TURBULENT PLASMA:
From Fusion Power Plants to Intergalactic Space and Back Again

Alexander Schekochihin
(Rudolf Peierls Centre for Theoretical Physics,
University of Oxford
& NBIA)

TURBULENT PLASMA:

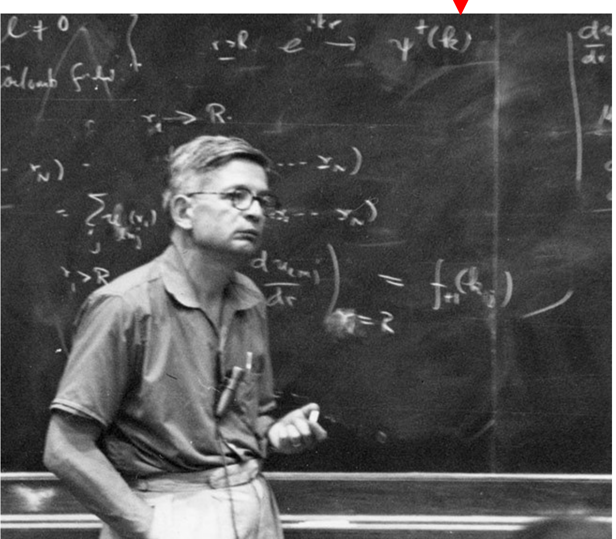
From Fusion Power Plants to Intergalactic Space and Back Again

Alexander Schekochihin
(Rudolf Peierls Centre for Theoretical Physics,
University of Oxford
& NBIA)

TURBULENT PLASMA:

From Fusion Power Plants to Intergalactic Space and Back Again

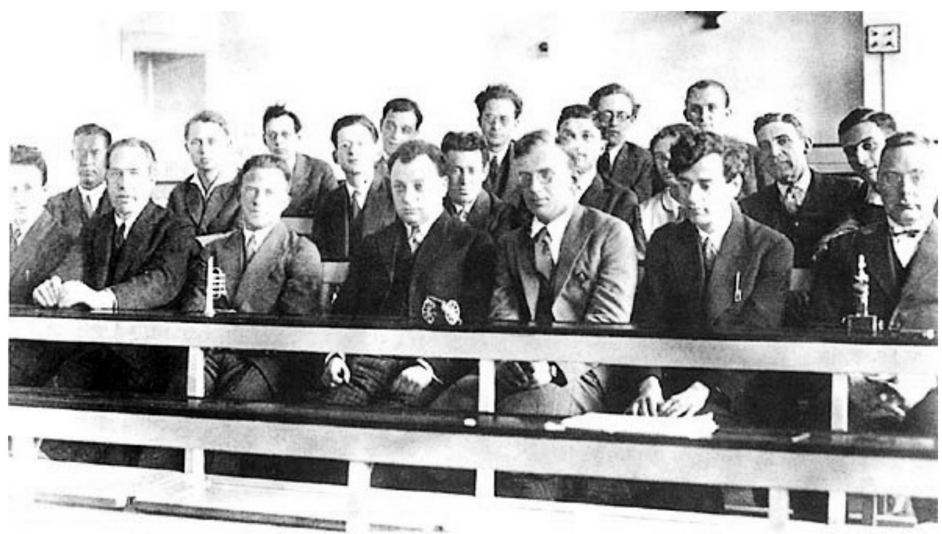
Alexander Schekochihin
(Rudolf Peierls Centre for Theoretical Physics,
University of Oxford
& NBIA)



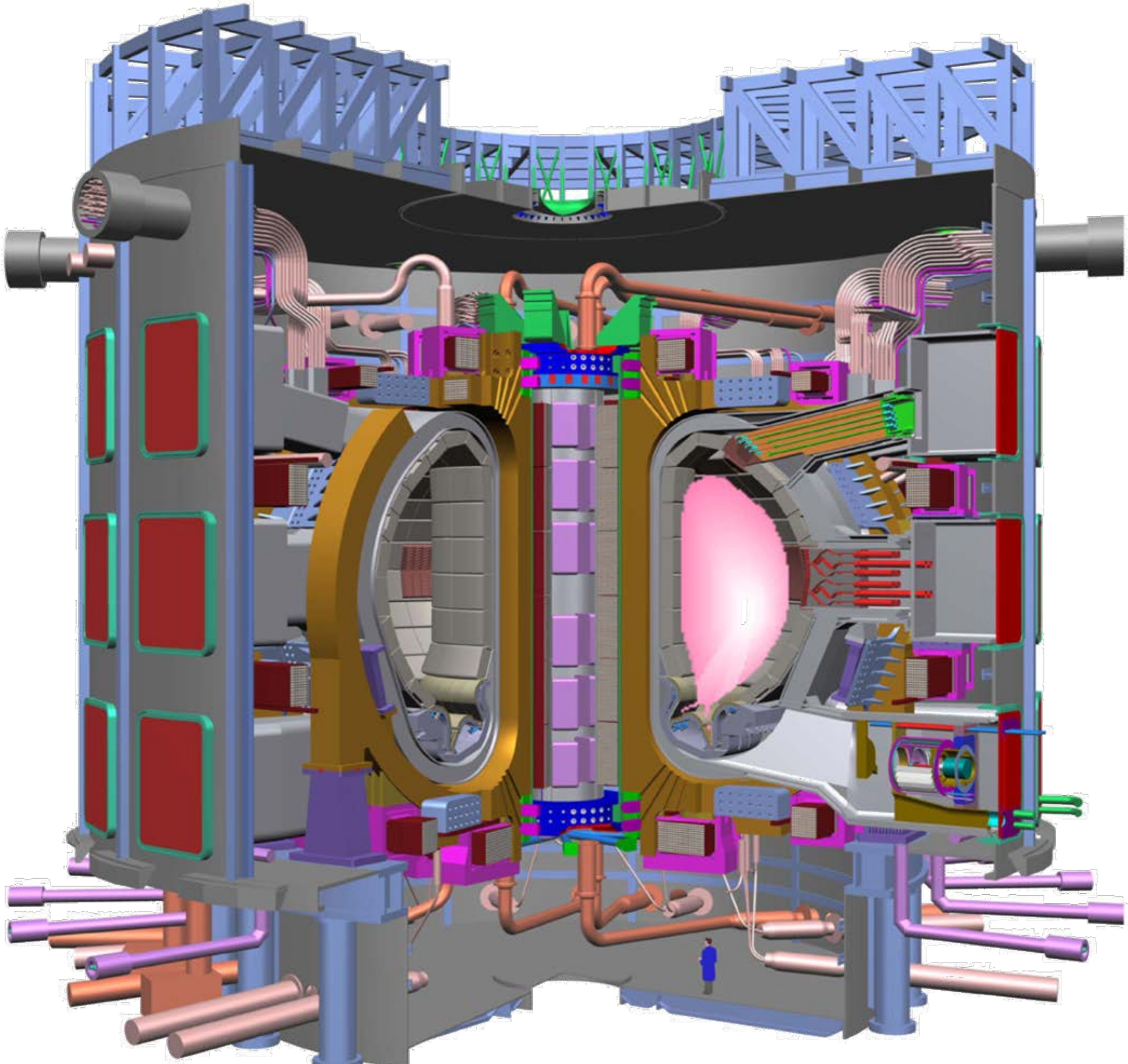
Credo

I am a ***theoretical physicist****.*

This means that I believe in the power of ***mathematical physics***
to predict ***reality***
and
in the ***universality*** *of certain essential features of that reality:*
i.e., that, in order to describe a given system,
we need not have a completely new theory every time,
but rather identify ***fundamental "building blocks" and principles***
that make up the system and govern its behavior.



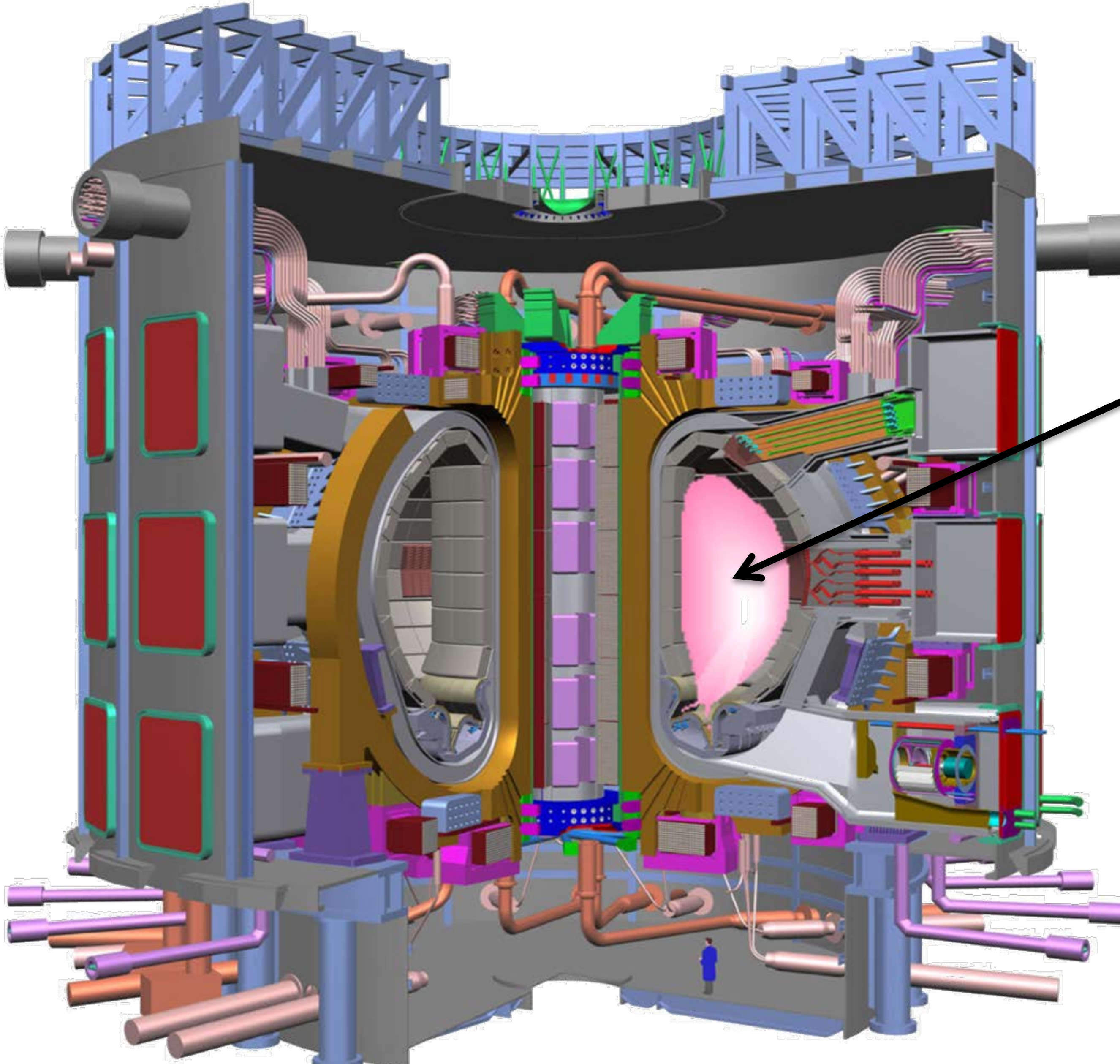
The System



Power can be extracted from **fusion** of hydrogen atoms.
For that, ionized hydrogen gas (**plasma**) must be held together and heated to high temperature.
It is held in a magnetic cage.
It doesn't like being held...

[Image: ITER]

The Cage

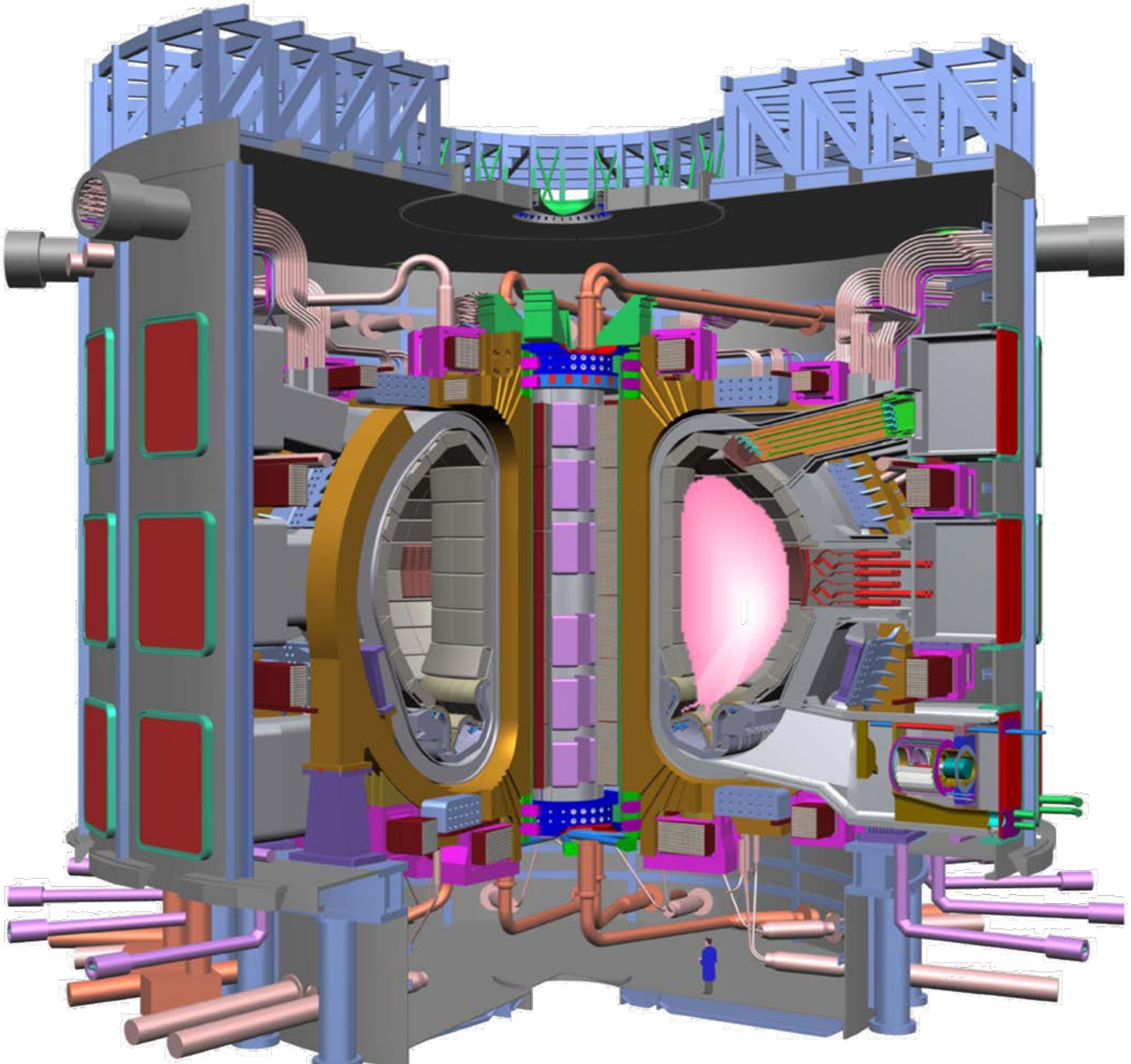


The Beast

It is held in a magnetic cage. It doesn't like being held...

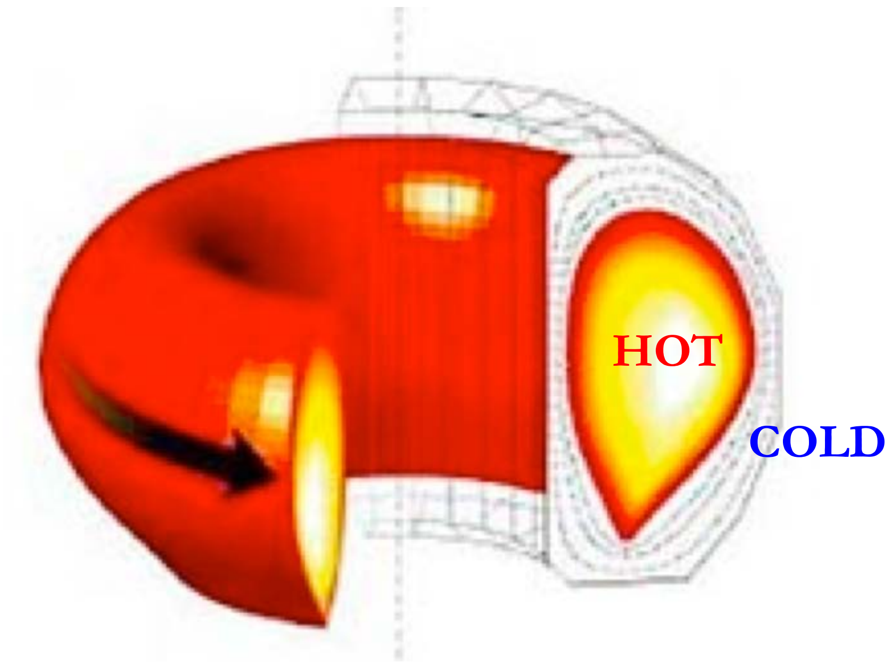
[Image: ITER]

Find Theoretical Physics Here!

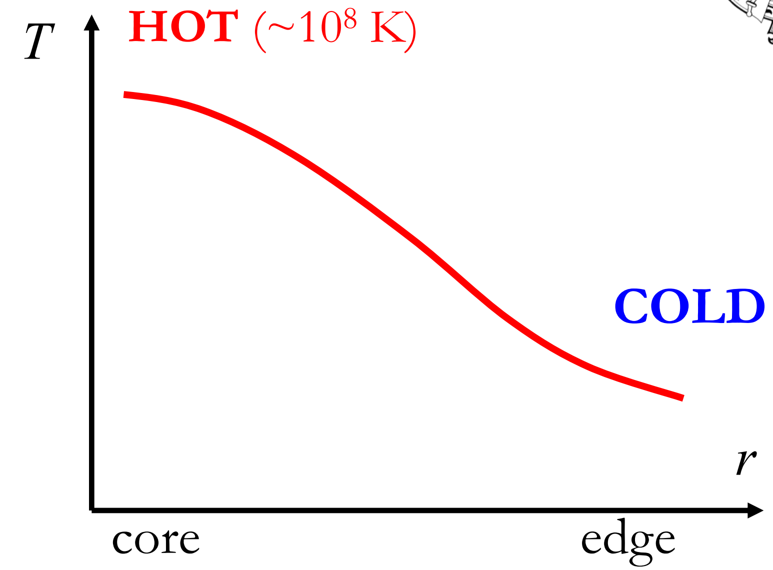


[Image: ITER]

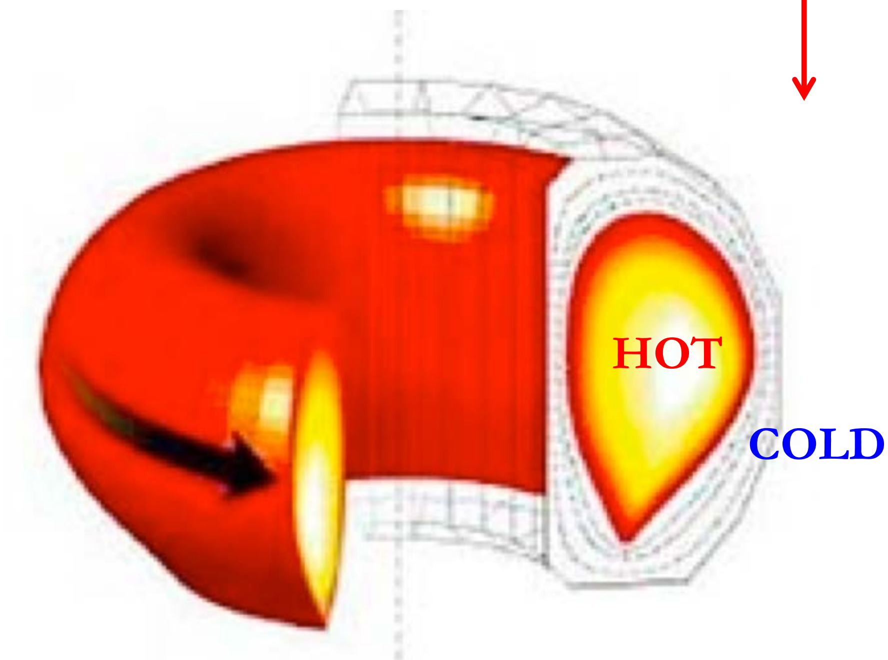
Find Theoretical Physics Here!



Find Theoretical Physics Here!



How hot does it get?
(How hot can we make it?)



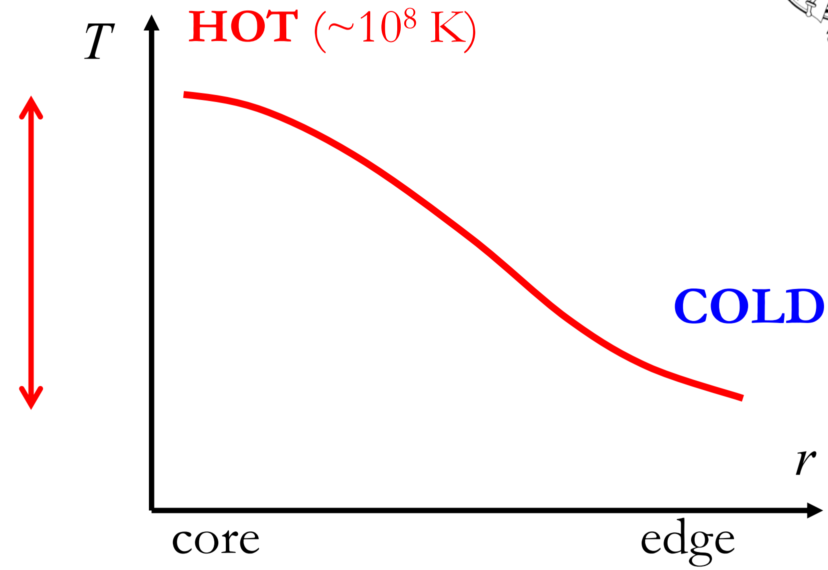
Undergraduate Physics: Heat Transport

How hot does it get?

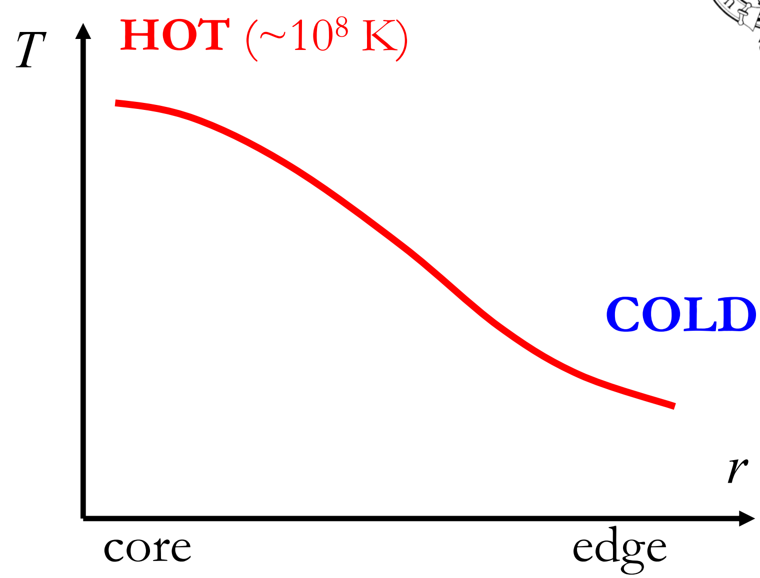
(How hot can we make it?)

2nd-year UG physics: heat equation

$$\frac{\partial T}{\partial t} = \underbrace{D}_{\text{heat diffusivity}}\Delta T + \underbrace{S}_{\text{sources – sinks (heating – cooling)}}$$



Undergraduate Physics: Heat Transport



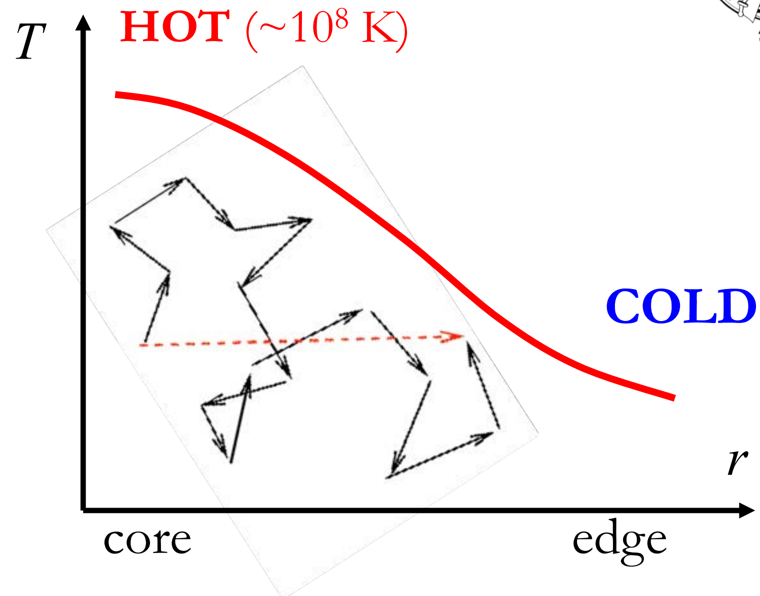
How hot does it get?
(How hot can we make it?)

2nd-year UG physics: heat equation

$\boxed{D\Delta T + S = 0}$ in steady state

Now solve this in a torus, knowing S and boundary conditions, get temperature profile $T(r)$, hand solution over to engineers, move on to thinking of dark matter, quantum entanglement, the brief history of time, etc…

Undergraduate Physics: Heat Transport



How hot does it get?
(How hot can we make it?)

2nd-year UG physics: heat equation

$$\boxed{D\Delta T + S = 0}$$ in steady state

Now solve this in a torus, knowing S and boundary conditions, get temperature profile $T(r)$, hand solution over to engineers, move on to thinking of dark matter, quantum entanglement, the brief history of time, etc…

Before we do that, what's D? It's a diffusion coefficient:

$$D \sim \frac{\langle \Delta x^2 \rangle}{\Delta t} \sim \langle v^2 \rangle \Delta t \sim c_s \lambda_{\mathrm{mfp}}$$ standard UG estimate

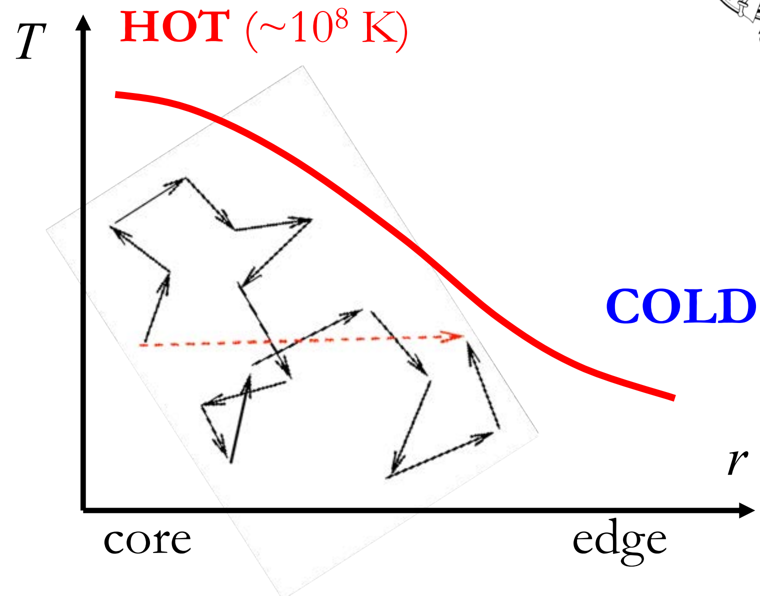
"random walk"

speed of sound

$v \sim c_s \sim (p/\rho)^{1/2}$

mean free path
between collisions

Undergraduate Physics: Heat Transport



How hot does it get?
(How hot can we make it?)

