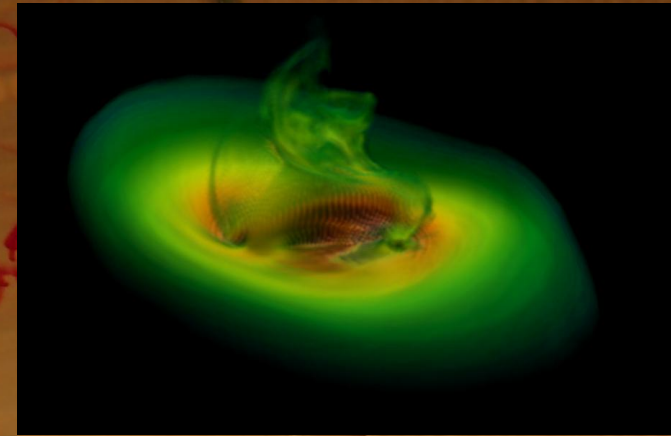
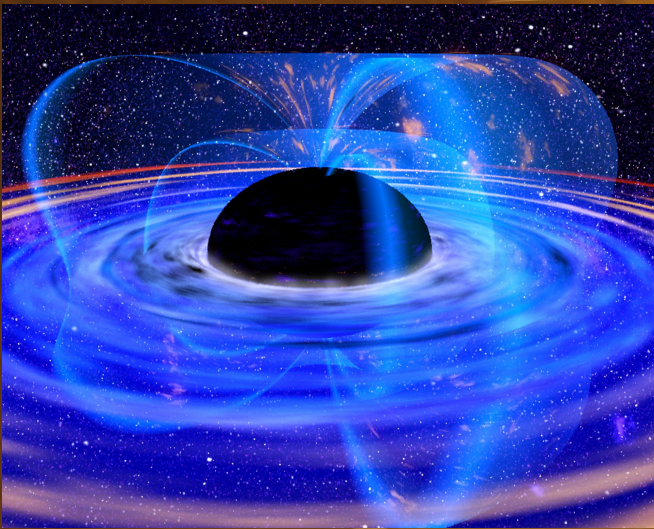
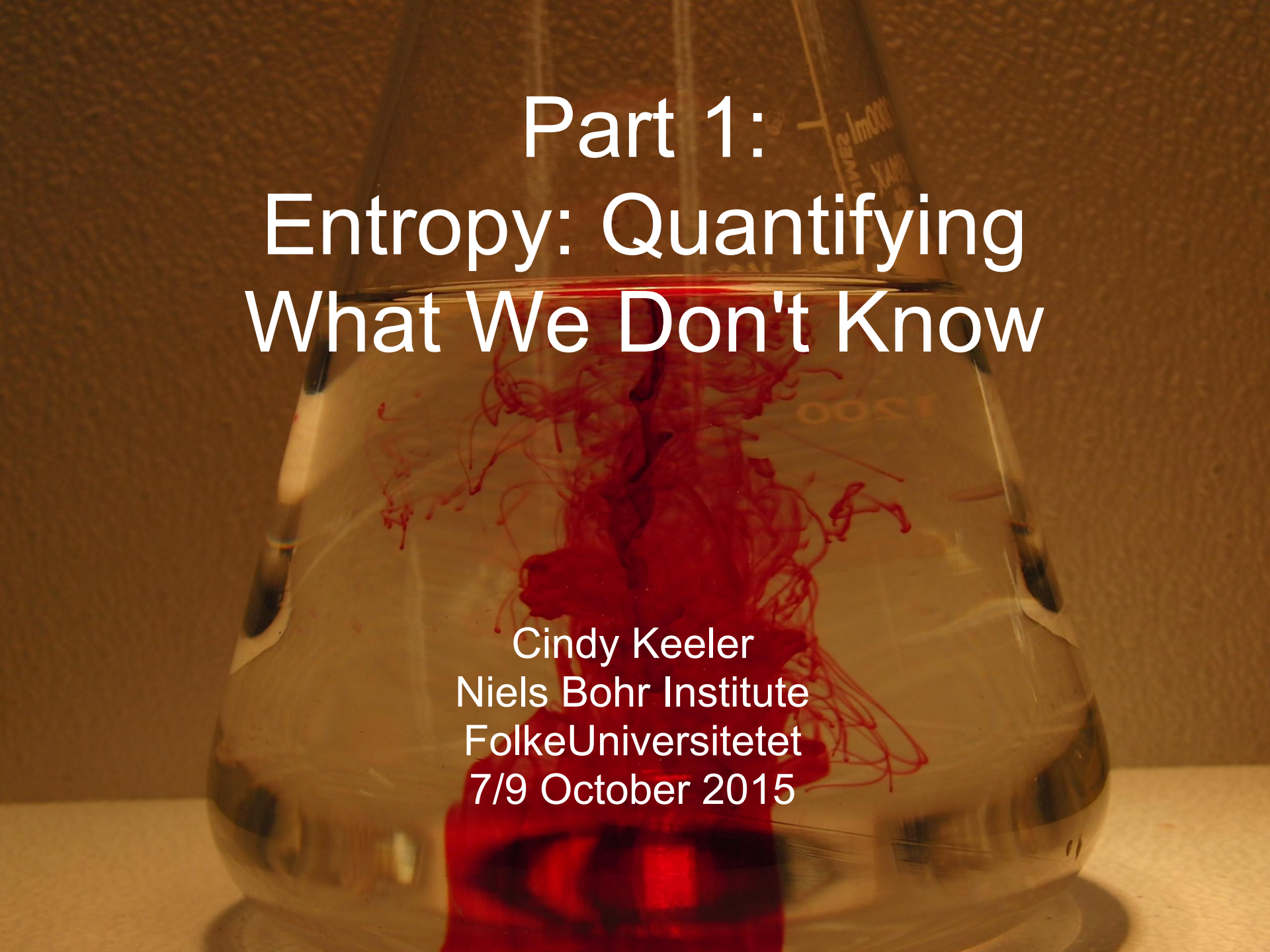


BLACK HOLES AND ENTROPY



Cindy Keeler
Niels Bohr Institute
FolkeUniversitetet
7/9 October 2015

A glass of water with red ink being poured in, creating a chaotic, fractal-like pattern. The ink is being poured from the bottom, creating a complex, branching structure that fills the water. The background is a textured, brownish surface.

