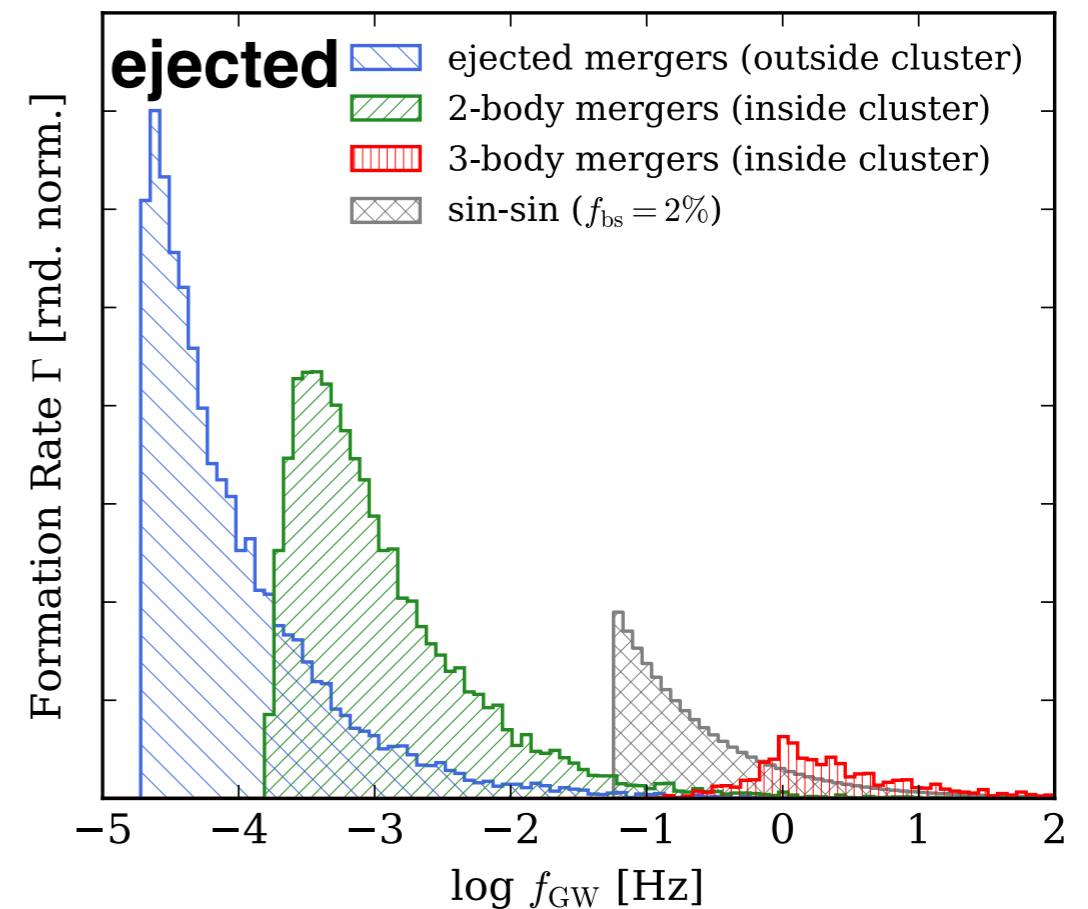
Merger Type: Ejected Merger



Merger Type: Ejected Merger



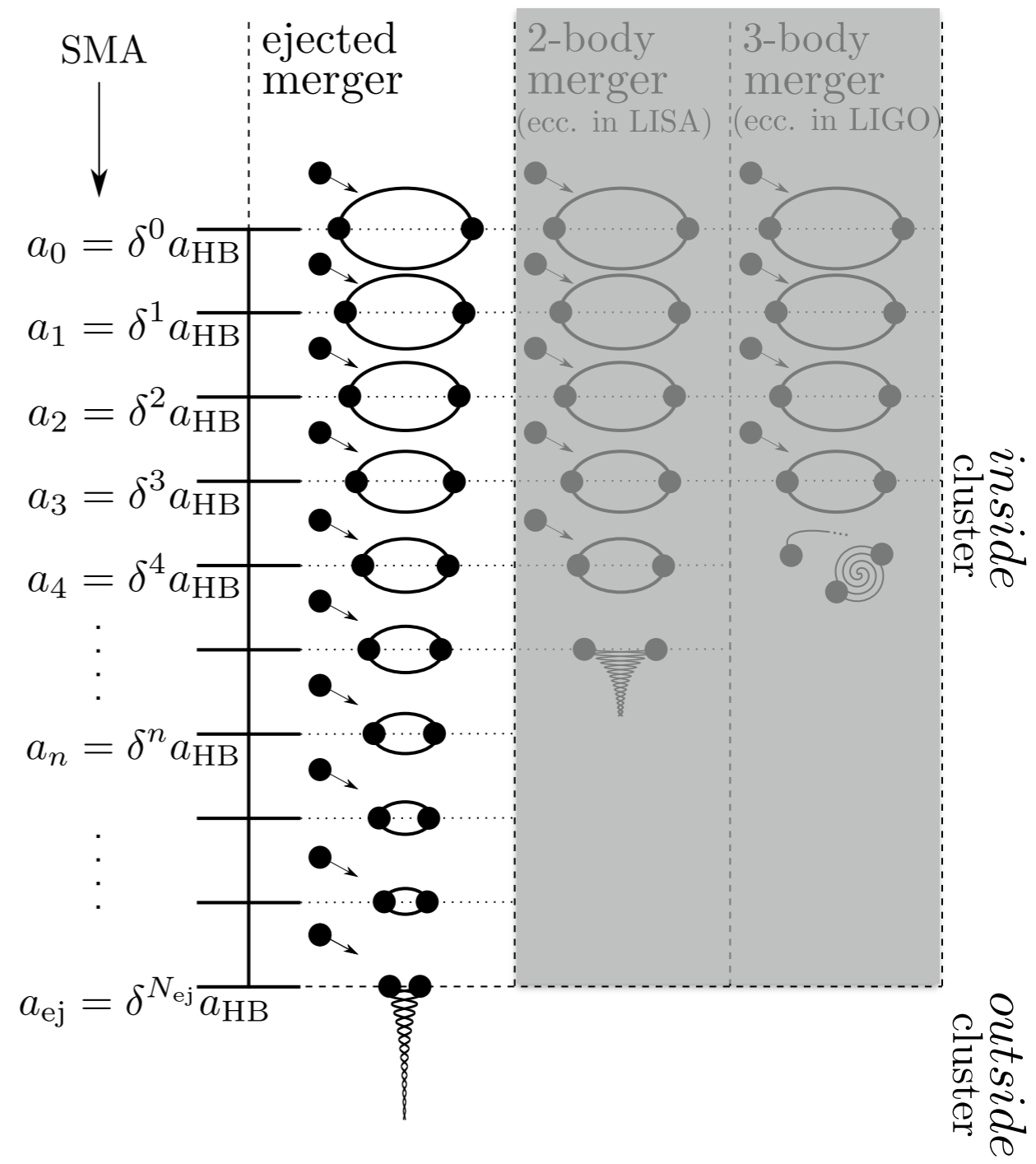
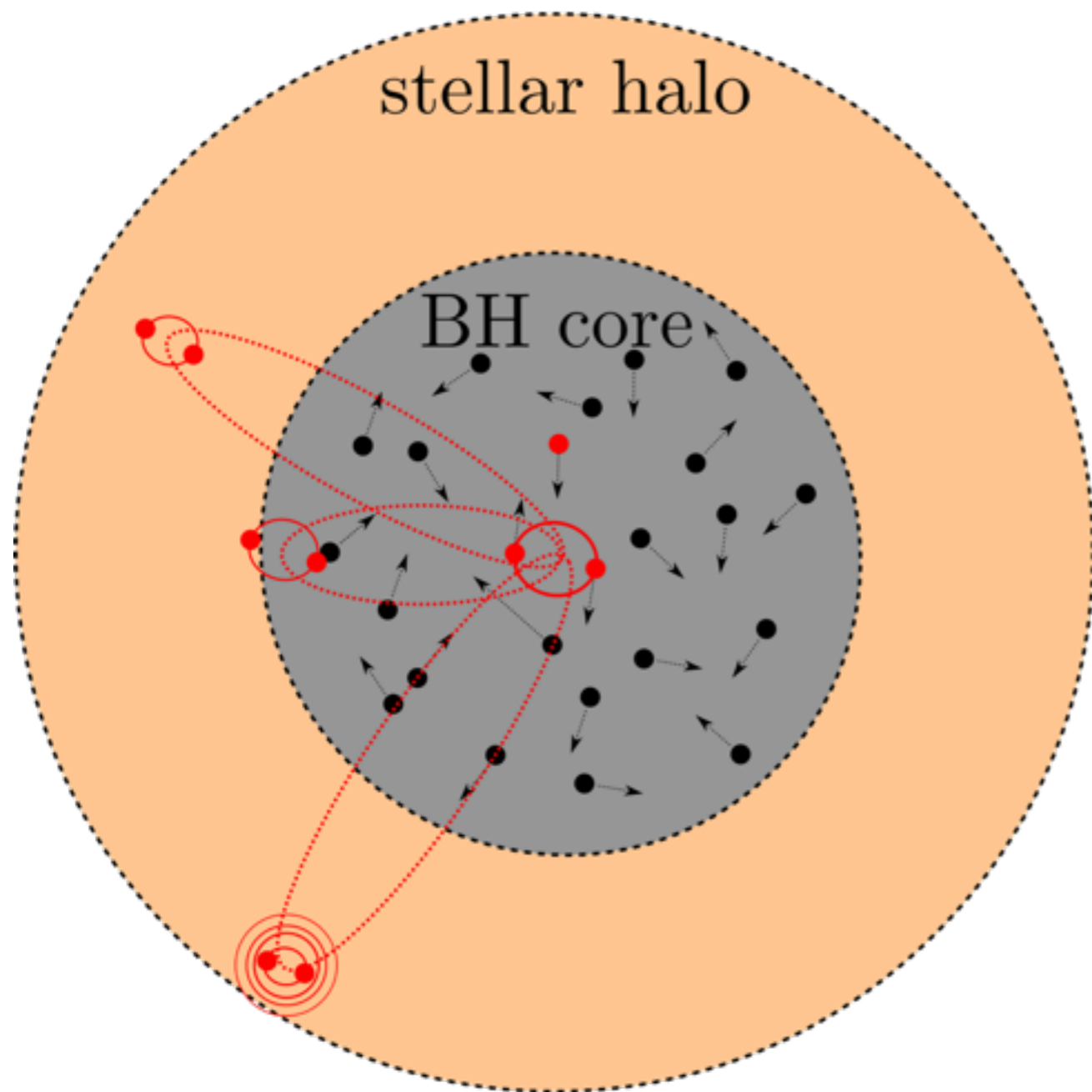
Circular GW sources



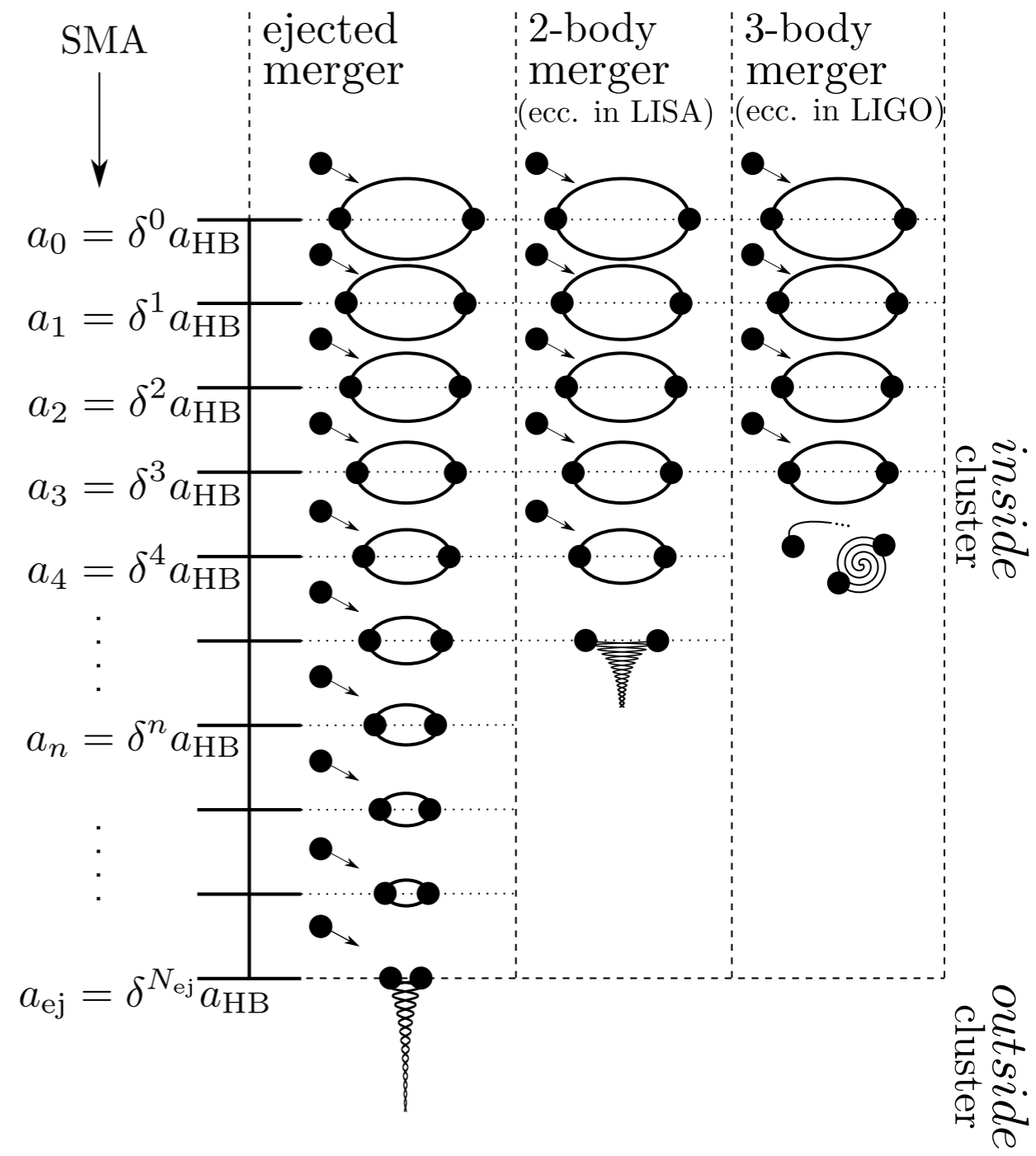
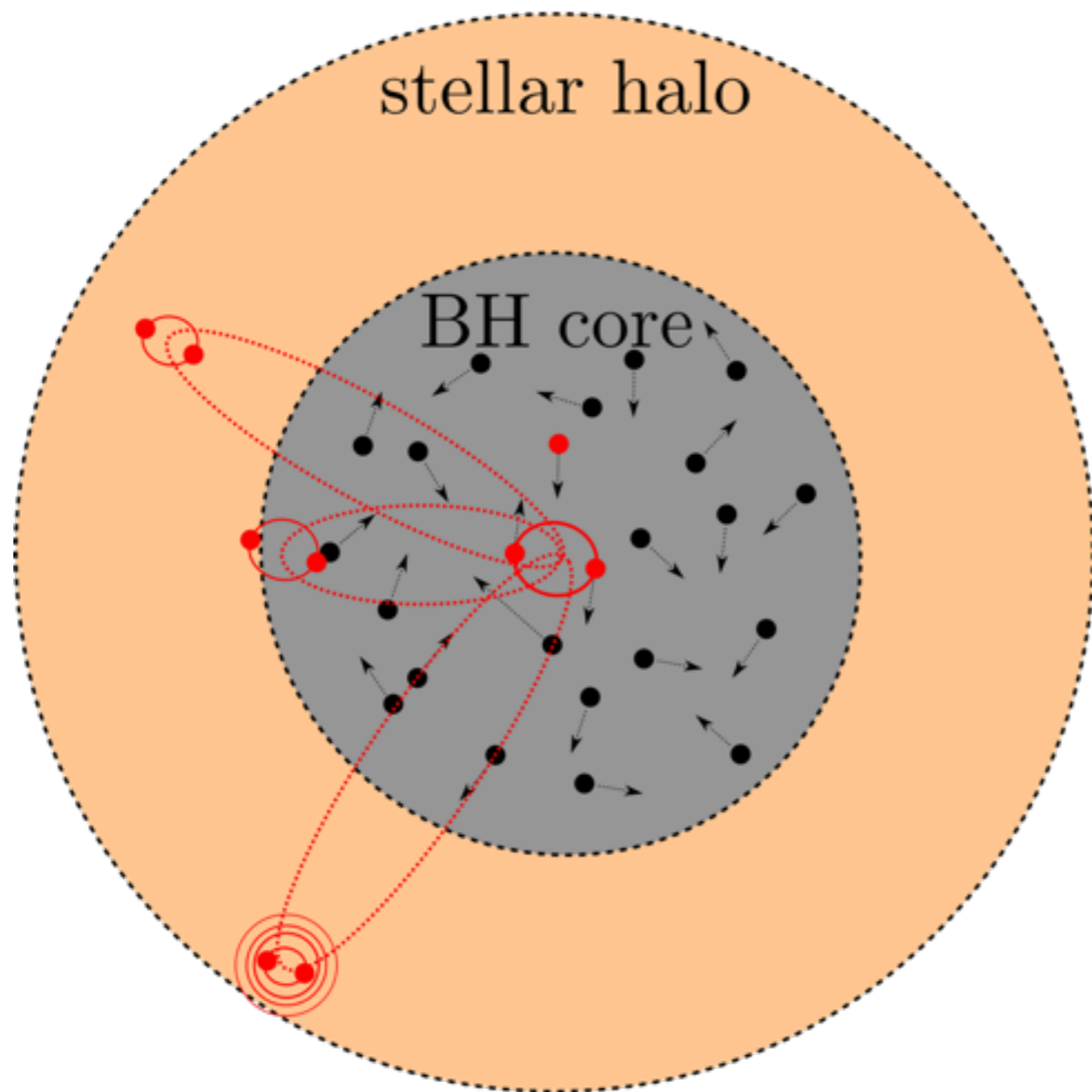
Important point: It is all about characteristic time scales!

$$f_{r,0}^{\text{peak}}(\mathcal{T}) \approx 2 \cdot 10^{-5} \text{ Hz} \left(\frac{\mathcal{T}}{10^{10} \text{ yrs}} \right)^{-3/7} \left(\frac{a}{0.5 \text{ au}} \right)^{3/14} \left(\frac{m}{30 M_{\odot}} \right)^{-11/14}$$