2nd-year UG physics: heat equation

$$\boxed{D\Delta T + S = 0}$$ in steady state

Now solve this in a torus, knowing *S* and boundary conditions, get temperature profile *T(r)*, hand solution over to engineers,
move on to thinking of dark matter, quantum entanglement, the brief history of time, etc…

Before we do that, what's *D?* It's a diffusion coefficient:

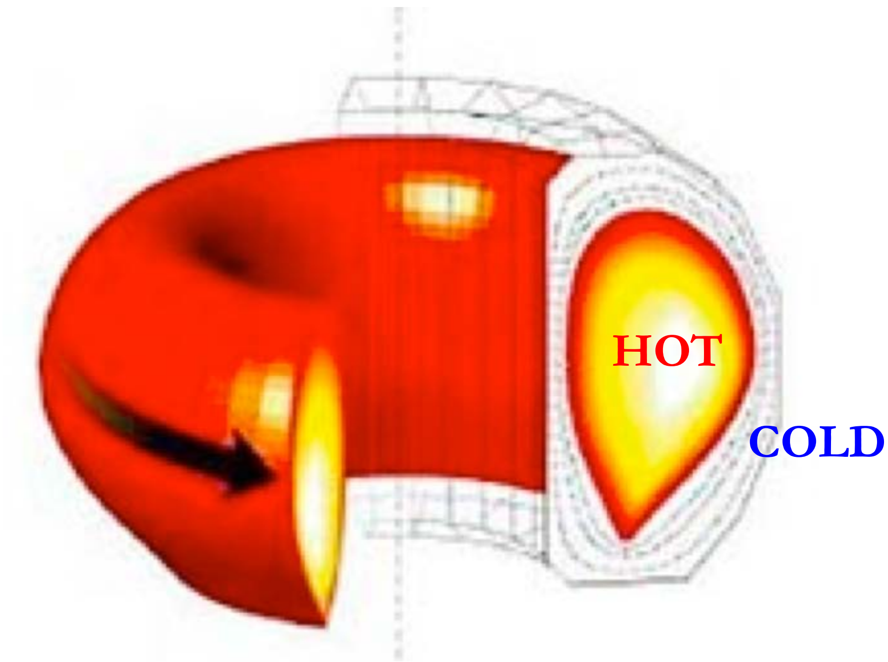
$$D \sim \frac{\langle \Delta x^2 \rangle}{\Delta t} \sim \frac{\rho_i^2}{\tau_c}$$ in a magnetised plasma

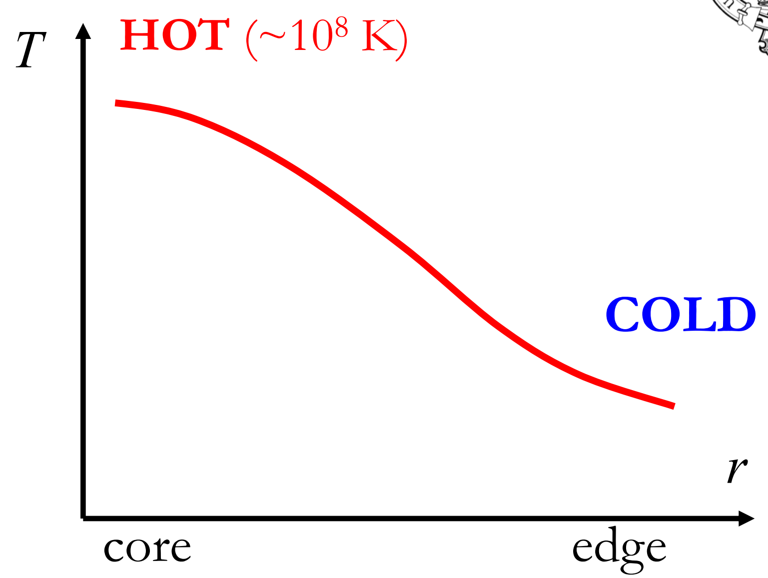
time between collisions $\tau_c \sim \lambda_{mfp}/c_s$

Larmor radius of the ions $\rho_i \sim c_s/\Omega_i$, $\Omega_i = eB/m_i c$

TOO SMALL TO EXPLAIN OBSERVED TRANSPORT!

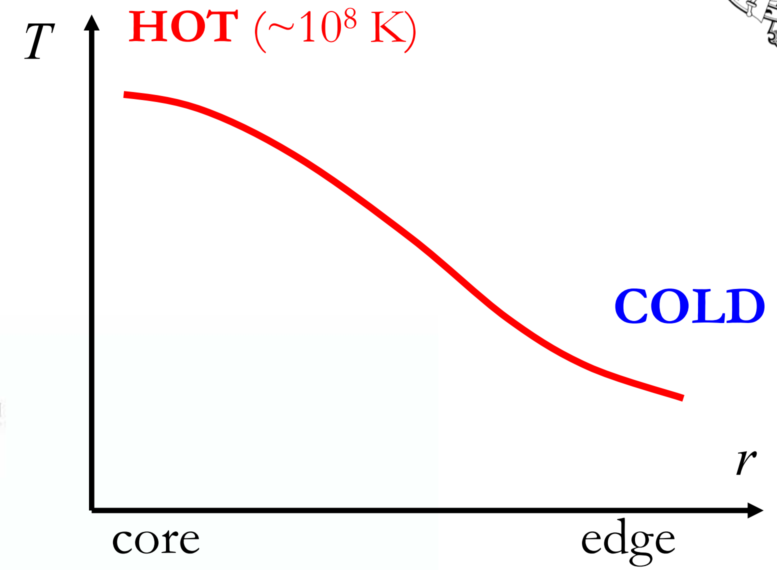
Look Closer…

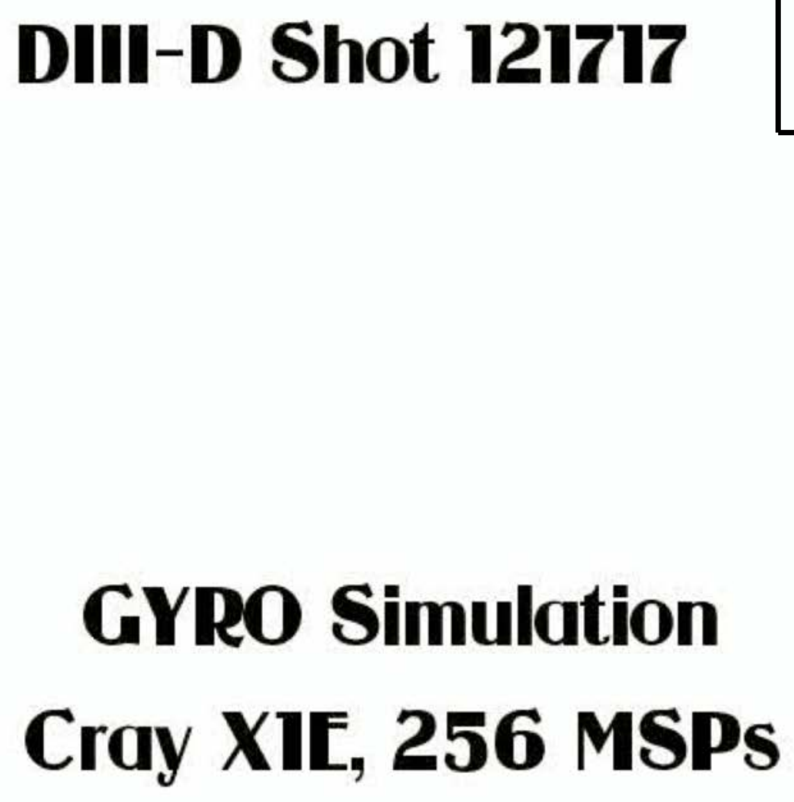




Look Closer…

Plasma in a tokamak is **turbulent** (nature dislikes gradients – lack of equilibrium! – and contrives to drive the system unstable)





Gyrokinetic simulation of the DIIID tokamak [R. Waltz & J. Candy, GA, San Diego]

Heat Diffusion + Turbulence

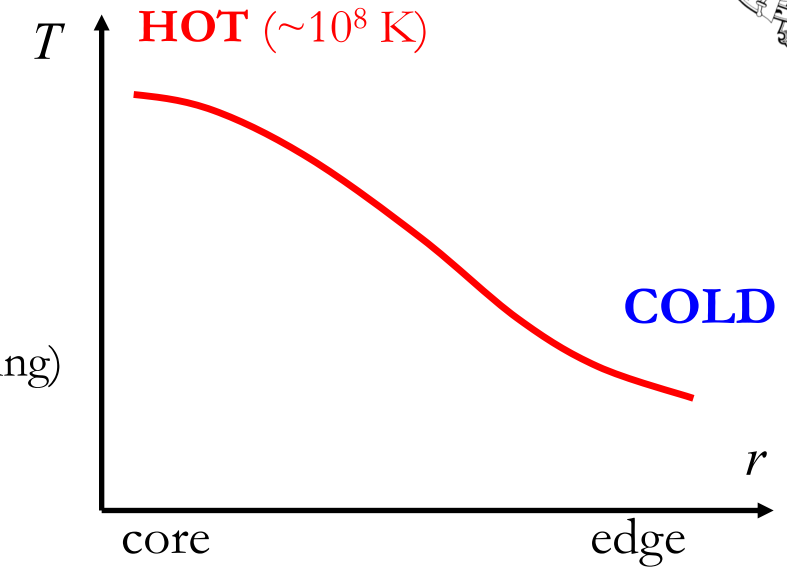
Heat equation in a moving medium:

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = D \Delta T + S$$

velocity of plasma motions (chaotic!)

heat diffusivity

sources – sinks (heating – cooling)



Heat Diffusion + Turbulence

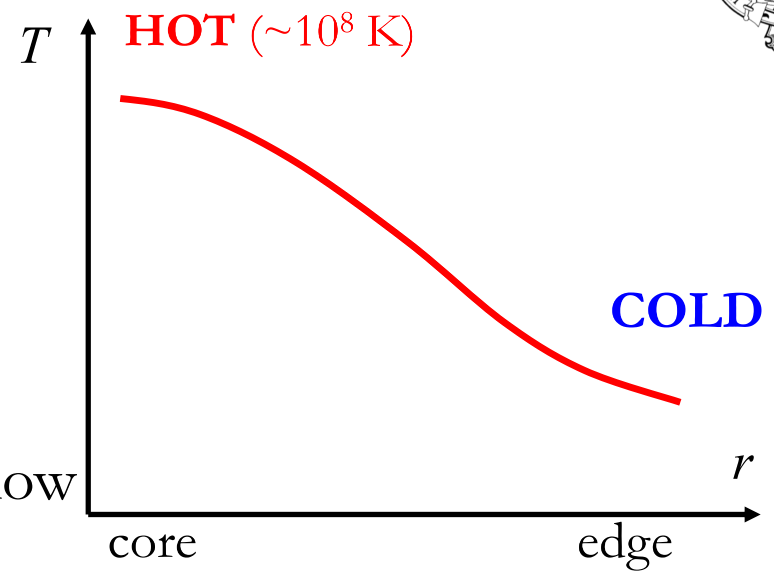
Heat equation in a moving medium:

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = D\Delta T + S$$

Mean profile: $\overline{T}(r) = \langle T\rangle$

$$T = \overline{T} + \delta T\,,\ \delta T \ll \overline{T}\,,$$

fluctuations are fast, mean quantities slow



Heat Diffusion + Turbulence

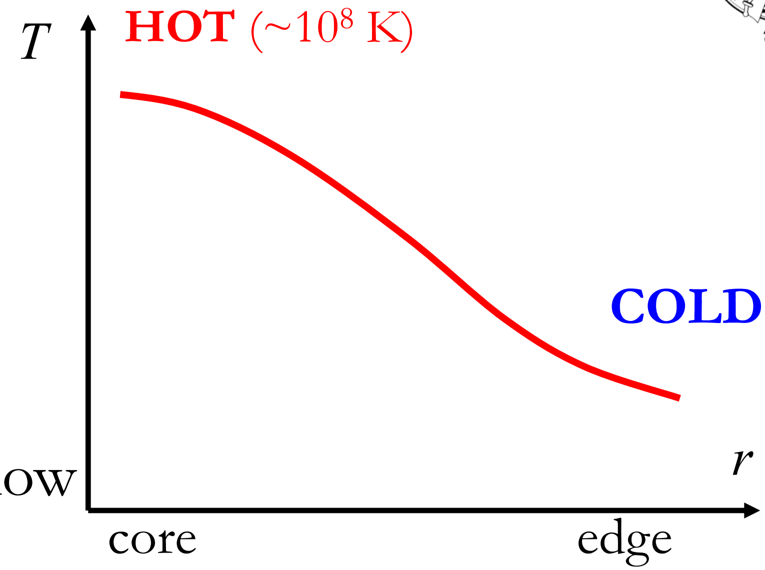
Heat equation in a moving medium:

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = D\Delta T + S$$

Mean profile: $\overline{T}(r) = \langle T\rangle$

$T = \overline{T} + \delta T\,,\ \delta T \ll \overline{T}\,,$

fluctuations are fast, mean quantities slow



Then, assuming $\nabla\cdot\mathbf{u} = 0$,

$$\frac{\partial \overline{T}}{\partial t} + \nabla\cdot\langle \mathbf{u}T\rangle = D\Delta\overline{T} + S$$

We need to know about fluctuations because

$\langle \mathbf{u}T\rangle = \langle\mathbf{u}\rangle\overline{T} + \langle \mathbf{u}\delta T\rangle = \langle \mathbf{u}\delta T\rangle$, assuming for now $\langle\mathbf{u}\rangle = 0$

Heat Diffusion + Turbulence

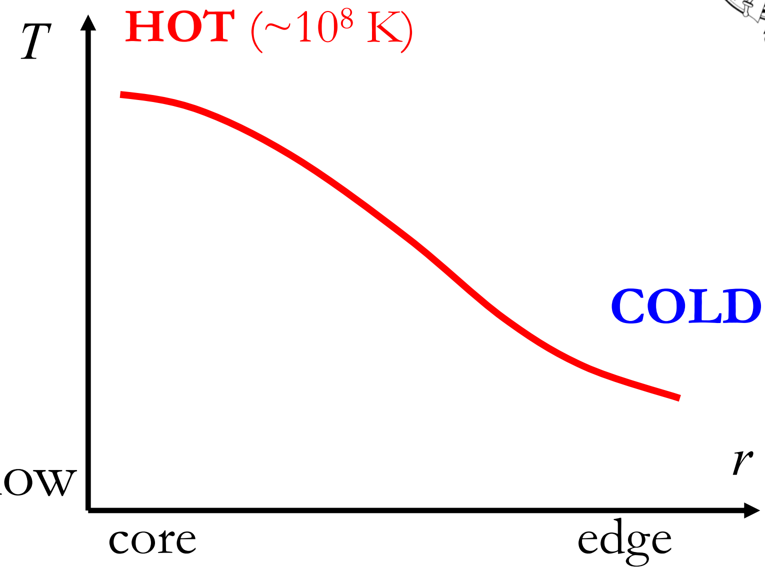
Heat equation in a moving medium:

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = D\Delta T + S$$

Mean profile: $\overline{T}(r) = \langle T\rangle$

$T = \overline{T} + \delta T\,,\ \delta T \ll \overline{T}\,,$

fluctuations are fast, mean quantities slow



Then, assuming $\nabla\cdot\mathbf{u} = 0$,

$$\frac{\partial \overline{T}}{\partial t} + \nabla\cdot\langle\mathbf{u}T\rangle = D\Delta\overline{T} + S$$

$$\langle\mathbf{u}(t)T(t)\rangle = \left\langle \mathbf{u}(t)\int_0^t dt'\,[-\mathbf{u}(t')\cdot\nabla \underbrace{T(t')}_{\approx\overline{T}(t)} + D\Delta \underbrace{T(t')}_{\approx\overline{T}(t)} + \underbrace{S(t')}_{\approx S(t)}]\right\rangle$$

Heat Diffusion + Turbulence

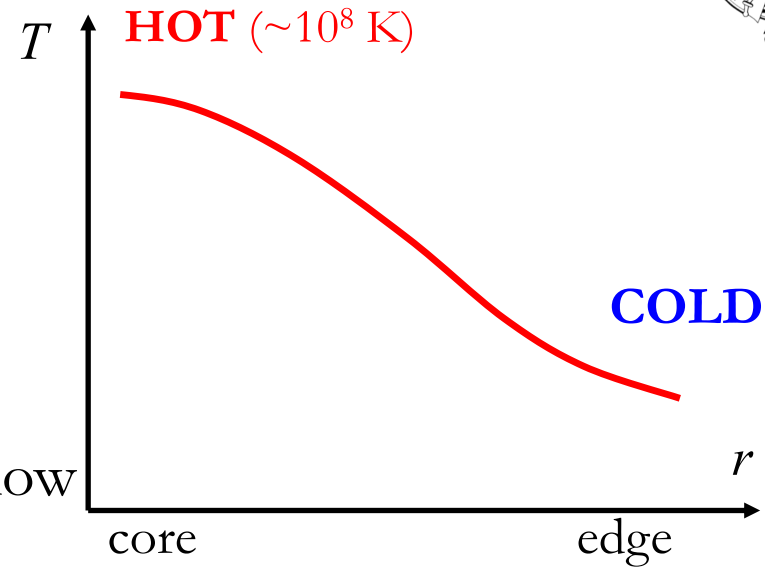
Heat equation in a moving medium:

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = D\Delta T + S$$

Mean profile: $\overline{T}(r) = \langle T \rangle$

$T = \overline{T} + \delta T\,,\ \delta T \ll \overline{T}\,,$

fluctuations are fast, mean quantities slow



Then, assuming $\nabla\cdot\mathbf{u} = 0$,

$$\frac{\partial \overline{T}}{\partial t} + \nabla\cdot\langle \mathbf{u}T\rangle = D\Delta\overline{T} + S$$

$$\langle \mathbf{u}(t)T(t)\rangle = \left\langle \mathbf{u}(t)\int_0^t dt'\,\Big[-\mathbf{u}(t')\cdot\underset{\approx\overline{T}(t)}{\nabla T(t')} + \underset{\approx\overline{T}(t)}{D\Delta T(t')} + \underset{\approx S(t)}{S(t')}\Big]\right\rangle$$

$$\langle \mathbf{u}(t)T(t)\rangle \approx -\left[\int_0^t dt'\,\langle \mathbf{u}(t)\mathbf{u}(t')\rangle\right]\cdot\nabla\overline{T}(t)$$ “turbulent heat flux”

Heat Diffusion + Turbulence

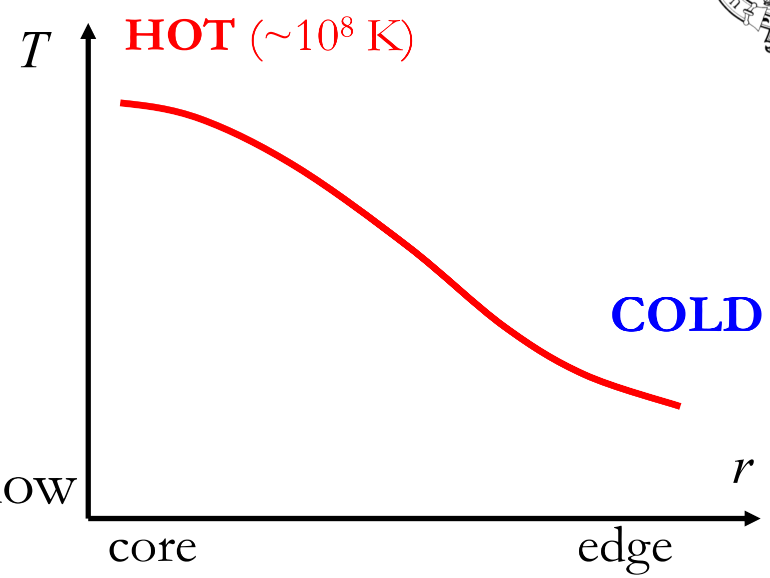
Heat equation in a moving medium:

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = D\Delta T + S$$

Mean profile: $\overline{T}(r) = \langle T\rangle$

$T = \overline{T} + \delta T\,,\ \delta T \ll \overline{T}\,,$

fluctuations are fast, mean quantities slow



Then, assuming $\nabla\cdot\mathbf{u} = 0$,

$$\frac{\partial \overline{T}}{\partial t} + \nabla\cdot\langle\mathbf{u}T\rangle = D\Delta\overline{T} + S$$

$$\frac{\partial \overline{T}}{\partial t} \approx \nabla\cdot\left[\int_0^t dt'\,\langle\mathbf{u}(t)\mathbf{u}(t')\rangle\right]\cdot\nabla\overline{T} + D\Delta\overline{T} + S$$

$$\langle\mathbf{u}(t)T(t)\rangle \approx -\left[\int_0^t dt'\,\langle\mathbf{u}(t)\mathbf{u}(t')\rangle\right]\cdot\nabla\overline{T}(t)$$ “turbulent heat flux”

Turbulent Transport

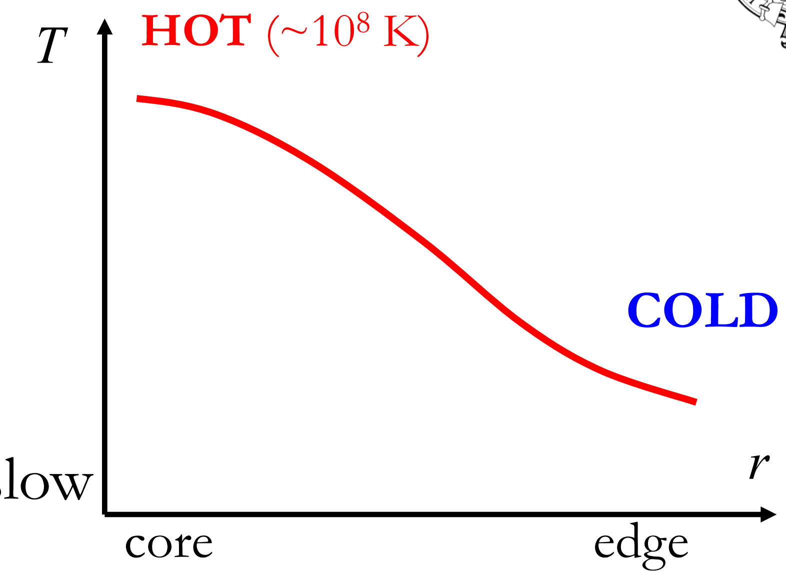
Heat equation in a moving medium:

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = D\Delta T + S$$

Mean profile: $\overline{T}(r) = \langle T \rangle$

$$T = \overline{T} + \delta T\,,\ \delta T \ll \overline{T}\,,$$

fluctuations are fast, mean quantities slow



Then, assuming $\nabla\cdot\mathbf{u} = 0$,

$$\frac{\partial \overline{T}}{\partial t} + \nabla\cdot\langle \mathbf{u}T\rangle = D\Delta\overline{T} + S$$

$$\frac{\partial \overline{T}}{\partial t} \approx \nabla\cdot\left[\int_0^t dt'\langle \mathbf{u}(t)\mathbf{u}(t')\rangle\right]\cdot\nabla\overline{T} + D\Delta\overline{T} + S$$

$$= \frac{1}{r}\frac{\partial}{\partial r} r\left[D^{(\mathrm{turb})} + D\right]\frac{\partial \overline{T}}{\partial r} + S$$

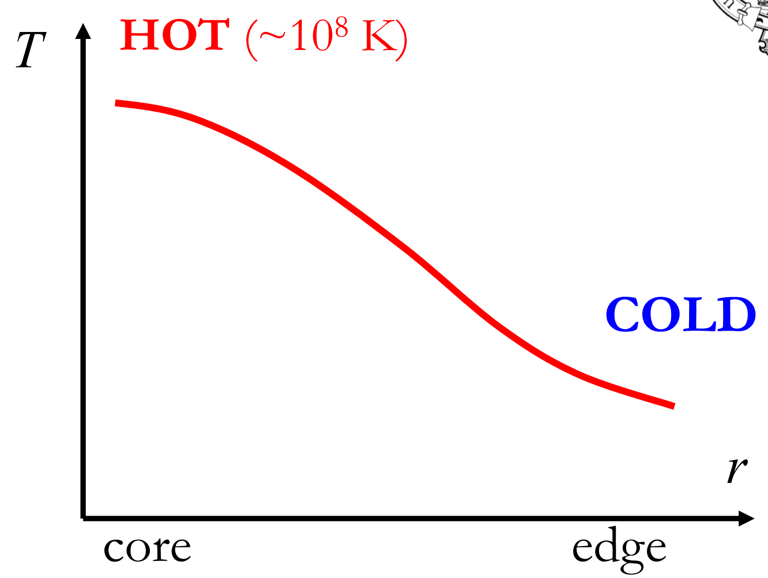
"turbulent diffusion" $D^{(\mathrm{turb})} = \int_0^t dt'\langle u_r(t)u_r(t')\rangle$

Turbulent Transport

So the "effective mean field theory" for our system looks like this:

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D \right] \frac{\partial \overline{T}}{\partial r} + S$$

$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle$$

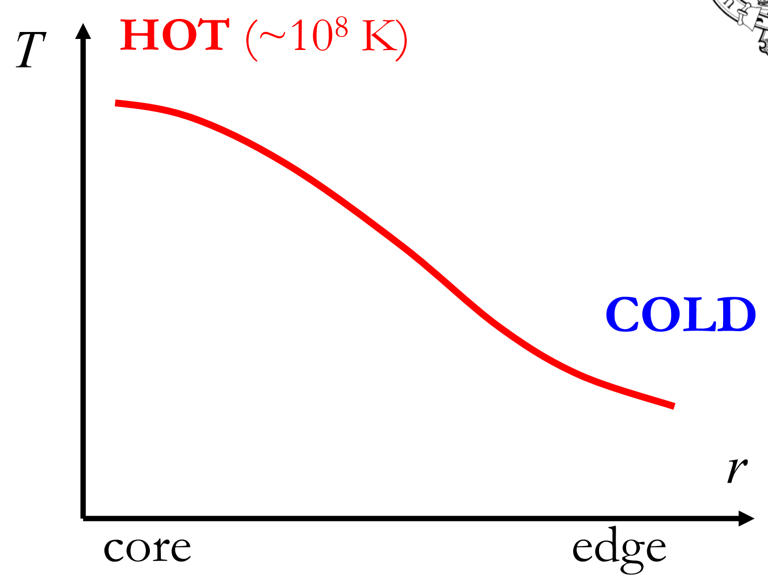


Turbulent Transport

So the "effective mean field theory" for our system looks like this:

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D \right] \frac{\partial \overline{T}}{\partial r} + S$$

$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle$$



These ideas are universal: e.g., if you are a (plasma) astrophysicist, you know that the largest plasma objects are clusters of galaxies (containing mostly dark matter and hot, diffuse plasma, not galaxies):

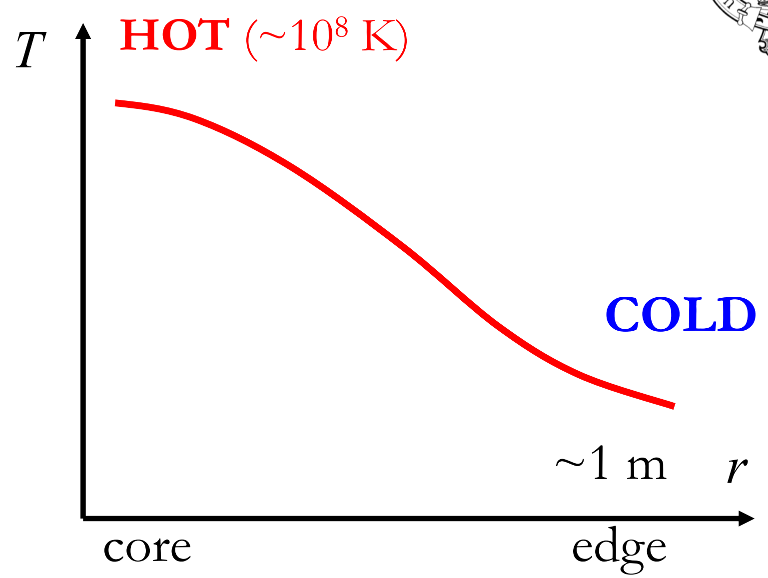
(Abell 262
in optical,
http://www.atlasoftheuniverse.com/superc/perpsc.html)

Turbulent Transport

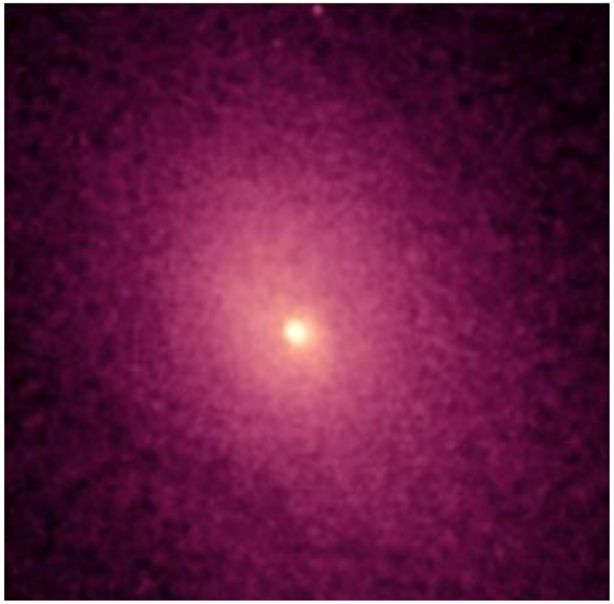
So the "effective mean field theory" for our system looks like this:

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D\right] \frac{\partial \overline{T}}{\partial r} + S$$

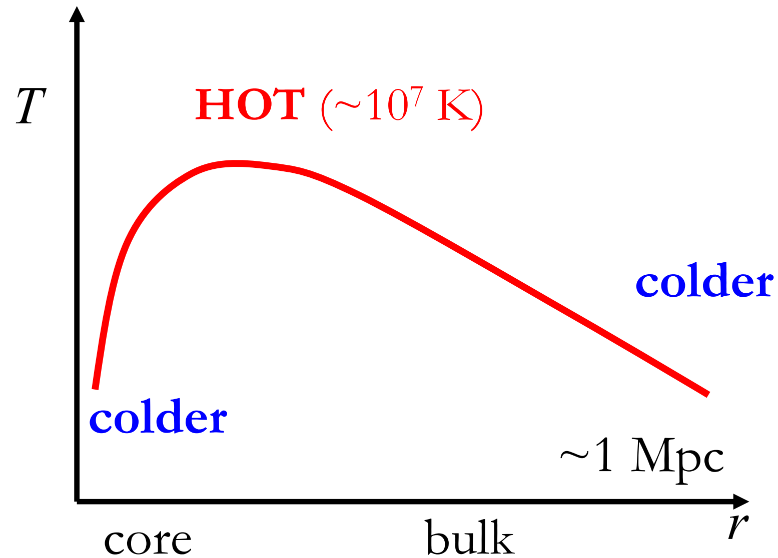
$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle$$



These ideas are universal: e.g., if you are a (plasma) astrophysicist, you know that the largest plasma objects are clusters of galaxies (containing mostly dark matter and hot, diffuse plasma, not galaxies):



(Abell 2019 in X-ray, Image: Chandra)

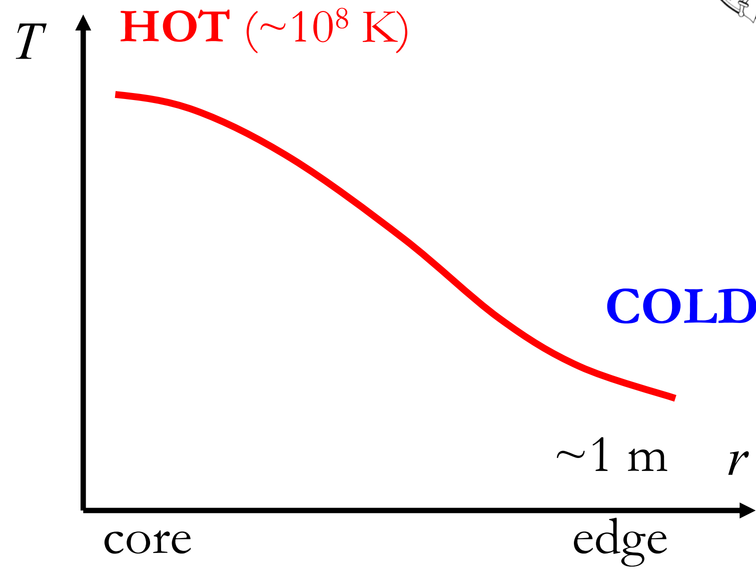


Turbulent Transport

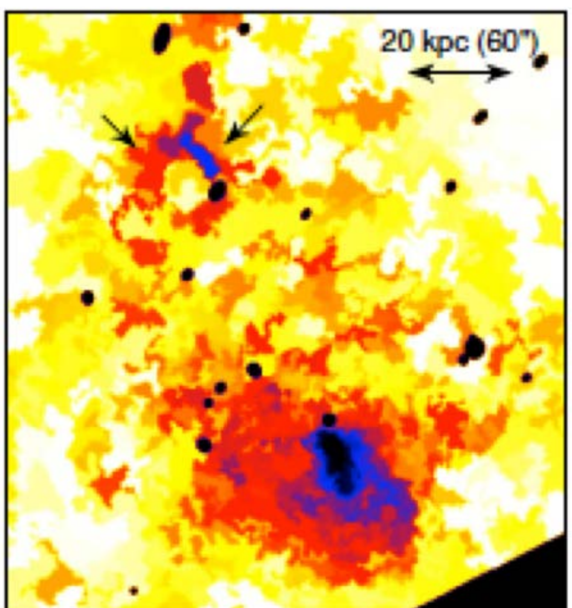
So the "effective mean field theory" for our system looks like this:

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D \right] \frac{\partial \overline{T}}{\partial r} + S$$

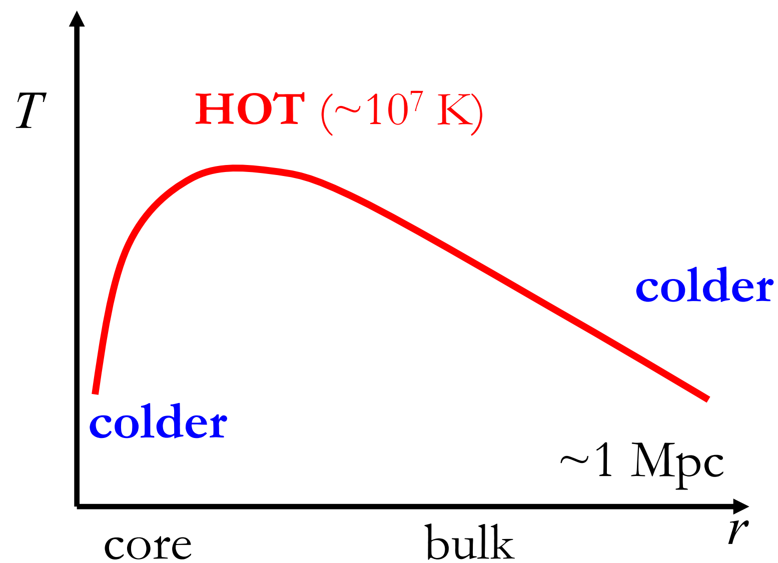
$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle$$



These ideas are universal: e.g., if you are a (plasma) astrophysicist, you know that the largest plasma objects are clusters of galaxies (containing mostly dark matter and hot, diffuse plasma, not galaxies):



(temperature fluctuation map of Abell 262, J. Sanders et al. 2009)

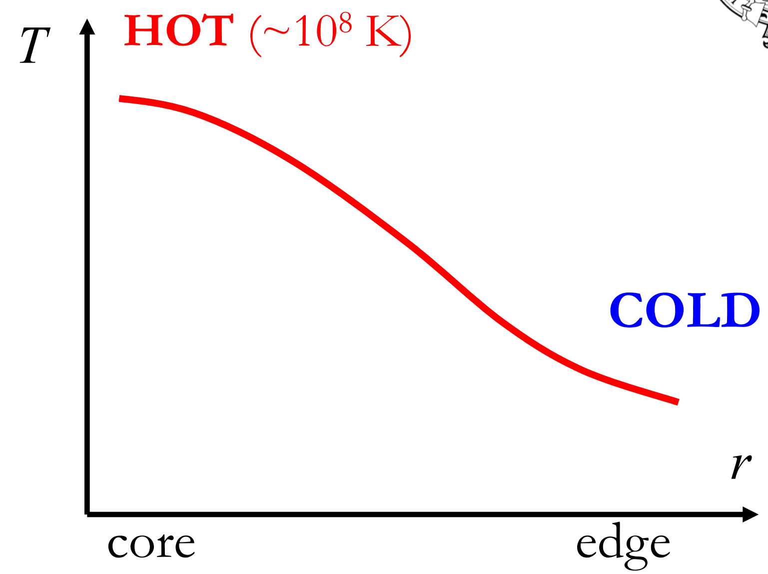


Turbulent Transport

So the "effective mean field theory" for our system looks like this:

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D \right] \frac{\partial \overline{T}}{\partial r} + S$$

$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle \gg D$$



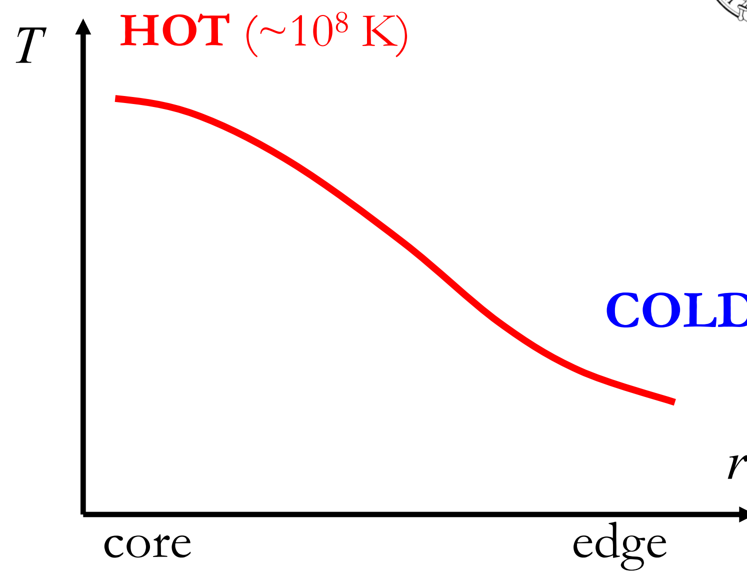
Turbulent transport is much faster than collisional: nature is impatient and will not wait for slow collisions to relax the system to equilibrium!

Turbulent Transport

So the "effective mean field theory" for our system looks like this:

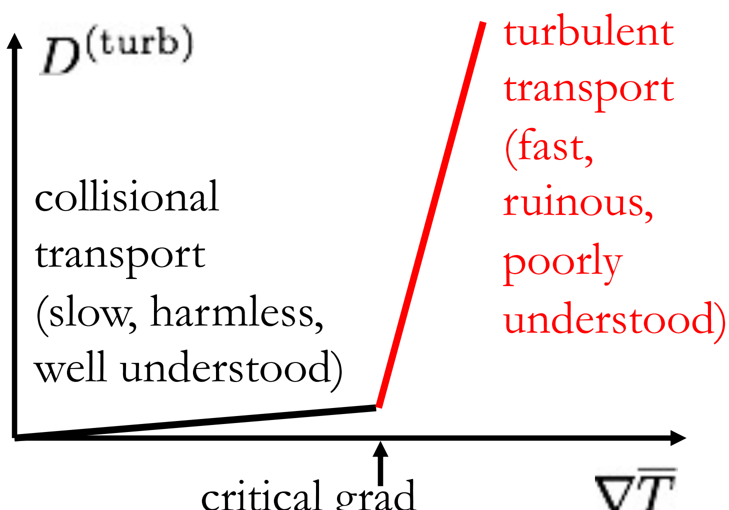
$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D \right] \frac{\partial \overline{T}}{\partial r} + S$$

$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle \gg D$$



Turbulent transport is much faster than collisional: nature is impatient and will not wait for slow collisions to relax the system to equilibrium!

We want to be able to predict $D^{(\mathrm{turb})}$ as a function of everything:
local equilibrium quantities (e.g., $\nabla \overline{T}$),
configuration of the magnetic cage,
energy and momentum inputs…

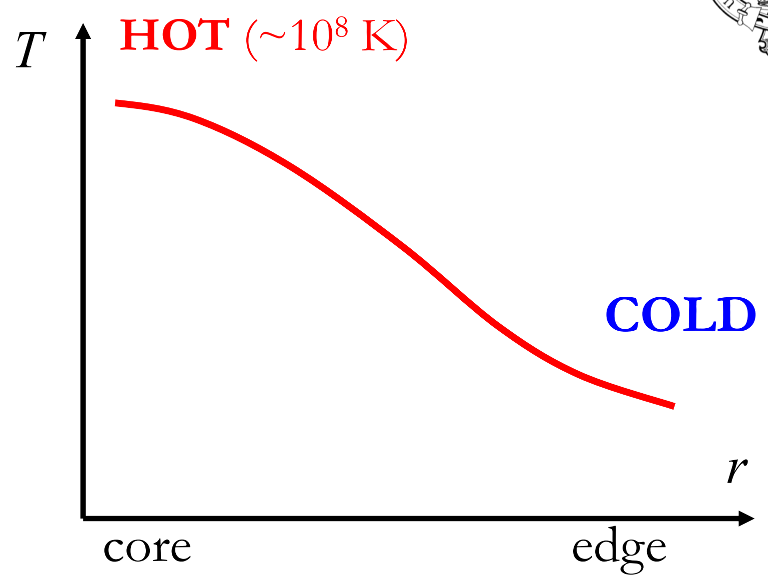


Turbulent Transport

So the "effective mean field theory" for our system looks like this:

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D\right] \frac{\partial \overline{T}}{\partial r} + S$$

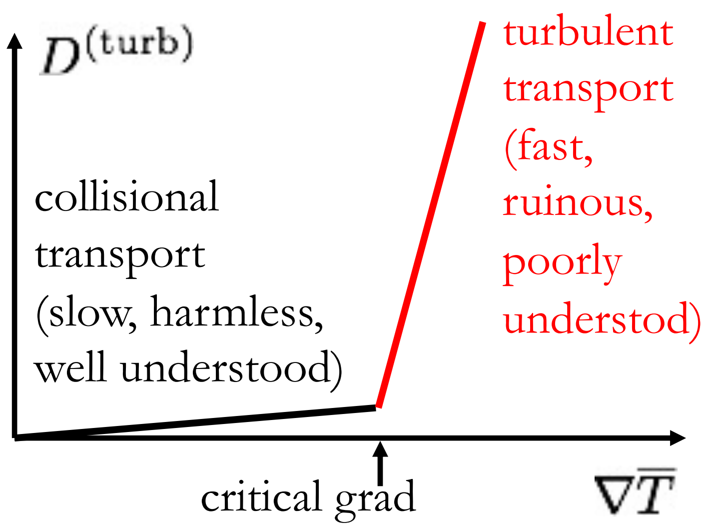
$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle \gg D$$



Turbulent transport is much faster than collisional: nature is impatient and will not wait for slow collisions to relax the system to equilibrium!

So turbulence is the enemy.
In order to kill it,
we must understand it

(also because it's a challenge and we must meet it to keep our self-respect as a species)

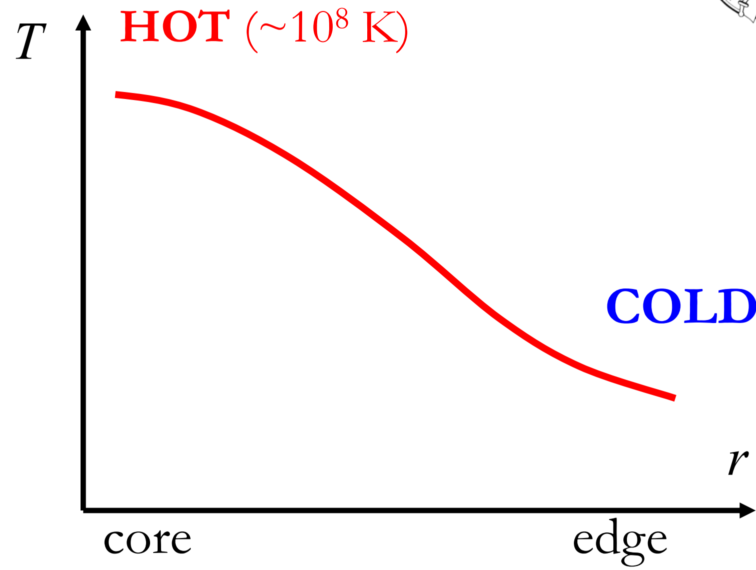


Turbulent Transport

So the "effective mean field theory" for our system looks like this:

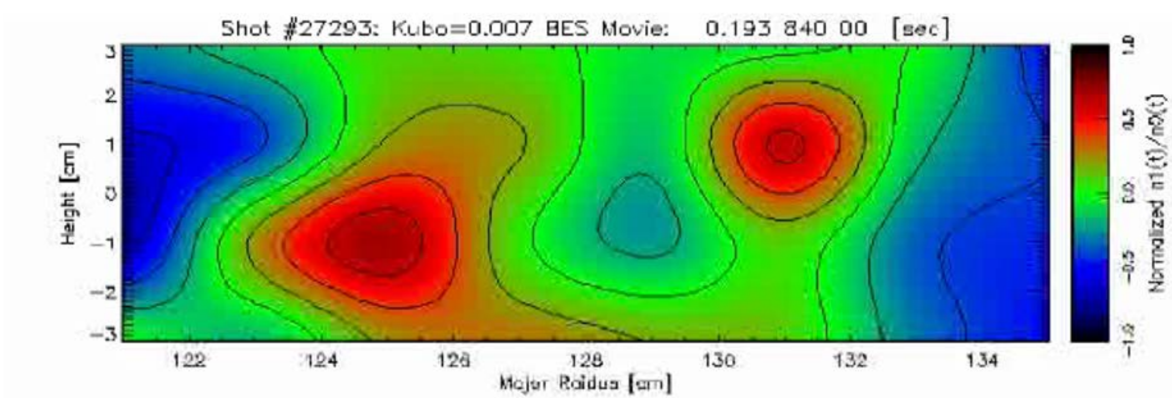
$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D \right] \frac{\partial \overline{T}}{\partial r} + S$$

$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle \gg D$$



Turbulent transport is much faster than collisional: nature is impatient and will not wait for slow collisions to relax the system to equilibrium!

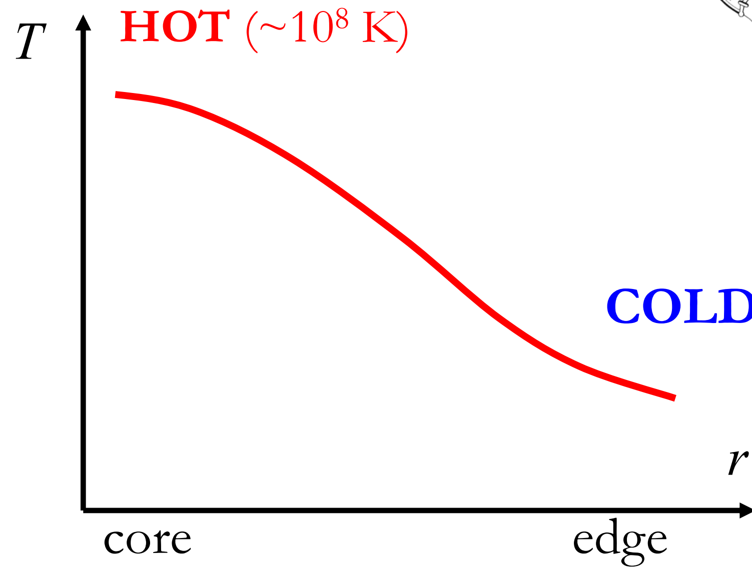
So turbulence is the enemy.
In order to kill it,
we must understand it
(also because it's a challenge and we must meet it to keep our self-respect as a species)



BES image of density fluctuations in MAST
[Movie: Y.-c. Ghim, Oxford]

Turbulent Transport

So the "effective mean field theory" for our system looks like this:



$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D\right] \frac{\partial \overline{T}}{\partial r} + S$$

$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle \gg D$$

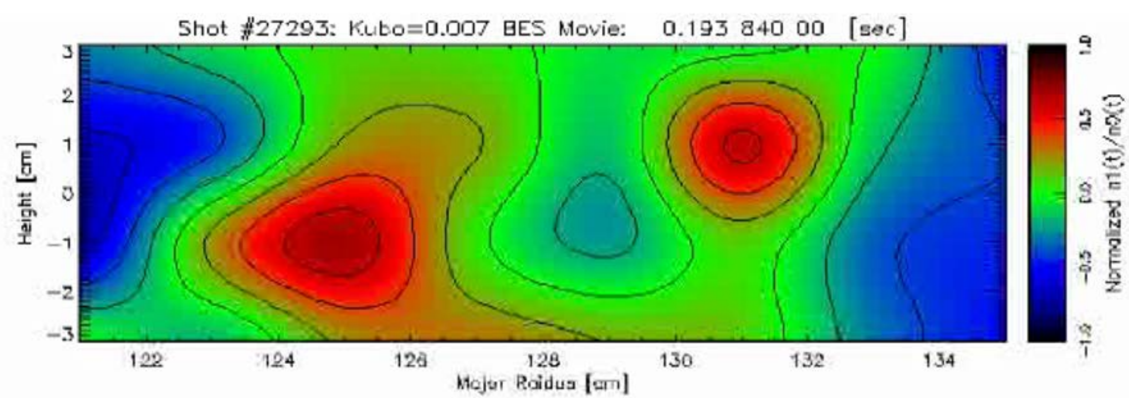
$$D^{(\mathrm{turb})} \sim u^2 \tau_{\mathrm{corr}}$$

$$\tau_{\mathrm{corr}} \sim \frac{\ell}{u}$$

ℓ ← eddy size

u ← eddy turnover velocity

τ_{corr}: eddy turnover time



BES image of density fluctuations in MAST
[Movie: Y.-c. Ghim, Oxford]

Turbulent Transport

So the "effective mean field theory" for our system looks like this:

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left[D^{(\mathrm{turb})} + D\right] \frac{\partial \overline{T}}{\partial r} + S$$

$$D^{(\mathrm{turb})} = \int_0^t dt' \langle u_r(t) u_r(t') \rangle \gg D$$

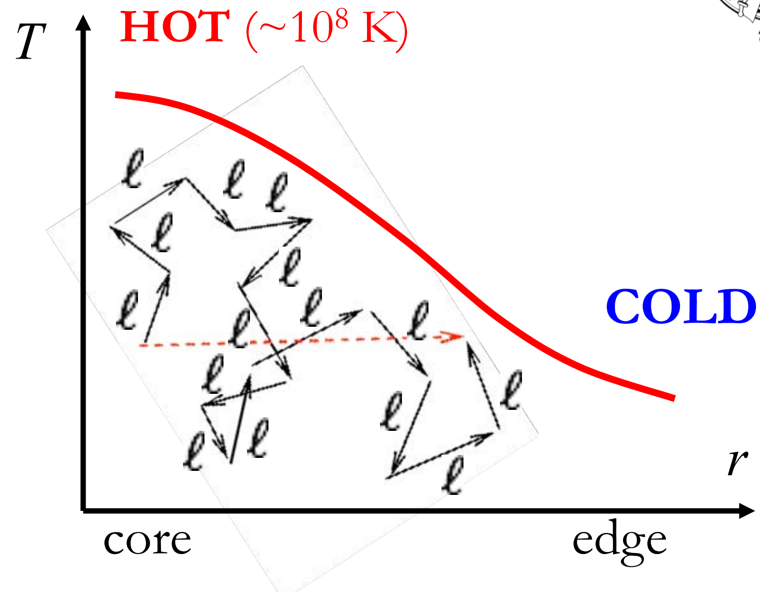
$$D^{(\mathrm{turb})} \sim u^2 \tau_{\mathrm{corr}} \sim \frac{\ell^2}{\tau_{\mathrm{corr}}} \sim u\ell$$

$$\tau_{\mathrm{corr}} \sim \frac{\ell}{u}$$

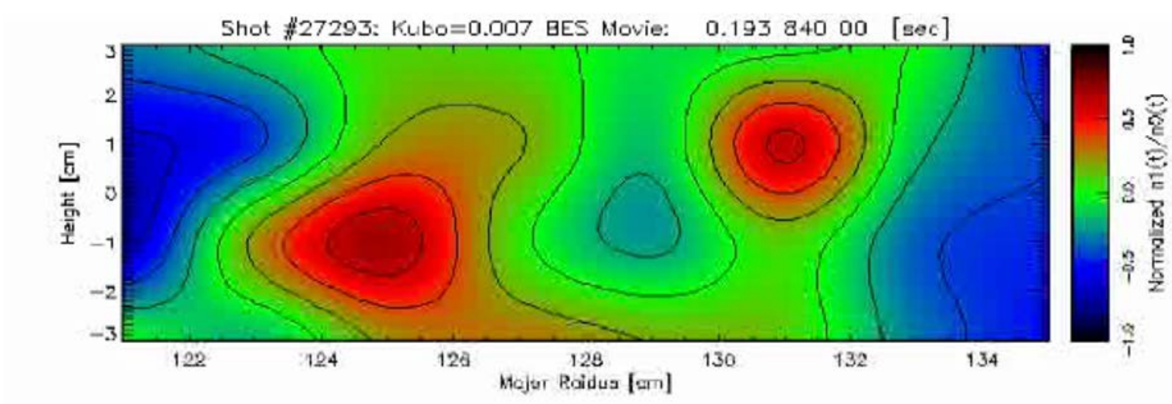
ℓ ← eddy size

u ← eddy turnover velocity

τ_{corr} ↑ eddy turnover time



random walk again, but now particles carrying energy hop from eddy to eddy



BES image of density fluctuations in MAST
[Movie: Y.-c. Ghim, Oxford]

Turbulence

I have started drawing on some notions to do with the nature of ***turbulence****.*

I shall now attempt a very basic and non-rigorous
Introduction to Turbulence…

La turbolenza (how it all started)

Leonardo da Vinci
(1452-1519)

La turbolenza (how it all started)

"Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion."