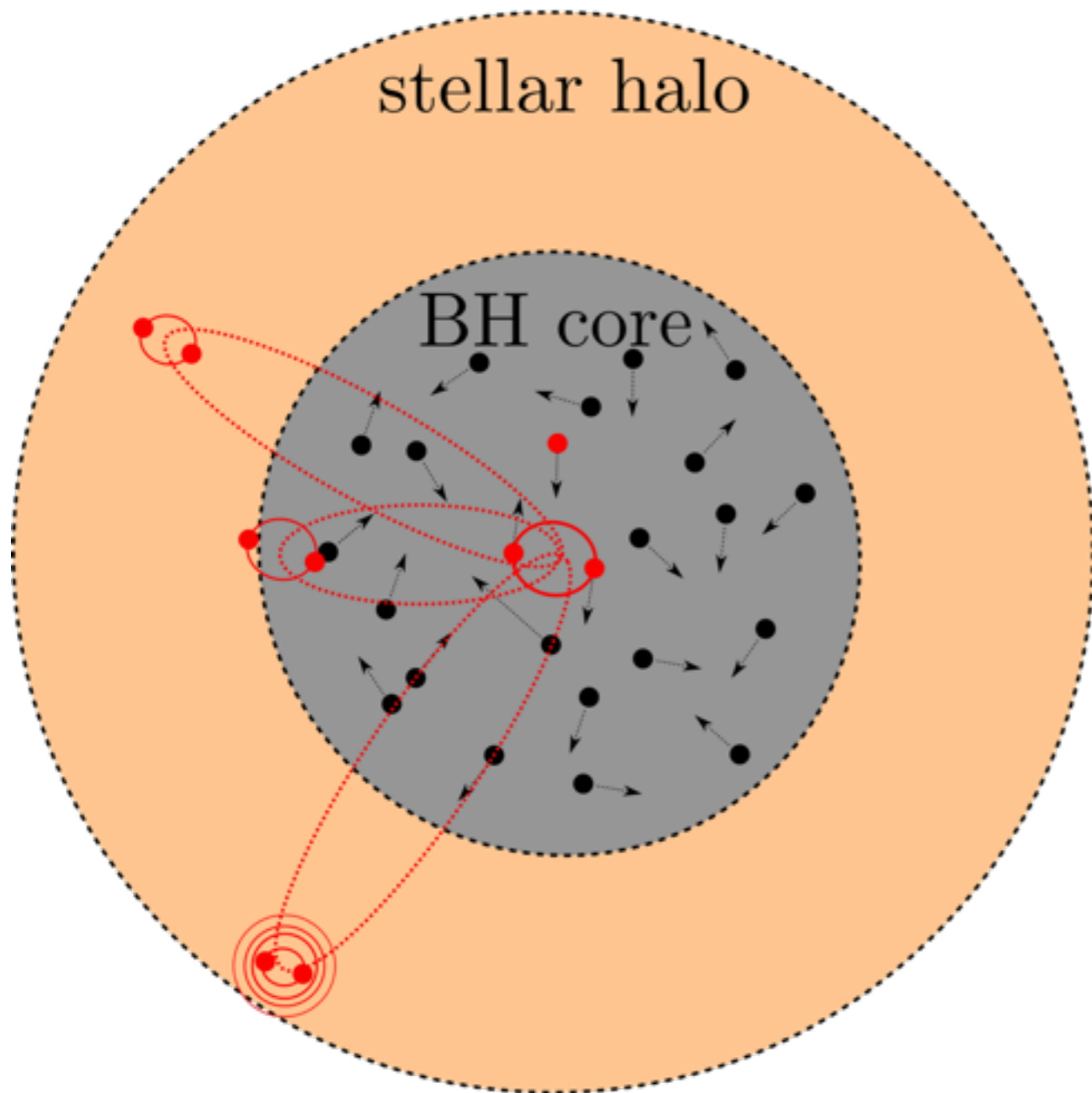
Merger Type: 2-body Merger



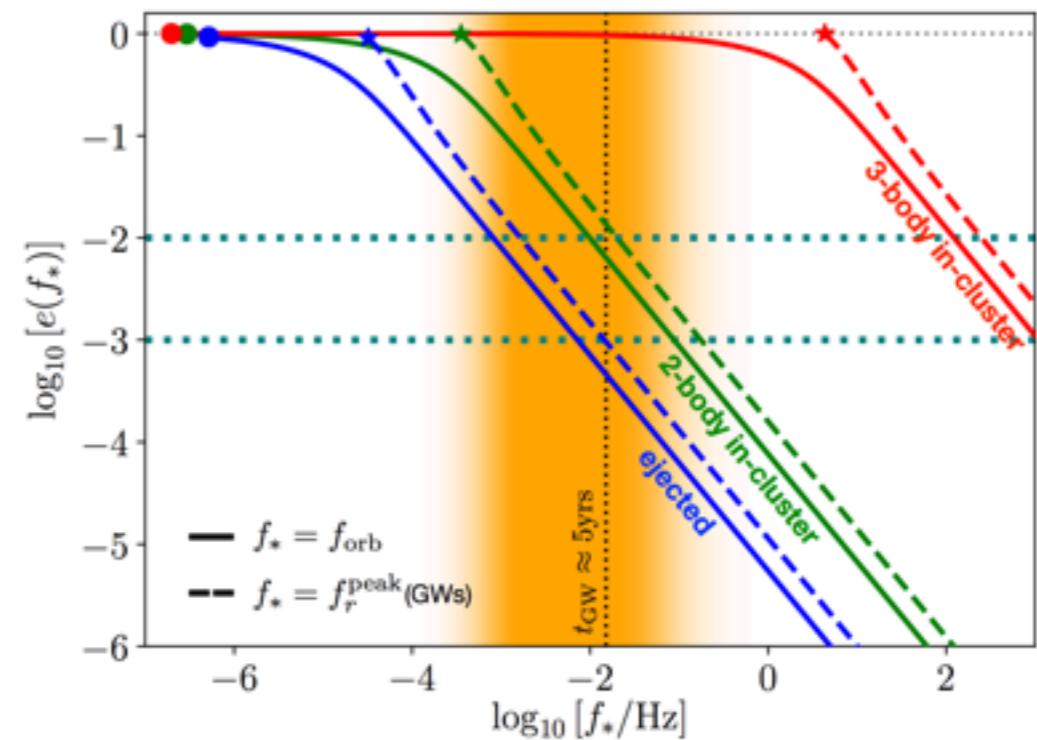
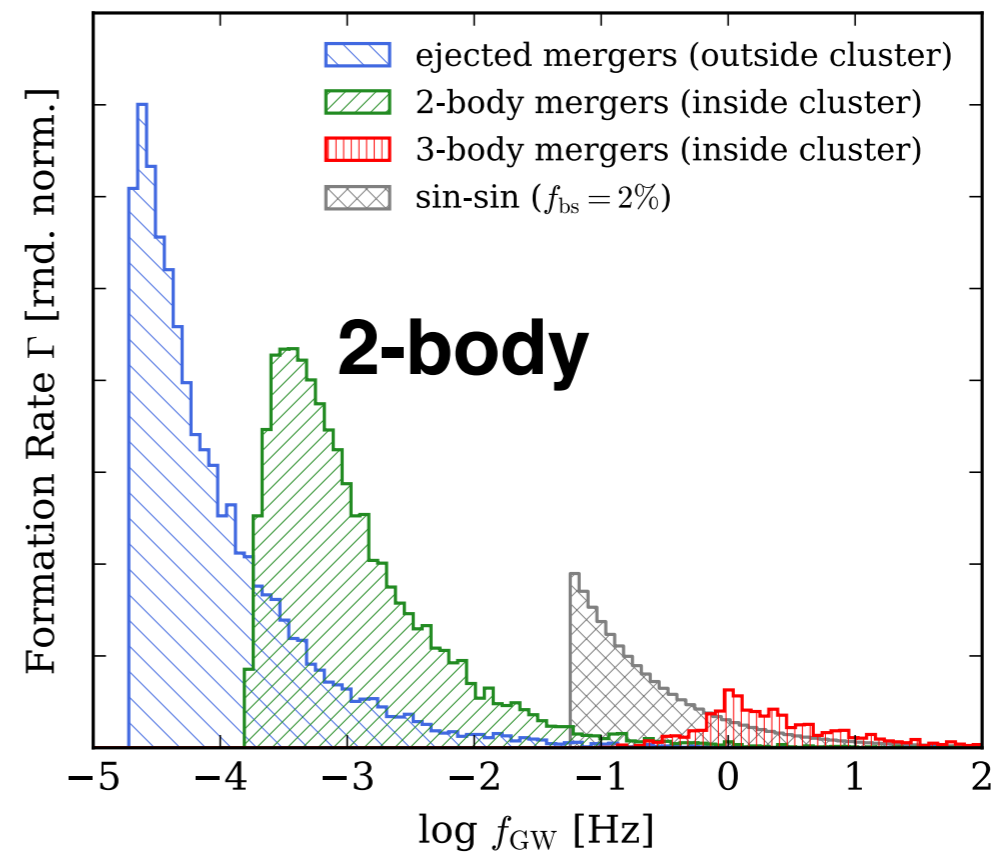
Merger Type: 2-body Merger



Merger Type: 2-body Merger



Eccentric LISA sources

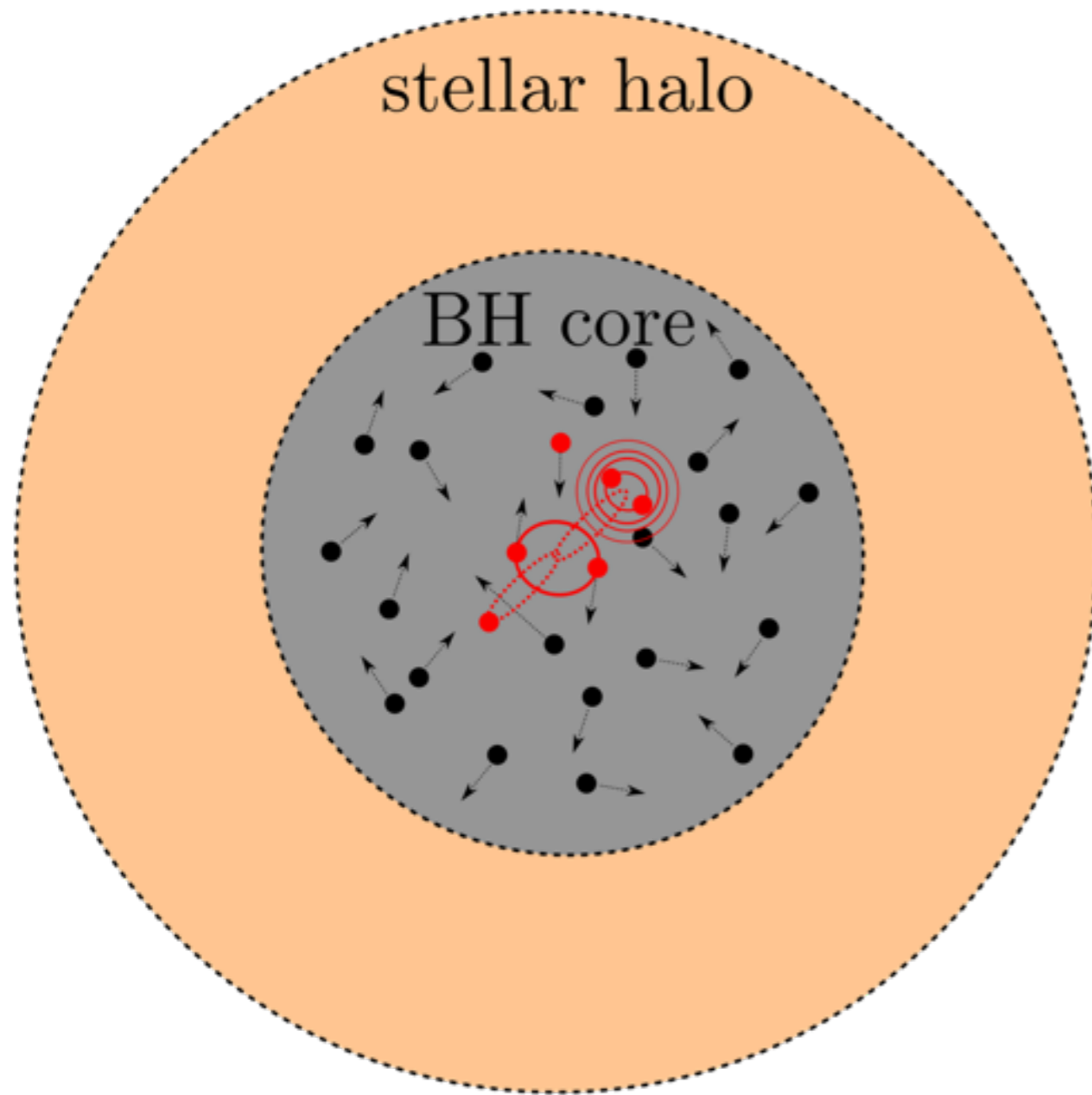


Consider again the characteristic time-scale argument:

$$f_{r,0}^{\text{peak}}(\mathcal{T}) \approx 2 \cdot 10^{-5} \text{ Hz} \left(\frac{\mathcal{T}}{10^{10} \text{ yrs}} \right)^{-3/7} \left(\frac{a}{0.5 \text{ au}} \right)^{3/14} \left(\frac{m}{30 M_{\odot}} \right)^{-11/14}$$

series of papers: Samsing, D'Orazio

Merger Type: 3-body Merger

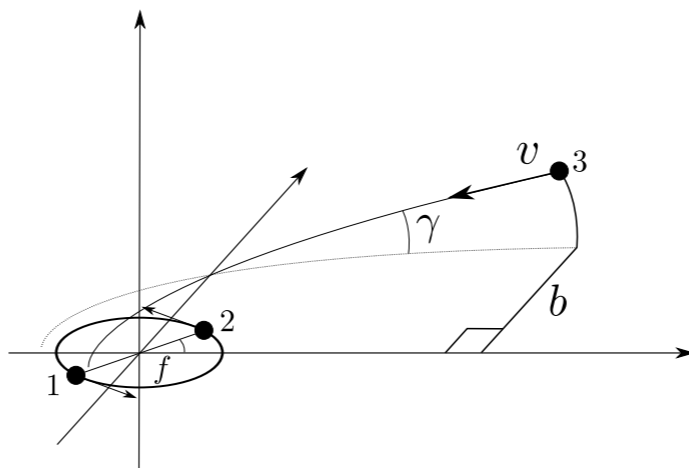
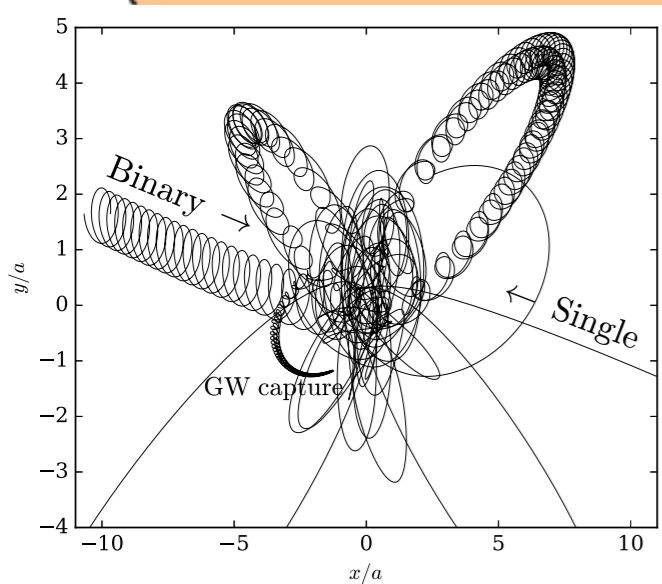
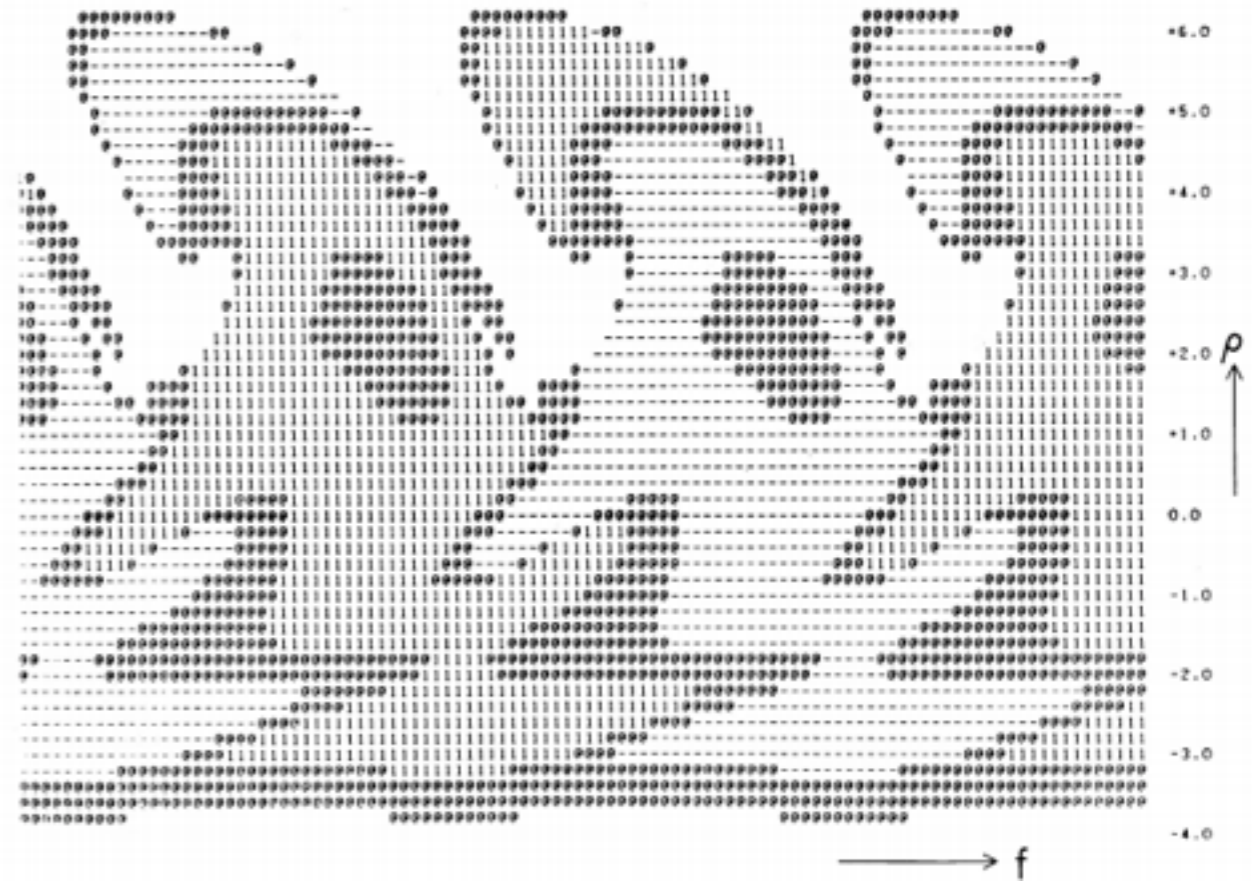
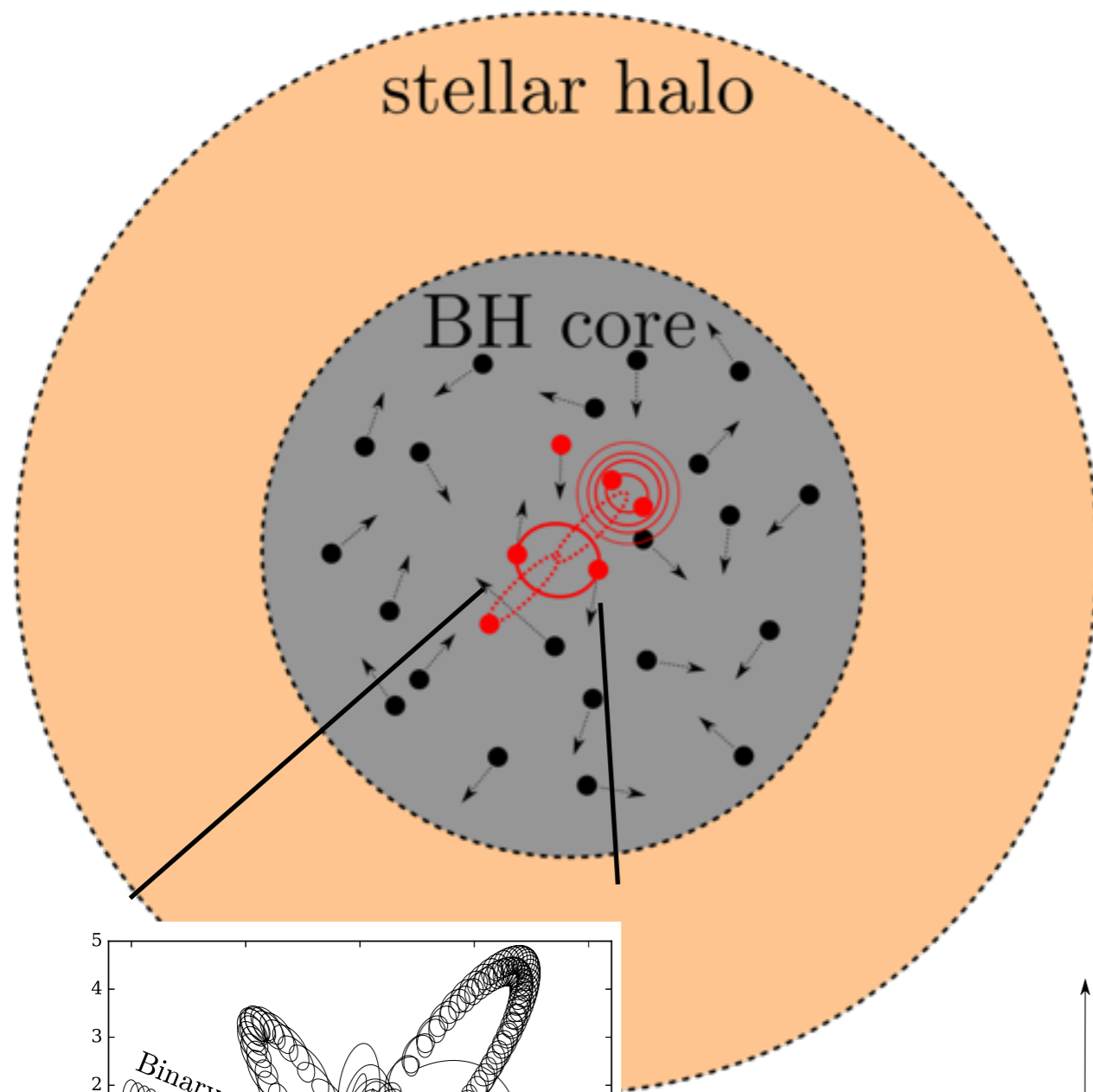


Post-Newtonian



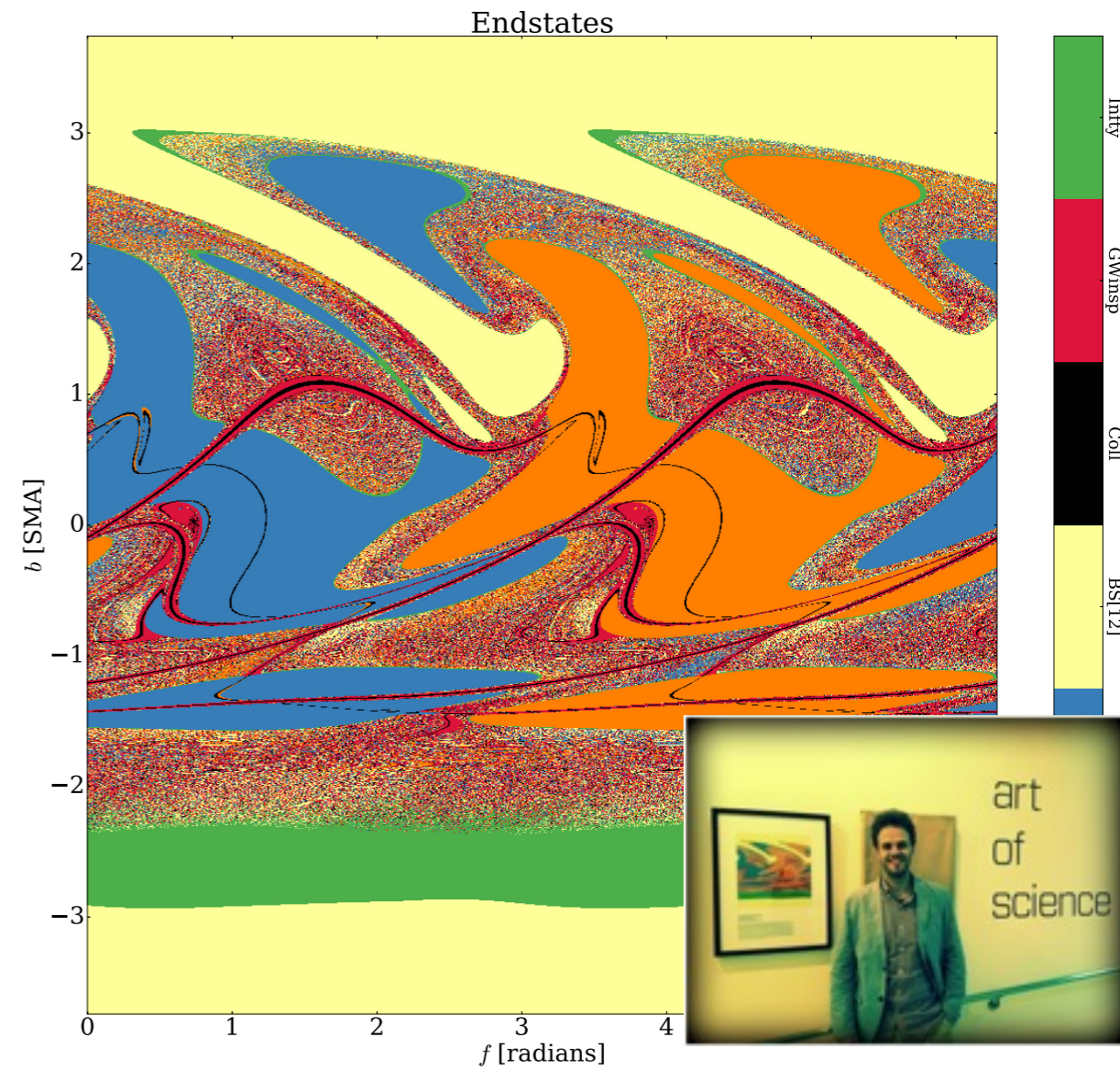
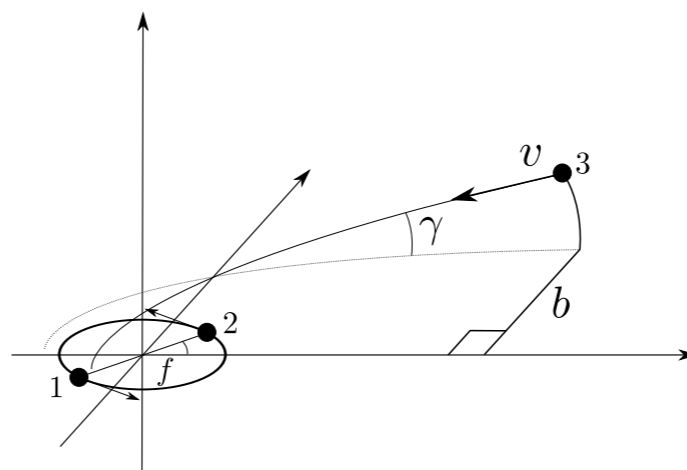
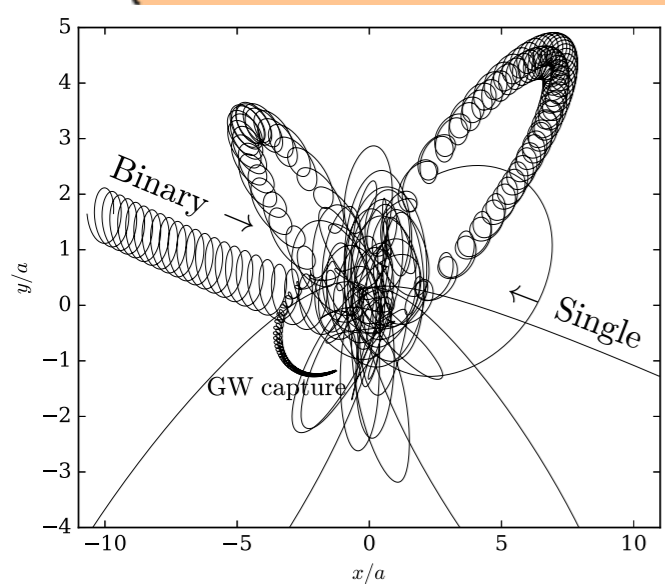
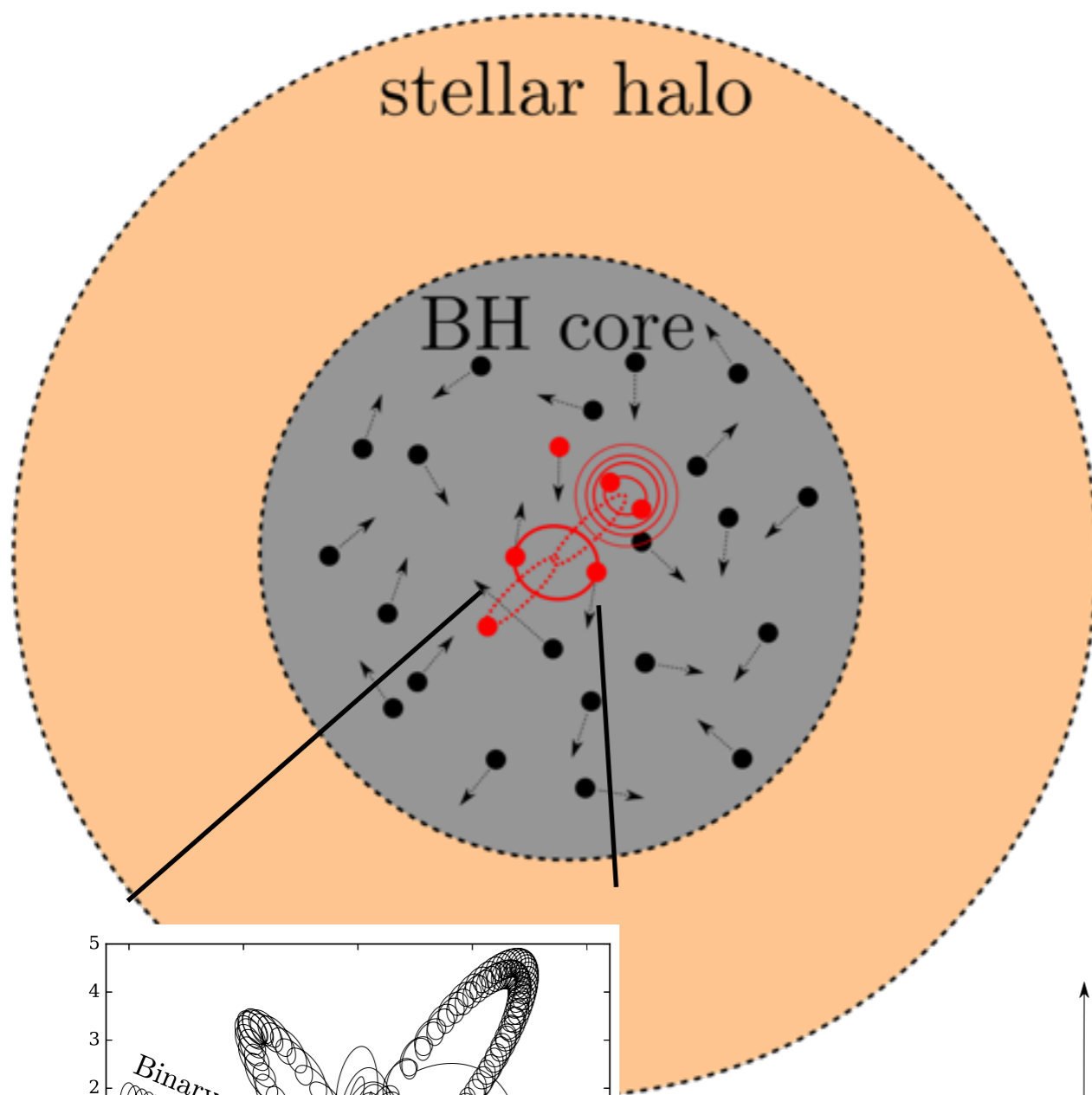
Merger Type: 3-body Merger

Topology

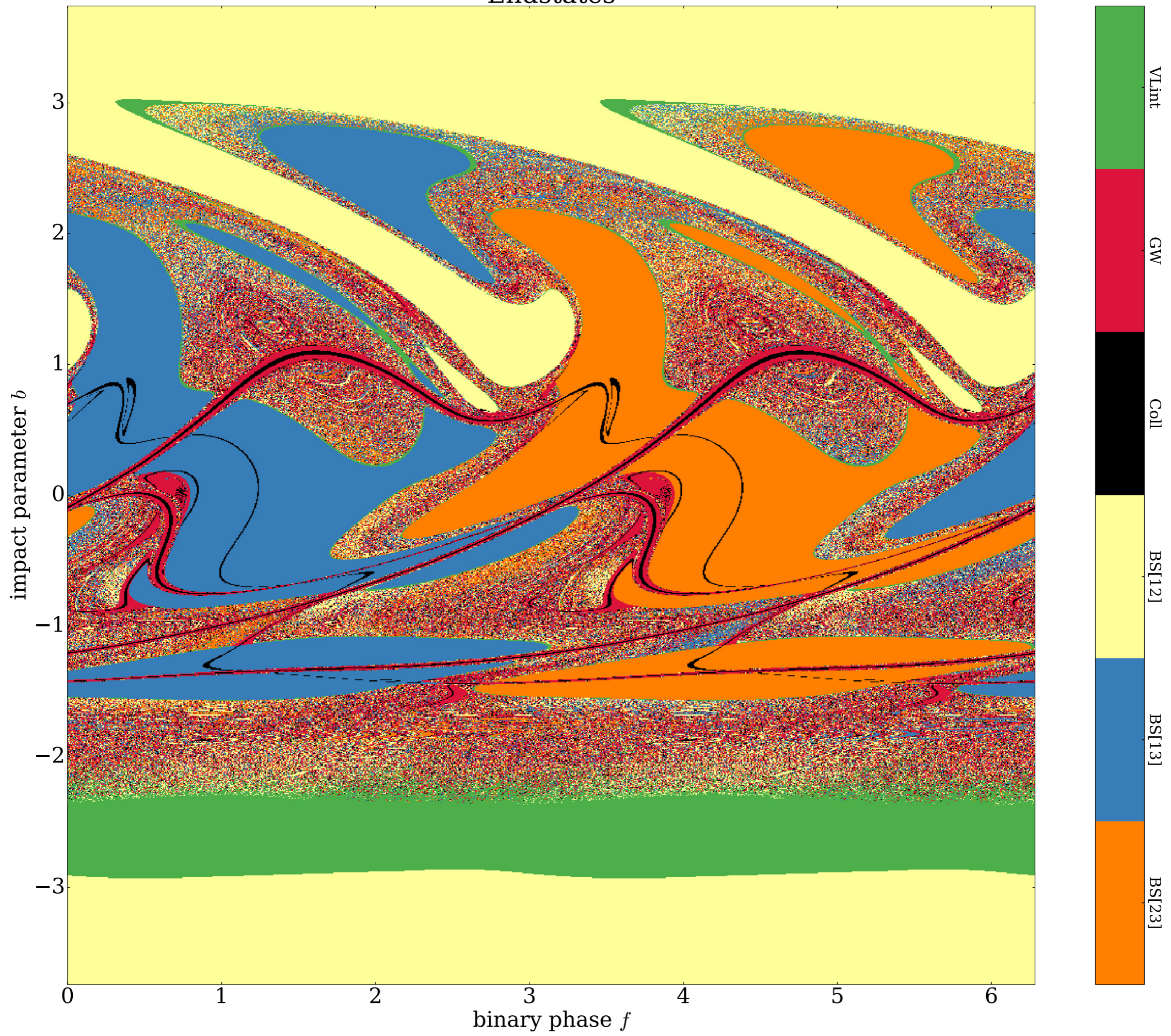


Merger Type: 3-body Merger

Topology

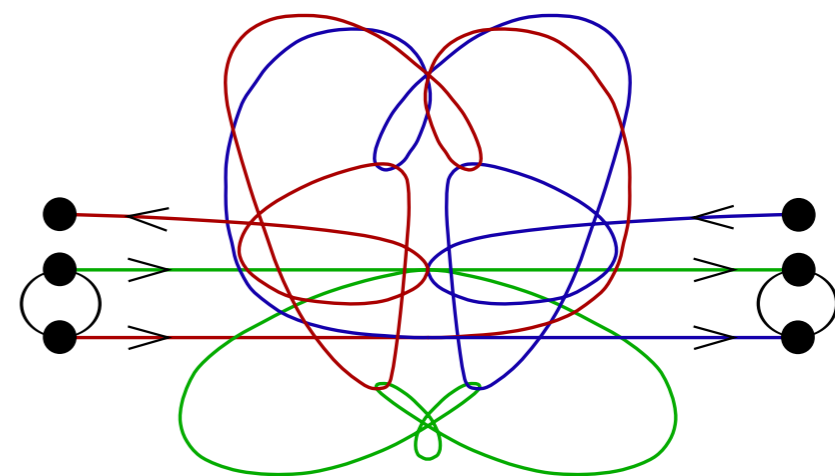
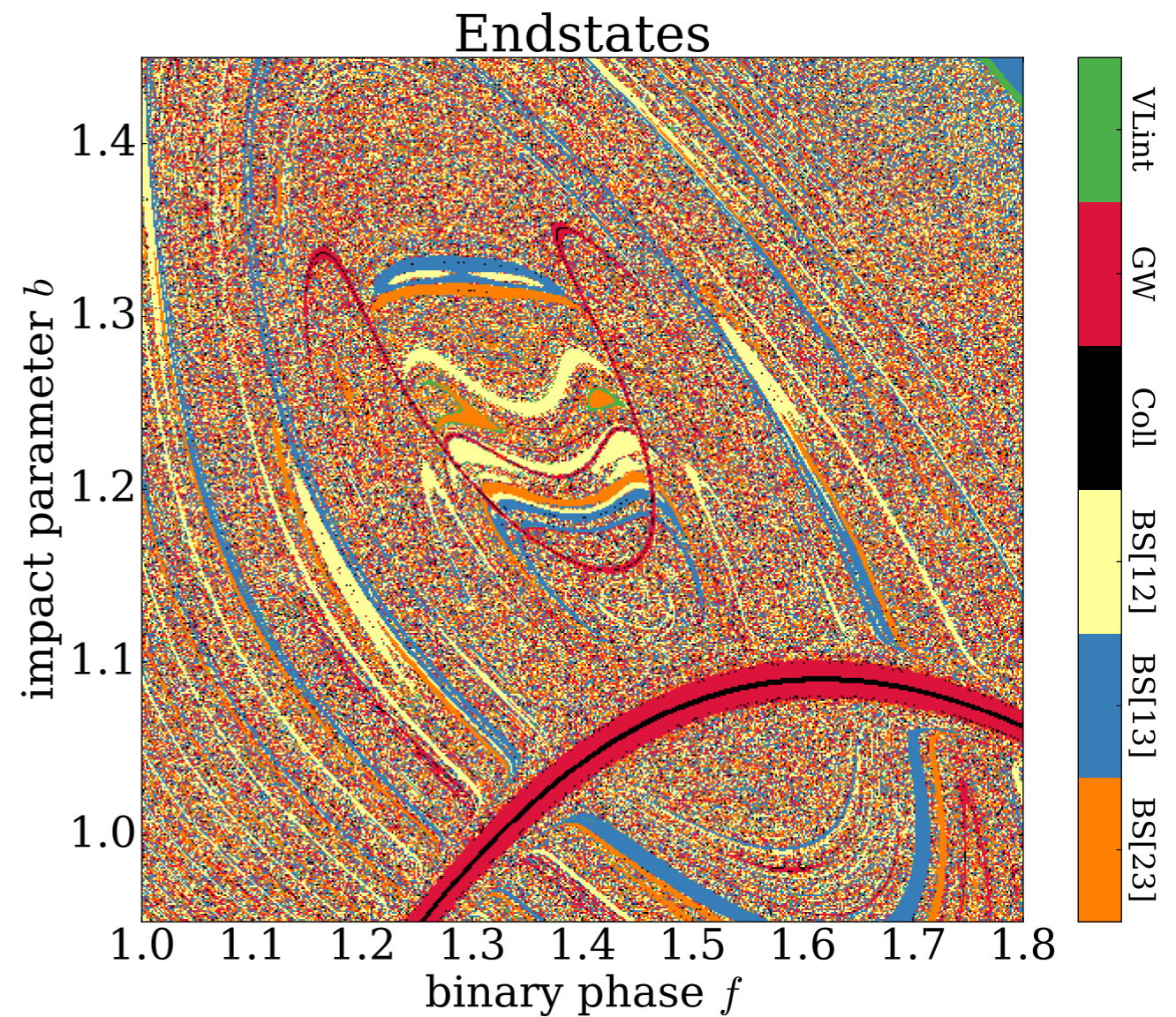
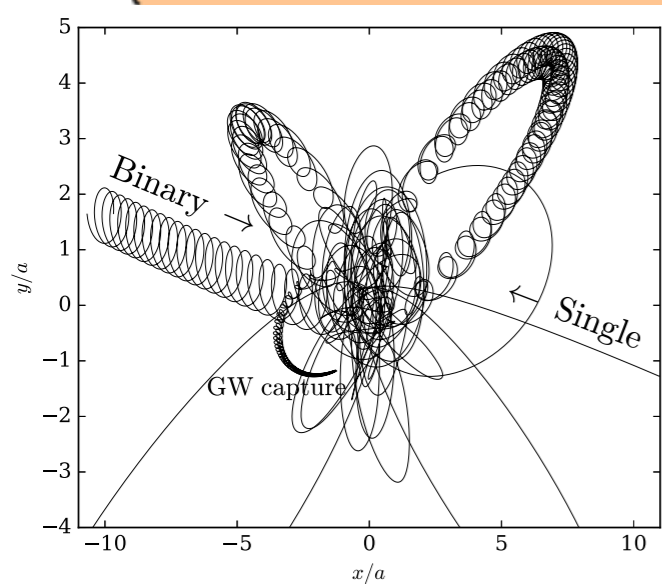
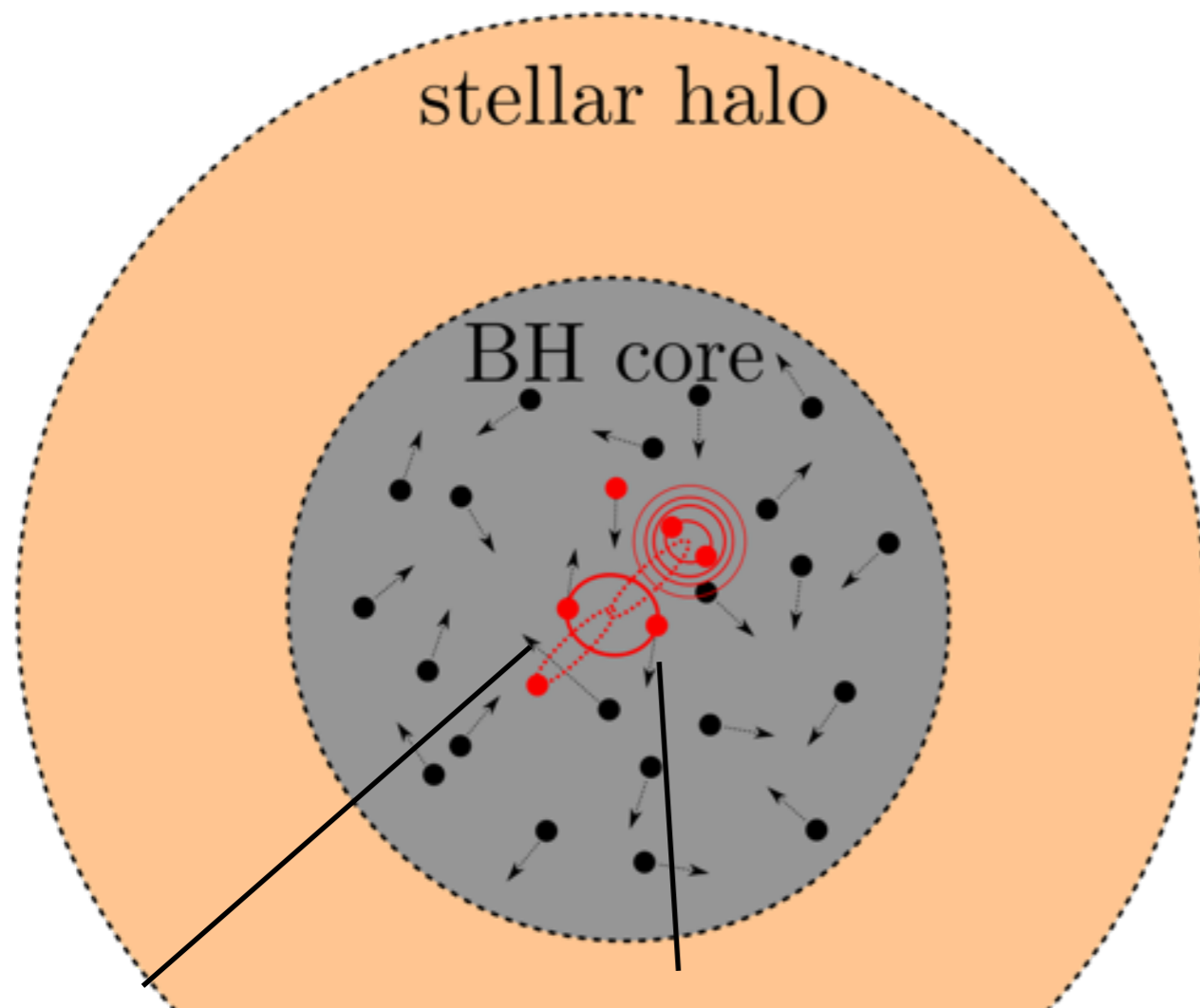