Leonardo da Vinci
(1452-1519)

La turbolenza (how it all started)

"Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion."

Leonardo da Vinci
(1452-1519)

La turbolenza (how it all started)

"Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion."

Leonardo da Vinci
(1452-1519)

La turbolenza (how it all started)

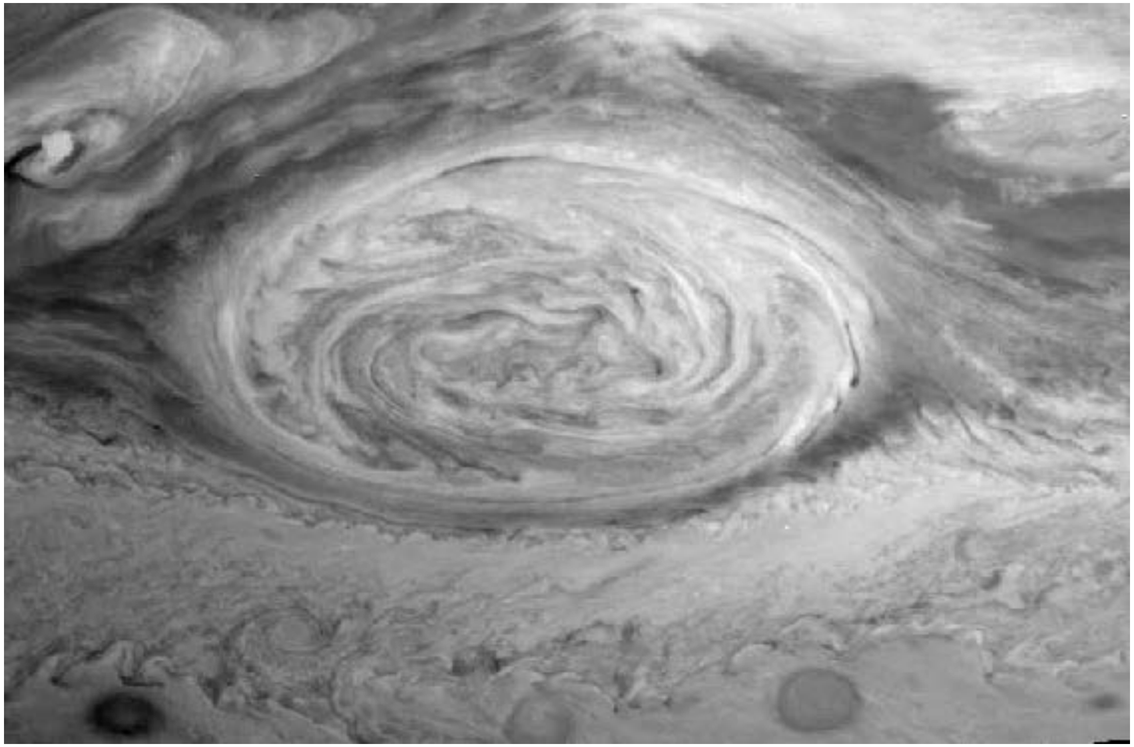
So, the basic idea is that a mean, laminar flow breaks up into disordered eddy-like motions

A Universal Phenomenon…

So, the basic idea is that a mean, laminar flow breaks up into disordered eddy-like motions

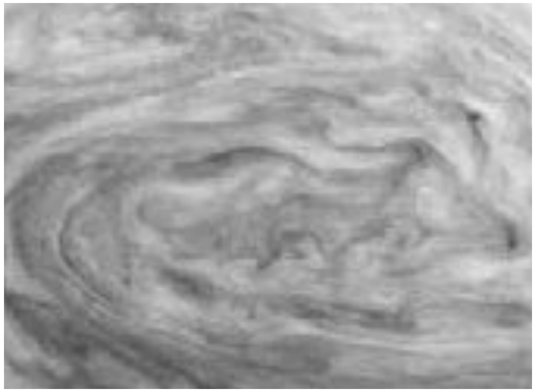
Turbulence in the wake of
Virgin Atlantic Airbus A340 descending to LHR
[Image: Greg Bajor on flickr, 2011]

A Universal Phenomenon…

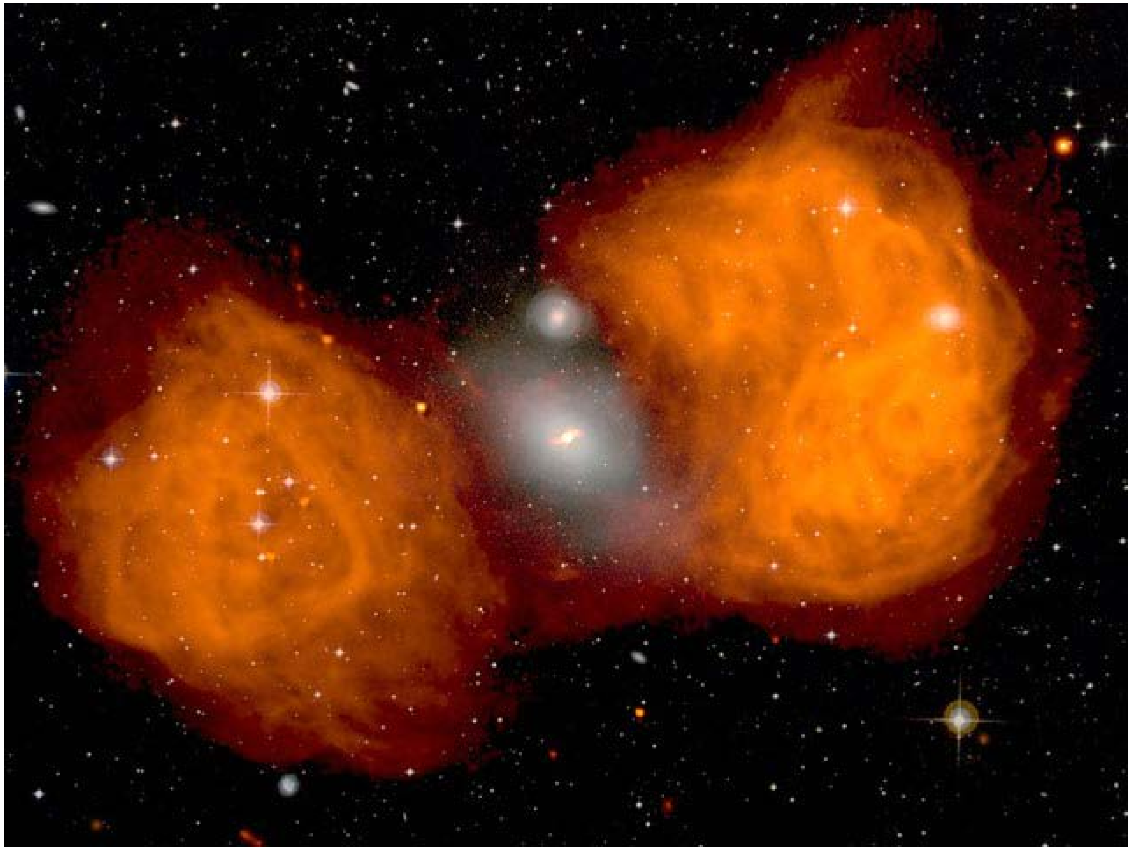


The Great Red Spot of Jupiter
[Image: Galileo, near-infrared (756 nm), 26 June 1996]

So, the basic idea is that a mean, laminar flow breaks up into disordered eddy-like motions



A Universal Phenomenon…



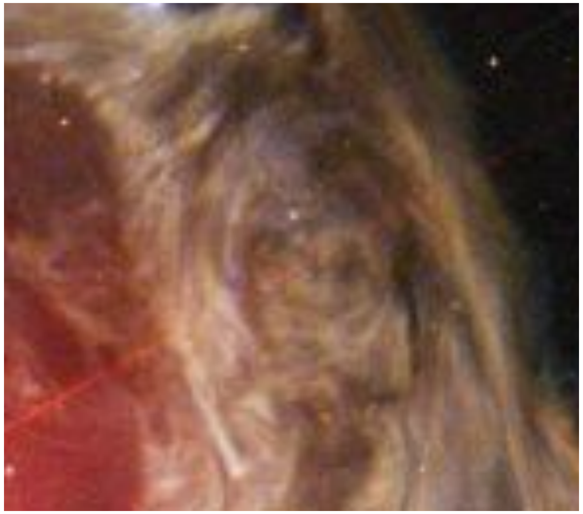
Radio Lobes of Fornax A (10^6 light years across)
[Image: Ed Fomalont (NRAO) et al.,
VLA, NRAO, AUI, NSF]

So, the basic idea is that a mean, laminar flow breaks up into disordered eddy-like motions

A Universal Phenomenon…

V838 Monocerotis, 20 000 light years away
[Image: Hubble, February 2004]

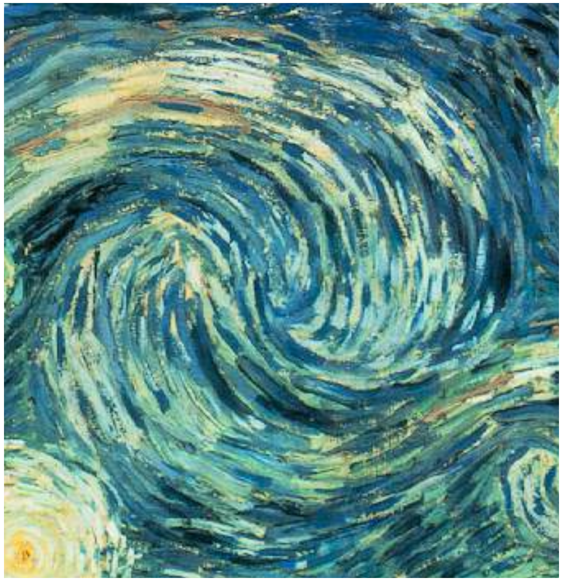
So, the basic idea is that a mean, laminar flow breaks up into disordered eddy-like motions



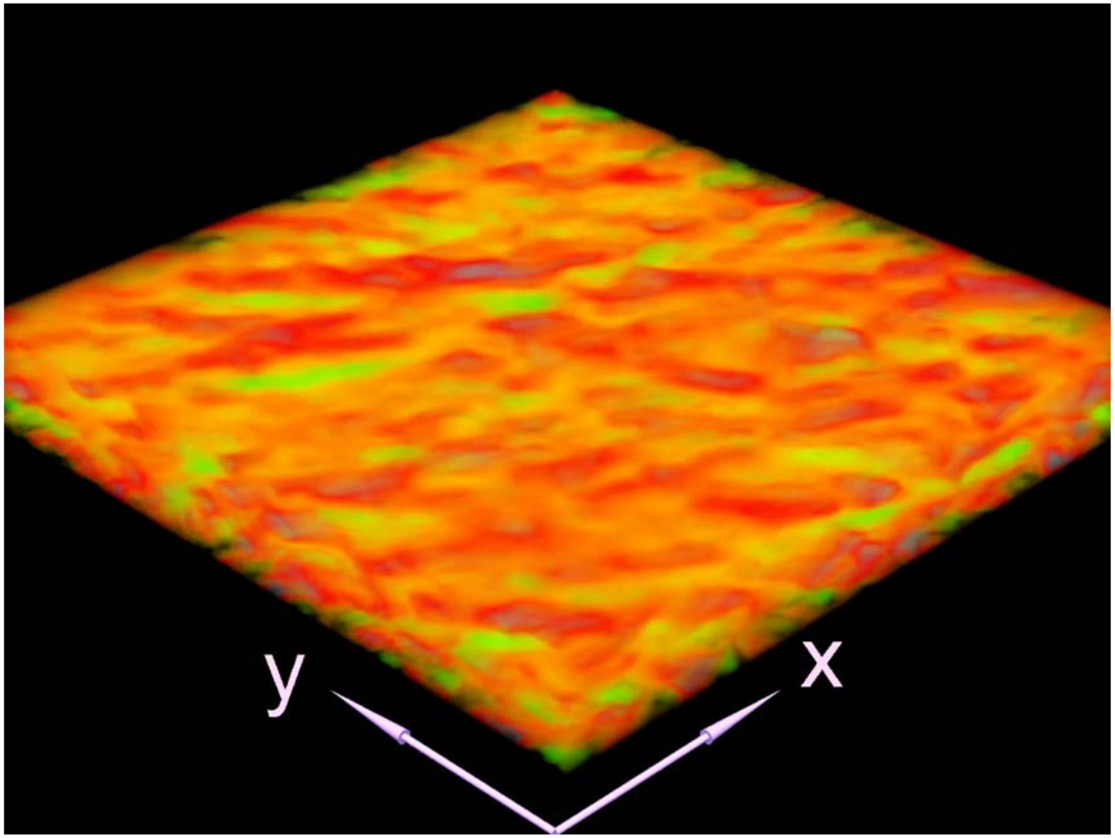
A Universal Phenomenon…

V. Van Gogh, *The Starry Night*, June 1889
(MoMA, NY)

So, the basic idea is that a mean, laminar flow breaks up into disordered eddy-like motions



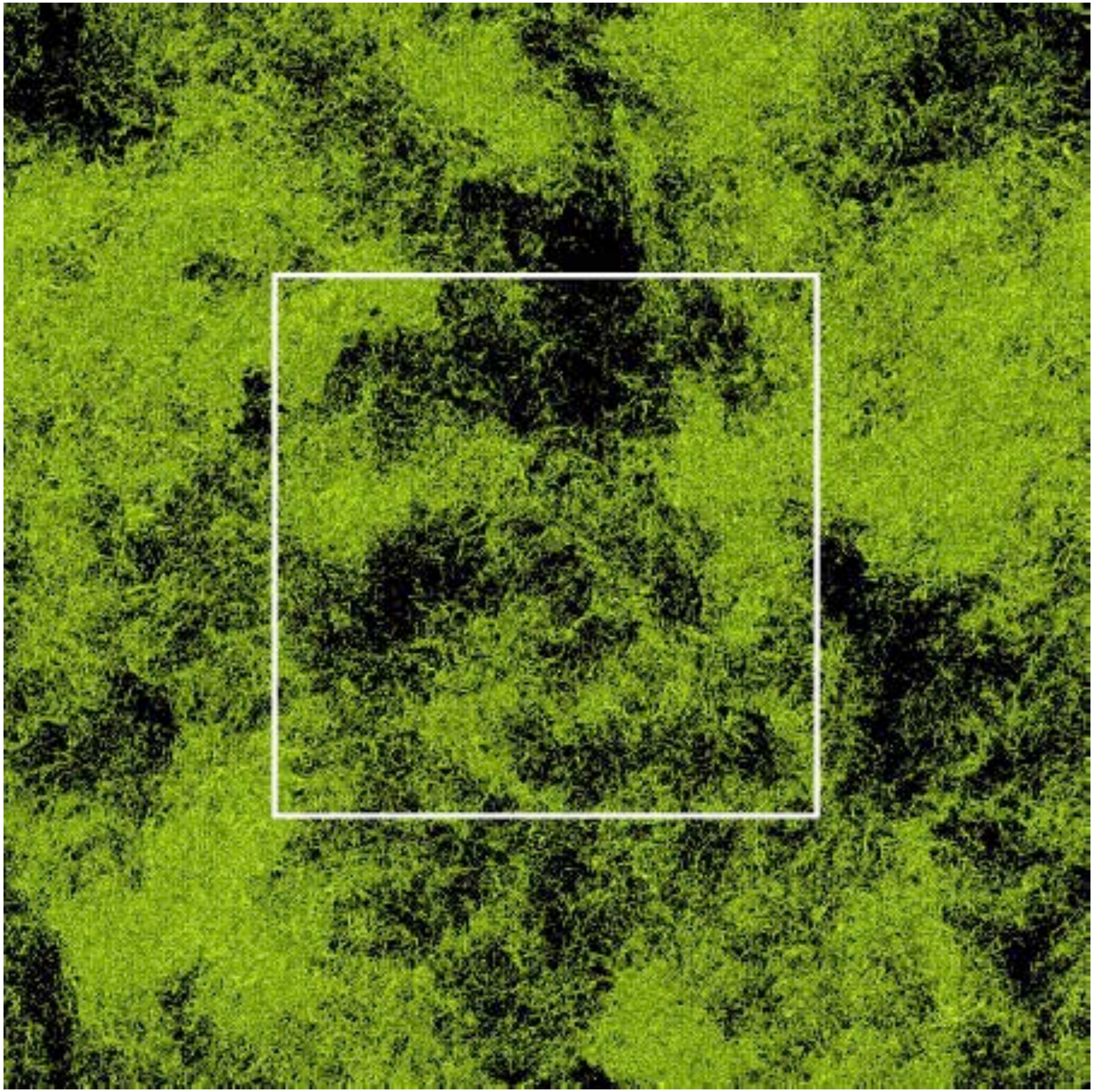
A Universal Phenomenon…



So, the basic idea is that a mean, laminar flow breaks up into disordered eddy-like motions

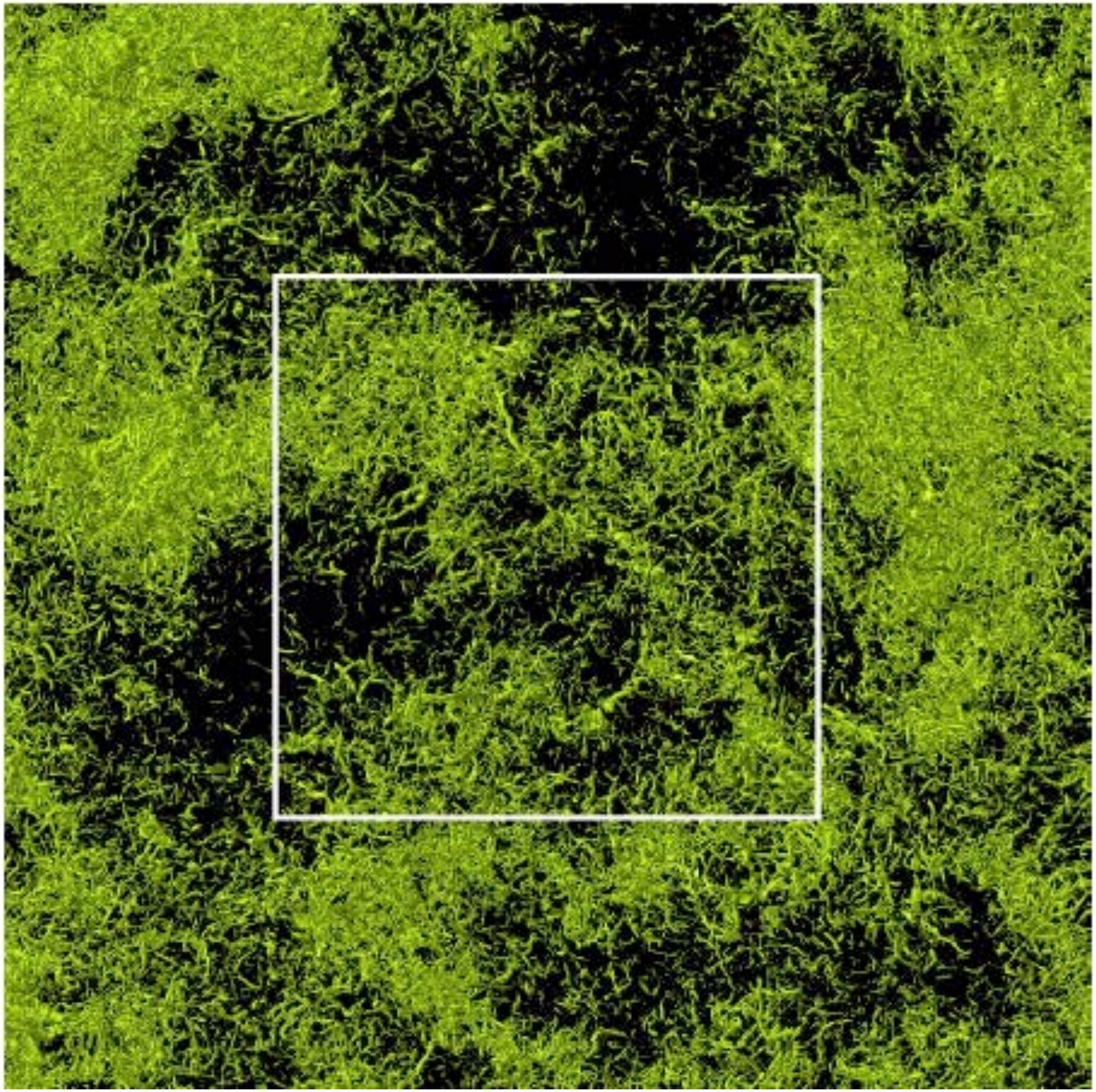
Gyrokinetic simulation of tokamak turbulence
[E. Highcock, Oxford]

What the Structure of These Fluctuations?



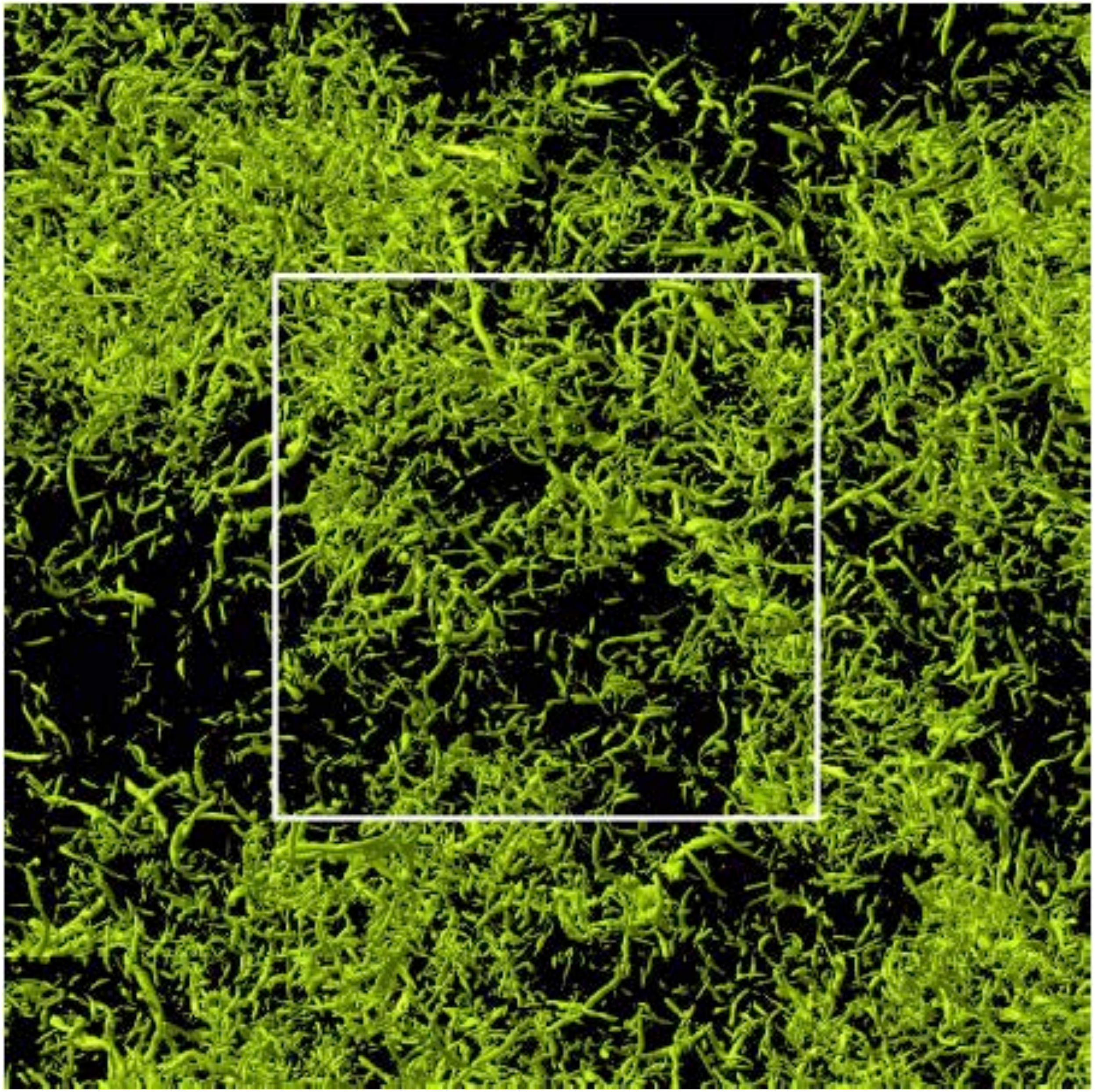
[Image: Earth Simulator, 4096^3, isovorticity surfaces; Y. Kaneda]