Endstates



Merger Type: 3-body Merger

Topology



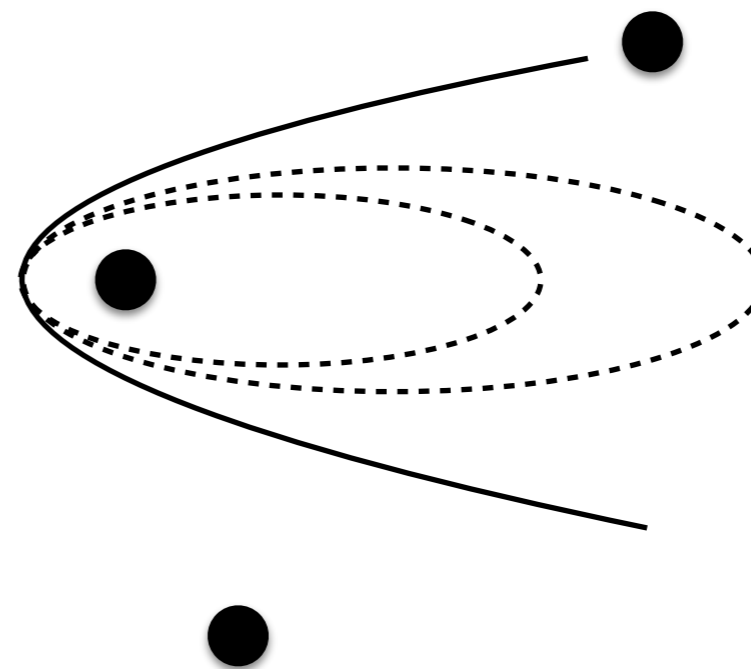
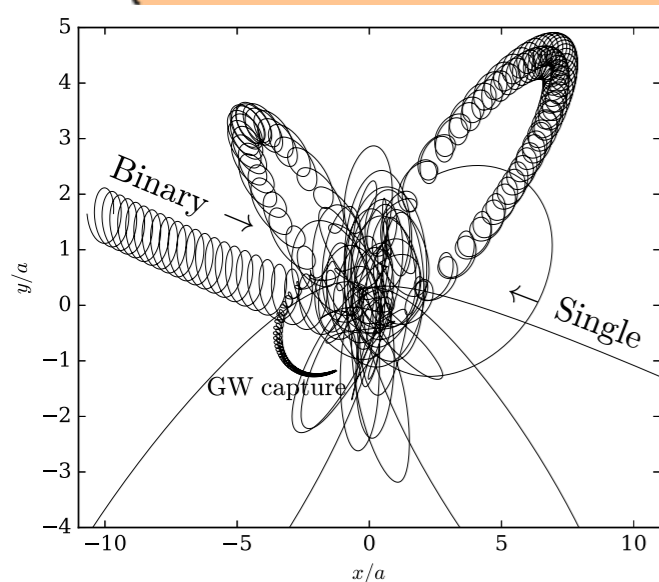
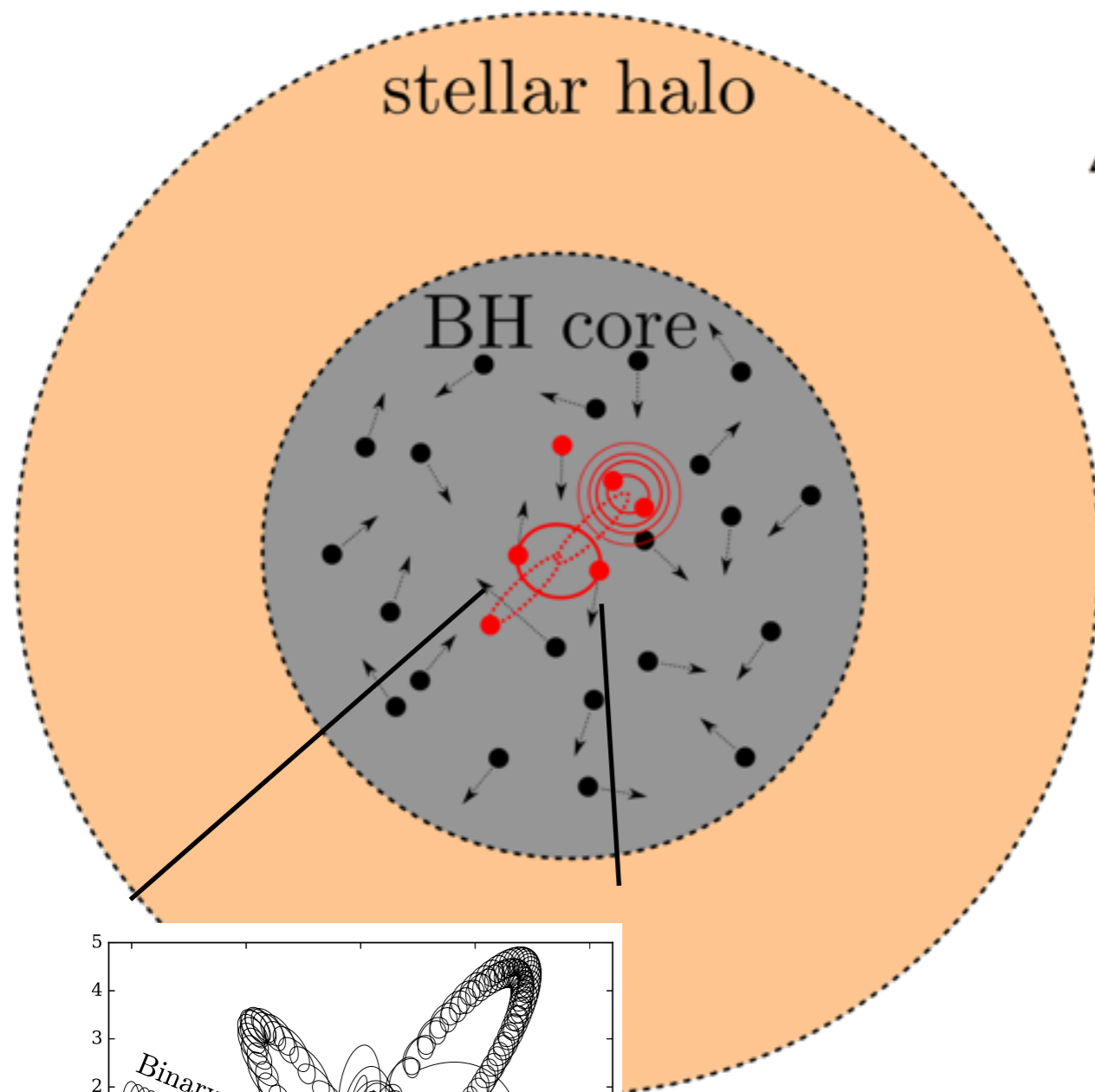
Merger Type: 3-body Merger

3-body GW capture:

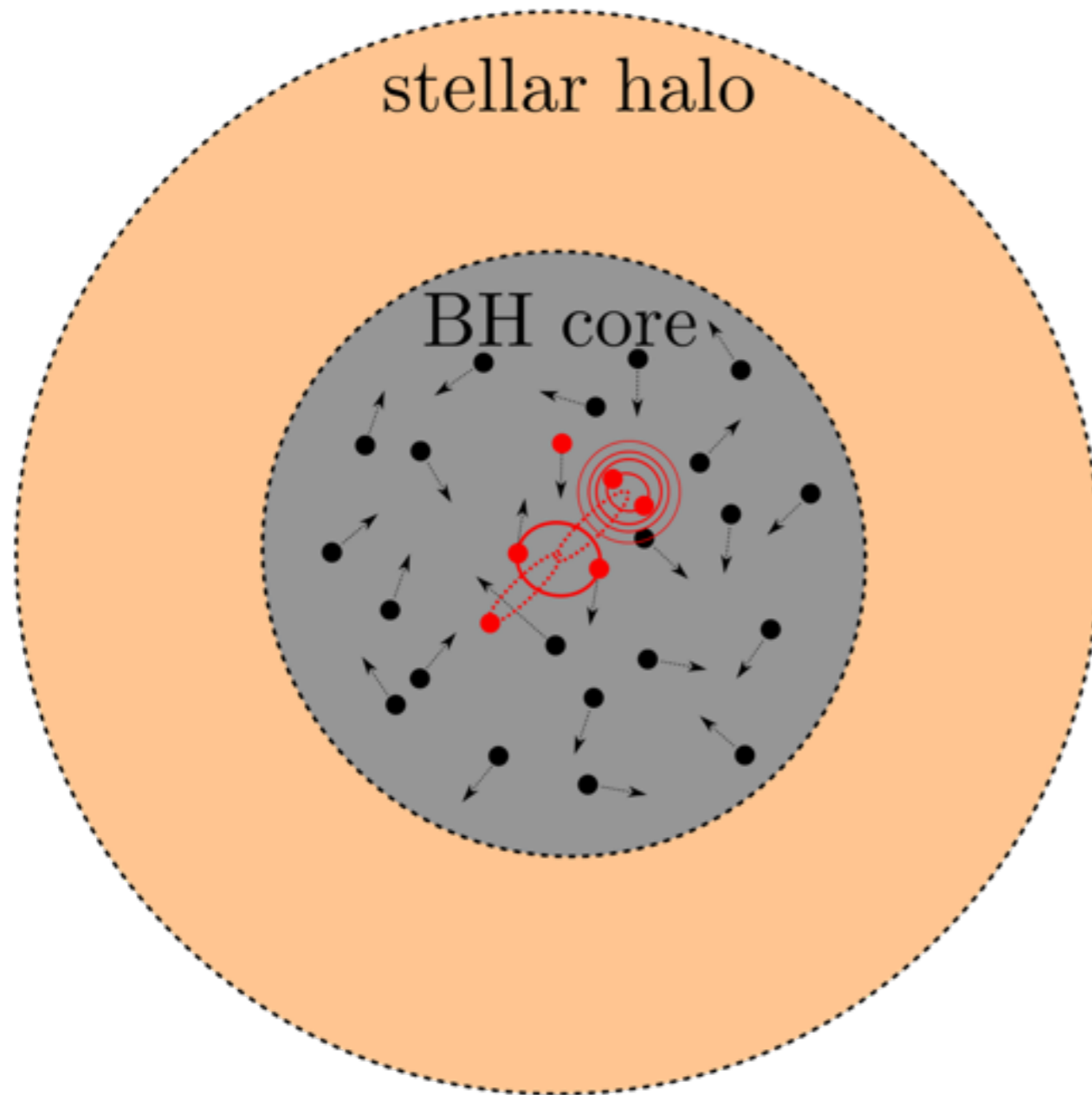
$$E_B \approx Gm^2/(2a)$$

$$\Delta E_p \approx (85\pi/12)G^{7/2}c^{-5}m^{9/2}r_p^{-7/2}$$

$$\mathcal{R}_{bs} \approx \left(\frac{85\pi}{24\sqrt{2}}\right)^{2/7} \times \mathcal{R}_m \left(\frac{a}{\mathcal{R}_m}\right)^{2/7}$$

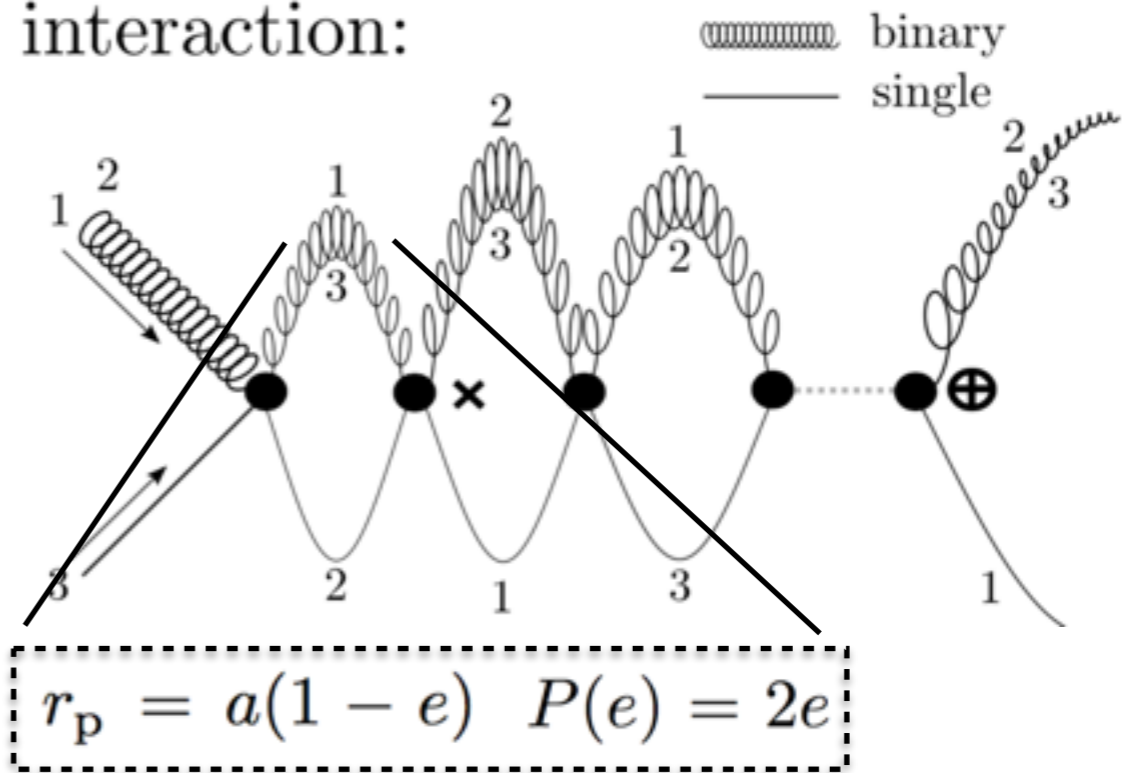


Merger Type: 3-body Merger



3-body Ecc. Probability:

interaction:



$$P_{\text{EM}}(a) \approx \frac{2r_{\text{EM}}}{a} \times N_{\text{IMS}}$$

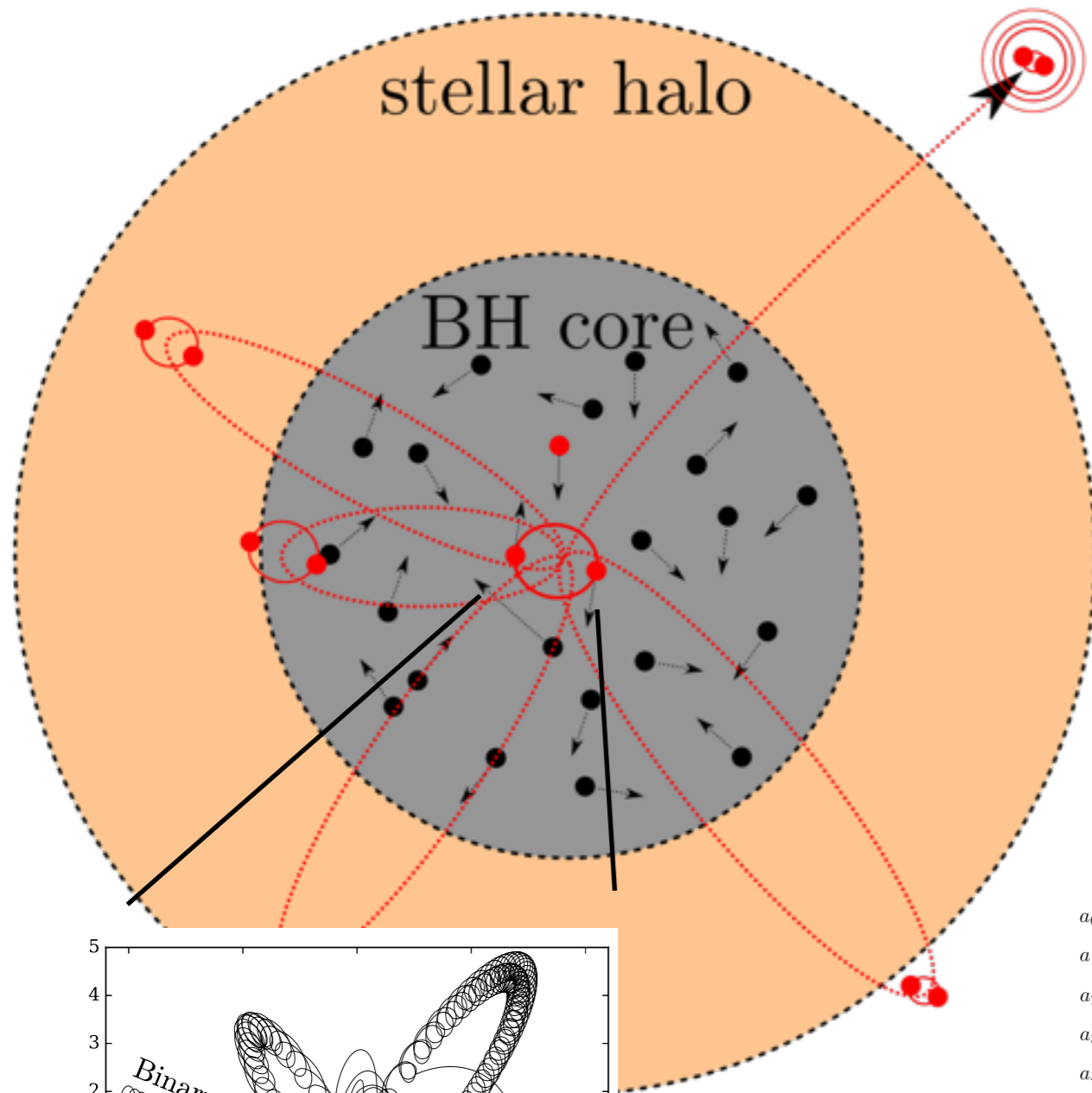
$$r_{\text{EM}} \approx \left(\frac{2Gm}{f^2\pi^2} \right)^{1/3} \frac{1}{2} \frac{1+e_f}{e_f^{12/19}} \left[\frac{425}{304} \left(1 + \frac{121}{304} e_f^2 \right)^{-1} \right]^{870/2299}$$

$$P_{\text{EM}}(a_{\text{in}}, a_{\text{ej}}) = \frac{1}{1 - \delta} \int_{a_{\text{ej}}}^{a_{\text{in}}} \frac{P_{\text{EM}}(a)}{a} da \approx \frac{P_{\text{EM}}(a_{\text{ej}})}{1 - \delta}$$

Merger Type: 3-body Merger

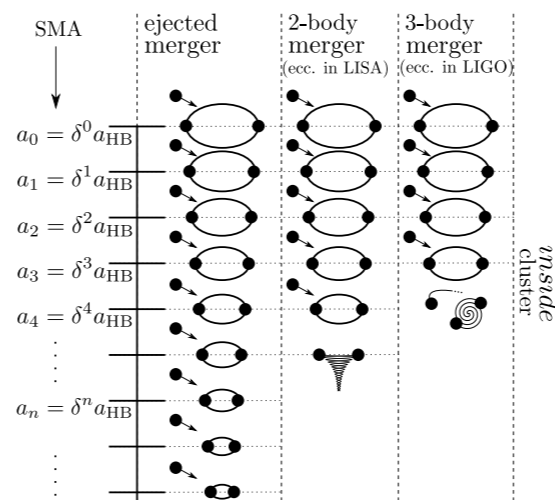
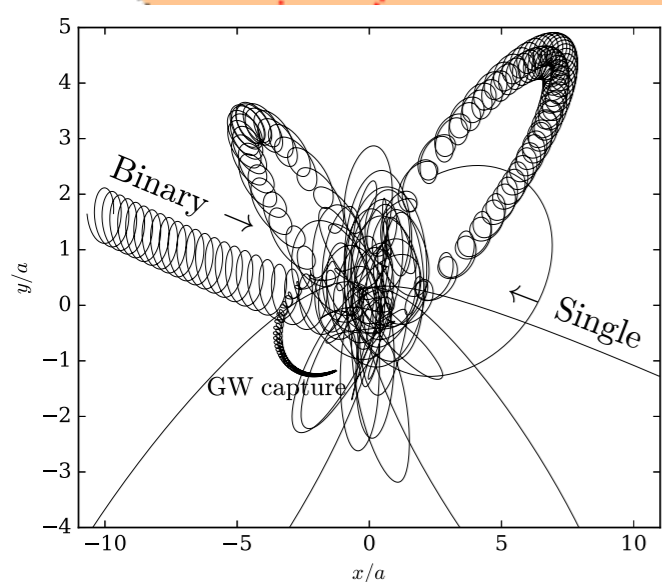
Eccentric Black Hole Mergers Forming in Globular Clusters
Authors: Johan Samsing

Newtonian codes underestimate the rate by more than a factor of 100!

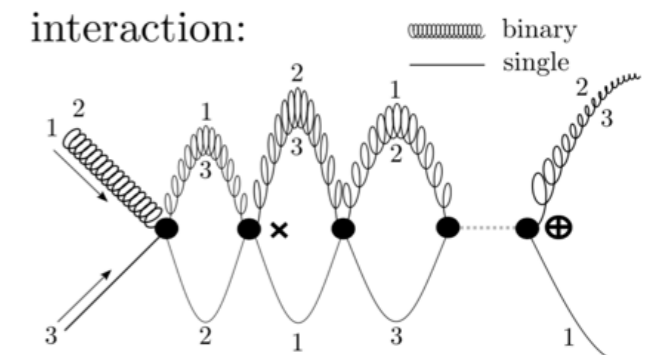


number of tries

- GR: **1x1 = 1**
 +GR: **20x20 = 400**

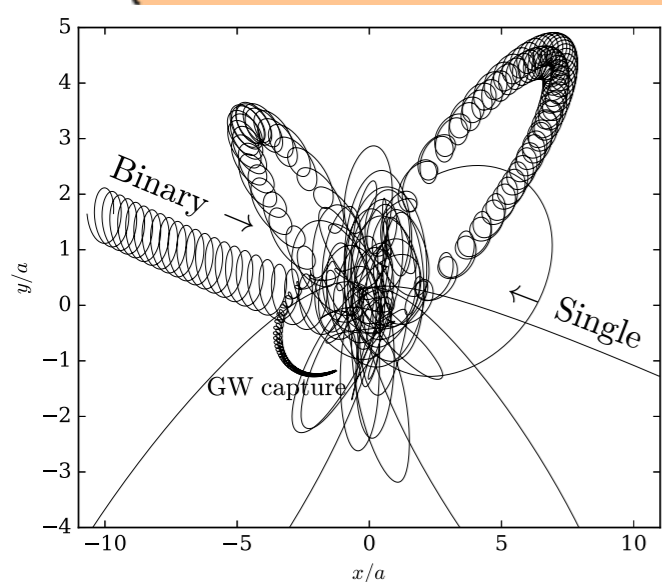
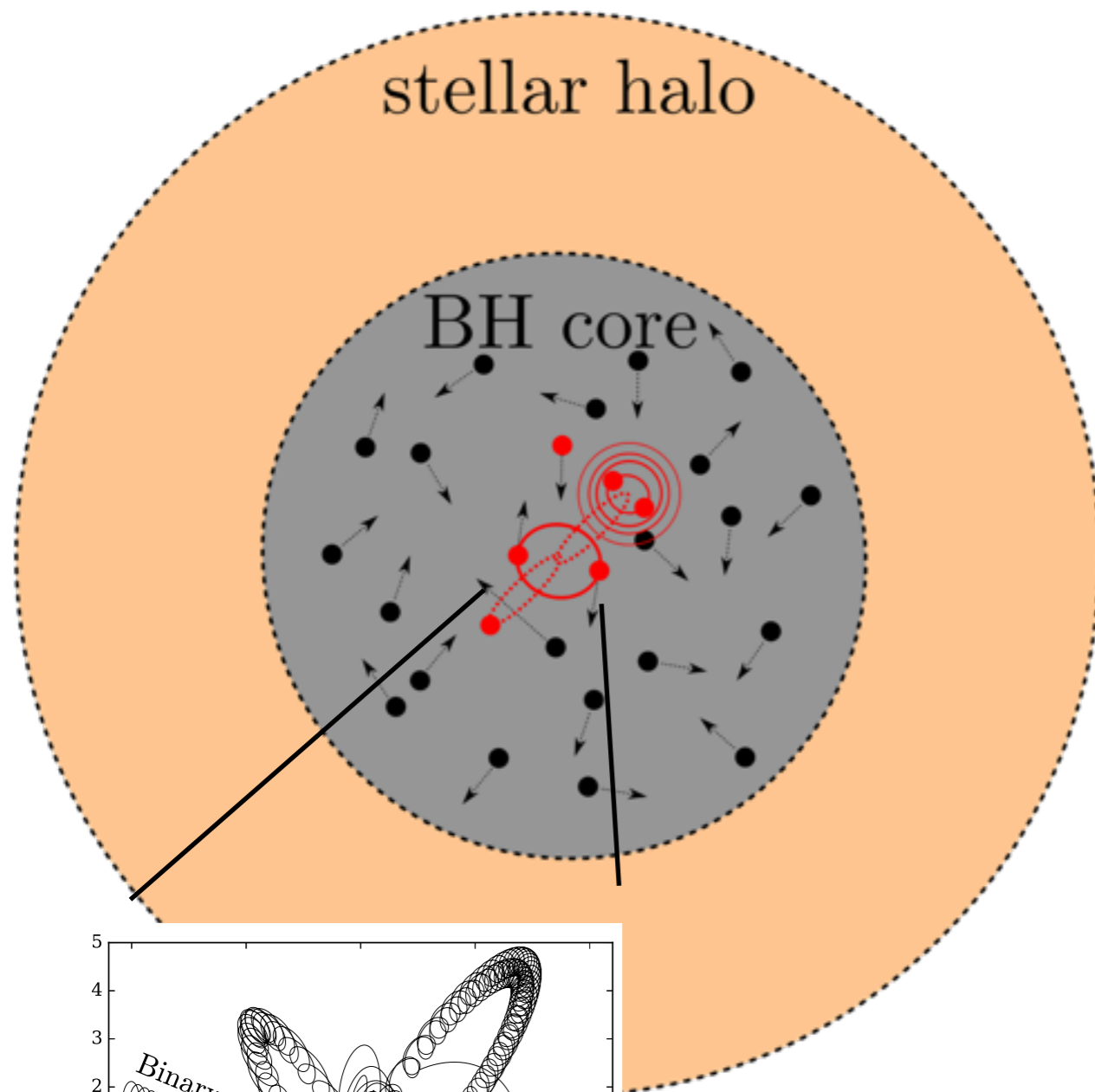


X



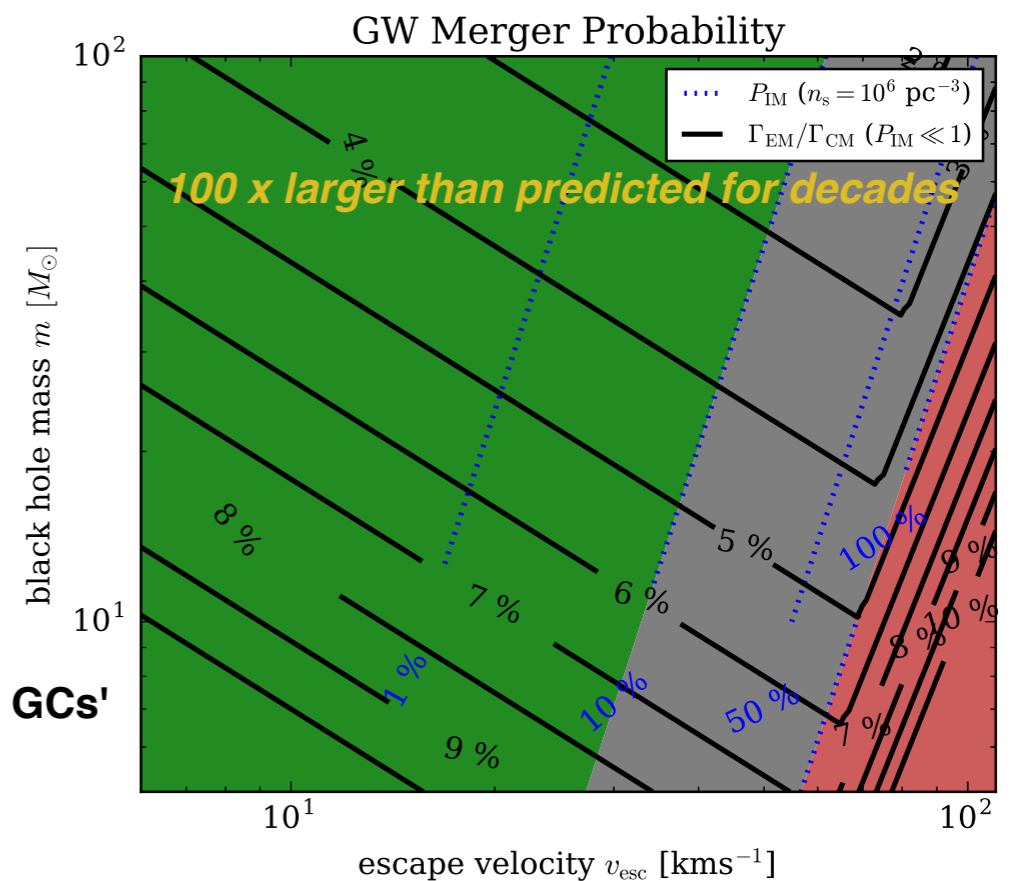
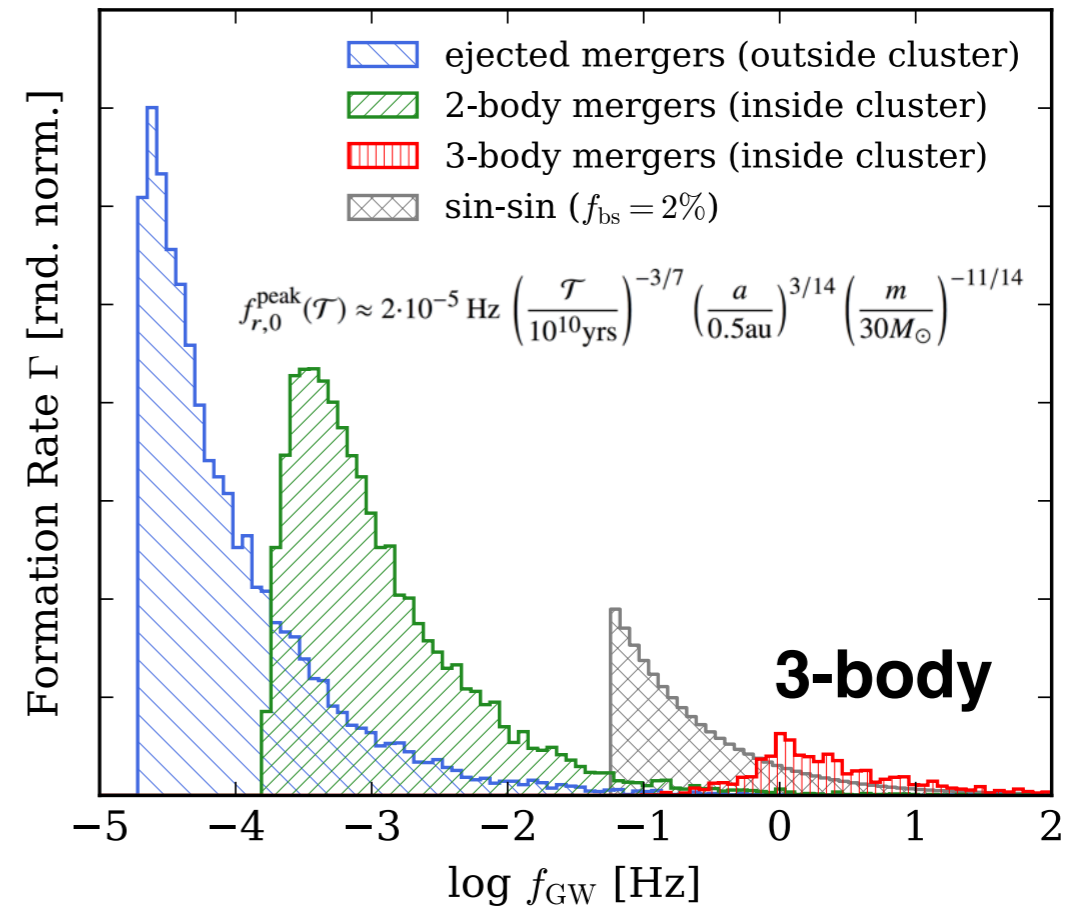
$$N_{bs}(a_{in}, a_{ej}) = \int_{a_{ej}}^{a_{in}} \frac{1}{1-\delta} \frac{1}{a} da = \frac{1}{1-\delta} \ln \left(\frac{a_{in}}{a_{ej}} \right) \quad N_{IMS} \approx (\max(a_{IMS})/a)^{7/2}$$

Merger Type: 3-body Merger

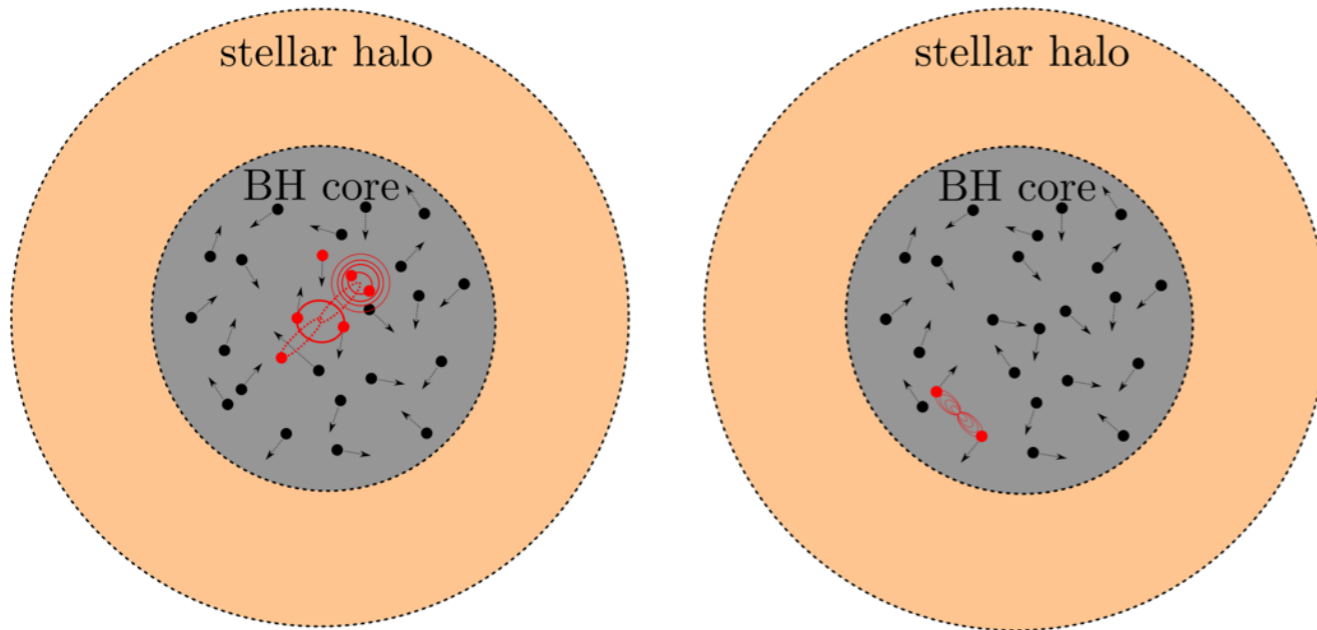


'Eccentric BHs forming in GCs'
Samsing, 18.

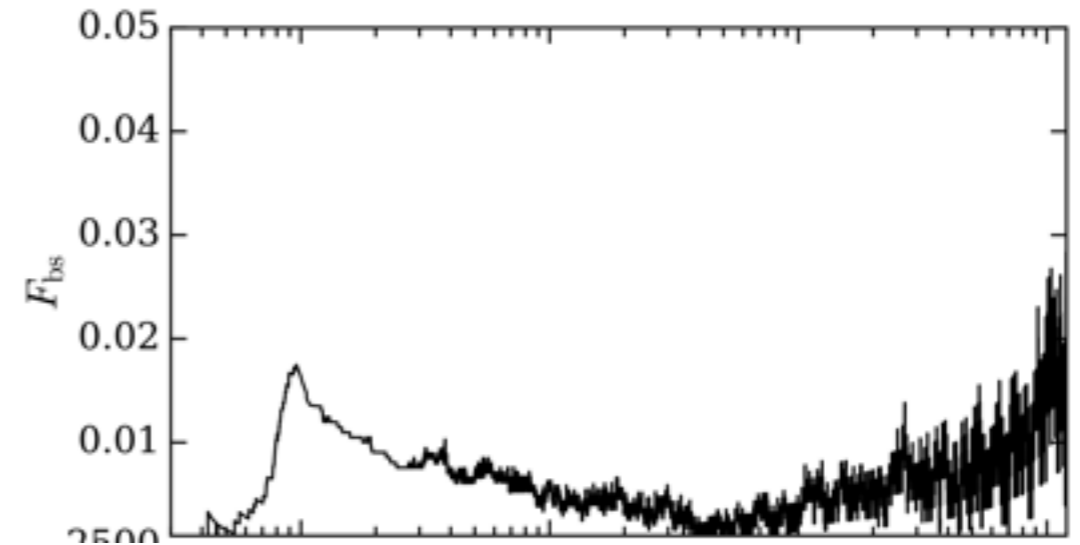
Eccentric LIGO sources



Comparing binary-single and single-single:



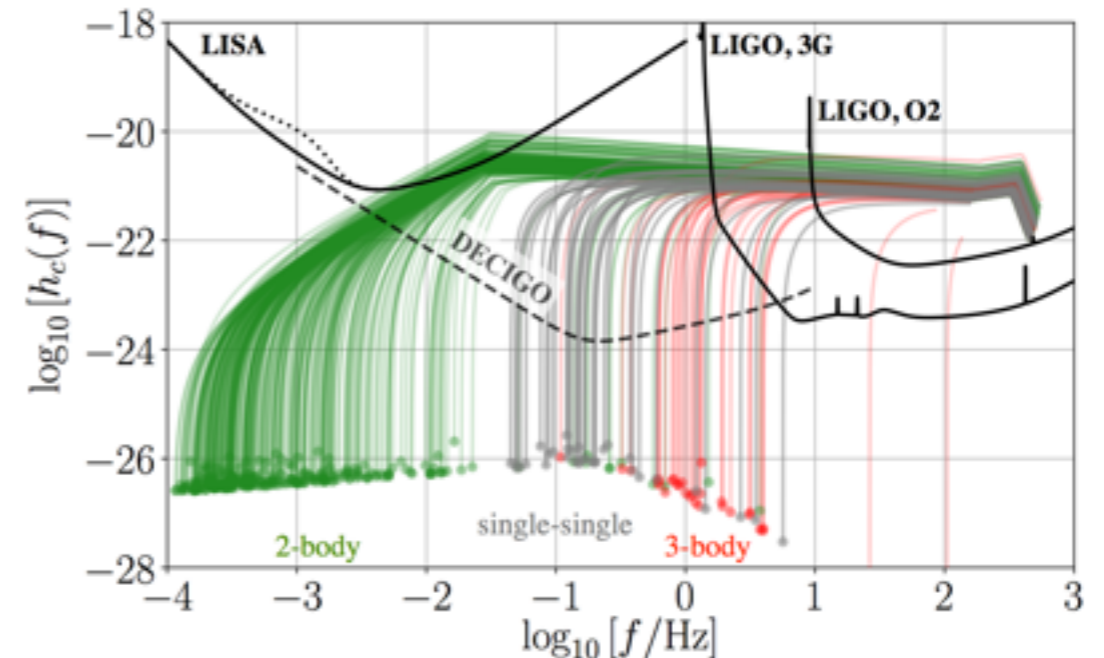
binary fraction



uniform sphere:

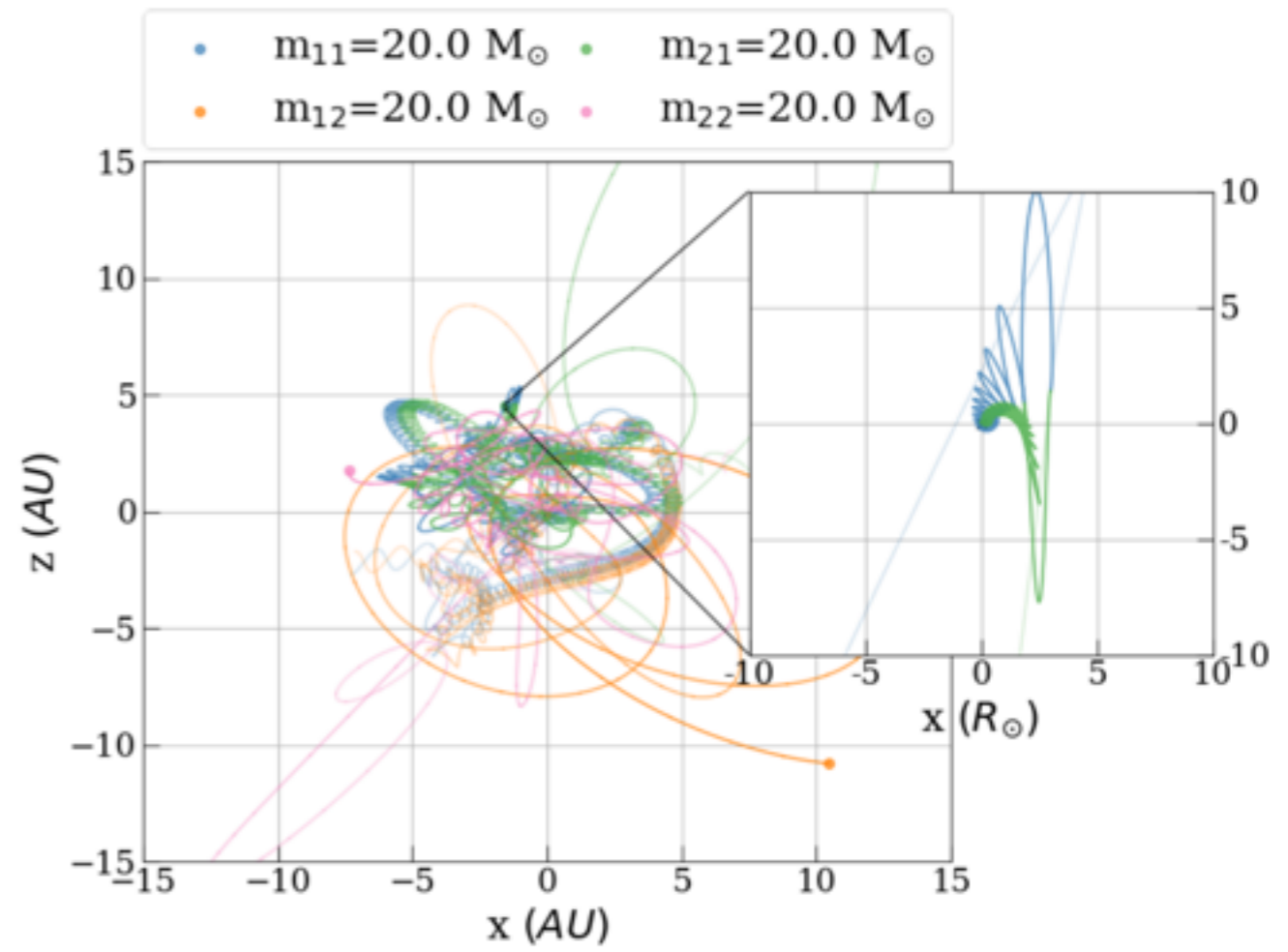
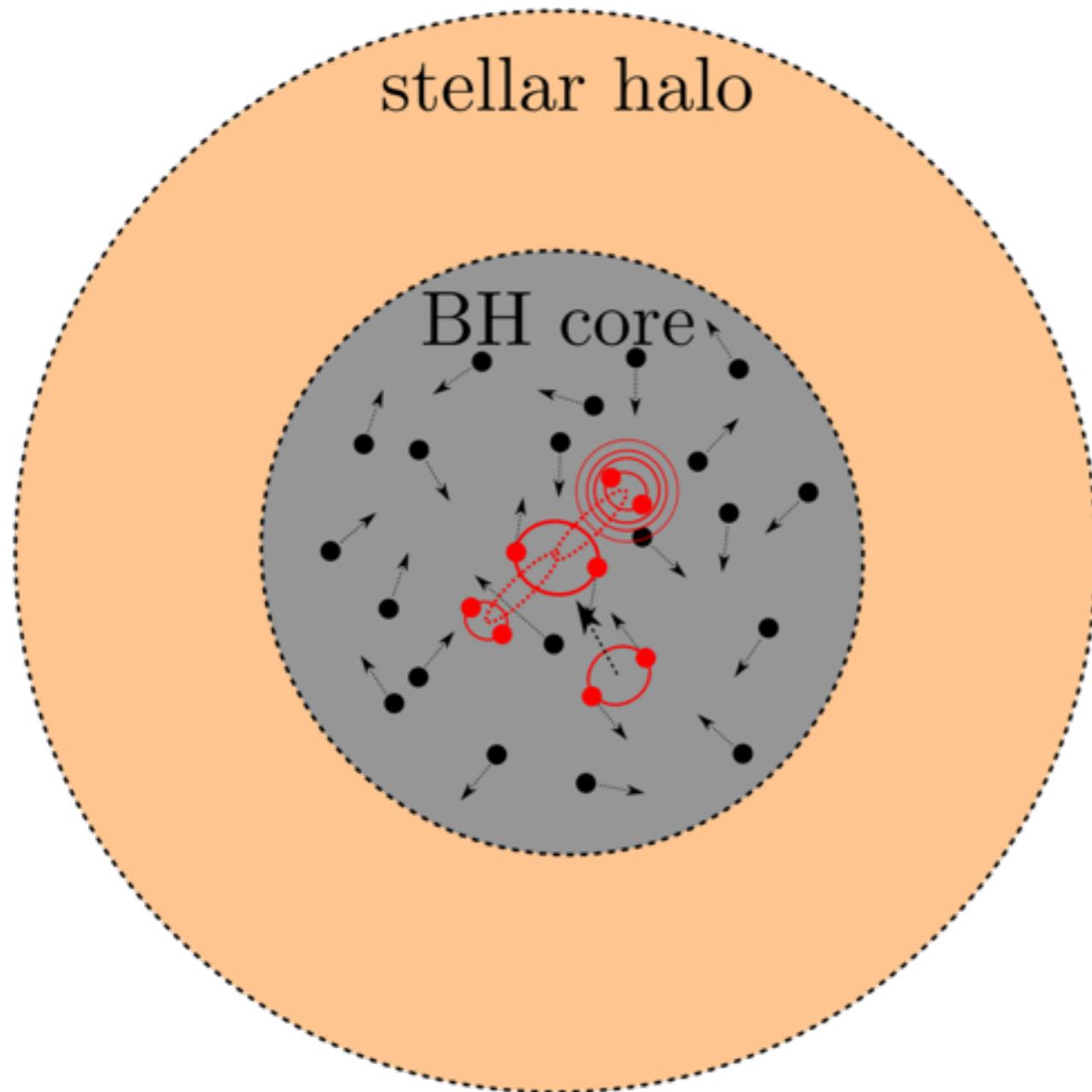
$$\frac{\Gamma_{bs}}{\Gamma_{ss}} \approx 6F_{bs}\mathcal{N} \times \left(\frac{\phi - 1}{12f_{ed}^2} \right)^{2/7} \frac{7}{5}$$