Turbulence is Multiscale Disorder



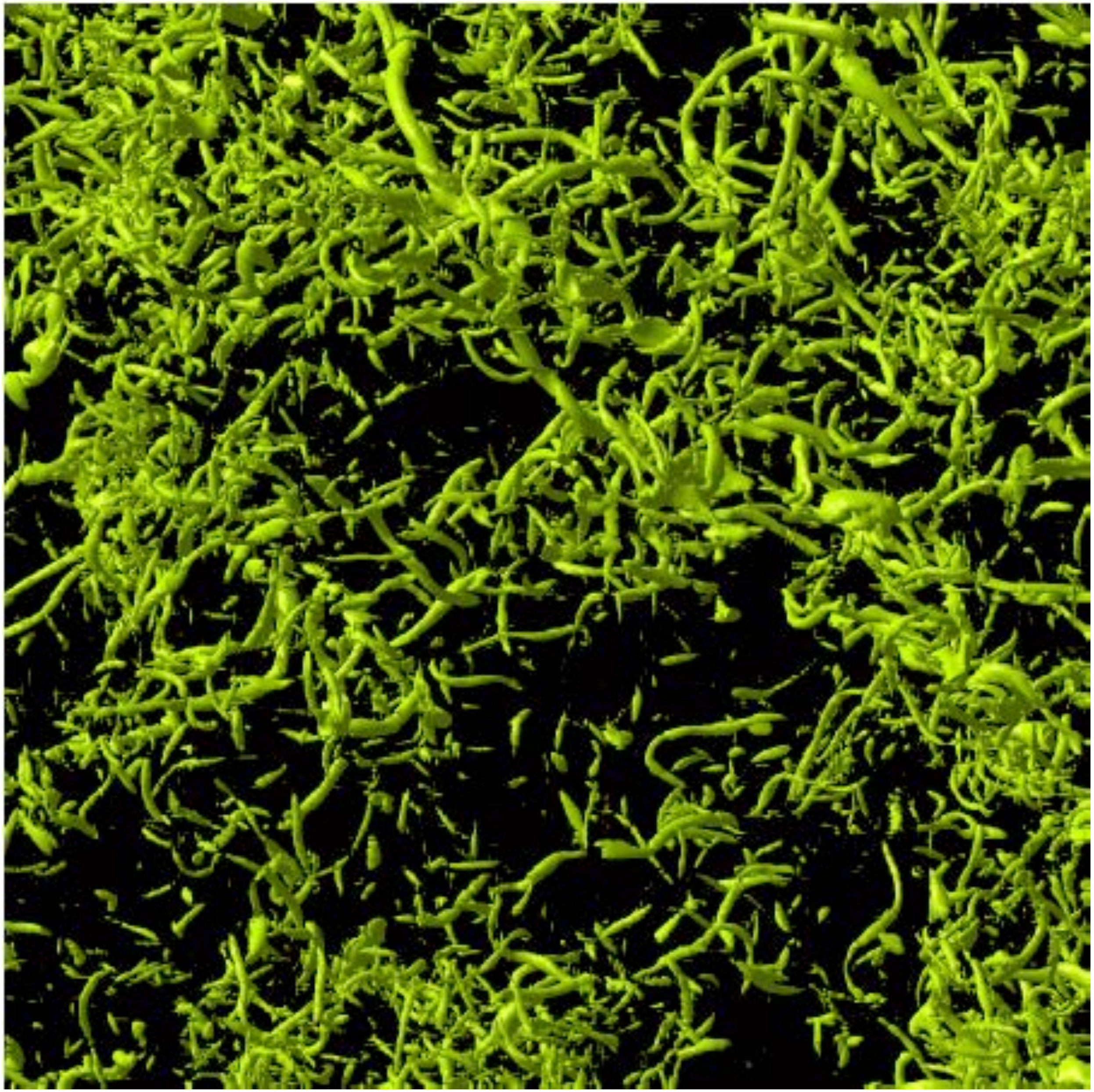
[Image: Earth Simulator, 4096^3, isovorticity surfaces; Y. Kaneda]

Turbulence is Multiscale Disorder



[Image: Earth Simulator, 4096^3, isovorticity surfaces; Y. Kaneda]

Turbulence is Multiscale Disorder

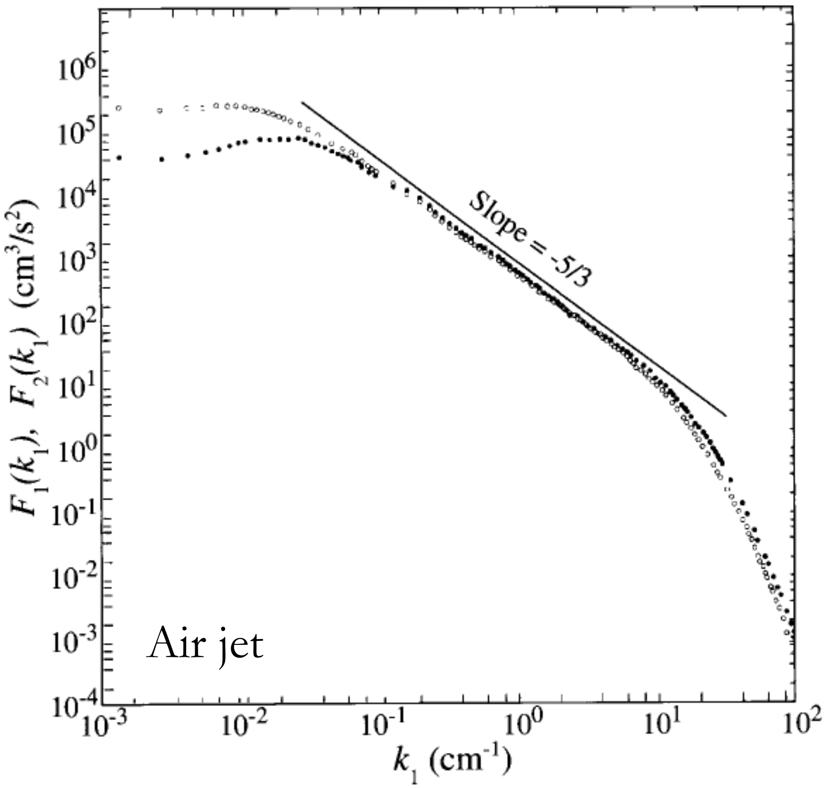


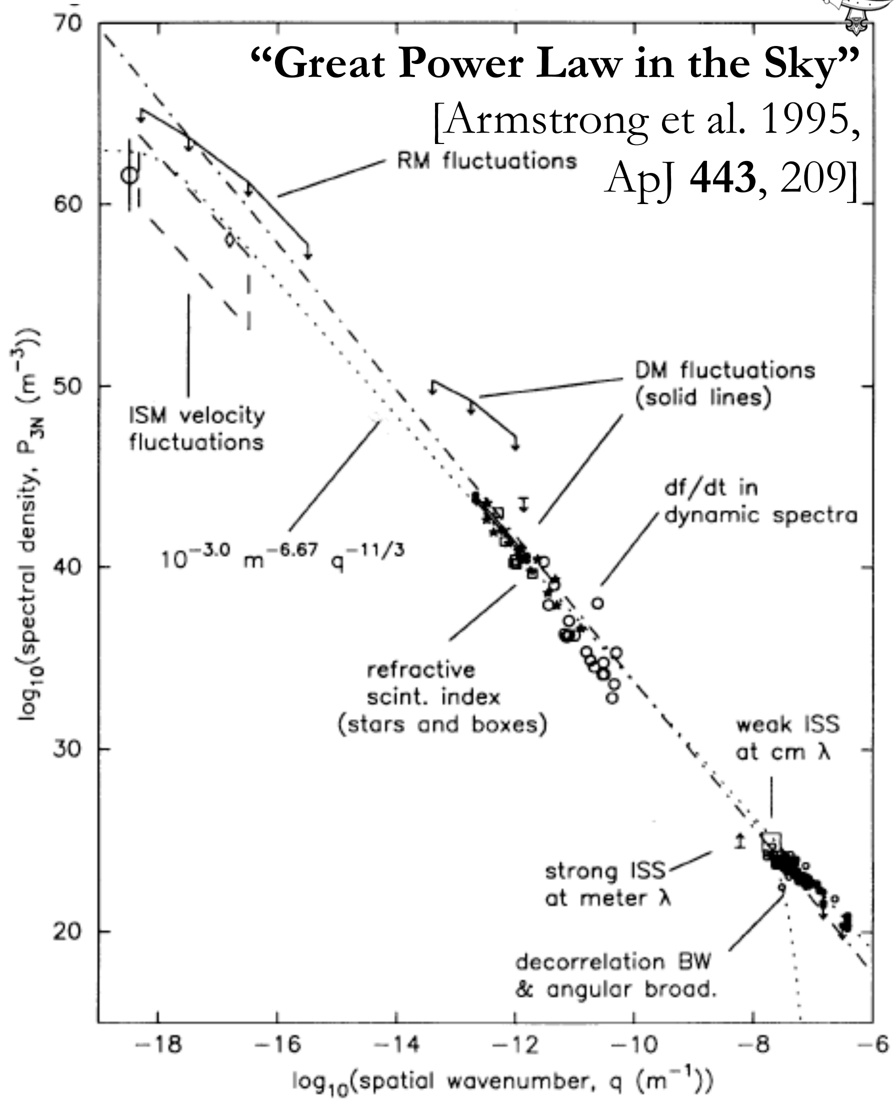
[Image: Earth Simulator, 4096^3, isovorticity surfaces; Y. Kaneda]

Turbulence is Multiscale Disorder

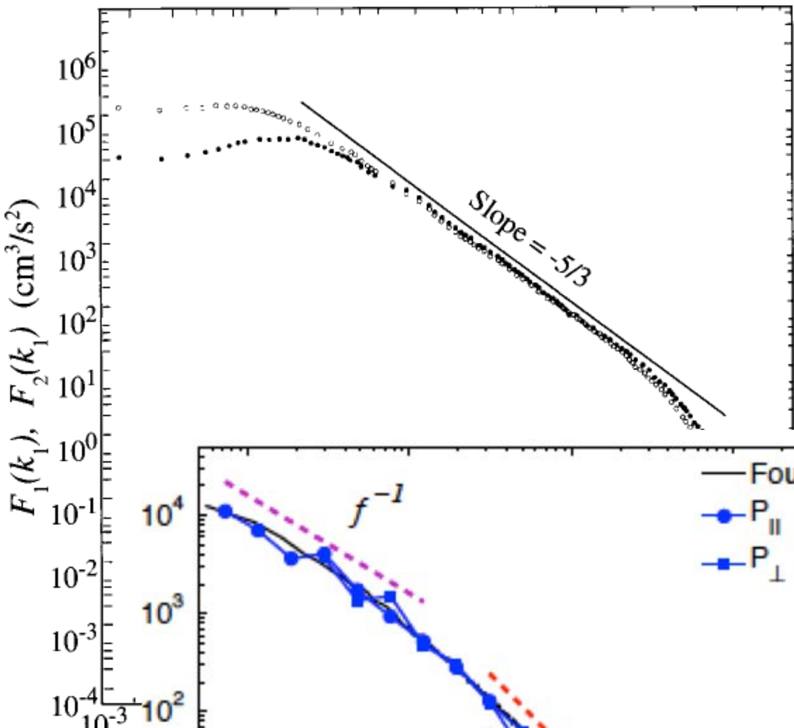
Gyrokinetic simulation of tokamak turbulence
[R. Waltz & J. Candy, GA, San Diego]

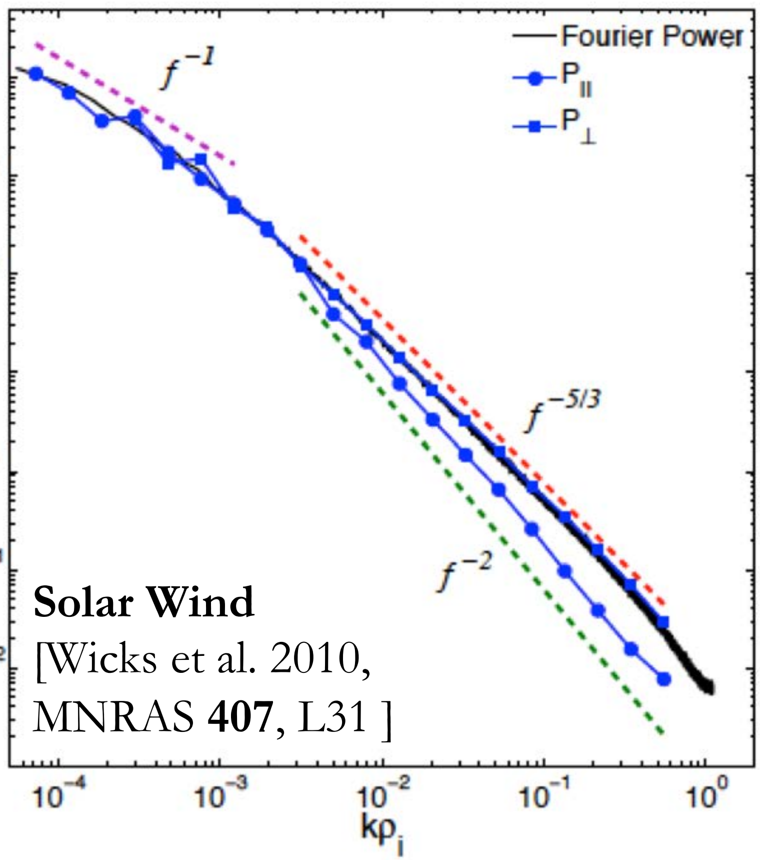
Spectra: Power Laws Galore



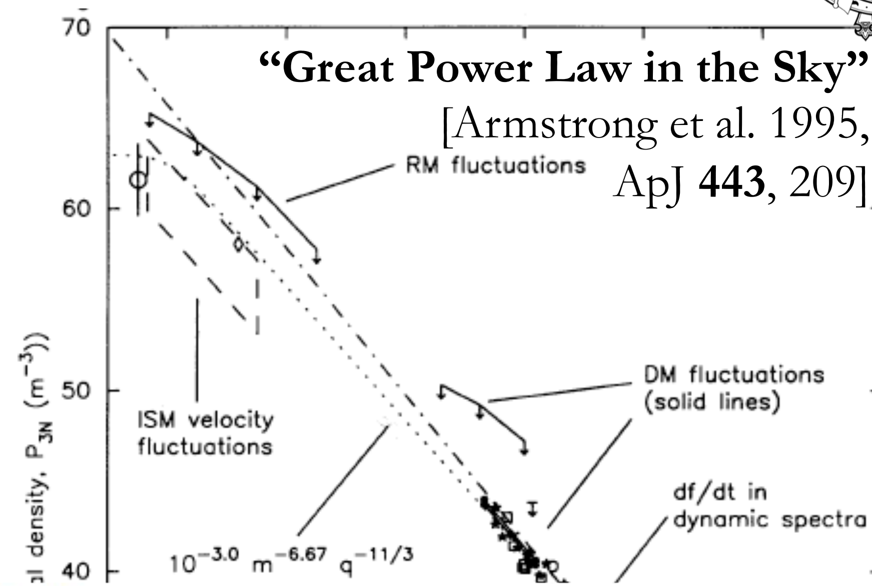


Spectra: Power Laws Galore

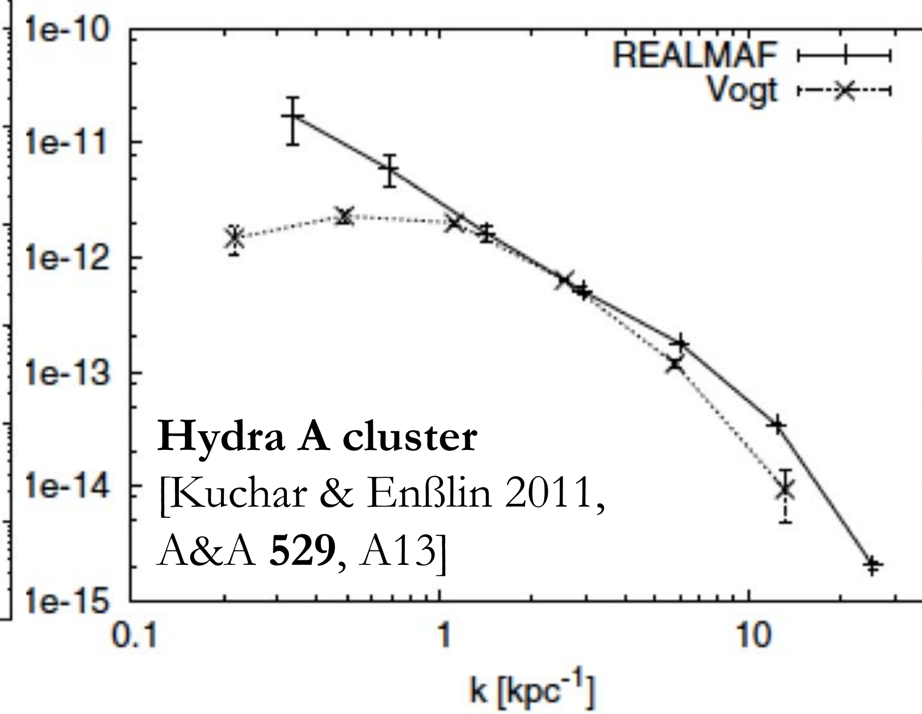




Solar Wind
[Wicks et al. 2010, MNRAS **407**, L31]

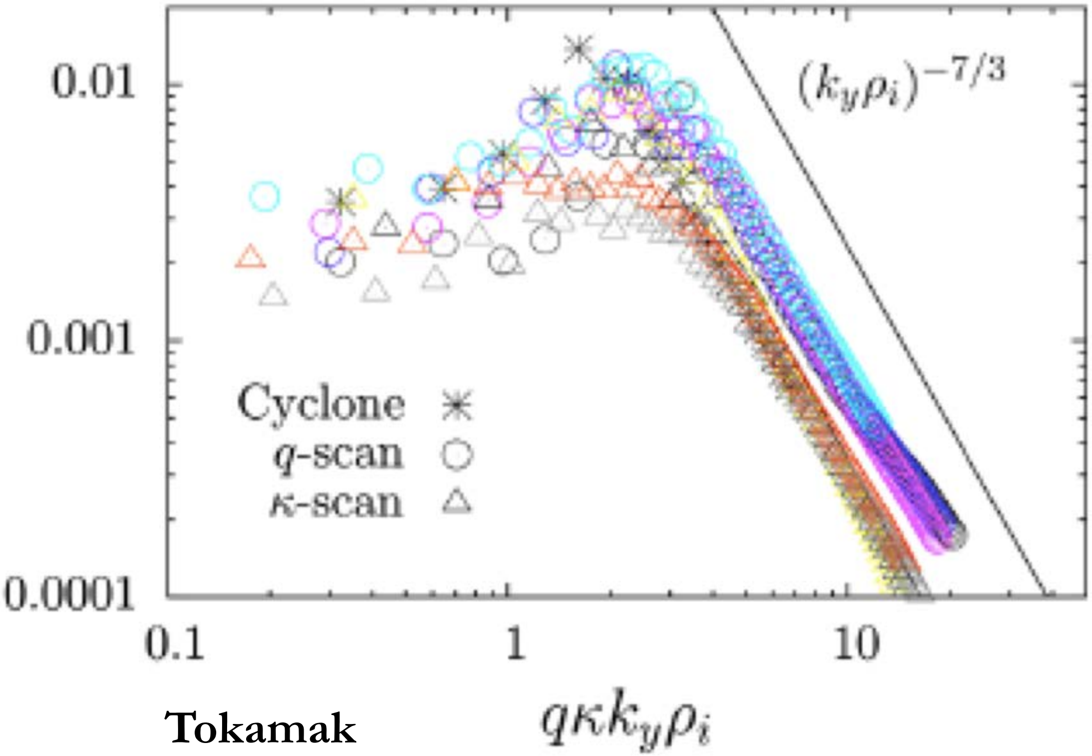


"Great Power Law in the Sky"
[Armstrong et al. 1995, ApJ **443**, 209]



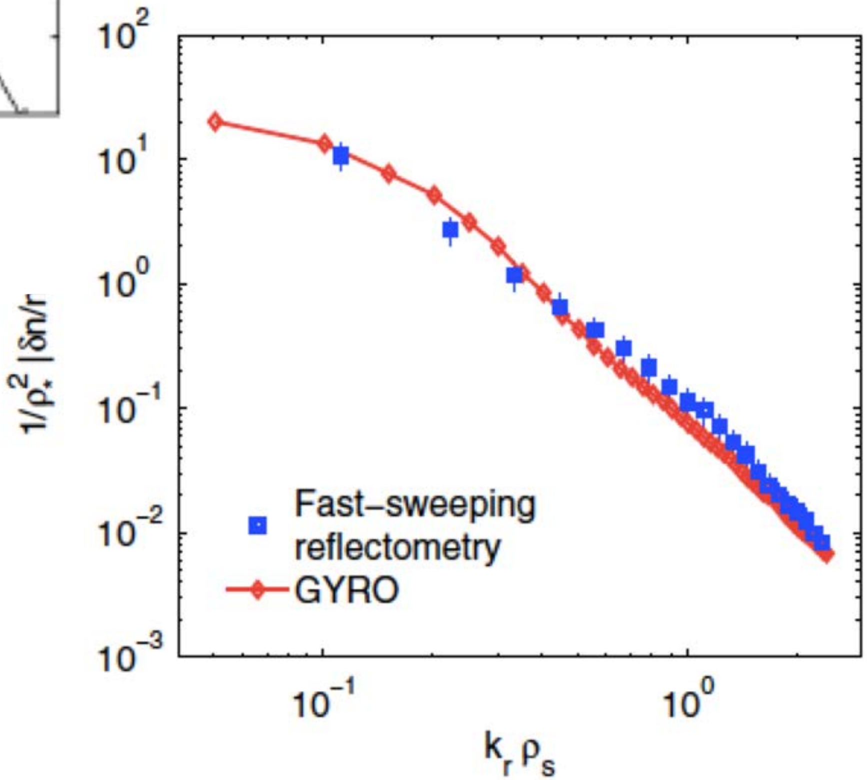
Hydra A cluster
[Kuchar & Enßlin 2011, A&A **529**, A13]

Spectra: Power Laws Galore



Tokamak simulation
[Barnes et al. 2011, PRL **107**, 115003]

TORE SUPRA experiment
[Casati et al. 2011, PRL **102**, 165005]



Why Is Turbulence Multiscale?

Fundamentally, it is about the way in which a nonlinear system processes **energy** *injected into it.*

I will provide a simple example of how that works…

Why Is Turbulence Multiscale?

Navier-Stokes Equation: $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{u} + \mathbf{f}$$

dissipation (viscosity)

injection (some mechanism for which this is a stand-in)

*Fundamentally, it is about the way in which a nonlinear system processes **energy** injected into it.*

I will provide a simple example of how that works…

Why Is Turbulence Multiscale?

Navier-Stokes Equation: $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{u} + \mathbf{f}$$

dissipation (viscosity)

injection (some mechanism for which this is a stand-in)

Kinetic energy:

$$\mathcal{E} = \frac{1}{2}\int \frac{d^3\mathbf{r}}{V}\,\rho|\mathbf{u}|^2$$

Why Is Turbulence Multiscale?

Navier-Stokes Equation: $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \underbrace{\nu\nabla^2\mathbf{u}}_{\text{dissipation}} + \underbrace{\mathbf{f}}_{\text{injection}}$$

Kinetic energy:

$$\mathcal{E} = \frac{1}{2}\int \frac{d^3\mathbf{r}}{V}\,\rho|\mathbf{u}|^2$$

$$\frac{d\mathcal{E}}{dt} = P_{\text{inj}} - P_{\text{diss}}$$

injected power

$$P_{\text{inj}} = \int \frac{d^3\mathbf{r}}{V}\,\rho\mathbf{u}\cdot\mathbf{f}$$

dissipated power

$$P_{\text{diss}} = \nu\int \frac{d^3\mathbf{r}}{V}\,\rho|\nabla\mathbf{u}|^2$$

Why Is Turbulence Multiscale?

Navier-Stokes Equation: $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \underbrace{\nu\nabla^2\mathbf{u}}_{\text{dissipation}} + \underbrace{\mathbf{f}}_{\text{injection}}$$

Kinetic energy:

$$\mathcal{E} = \frac{1}{2}\int\frac{d^3\mathbf{r}}{V}\rho|\mathbf{u}|^2$$

$$\frac{d\mathcal{E}}{dt} = P_{\text{inj}} - P_{\text{diss}} = 0$$

Steady state:

injected power

$$\boxed{P_{\text{inj}} = \int\frac{d^3\mathbf{r}}{V}\rho\mathbf{u}\cdot\mathbf{f}} = \boxed{P_{\text{diss}} = \nu\int\frac{d^3\mathbf{r}}{V}\rho|\nabla\mathbf{u}|^2}$$

dissipated power

Why Is Turbulence Multiscale?

Navier-Stokes Equation: $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{u} + \mathbf{f}$$

dissipation ($\nu\nabla^2\mathbf{u}$) injection ($\mathbf{f}$)

Kinetic energy:

$$\mathcal{E} = \frac{1}{2}\int\frac{d^3\mathbf{r}}{V}\rho|\mathbf{u}|^2$$

$$\frac{d\mathcal{E}}{dt} = P_{\rm inj} - P_{\rm diss} = 0$$

Steady state:

injected power

dissipated power

$$\boxed{P_{\rm inj} = \int\frac{d^3\mathbf{r}}{V}\rho\mathbf{u}\cdot\mathbf{f}} = \boxed{P_{\rm diss} = \nu\int\frac{d^3\mathbf{r}}{V}\rho|\nabla\mathbf{u}|^2}$$

$$P_{\rm inj} \sim \frac{\rho u_{\rm rms}^3}{L}$$

depends on outer-scale quantities only

$$P_{\rm diss} \sim \frac{\rho\nu u_{\rm rms}^2}{L^2}$$

if estimated at outer scale

Why Is Turbulence Multiscale?

Navier-Stokes Equation: $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \underbrace{\nu\nabla^2\mathbf{u}}_{\text{dissipation}} + \underbrace{\mathbf{f}}_{\text{injection}}$$

Kinetic energy:

$$\mathcal{E} = \frac{1}{2}\int\frac{d^3\mathbf{r}}{V}\rho|\mathbf{u}|^2$$

$$\frac{d\mathcal{E}}{dt} = P_{\rm inj} - P_{\rm diss} = 0$$

Steady state:

$$\boxed{\text{injected power} \quad P_{\rm inj} = \int\frac{d^3\mathbf{r}}{V}\rho\mathbf{u}\cdot\mathbf{f}} = \boxed{\text{dissipated power} \quad P_{\rm diss} = \nu\int\frac{d^3\mathbf{r}}{V}\rho|\nabla\mathbf{u}|^2}$$

$$P_{\rm inj} \sim \frac{\rho u_{\rm rms}^3}{L}$$

depends on outer-scale quantities only

$$P_{\rm diss} \sim \frac{\rho\nu u_{\rm rms}^2}{L^2}$$

if estimated at outer scale

But then $$\frac{P_{\rm inj}}{P_{\rm diss}} \sim \frac{u_{\rm rms}L}{\nu} = \underbrace{\mathrm{Re}}_{\text{``Reynolds number''}} \gg 1$$ imbalance!

Why Is Turbulence Multiscale?

Navier-Stokes Equation: $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{u} + \mathbf{f}$$

dissipation injection

Kinetic energy:

$$\mathcal{E} = \frac{1}{2}\int\frac{d^3\mathbf{r}}{V}\rho|\mathbf{u}|^2$$

$$\frac{d\mathcal{E}}{dt} = P_{\rm inj} - P_{\rm diss} = 0$$

Steady state:

injected power

dissipated power

$$\boxed{P_{\rm inj} = \int\frac{d^3\mathbf{r}}{V}\rho\mathbf{u}\cdot\mathbf{f}} = \boxed{P_{\rm diss} = \nu\int\frac{d^3\mathbf{r}}{V}\rho|\nabla\mathbf{u}|^2}$$

$$P_{\rm inj} \sim \frac{\rho u_{\rm rms}^3}{L} \qquad \cancel{P_{\rm diss} \sim \frac{\rho\nu u_{\rm rms}^2}{L^2}}$$

To balance dissipation with power injection, turbulence makes small scales

How small is an easy dimensional guess:

$$\ell_\nu \sim (\rho\nu^3/P_{\rm inj})^{1/4} \sim L\mathrm{Re}^{-3/4}$$

"Kolmogorov scale"

Why Is Turbulence Multiscale?