f distribution

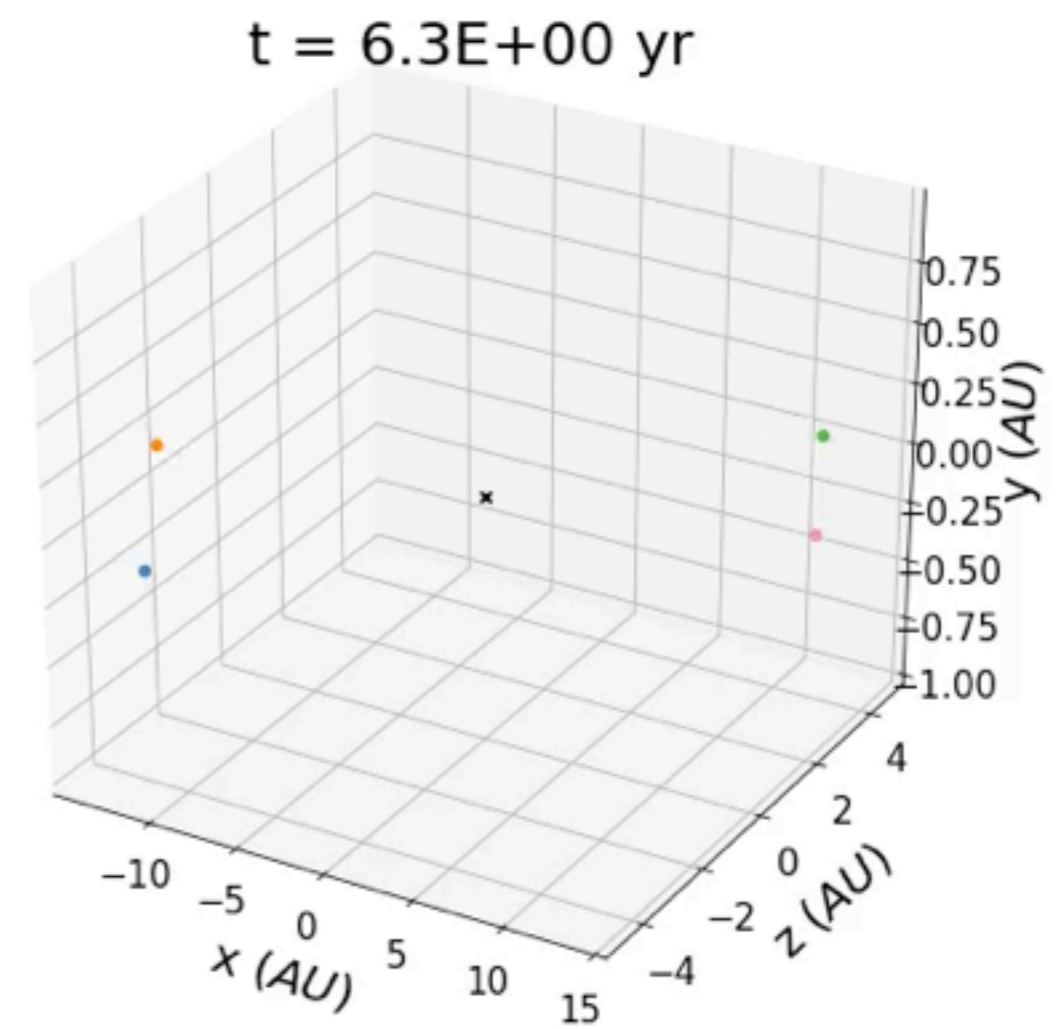
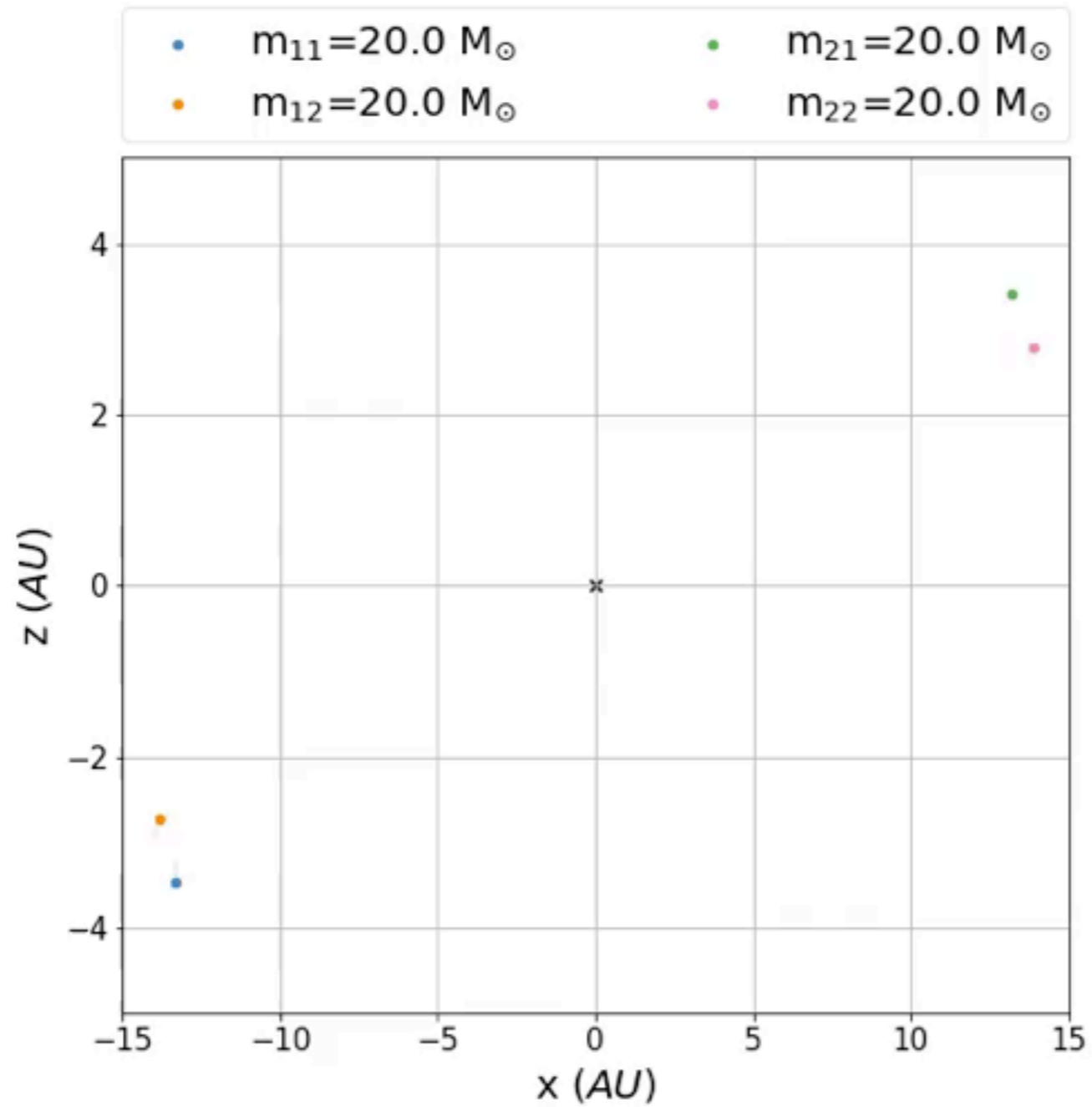
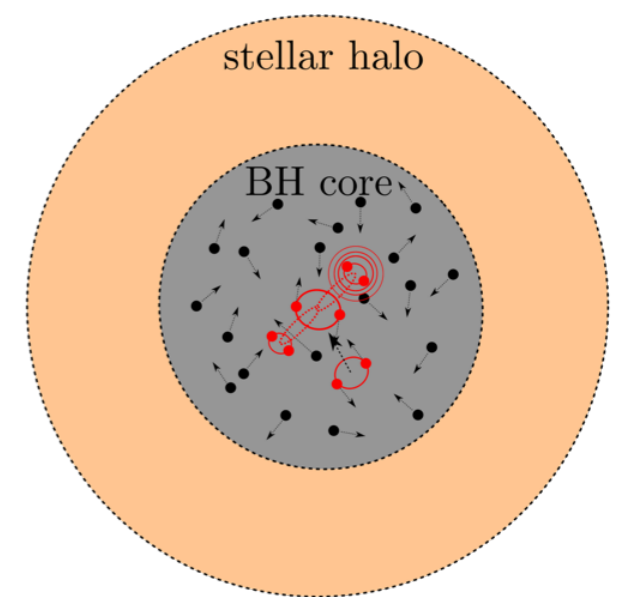


Can we here probe the BH core properties?

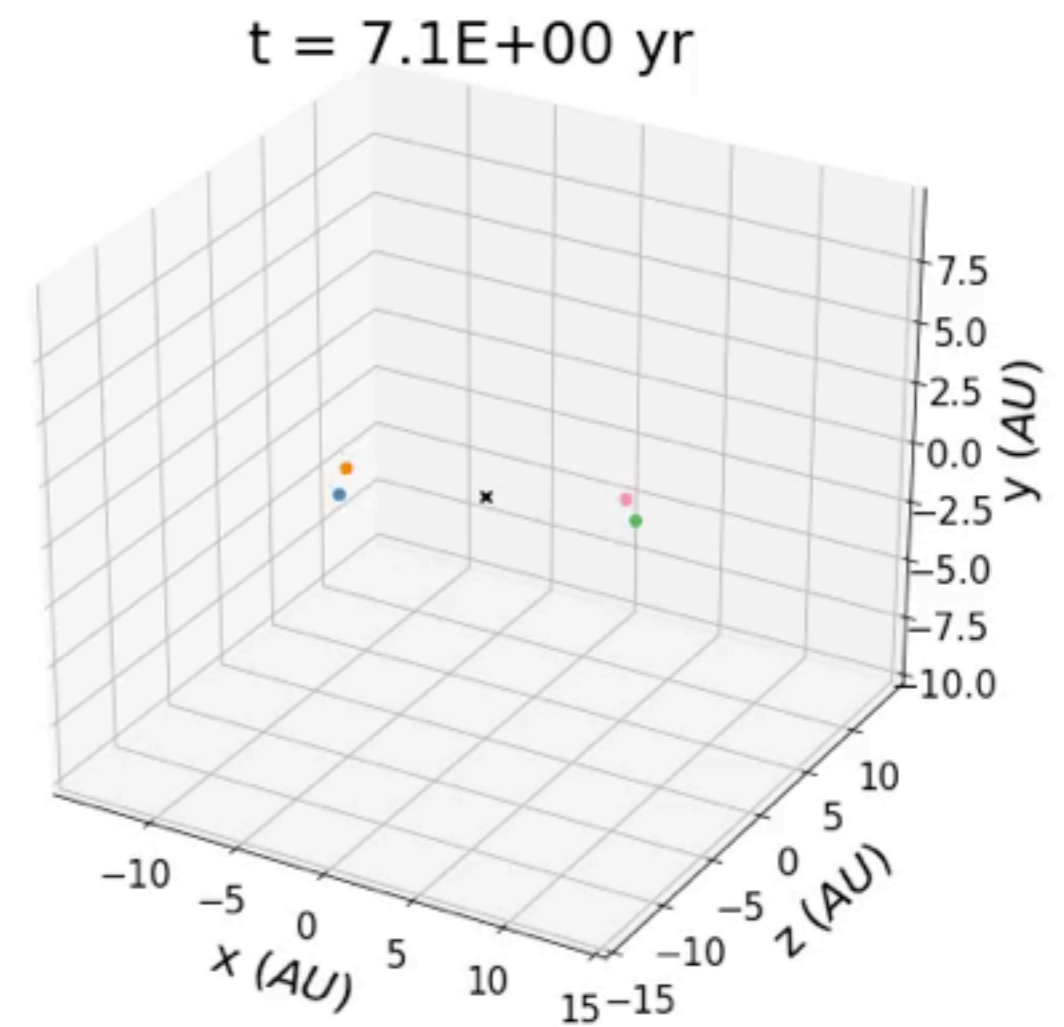
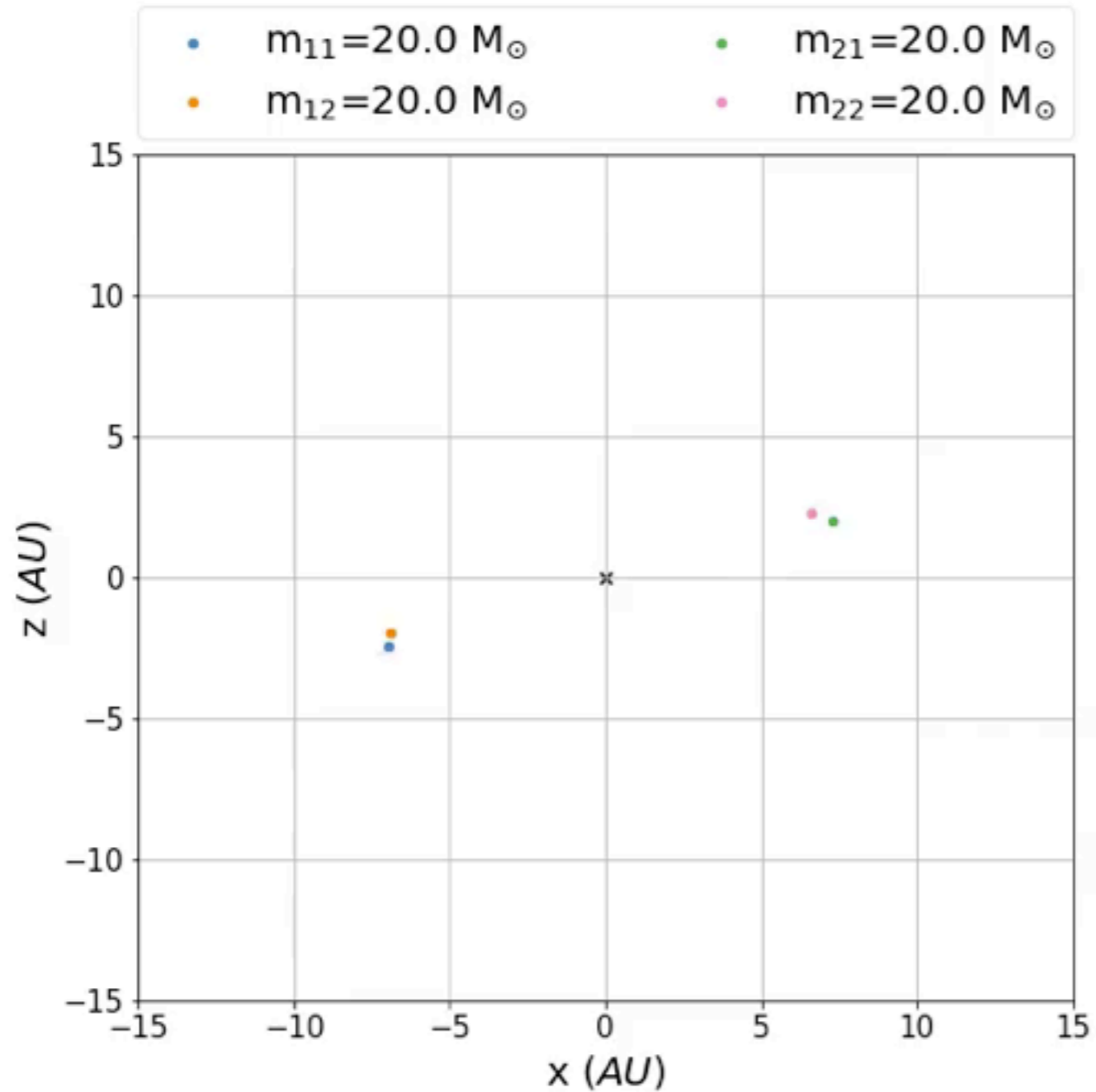
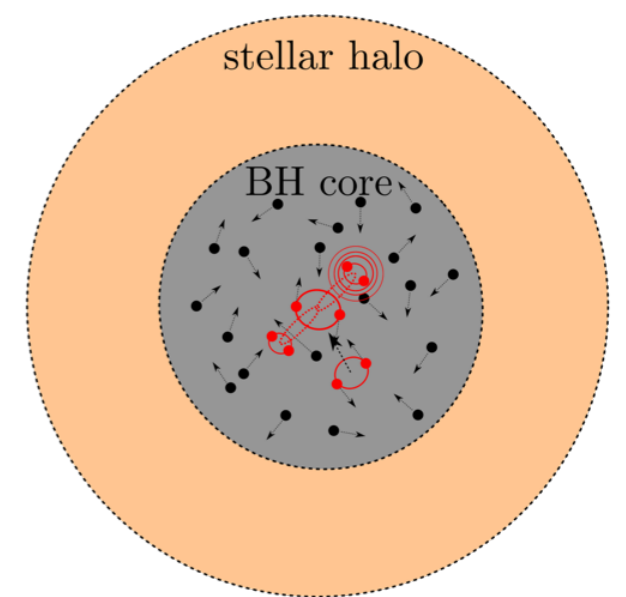
Merger Type: 4-body Merger

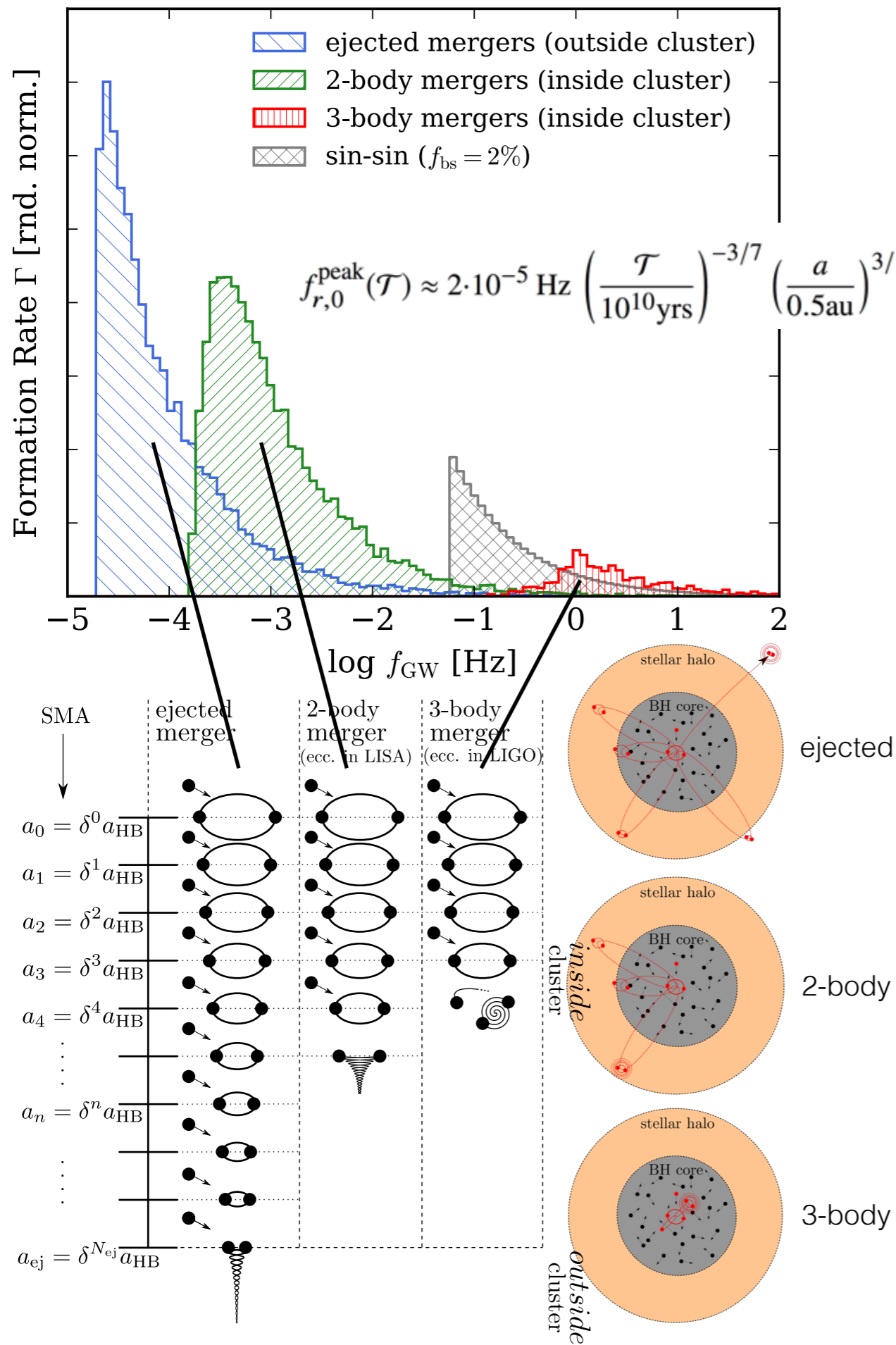


Merger Type: 4-body Merger



Merger Type: 4-body Merger





Peak Normalizations:

$$P_i \approx F_i \times \left(\frac{\tau_i(a_{\text{ej}})}{t_{\text{GW}}^{e=0}(a_{\text{ej}})} \right)^{2/7}$$

$$F_{\text{in}} \approx (7/10)/(1 - \delta) \approx 3$$

$$F_{\text{GW}} \approx (7/5)/(1 - \delta) \times N_{\text{MS}} \approx 120$$

$$a_{\text{ej}} \sim 0.5 \text{ AU} \quad M \sim 30 M_{\odot}$$

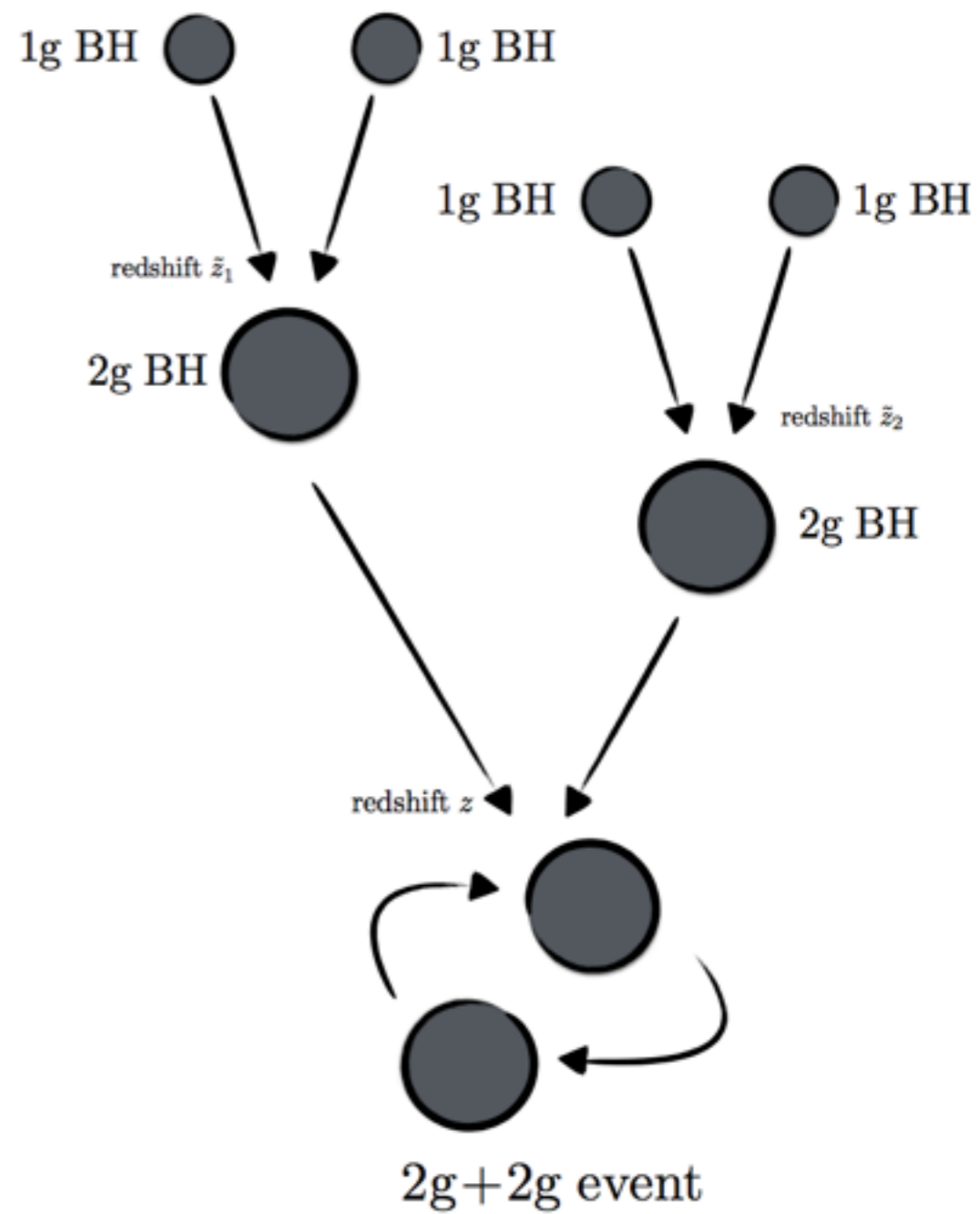
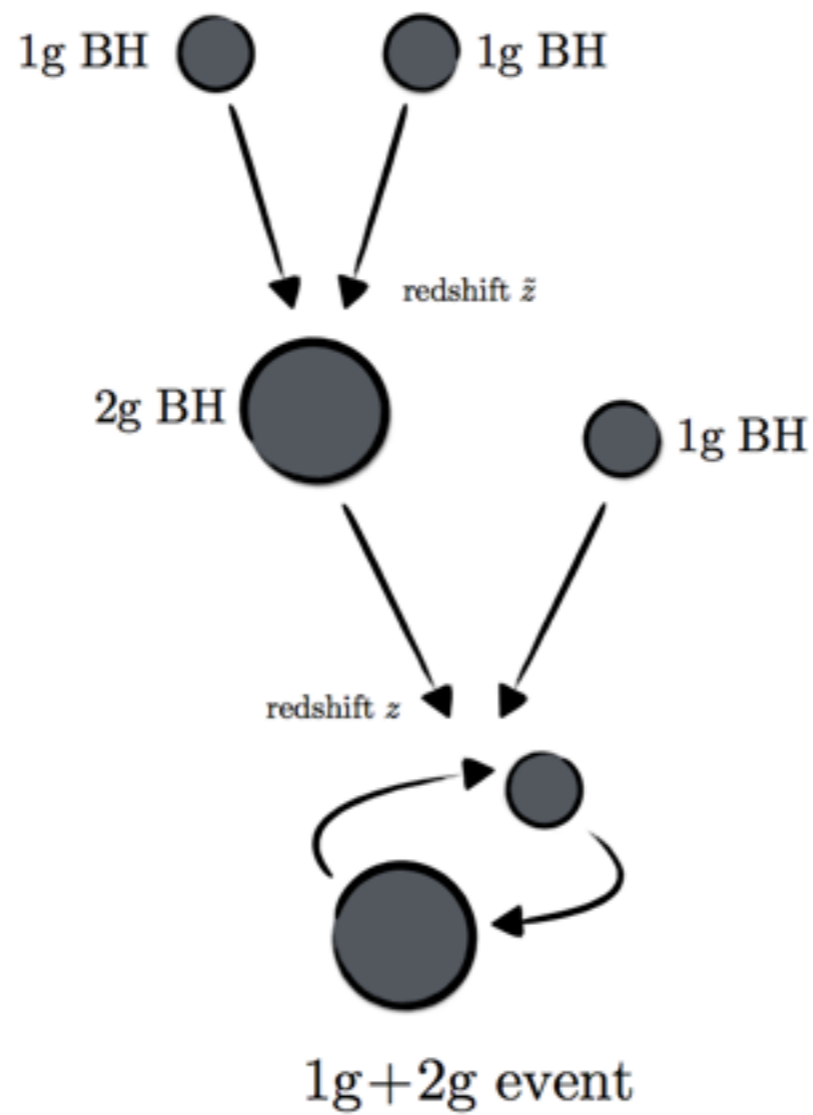
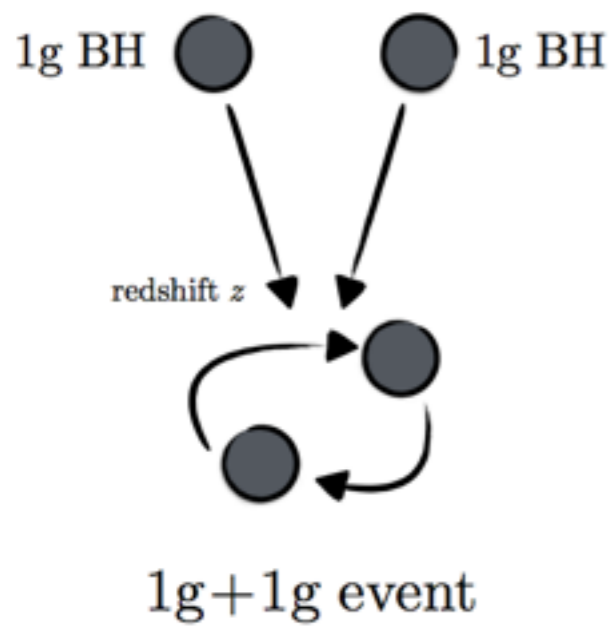
$$\tau_{\text{in}} \sim 10^7 \text{ years} \quad \tau_{\text{GW}} \sim 0.1 \text{ year}$$

$$P_{\text{in}} \approx 0.15 \quad P_{\text{GW}} \approx 0.03$$

$$P(t_{\text{GW}}(a_{\text{ej}}) < T_{\text{H}}) \approx 0.35$$

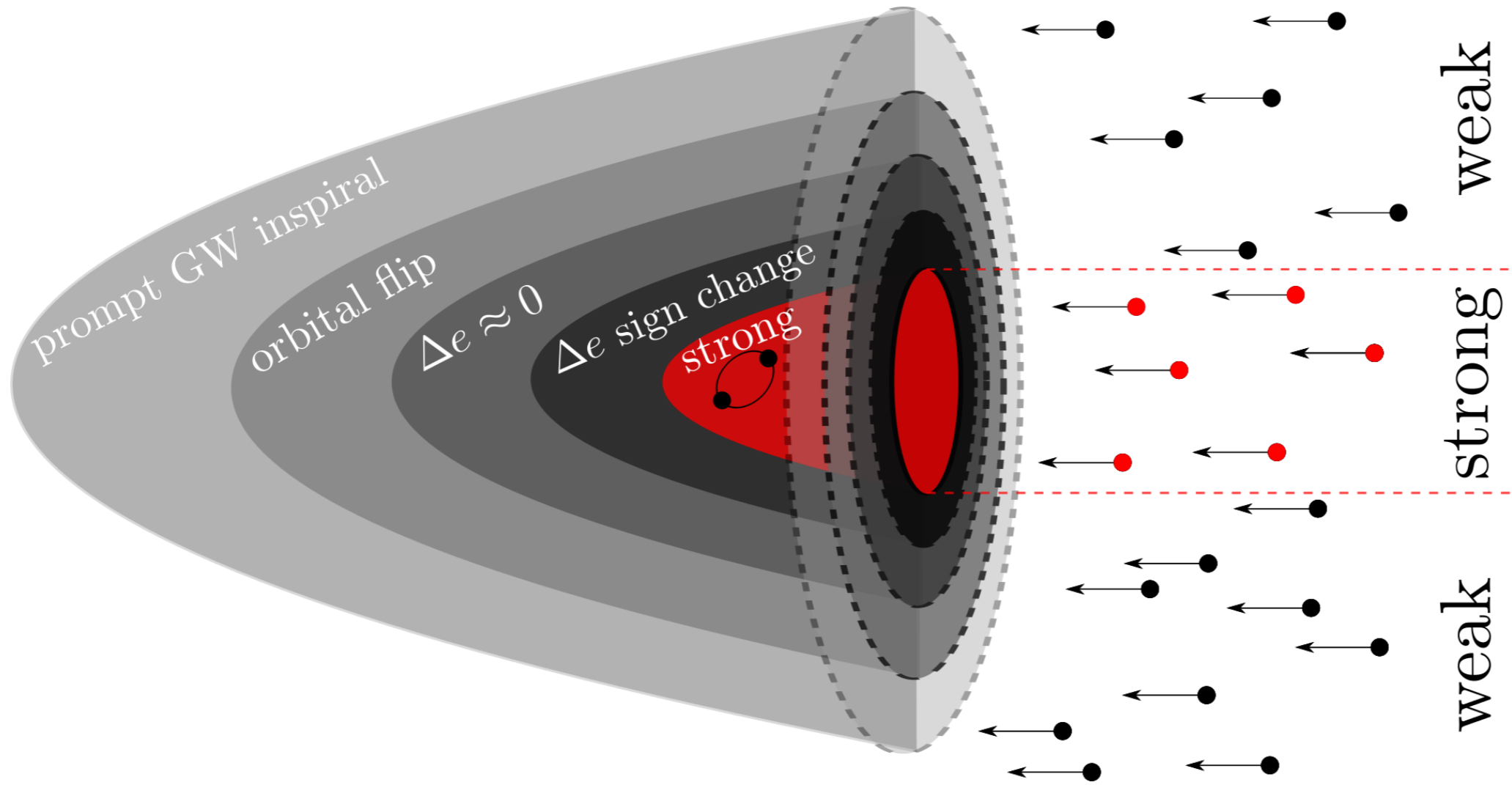
$$0.82 \times 0.35 \approx 0.3$$

start to reach 10% high ecc. LIGO mergers



Merger Type: Secular-processes

- work done with Adrian Hamers (IAS)



Merger Type: Secular-processes

Why is this important?

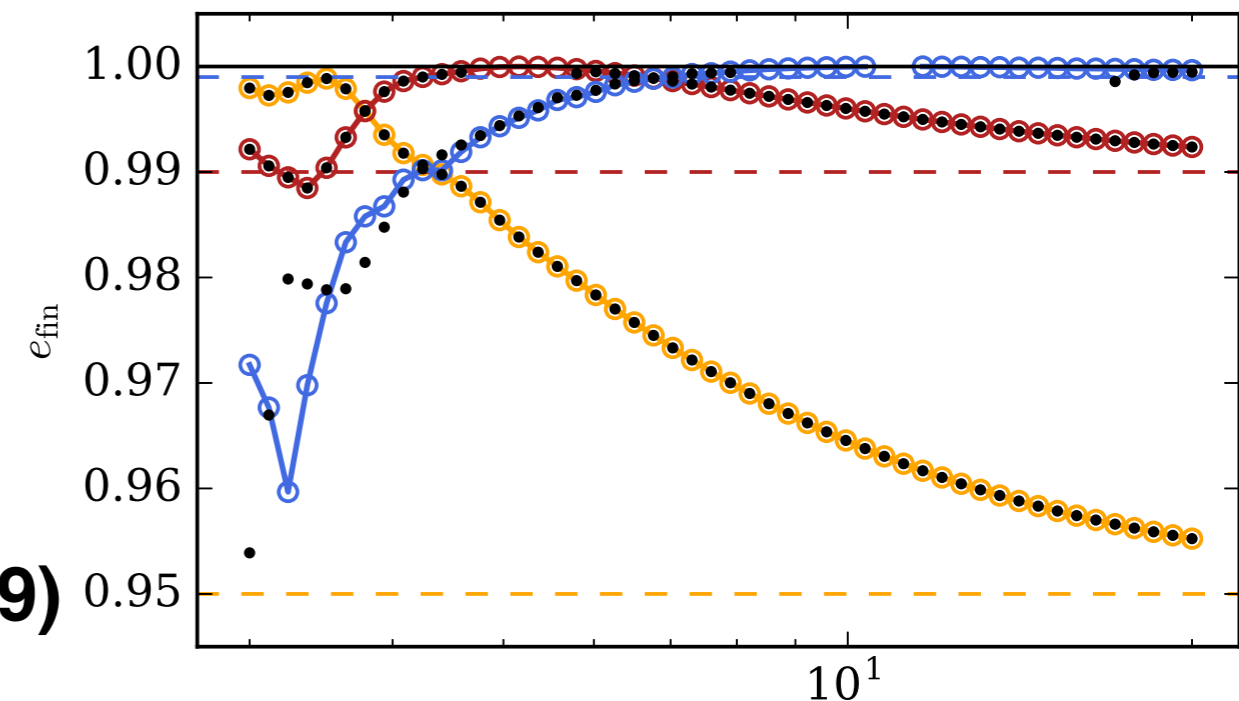
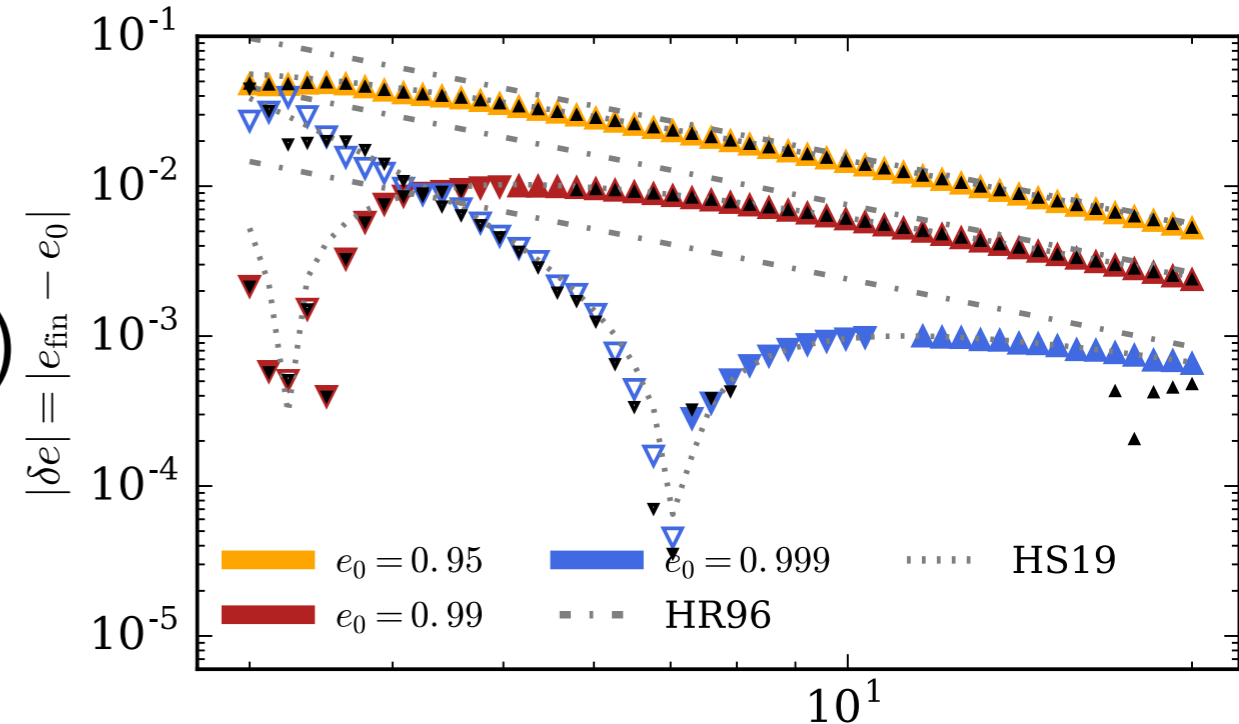
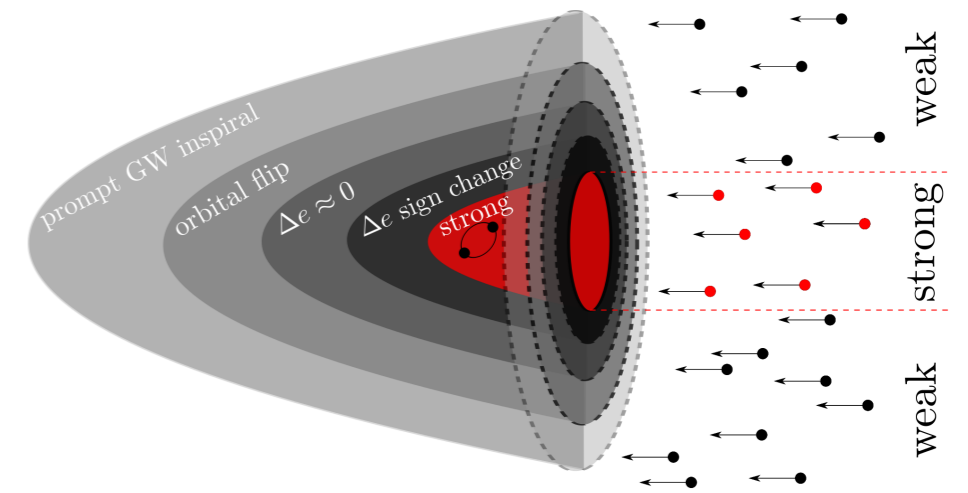
1.order (Heggie, Rasio 96)

$$\delta e = -\frac{15\pi}{16} \left(\frac{2m_3^2 a^3}{M_{123} M_{12} r_p^3} \right)^{1/2} e \sqrt{1-e^2} \sin 2\Omega \sin^2 i$$

2.order 2x(Hamers, Samsing 19)

$$\Delta e_{\text{SO}} = \Delta e_{\text{FO}} + \epsilon^2 \frac{3}{512} \pi e_0 \left[-100 (1 - e_0^2) \sin 2\Omega \right. \\ \left. \left\{ (5 \cos i + 3 \cos 3i) \cos 2\omega + 6 \sin i \sin 2i \right\} \right. \\ \left. + 4 \cos 2i \left\{ 3\pi (81e_0^2 - 56) - 200 (1 - e_0^2) \right. \right. \\ \left. \left. \cos 2\Omega \sin 2\omega \right\} + 3\pi \left\{ 200e_0^2 \sin^4 i \cos 4\Omega \right. \right. \\ \left. \left. + 8 (16e_0^2 + 9) \sin^2 2i \cos 2\Omega \right. \right. \\ \left. \left. + (39e_0^2 + 36) \cos 4i - 299e_0^2 + 124 \right\} \right],$$

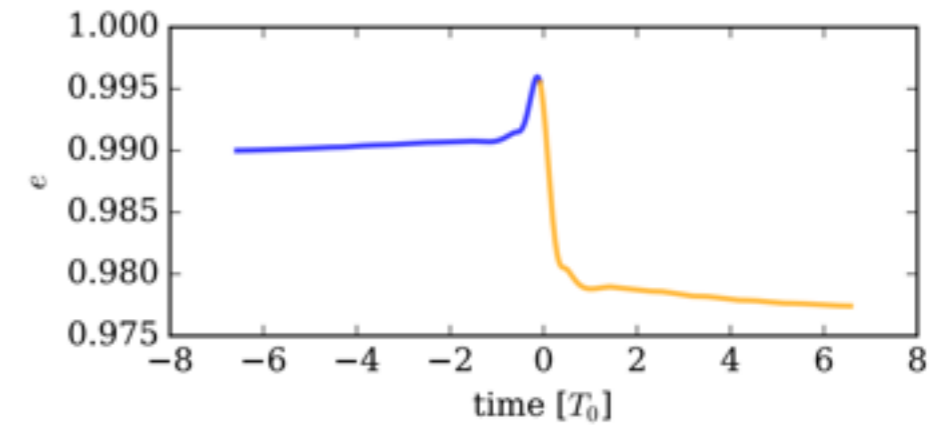
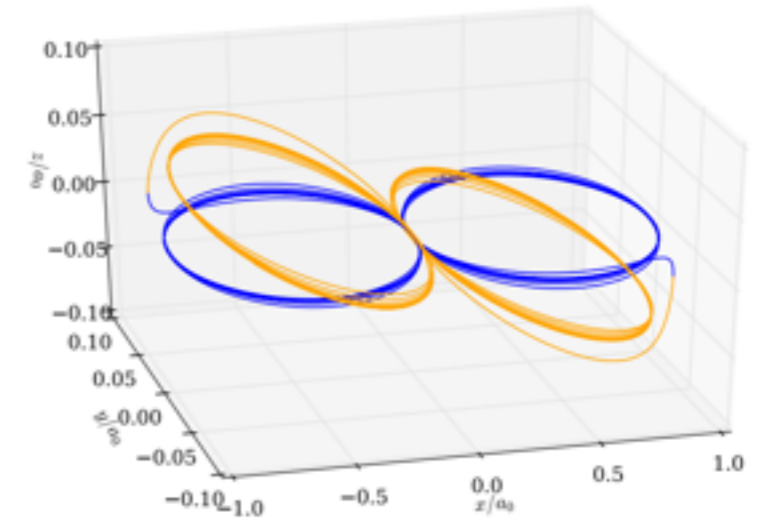
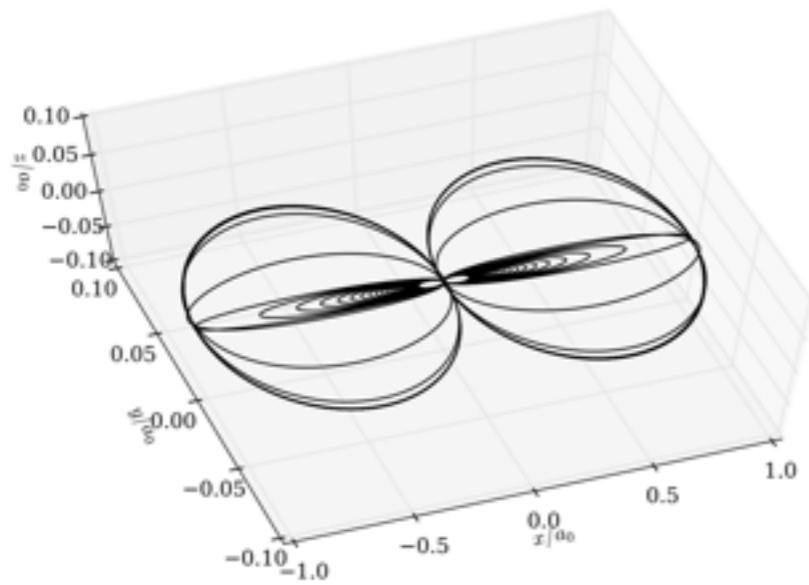
and 1x(Samsing, Hamers, Types 19)



Merger Type: Secular-processes

PN effects?