Navier-Stokes Equation: $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla p}{\rho} + \underbrace{\nu\nabla^2\mathbf{u}}_{\text{dissipation}} + \underbrace{\mathbf{f}}_{\text{injection}}$$

Kinetic energy:

$$\mathcal{E} = \frac{1}{2}\int \frac{d^3\mathbf{r}}{V}\,\rho|\mathbf{u}|^2 \qquad \frac{d\mathcal{E}}{dt} = P_{\rm inj} - P_{\rm diss} = 0$$

Steady state: $$\boxed{\underbrace{P_{\rm inj} = \int \frac{d^3\mathbf{r}}{V}\,\rho\mathbf{u}\cdot\mathbf{f}}_{\text{injected power}}} = \boxed{\underbrace{P_{\rm diss} = \nu\int \frac{d^3\mathbf{r}}{V}\,\rho|\nabla\mathbf{u}|^2}_{\text{dissipated power}}}$$

$$P_{\rm inj} \sim \frac{\rho u_{\rm rms}^3}{L} \qquad \cancel{P_{\rm diss} \sim \frac{\rho\nu u_{\rm rms}^2}{L^2}}$$

To balance dissipation with power injection, turbulence makes small scales

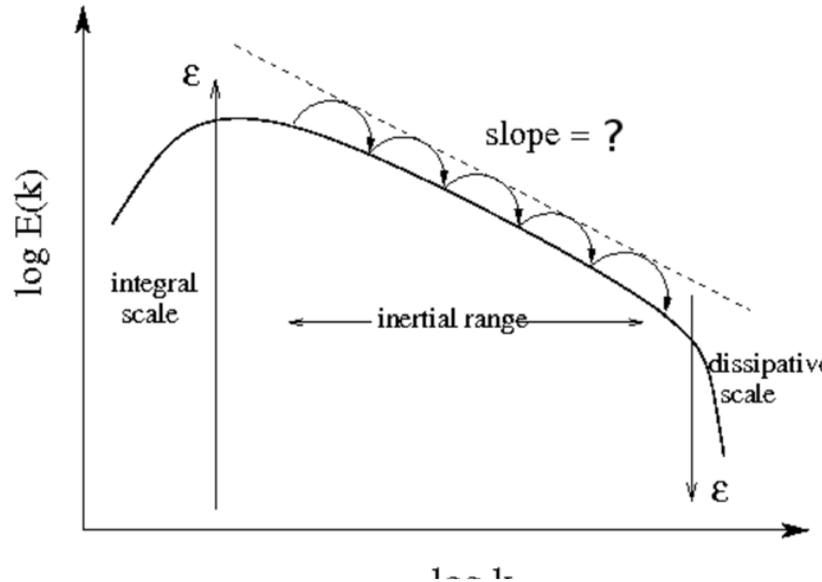
How small is an easy dimensional guess:

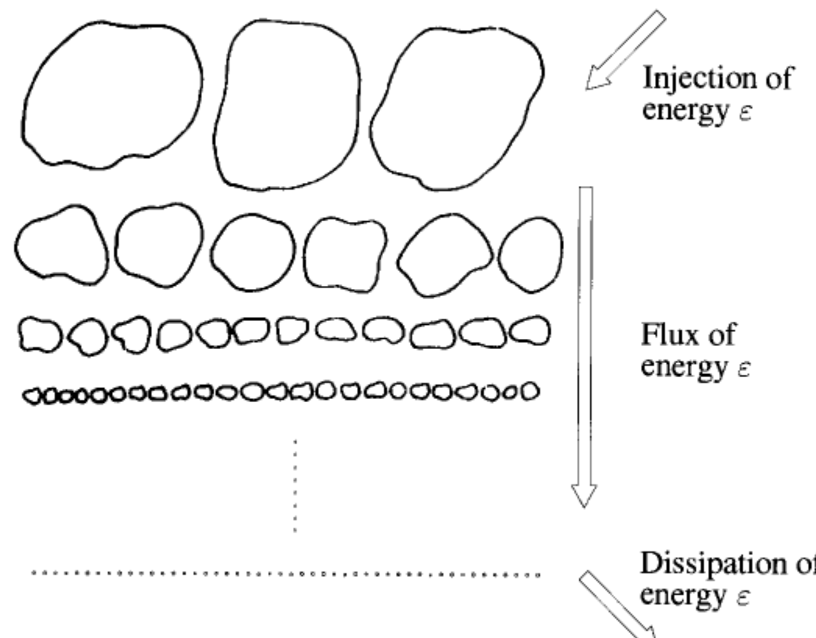
$$\underbrace{L}_{\text{injection}} \gg \underbrace{\ell}_{\text{"inertial range"}} \gg \underbrace{\ell_\nu}_{\text{dissipation}} \sim (\rho\nu^3/P_{\rm inj})^{1/4} \sim L\mathrm{Re}^{-3/4}$$

"Kolmogorov scale"

The Richardson Cascade

Lewis Fry Richardson F.R.S.
(1881-1953)





Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.

1922

The Jonathan Swift Cascade

Lewis Fry Richardson F.R.S.
(1881-1953)

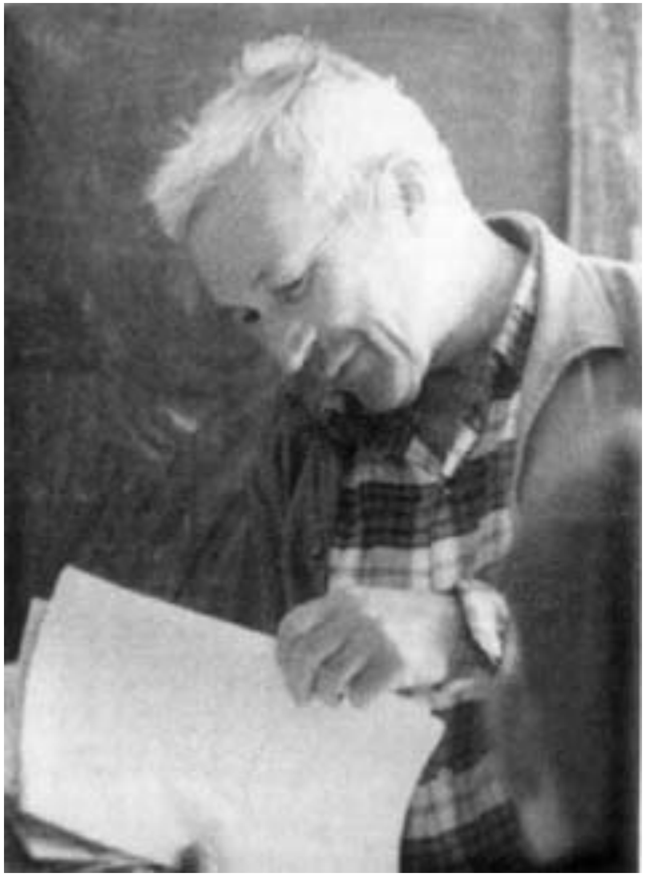
Big whorls have little whorls
  That feed on their velocity,
And little whorls have lesser whorls
  And so on to viscosity.

1922

Jonathan Swift
(1667-1745)

So, nat'ralists observe, a flea
Hath smaller fleas that on him prey;
And these have smaller yet to bite 'em,
And so proceed ad infinitum.
Thus every poet, in his kind,
Is bit by him that comes behind.

The Kolmogorov Cascade



A. N. Kolmogorov
(1903-1987)

- Universality *(no special systems)*
- Homogeneity *(no special locations)*
- Isotropy *(no special directions)*
- Locality *(no special scales)*

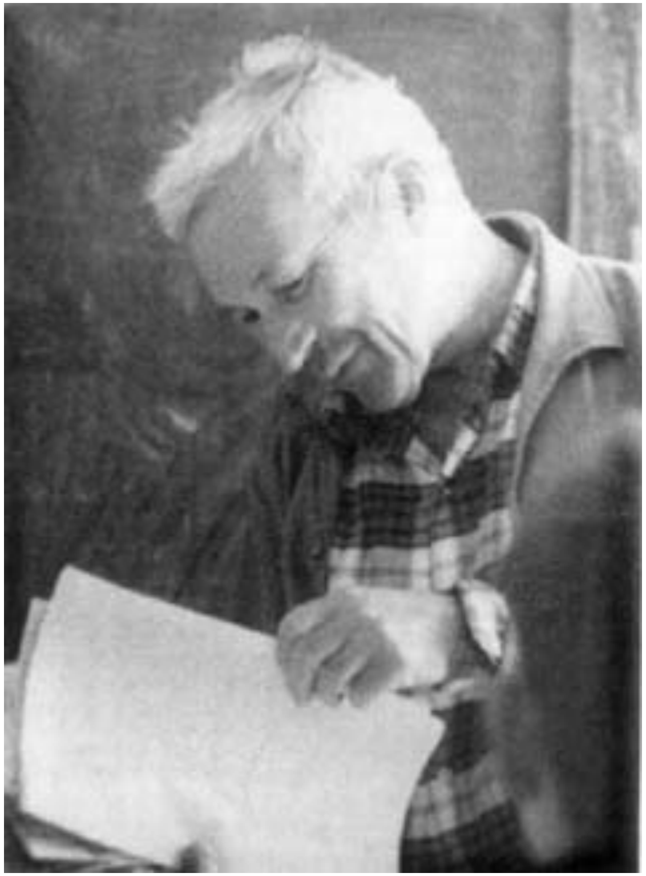
Any broken symmetries are restored in the inertial range…

We wish to predict $\delta u(\ell) = u(r+\ell) - u(r)$

At each scale, $\dfrac{\rho\,\delta u(\ell)^2}{\tau(\ell)} \sim P_{\rm inj} = \text{const}$

"cascade time"

The Kolmogorov Cascade



A. N. Kolmogorov
(1903-1987)

- Universality *(no special systems)*
- Homogeneity *(no special locations)*
- Isotropy *(no special directions)*
- Locality *(no special scales)*

Any broken symmetries are restored in the inertial range…

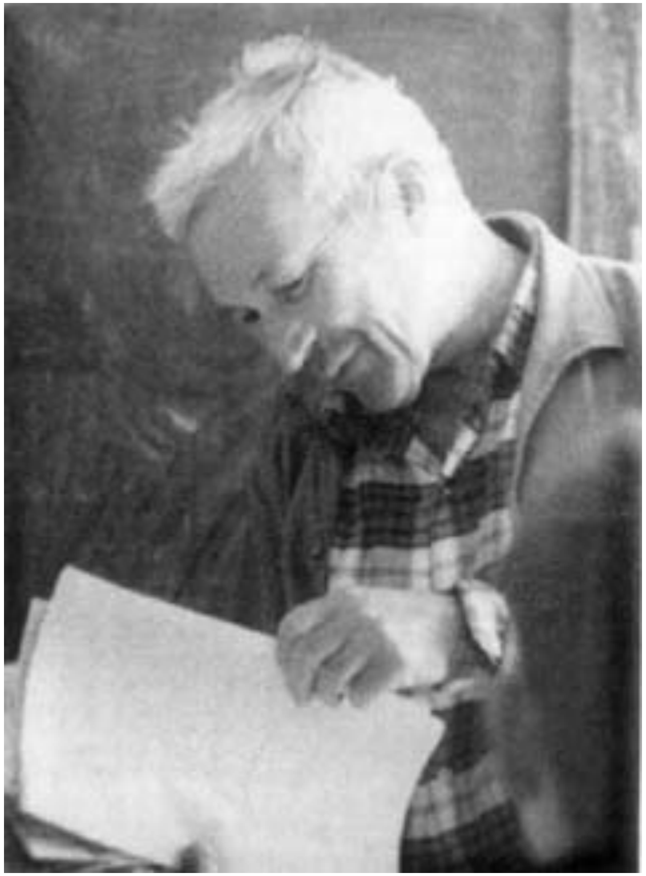
We wish to predict $\delta u(\ell) = u(r+\ell) - u(r)$

At each scale, $\dfrac{\rho\,\delta u(\ell)^2}{\tau(\ell)} \sim P_{\mathrm{inj}} = \mathrm{const}$

"cascade time"

Dimensionally, $\tau(\ell) \sim \ell/\delta u(\ell)$

The Kolmogorov Cascade



A. N. Kolmogorov
(1903-1987)

- Universality *(no special systems)*
- Homogeneity *(no special locations)*
- Isotropy *(no special directions)*
- Locality *(no special scales)*

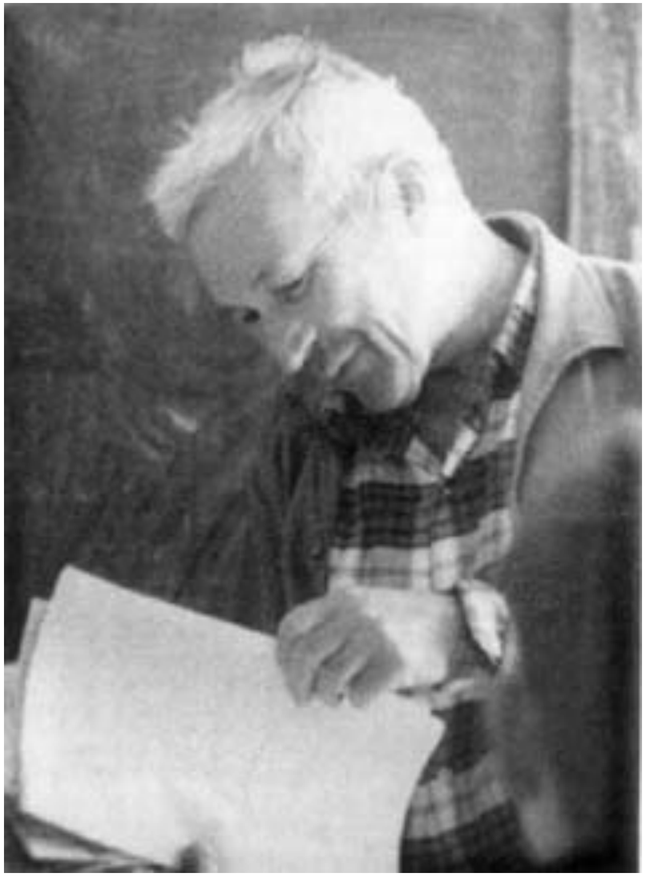
Any broken symmetries are restored in the inertial range…

We wish to predict $\delta u(\ell) = u(r+\ell) - u(r)$

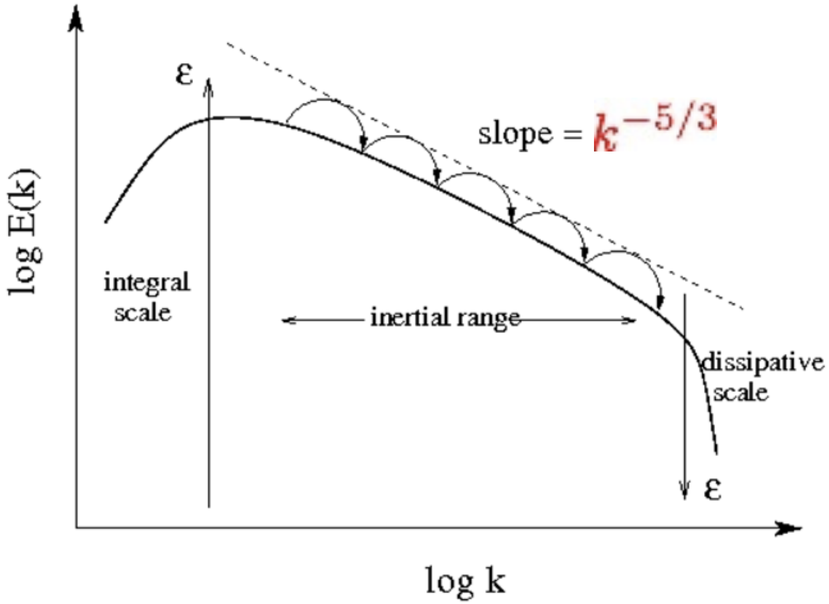
At each scale, $\dfrac{\rho\, \delta u(\ell)^3}{\ell} \sim P_{\text{inj}} = \text{const}$

Therefore, $\boxed{\delta u(\ell) \propto \ell^{1/3}}$ **K41**

The Kolmogorov Cascade



A. N. Kolmogorov
(1903-1987)



We wish to predict $\delta u(\ell) = u(r+\ell) - u(r)$

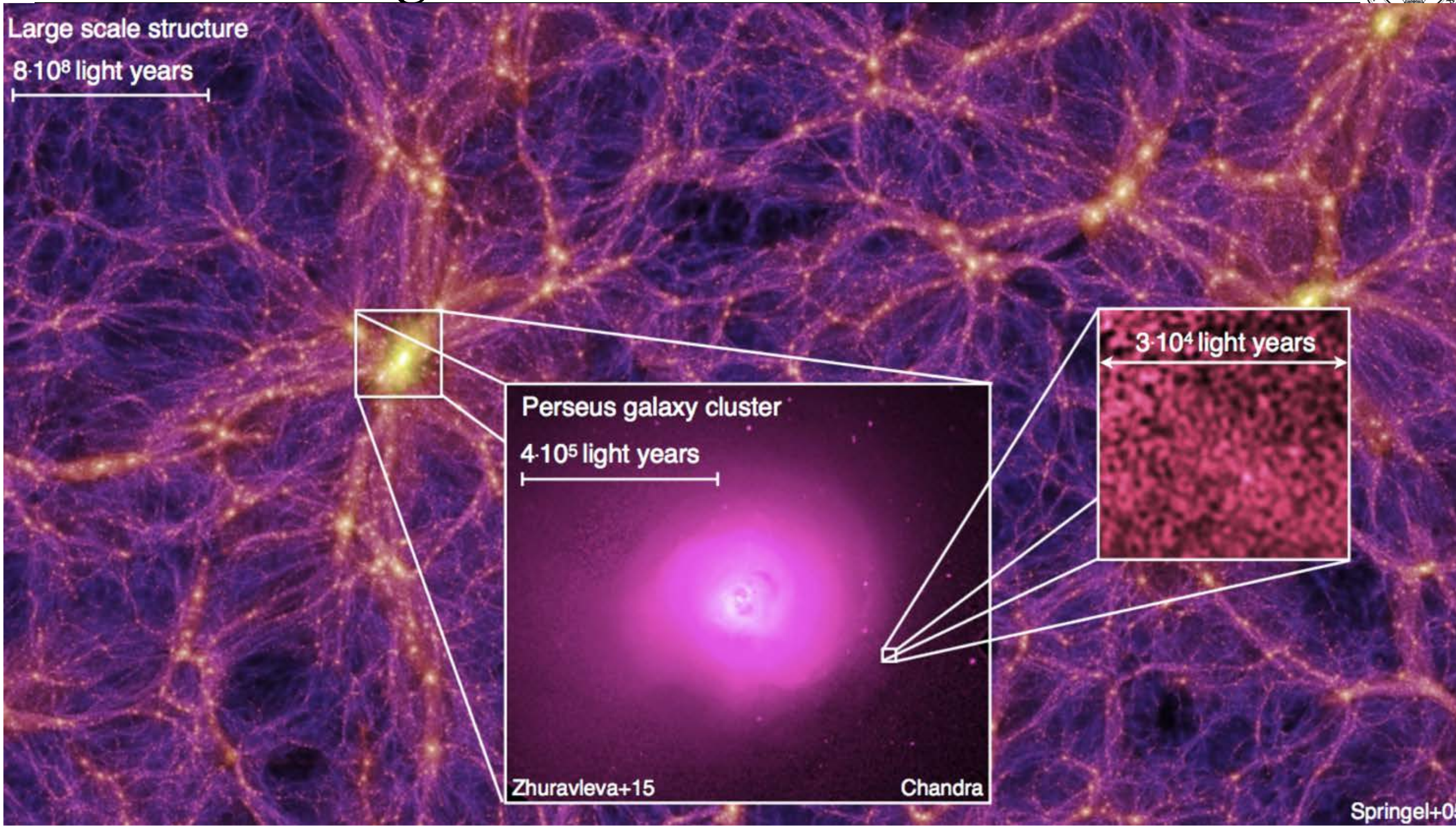
At each scale, $\dfrac{\rho\,\delta u(\ell)^3}{\ell} \sim P_{\rm inj} = \text{const}$

Therefore, $\boxed{\delta u(\ell) \propto \ell^{1/3}}$ **K41**

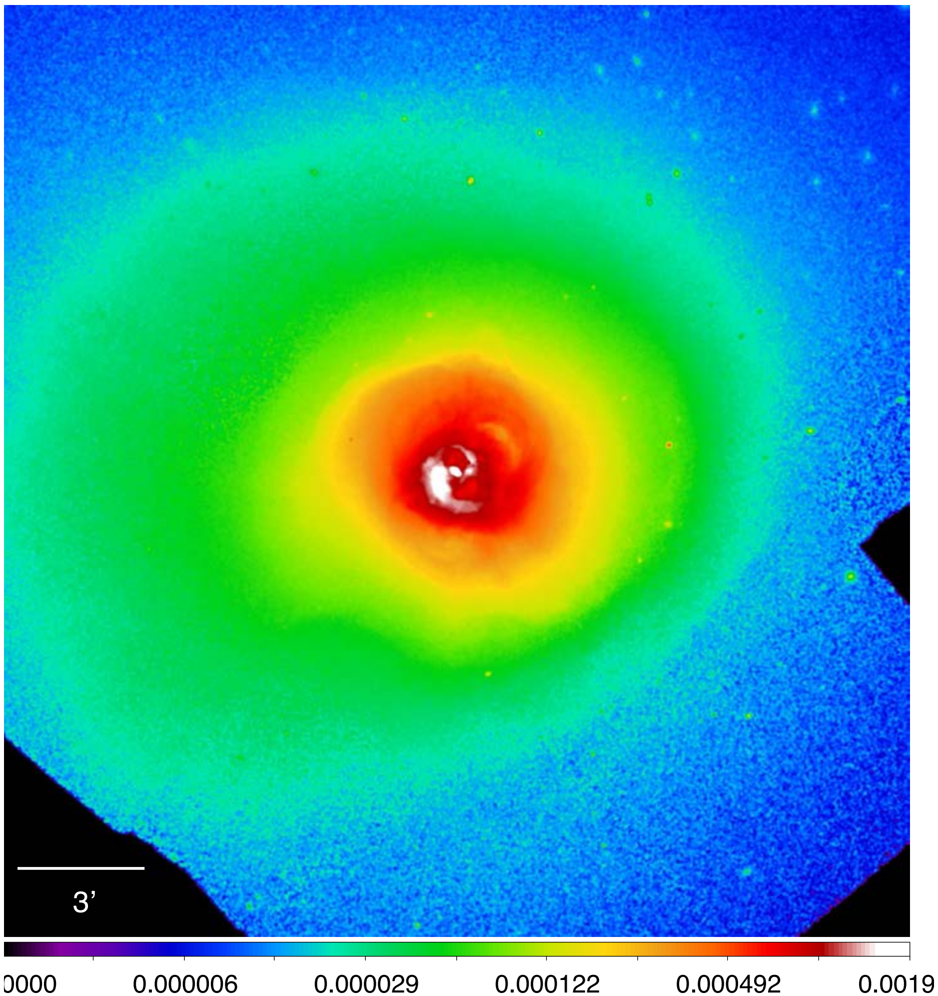
$$\delta u(\ell)^2 \sim \int_{1/\ell}^{\infty} dk E(k) \quad \Rightarrow \quad E(k) \propto k^{-5/3}$$

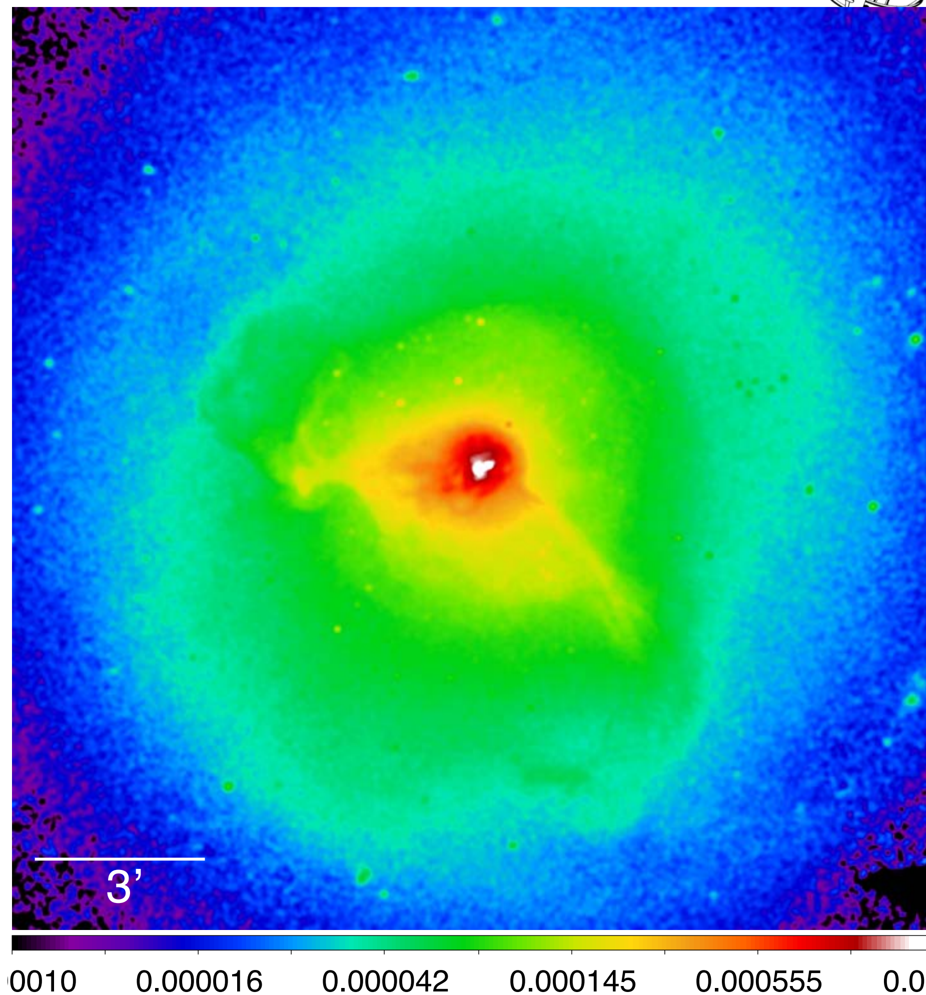
"Kolmogorov spectrum"