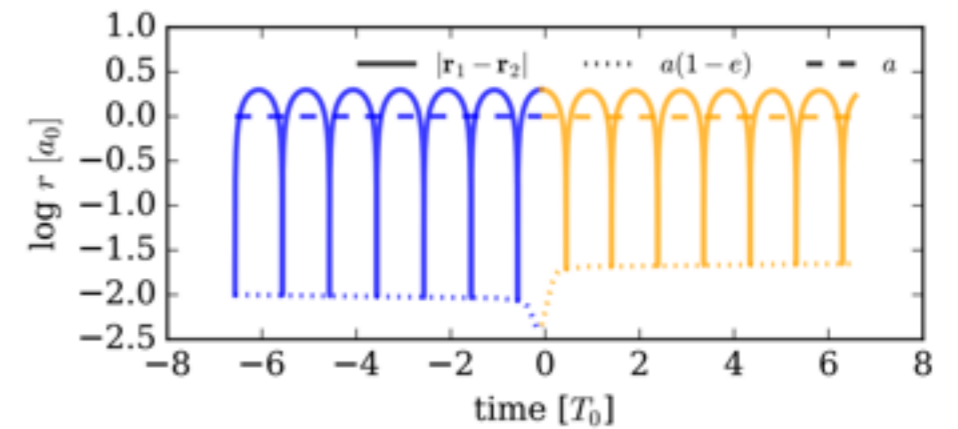
2.5



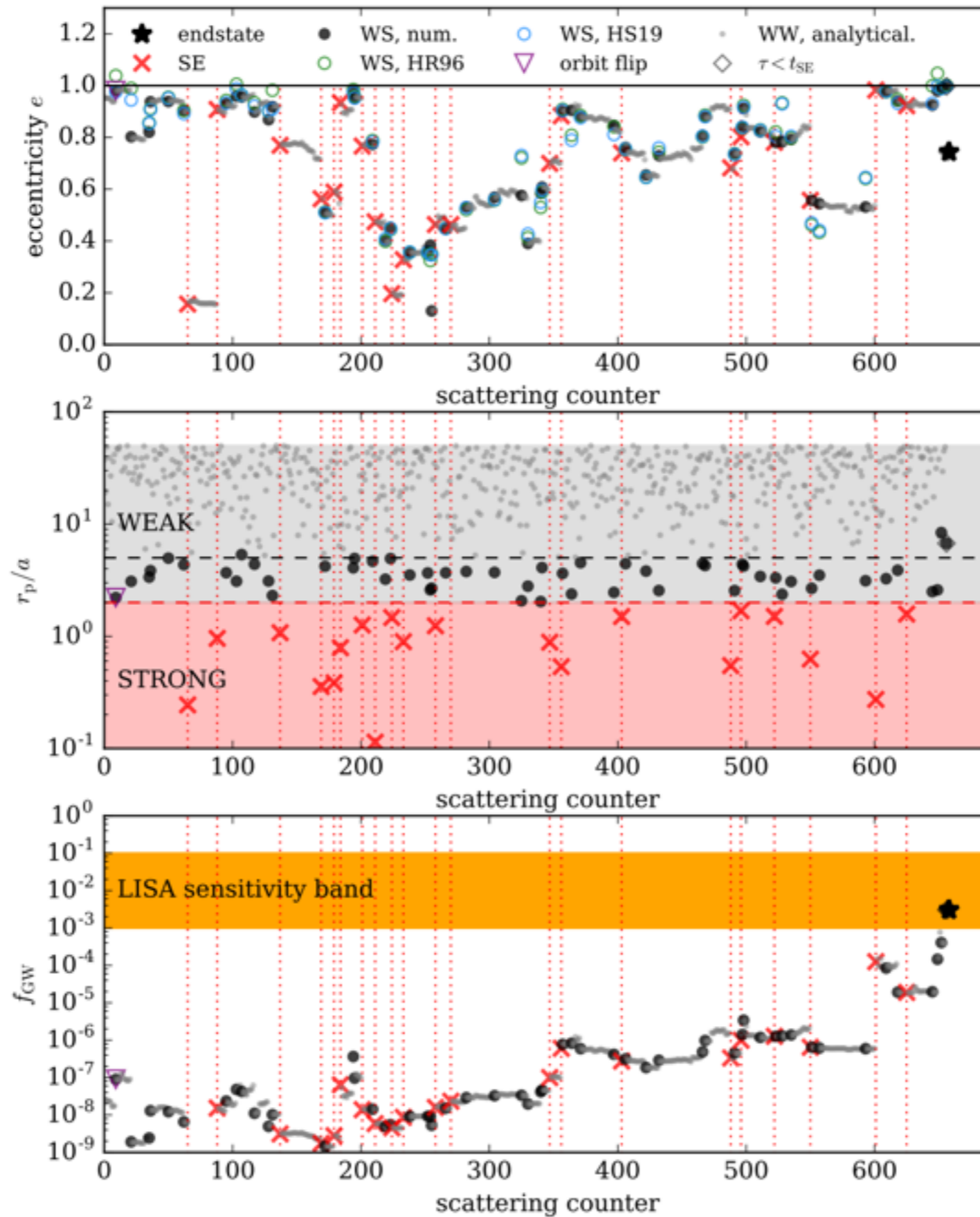
1,2

$$\frac{r_p}{a_0} \gtrsim \frac{a_0^{2/3}}{\mathcal{R}_m^{2/3}} (1 - e_0^2)^{2/3}$$

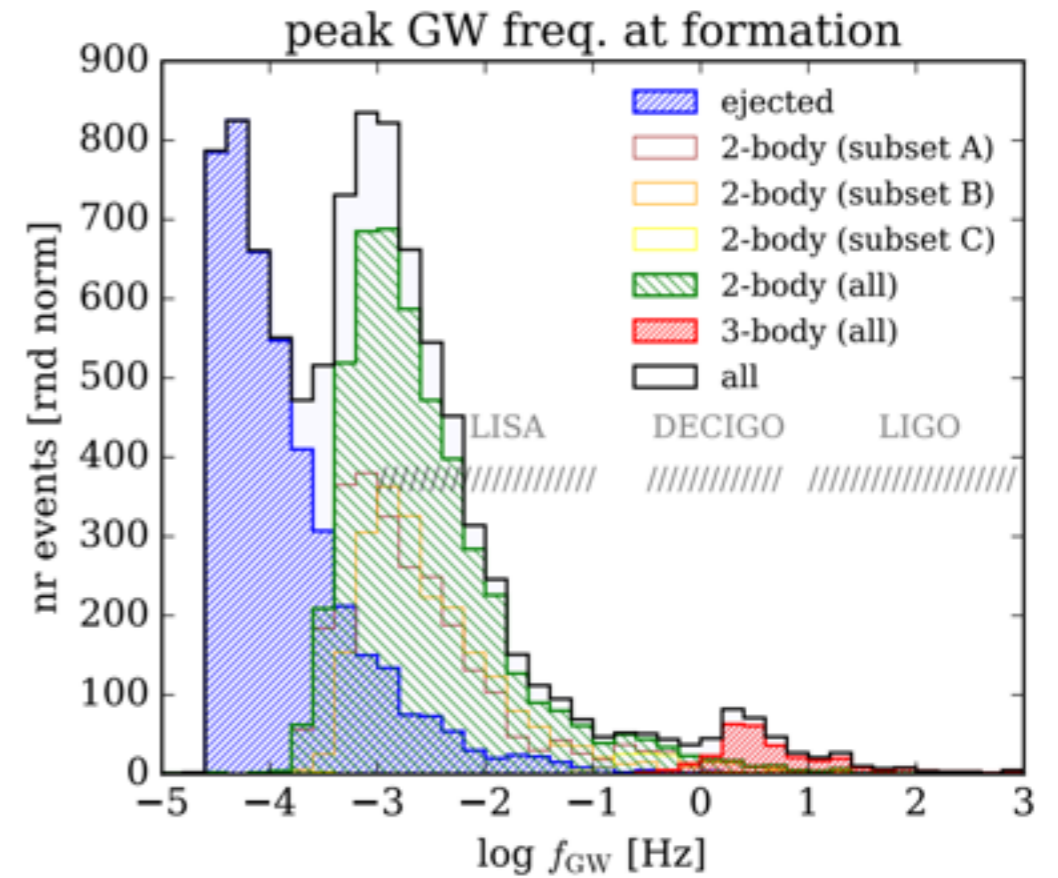
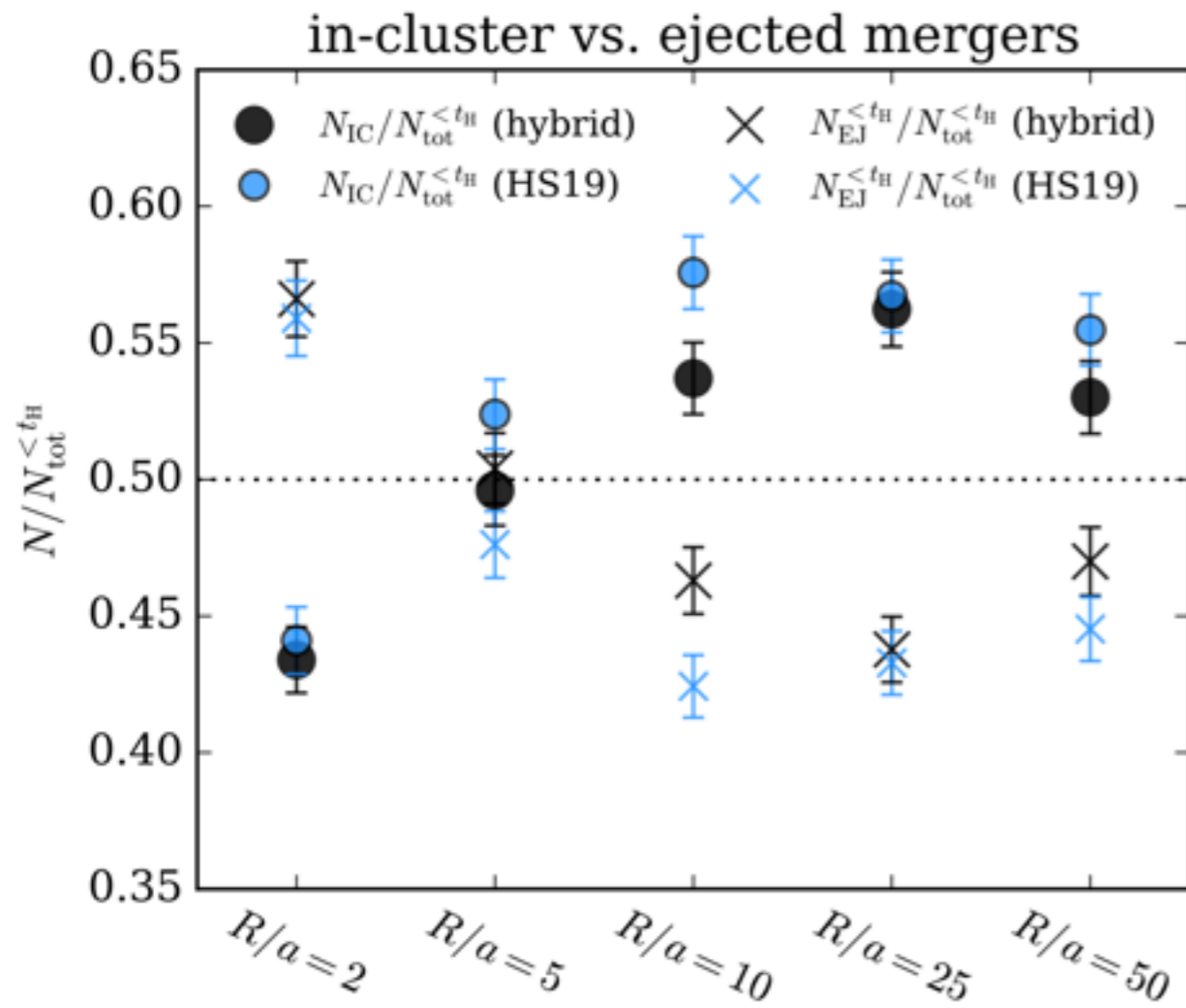
$$\gtrsim 10^3 \times \left(\frac{a_0}{0.5\text{AU}}\right)^{2/3} \left(\frac{m}{20M_\odot}\right)^{-2/3} \left(1 - (e_0/0.99)^2\right)^{2/3}$$



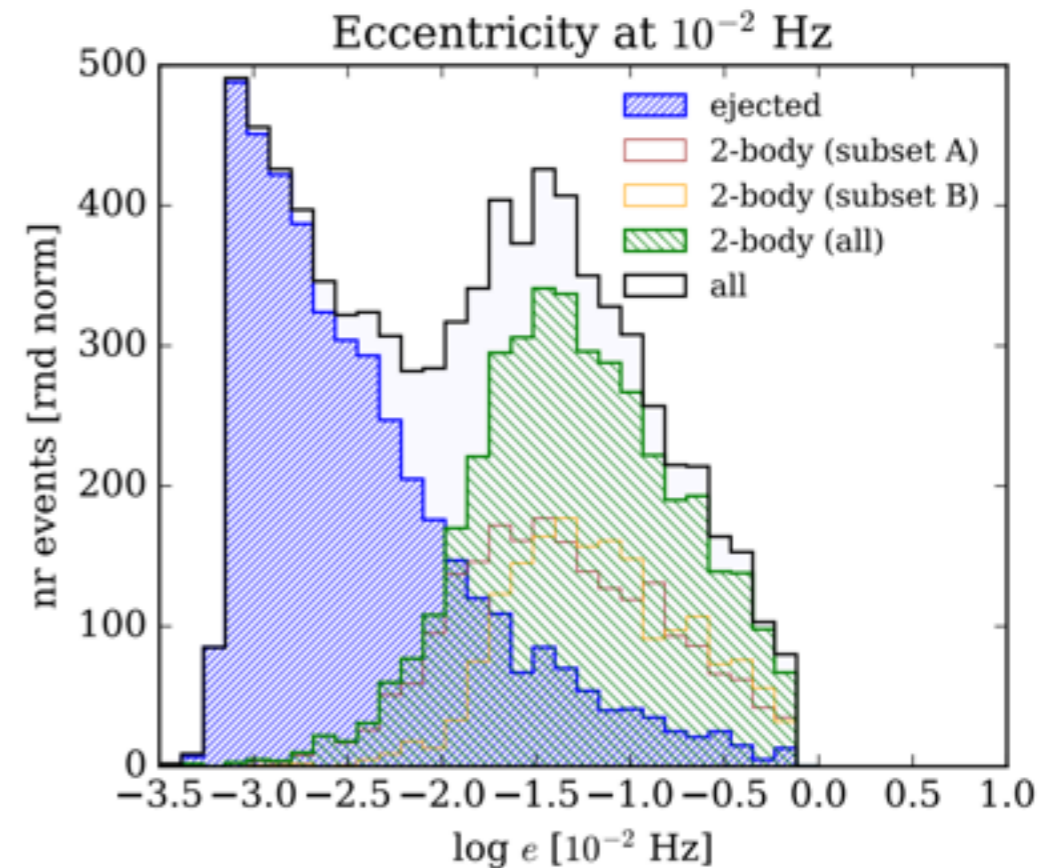
Merger Type: Secular-processes



Results from our MC code



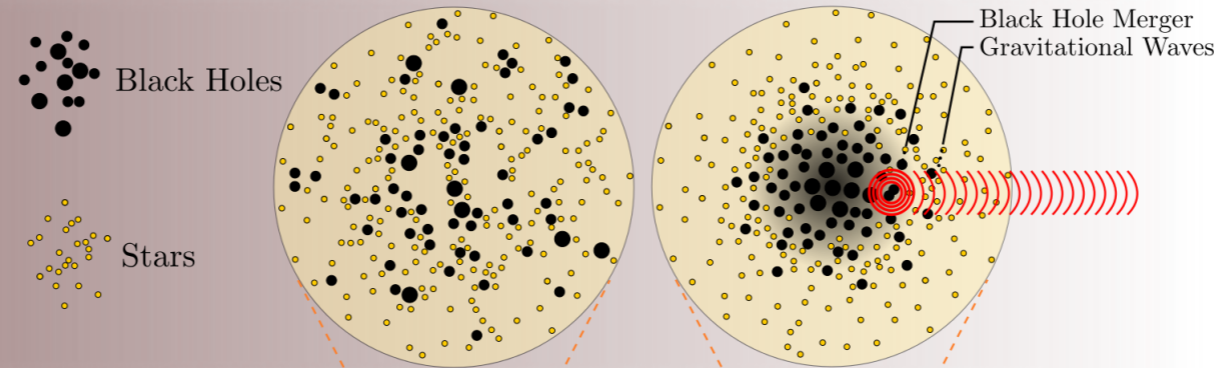
more BBHs are driven to merger!



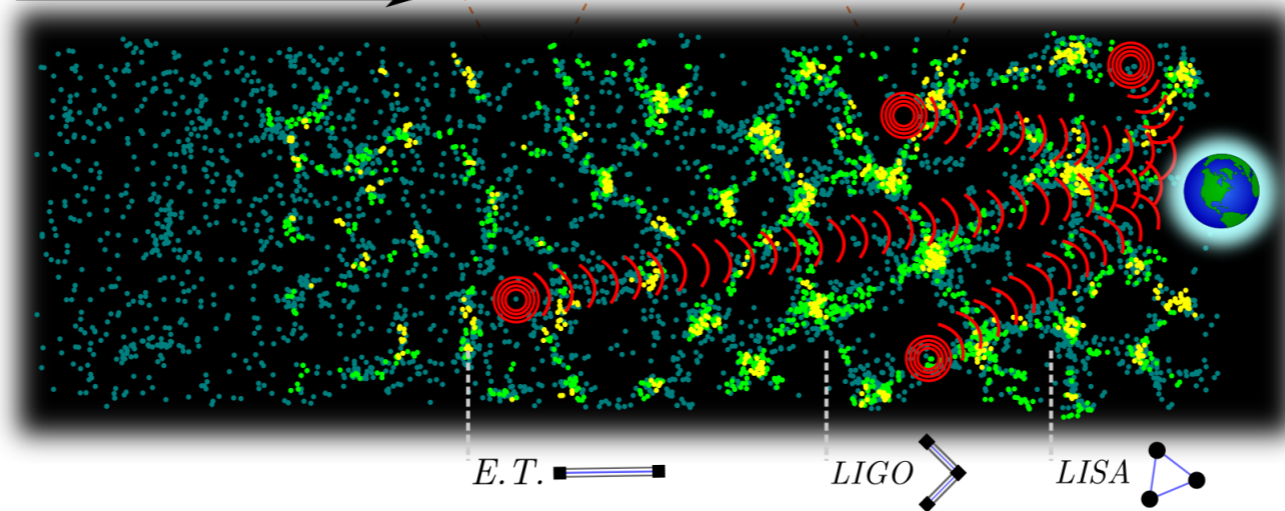
Future

Overview

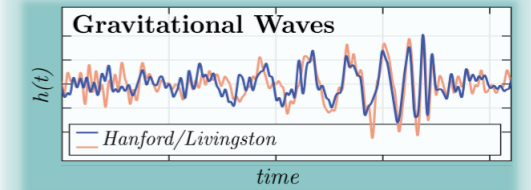
Cluster Evolution →



Cosmological Evolution →



Observables



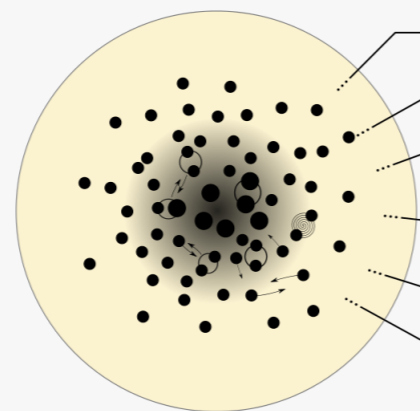
- Masses of the two BHs
- Spins of the two BHs
- Orbital Eccentricity
- Distance
- Perturbations, etc.

Implications

- Black Hole Formation
- Astrophysical Channels
- Relativistic Dynamics
- Tests of General Relativity
- Cosmological Evolution

Theoretical Modeling

Cluster Model

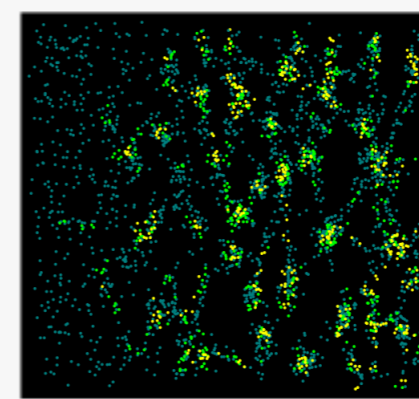


Interacting Components

- stellar 'fluid'
- single-single
- binary-single
- binary-binary
- GW evolution
- secular evolution

Evolve using Boltzmann eqs.

Cosmology Model



Interacting Components

- Dark Matter
- Stars + Gas

Evolve using N-body Codes



THANK YOU