The Largest Turbulent Pool in the Universe



Turbulence in Perseus & Virgo



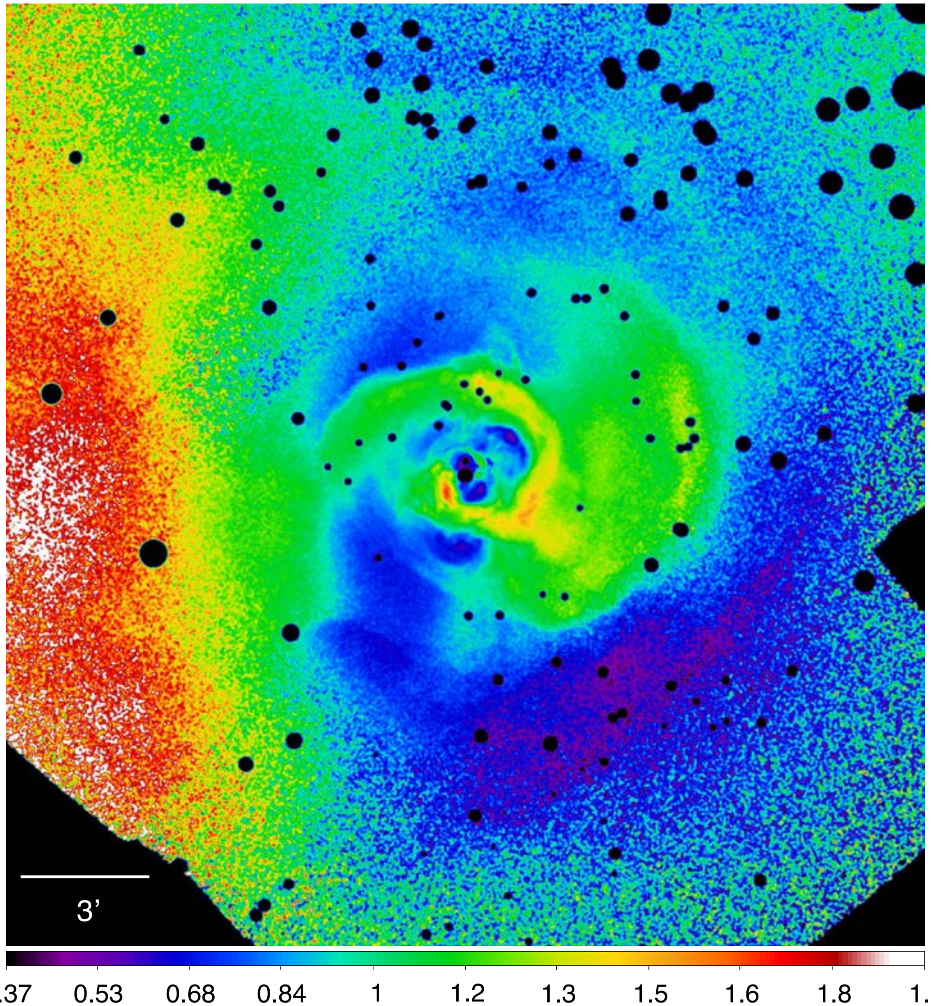


Perseus
1’=20 kpc

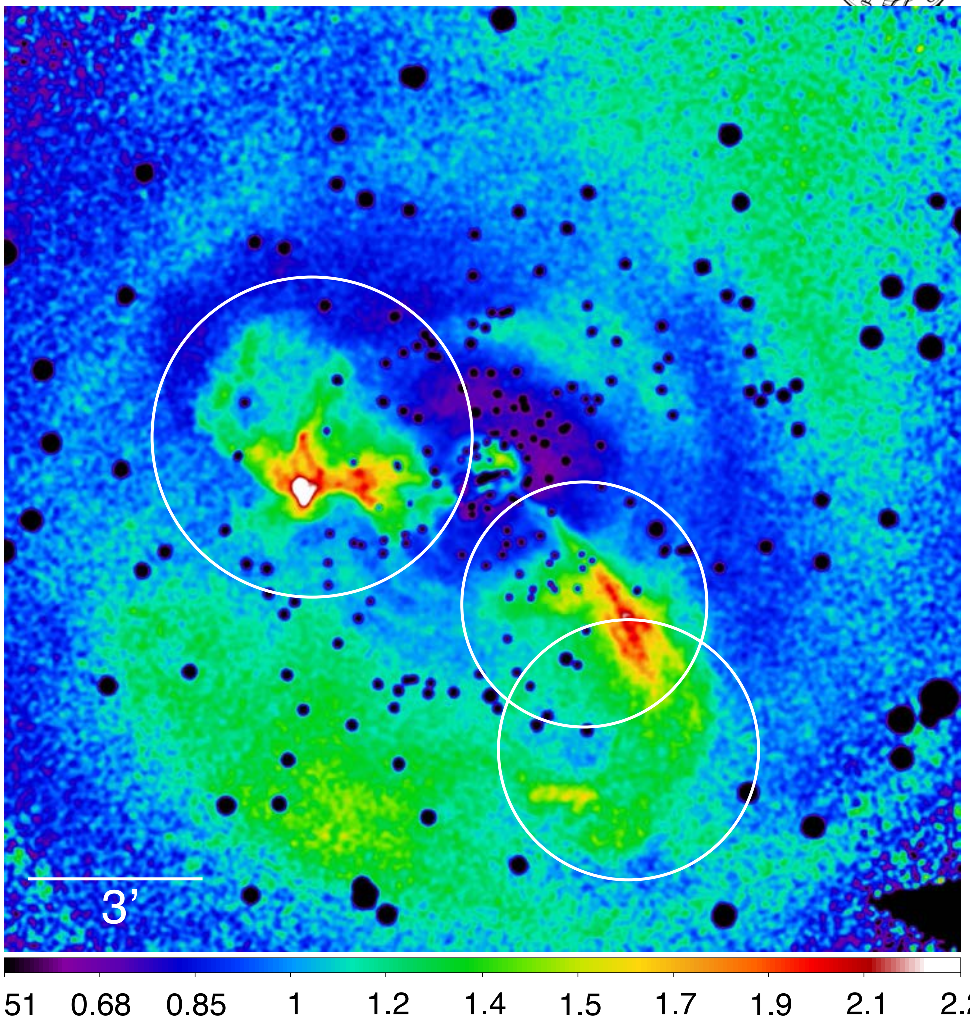
Virgo
1’=5 kpc

Zhuravleva, Churazov, AAS et al. 2014, Nature 515, 85 [arXiv:1410.6485]

Turbulence in Perseus & Virgo



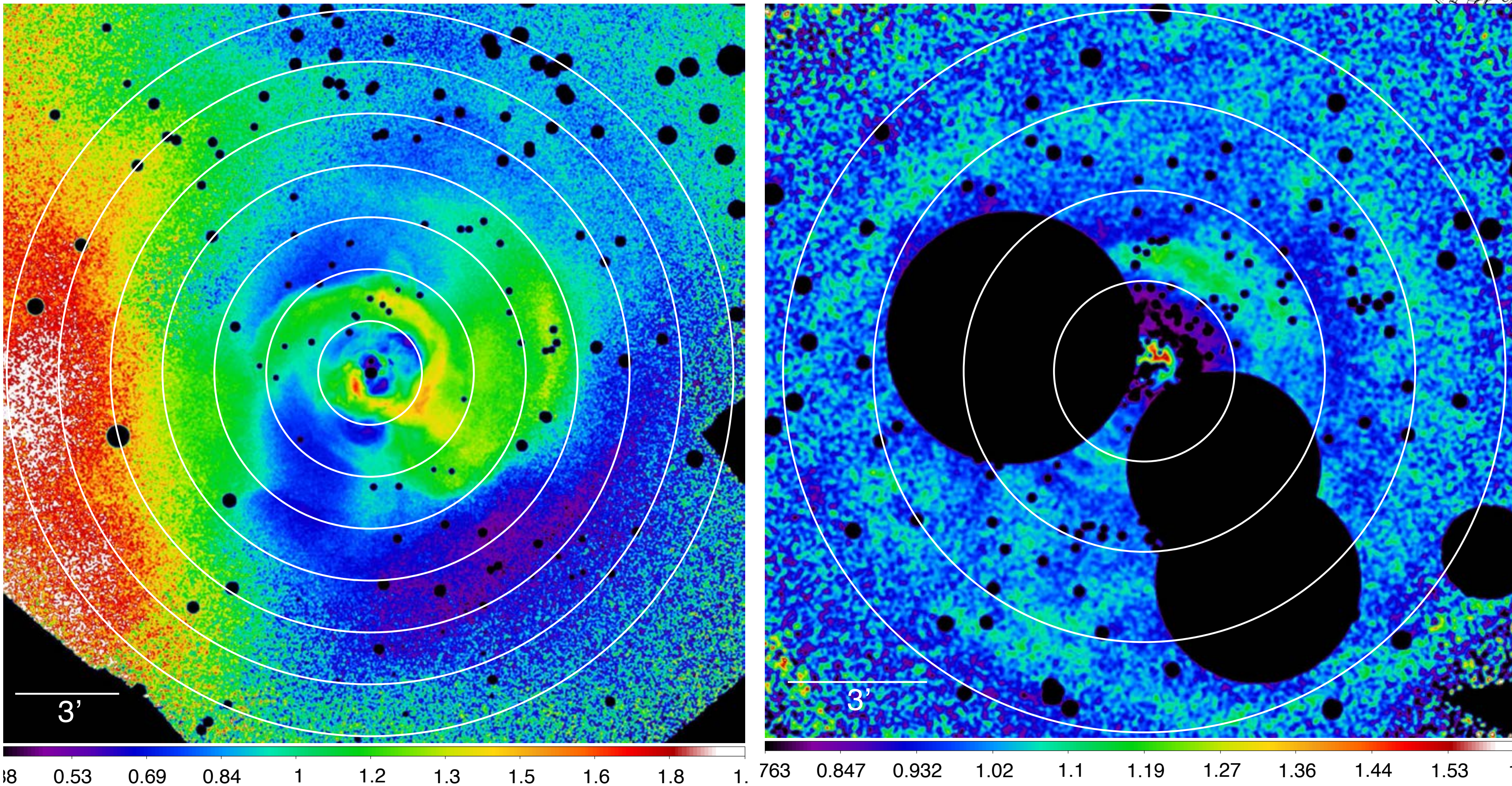
Perseus
1’=20 kpc



Virgo
1’=5 kpc

Zhuravleva, Churazov, AAS et al. 2014, Nature 515, 85 [arXiv:1410.6485]

Turbulence in Perseus & Virgo

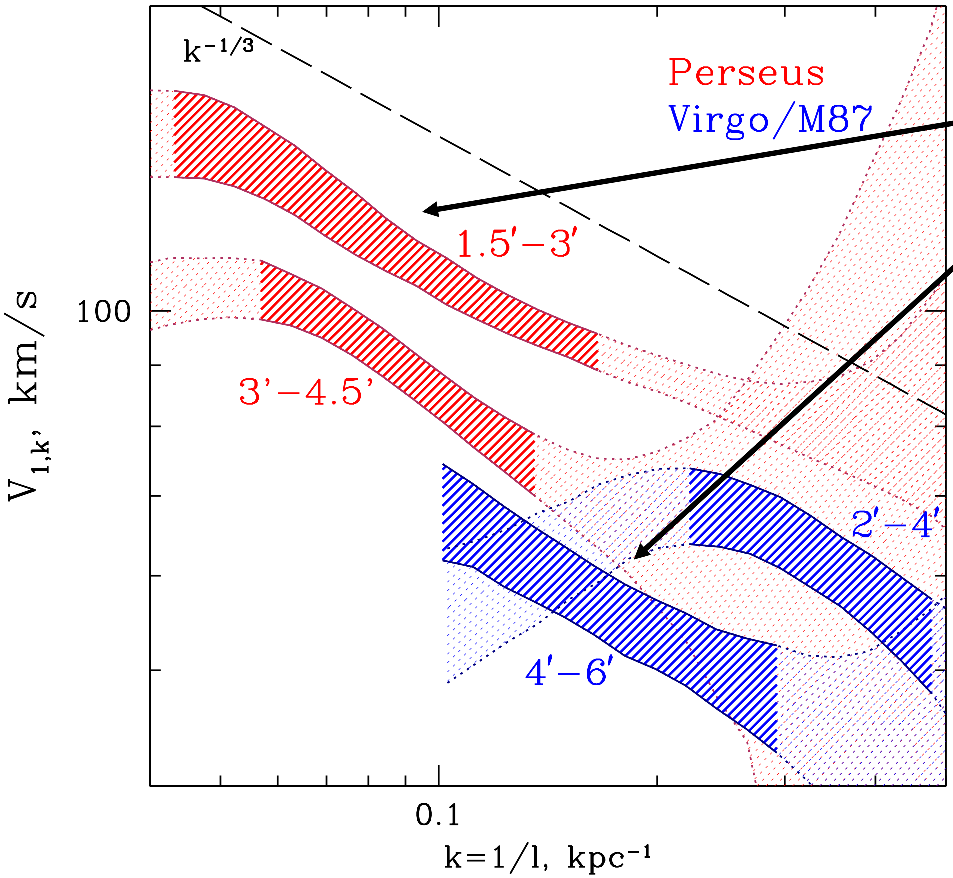


Perseus
1'=20 kpc

Virgo
1'=5 kpc

Zhuravleva, Churazov, AAS et al. 2014, Nature 515, 85 [arXiv:1410.6485]

Turbulence in Perseus & Virgo



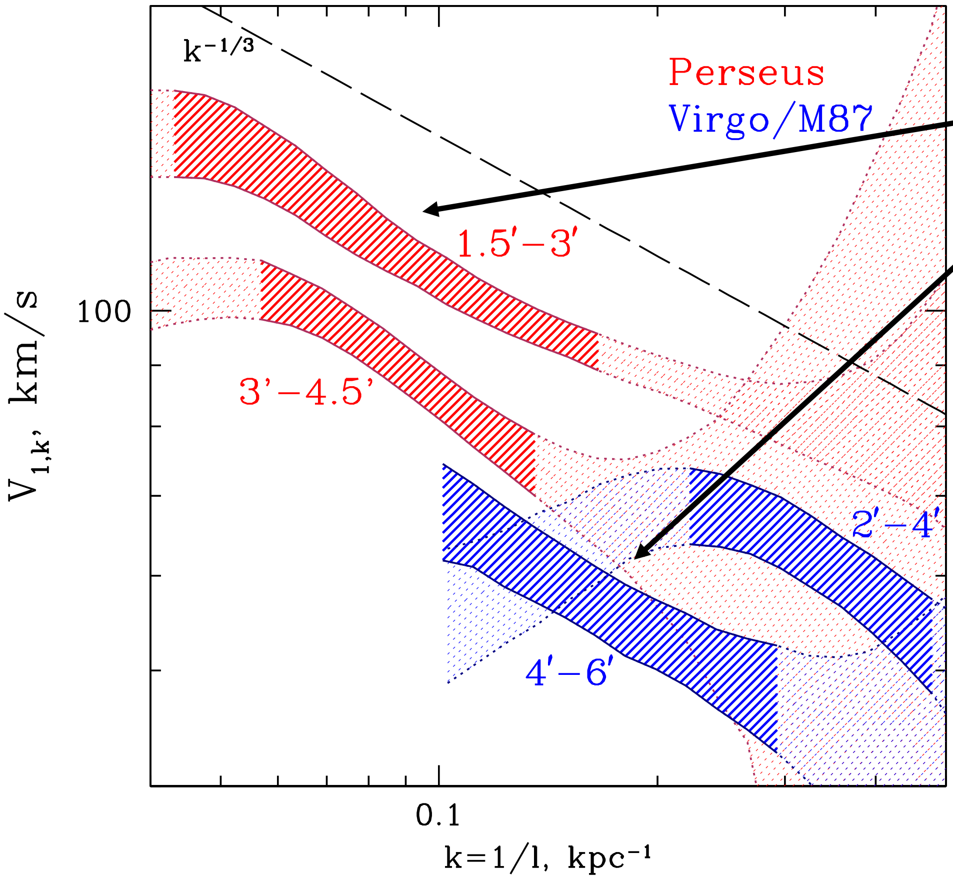
"Inertial range," consistent with Kolmogorov scaling

$$\delta u \sim P_{\rm inj}^{1/3} k^{-1/3}, \quad k = \ell^{-1}$$

Velocity spectral amplitudes inferred from density spectra

Zhuravleva, Churazov, AAS et al. 2014, Nature 515, 85 [arXiv:1410.6485]

Turbulence in Perseus & Virgo



“Inertial range,” consistent with Kolmogorov scaling

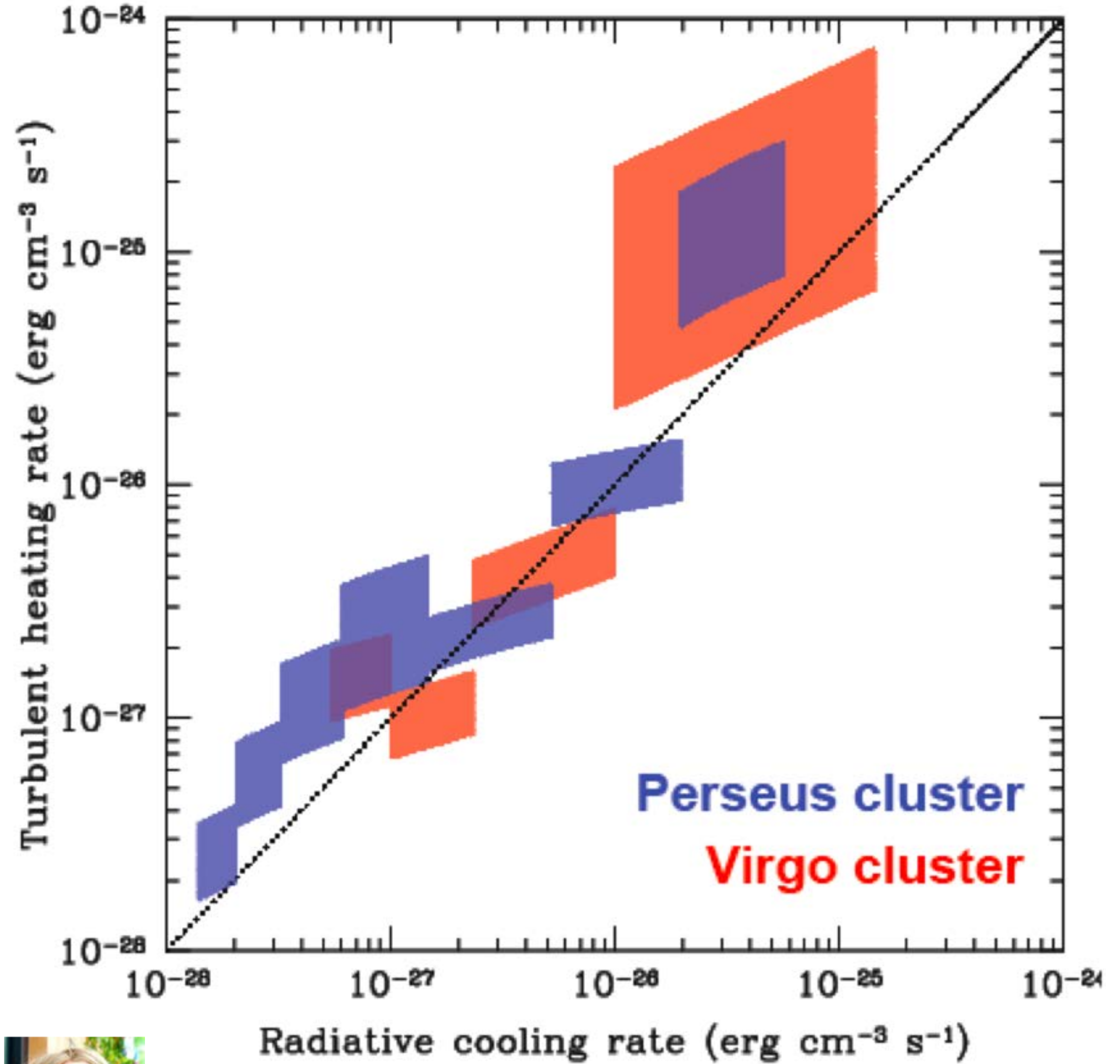
$$\delta u \sim P_{\rm inj}^{1/3} k^{-1/3}, \quad k = \ell^{-1}$$

Read this off from measured spectra

Velocity spectral amplitudes inferred from density spectra

Zhuravleva, Churazov, AAS et al. 2014, Nature 515, 85 [arXiv:1410.6485]

Turbulence in Perseus & Virgo



$$\delta u \sim P_{\rm inj}^{1/3} k^{-1/3}, \quad k = \ell^{-1}$$

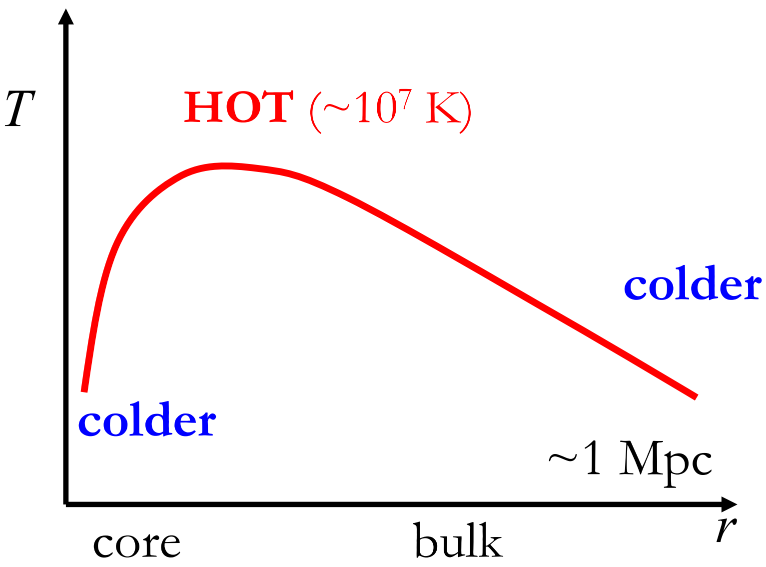
Read this off from measured spectra

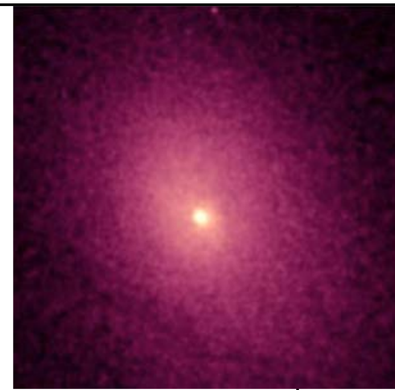
$$\boxed{P_{\rm inj} \sim \text{Radiative cooling}}$$

…so clusters are heated via turbulent cascade

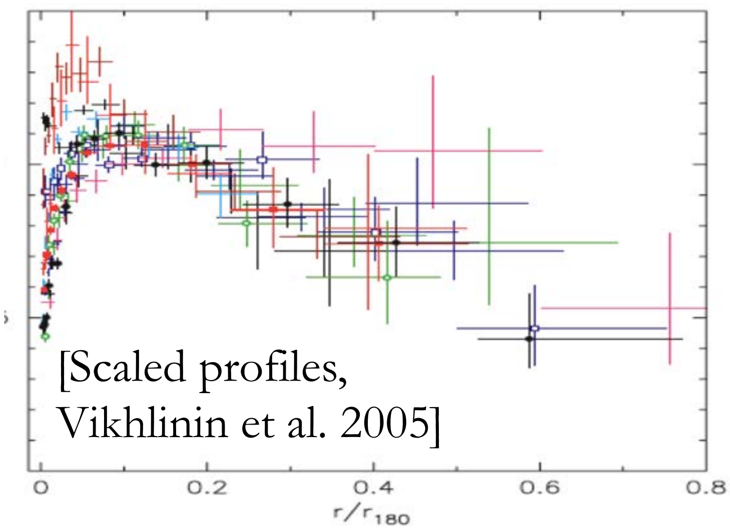
Zhuravleva, Churazov, AAS et al. 2014, Nature 515, 85 [arXiv:1410.6485]

Turbulence in Perseus & Virgo





This has bearing on how we explain observed cluster temperature profiles



[Scaled profiles, Vikhlinin et al. 2005]

$$\delta u \sim P_{\rm inj}^{1/3} k^{-1/3}, \quad k = \ell^{-1}$$

Read this off from measured spectra

$$P_{\rm inj} \sim \text{Radiative cooling}$$

…so clusters are heated via turbulent cascade

la Repubblica.it
Il mondo in diretta 24 ore su 24

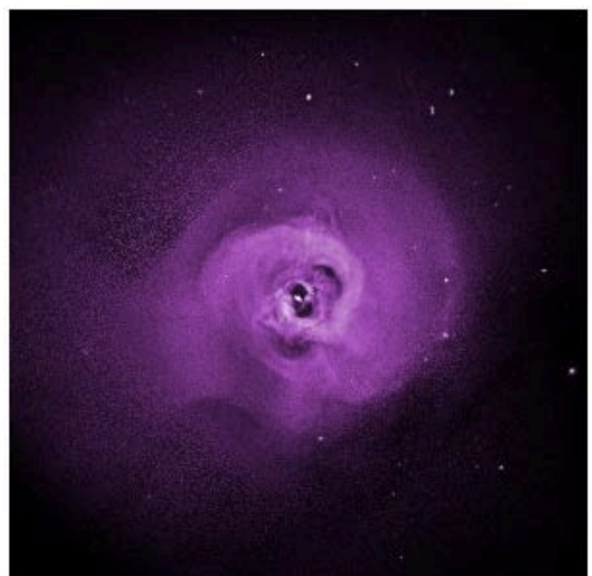
The Italian press had the right basic idea of how it all works…

Come si formano le galassie? Guardate nel caffellatte

Quando le stelle sono figlie del caos. I fenomeni turbolenti sono presenti ovunque nella nostra vita quotidiana. Ma nelle galassie più lontane, la turbolenza può persino influenzare la nascita di nuove stelle, come sostiene un nuovo studio su Nature

di MASSIMILIANO RAZZANO

 | 31 ottobre 2014



PROVATE a farci caso domattina, quando farete colazione. Dopo aver versato il caffè nel latte, vedrete formarsi dei piccoli vortici, dovuti a fenomeni di turbolenza creati dall'incontro dei due liquidi. Pensate che fenomeni turbolenti non troppo diversi governano non solo il destino del vostro caffellatte, ma anche quello delle future stelle. Lo sostiene un nuovo studio condotto da un team internazionale di astronomi, che hanno studiato l'emissione di raggi X di due giganteschi ammassi di

Zhuravleva, Churazov, AAS e

la Repubblica.it
Il mondo in diretta 24 ore su 24

The Italian press had
the right basic idea
of how it all works…

Come si formano le galassie? Guardate nel caffellatte

Quando le stelle sono figlie del caos. I fenomeni turbolenti sono presenti ovunque nella nostra vita quotidiana. Ma nelle galassie più lontane, la turbolenza può persino influenzare la nascita di nuove stelle, come sostiene un nuovo studio su Nature

di MASSIMILIANO RAZZANO

Lo leggo dopo | 31 ottobre 2014

PROVATE a farci caso domattina, quando farete colazione. Dopo aver versato il caffè nel latte, vedrete formarsi dei piccoli vortici, dovuti a fenomeni di turbolenza creati dall'incontro dei due liquidi. Pensate che fenomeni turbolenti non troppo diversi governano non solo il destino del vostro caffellatte, ma anche quello delle future stelle. Lo sostiene un nuovo studio condotto da un team internazionale di astronomi, che hanno studiato l'emissione di raggi X di due giganteschi ammassi di

Zhuravleva, Churazov, AAS e

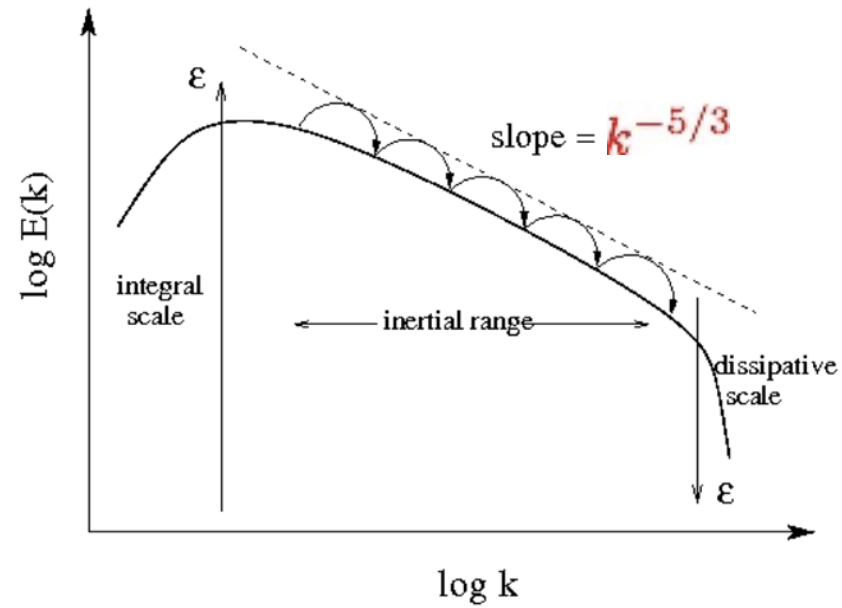
Back to Turbulent Transport…

Recall that
turbulent diffusivity is

$$D^{(\mathrm{turb})} \sim \delta u(\ell)\ell$$
$$\propto \ell^{4/3}$$

Thus, the largest-scale eddies make the largest contribution
to the turbulent transport

The interesting practical question is what that scale is and how fast these eddies are



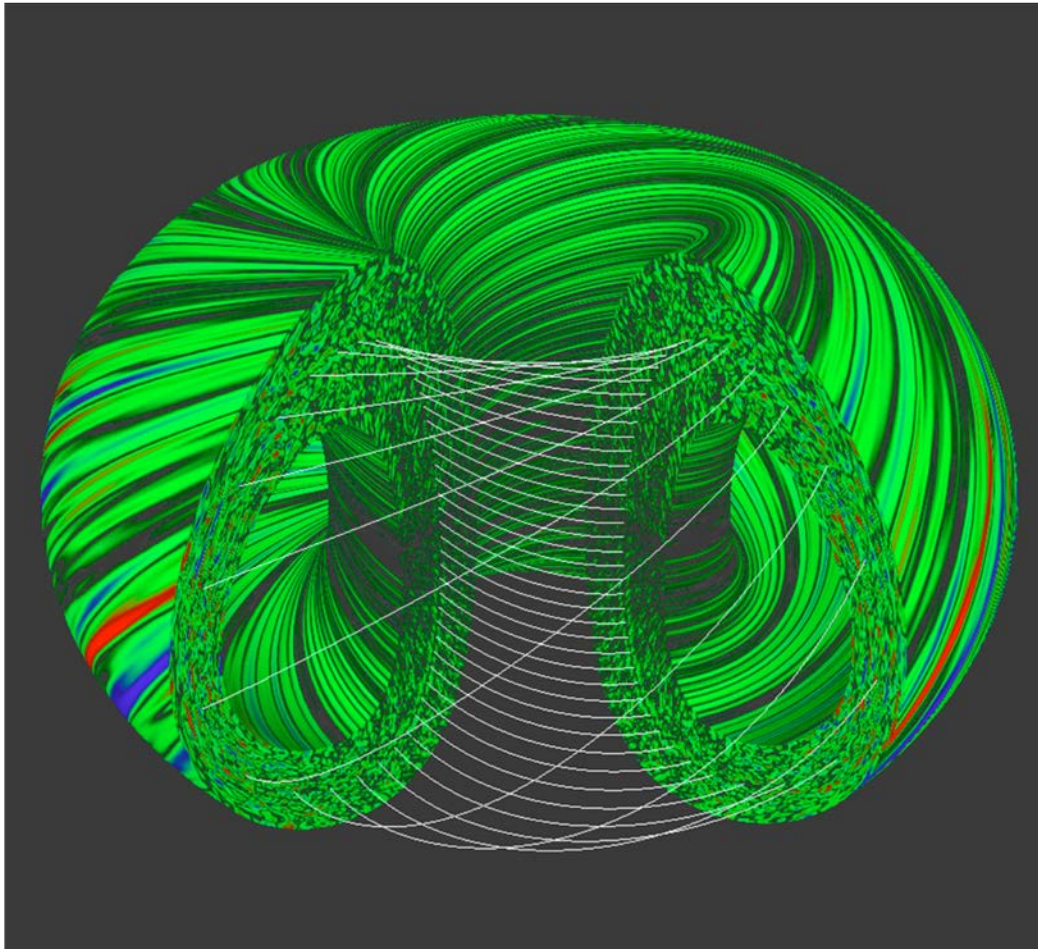
We wish to predict $\delta u(\ell) = u(r+\ell) - u(r)$

At each scale, $\dfrac{\rho\, \delta u(\ell)^3}{\ell} \sim P_{\mathrm{inj}} = \mathrm{const}$

Therefore, $\boxed{\delta u(\ell) \propto \ell^{1/3}}$ **K41**

Further Complications…

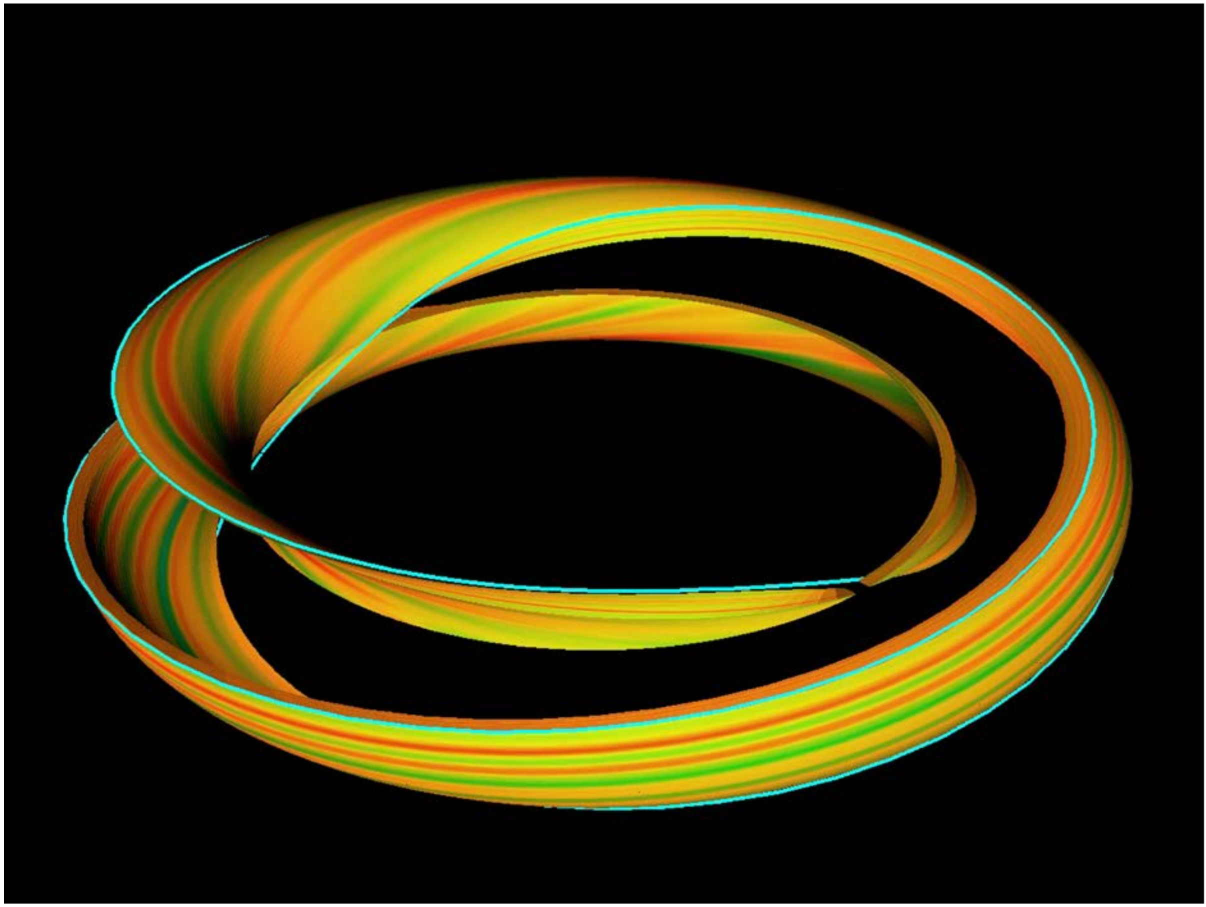
1. Turbulence in a tokamak is not homogeneous: conditions vary with radius, so we theorise/simulate locally on magnetic surfaces;



[Image: W. Dorland]

Further Complications…

1. Turbulence in a tokamak is not homogeneous: conditions vary with radius, so we theorise/simulate locally on magnetic surfaces; our "homogeneous box" is in fact a curvilinear flux tube:

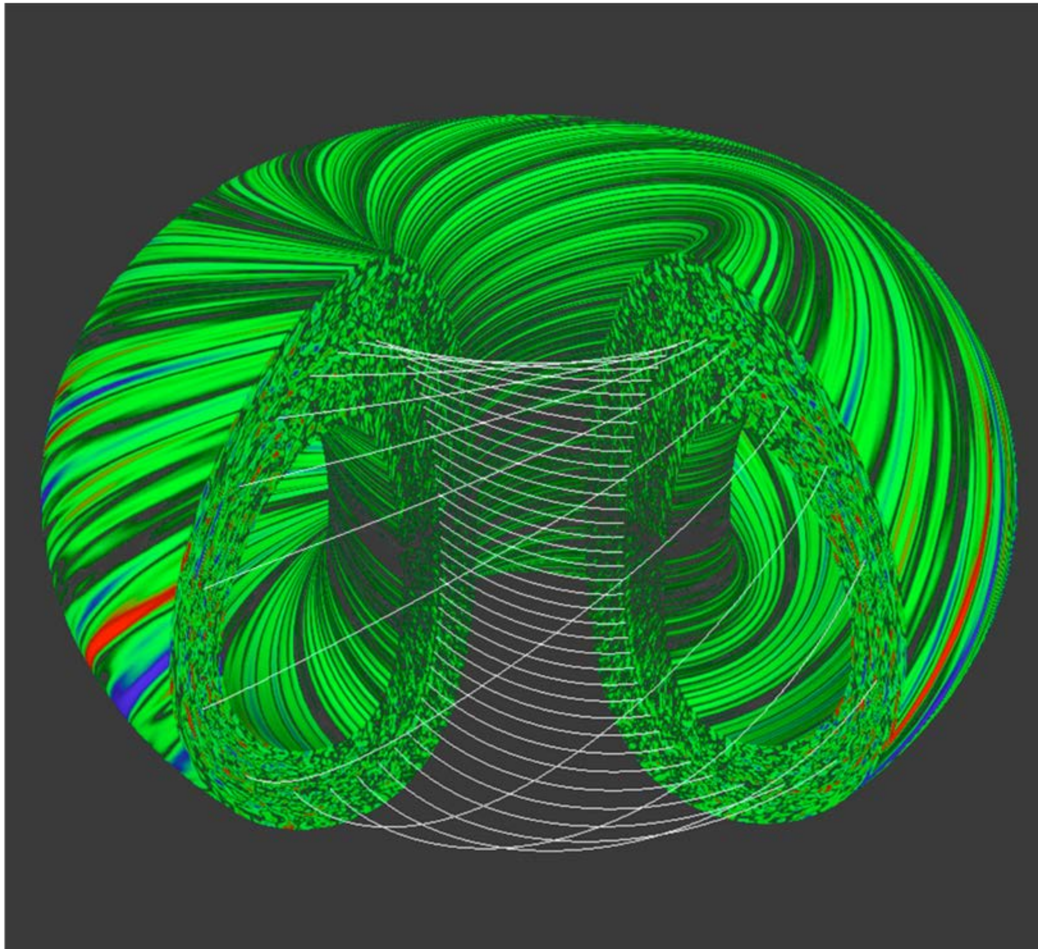


[Illustration:
E. Highcock, Oxford]

Further Complications…

1. Turbulence in a tokamak is not homogeneous: conditions vary with radius, so we theorise/simulate locally on magnetic surfaces; our "homogeneous box" is in fact a curvilinear flux tube

2. Turbulence in a tokamak is not isotropic:
everything is highly stretched along the magnetic field;
this requires some new theoretical concepts concerning the interplay of nonlinear energy cascade and linear wave propagation (along the magnetic field)



[Image: W. Dorland]

Further Complications…

3. Turbulence in a tokamak (and generally in plasmas) is not in a 3D space: in reality the plasma is described by a kinetic equation for the particle distribution function (PDF),

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f + \frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial f}{\partial \mathbf{v}} = C[f]$$

particle streaming — electric field — Lorentz force — collisions

The PDF $f(t, \mathbf{r}, \mathbf{v})$ is a field in a 6D phase space. In a turbulent system, small scales will develop not just in **r** but also in **v** (the $\mathbf{v}\cdot\nabla f$ term is a shear in phase space, leading to "phase mixing," i.e., formation of large gradients in velocity space). Thus we have to understand the cascade of energy (or, as it in fact turns out, entropy) in a 6D phase space.

Further Complications…

3. Turbulence in a tokamak (and generally in plasmas) is not in a 3D space: in reality the plasma is described by a kinetic equation for the particle distribution function (PDF),

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) \cdot \frac{\partial f}{\partial \mathbf{v}} = C[f]$$

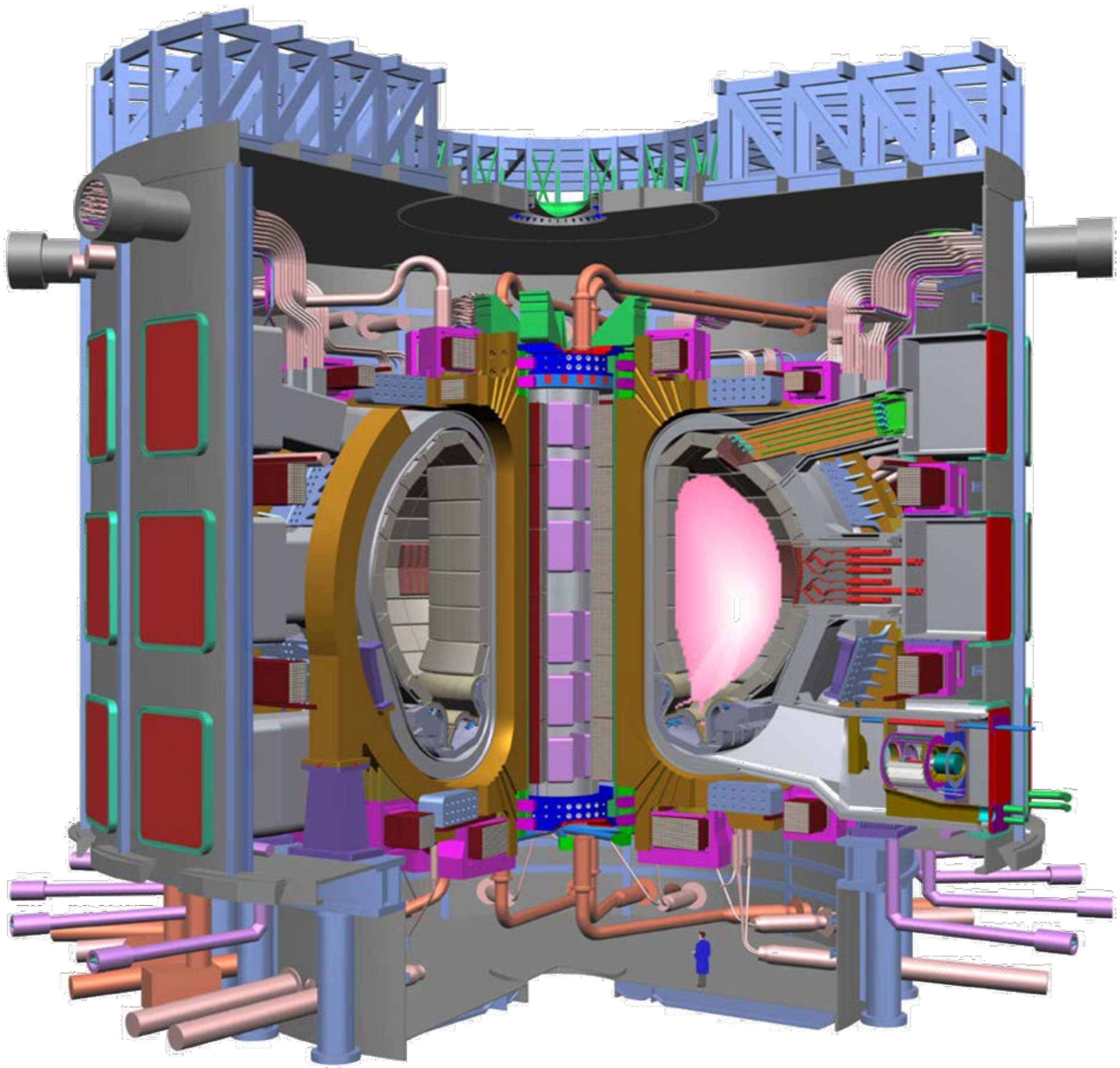
particle streaming   electric field   Lorentz force   collisions

The PDF $f(t, \mathbf{r}, \mathbf{v})$ is a field in a 6D phase space. In a turbulent system, small scales will develop not just in **r** but also in **v** (the $\mathbf{v} \cdot \nabla f$ term is a shear in phase space, leading to "phase mixing," i.e., formation of large gradients in velocity space). Thus we have to understand the cascade of energy (or, as it in fact turns out, entropy) in a 6D phase space.

4. You don't want to know what #4 is…

The Story So Far…

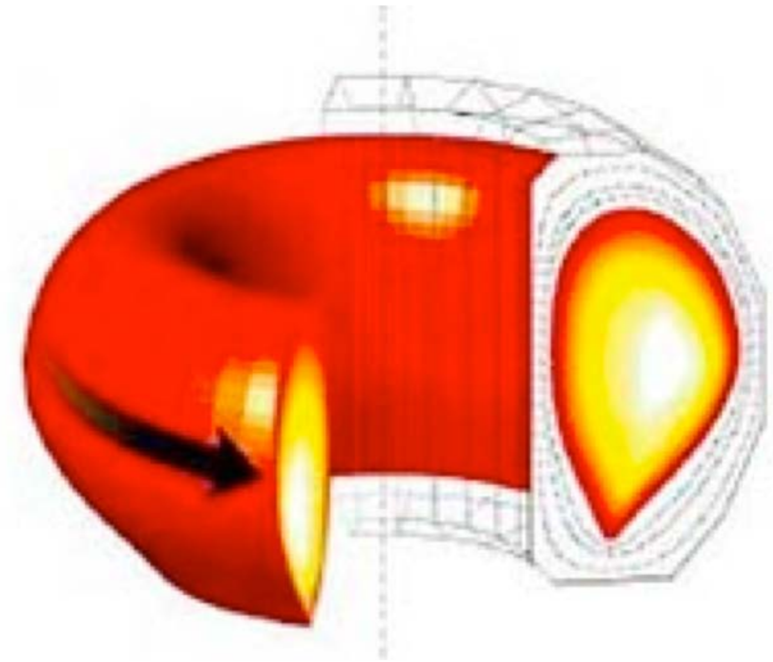
- We want to build a machine to tap the energy that fuels stars…



[Image: ITER]

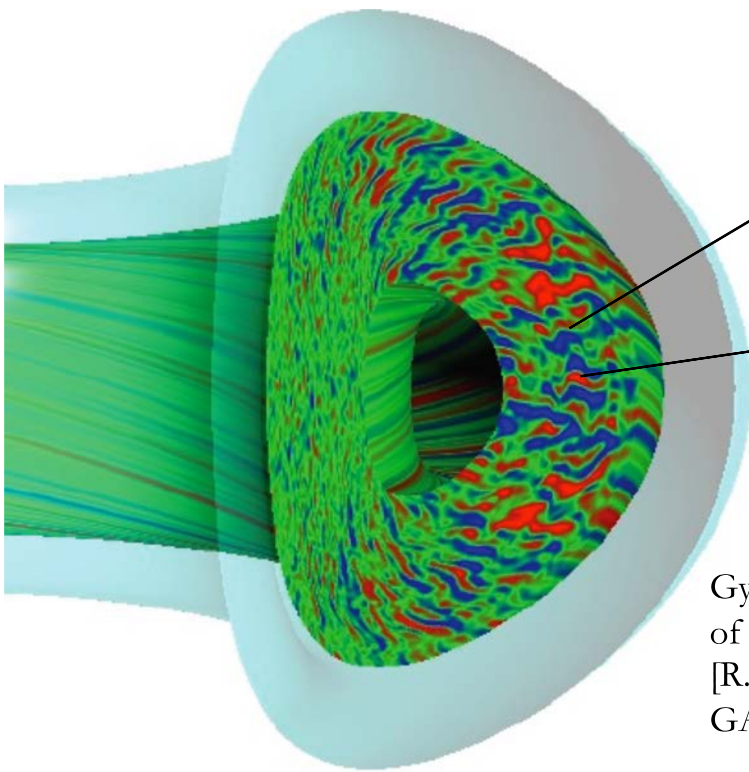
The Story So Far…

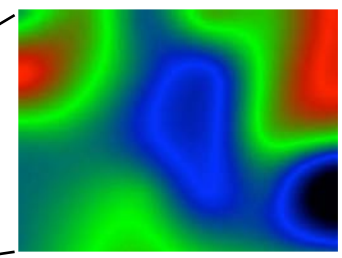
- We want to build a machine to tap the energy that fuels stars…
- Inside the machine, plasma is locked in a magnetic cage and kept out of equilibrium (hot inside, cold outside)…



The Story So Far…

- We want to build a machine to tap the energy that fuels stars…
- Inside the machine, plasma is locked in a magnetic cage and kept out of equilibrium (hot inside, cold outside)…
- It rattles its cage, breaks into whirls and swirls in its quest to regain equilibrium… **To keep it in and keep it hot, we must tame the nonlinear beast: turbulence…**





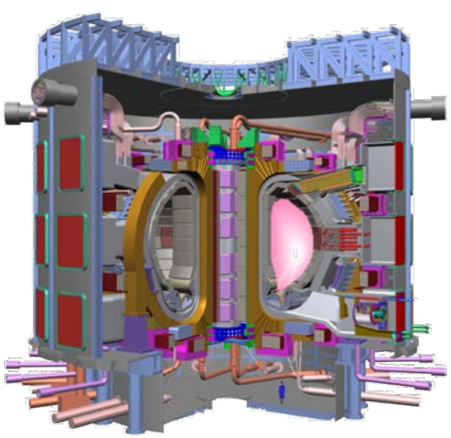
BES measurement
of density fluctuations
in the DIIID tokamak
[G. McKee, GA & UW Madison]

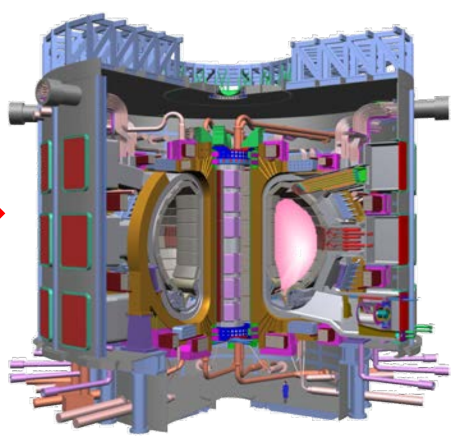
Gyrokinetic simulation
of the DIIID tokamak
[R. Waltz & J. Candy,
GA, San Diego]

The Story So Far…

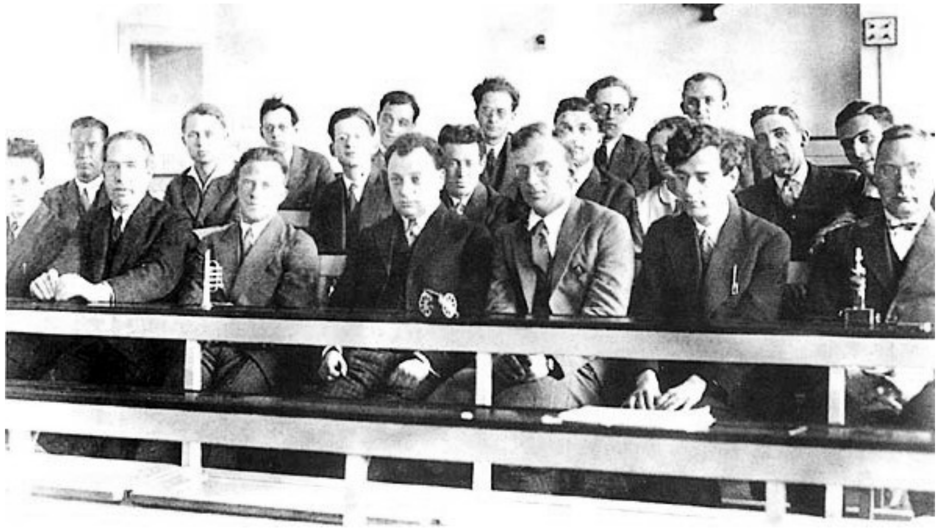
- We want to build a machine to tap the energy that fuels stars…
- Inside the machine, plasma is locked in a magnetic cage and kept out of equilibrium (hot inside, cold outside)…
- It rattles its cage, breaks into whirls and swirls in its quest to regain equilibrium… To keep it in and keep it hot, we must tame the nonlinear beast: turbulence…

- *But the* ***real reason*** *we're in this business is that we get to probe Nature's tricks – and find that, on a journey to sort out a fusion power plant, we can take a scenic route via a nearby galaxy cluster…*





The Story So Far…



- *But the **real reason** we're in this business is that we get to probe Nature's tricks – and find that, on a journey to sort out a fusion power plant, we can take a scenic route via a nearby galaxy cluster…*