Part 1: Entropy: Quantifying What We Don't Know

Cindy Keeler
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7/9 October 2015

What is Entropy?

- OED: "The name given to one of the quantitative elements which determine the thermodynamic condition of a portion of matter."
- "tendency of systems to settle into most common state"
- "measure of disorder"
Dansk Fremmedordbog: "et maal for uorden i et system"
- Wikipedia: "a measure of the amount of thermal energy that *cannot* be used to do work."

All of these definitions capture some aspects of entropy.

Entropy: the Formula

$$\log[\text{\# of accessible microstates}]$$

Log is useful for discussing large numbers: it gives us the exponent.

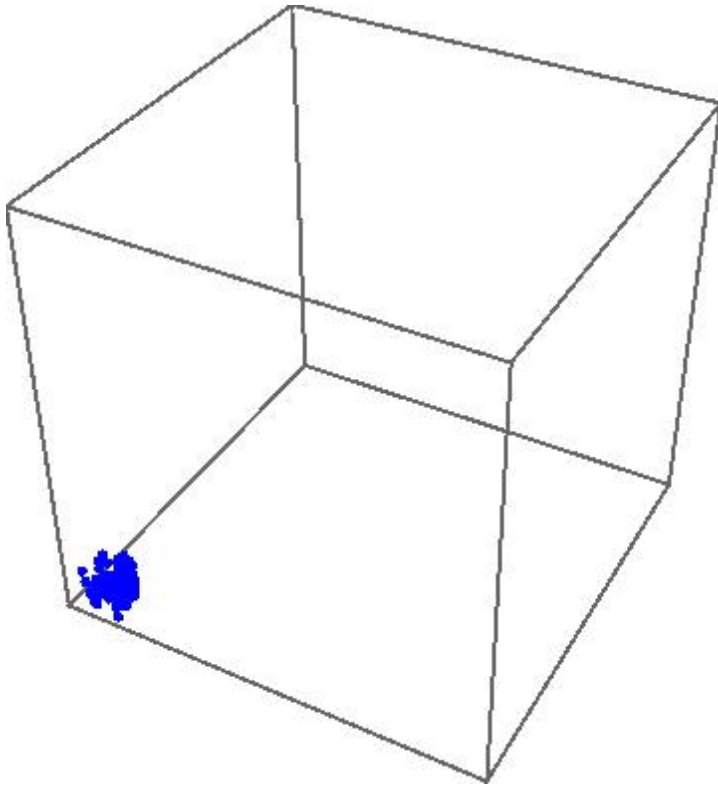
Largest Mersenne Prime: $2^{57885161} - 1$
this number is 17,425,170 digits long.

"accessible microstate" is a particular microscopic arrangement whose properties match your macroscopic requirements, such as:

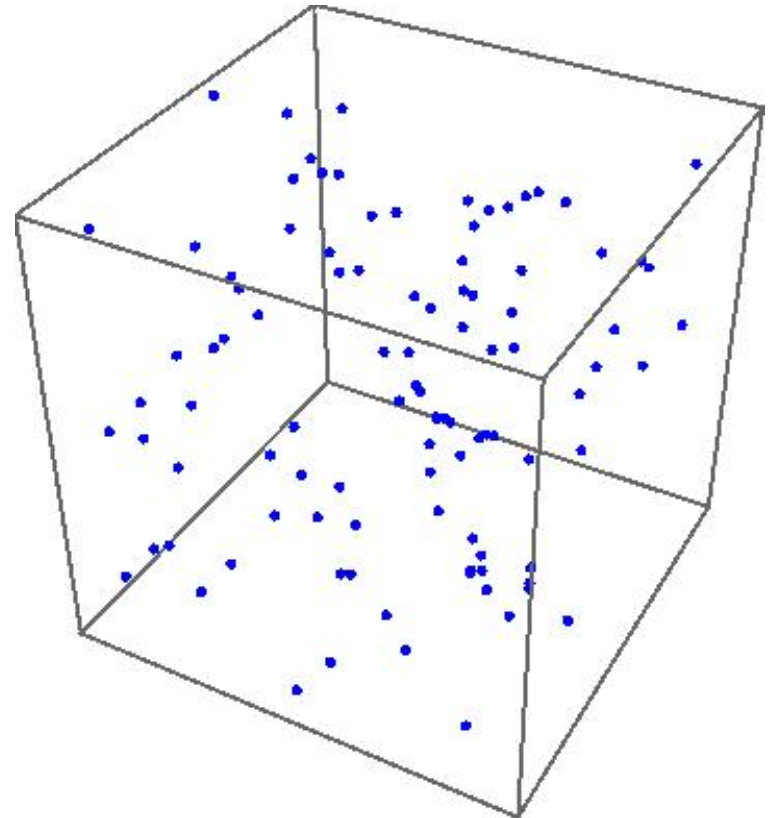
Pressure, Volume, Temperature, # of molecules, electric charge, magnetic field

Air Molecules Spread Out

100 molecules in a 10m cube. Model each one as a one angstrom cube.



$\sim 10^{330}$ ways to arrange in 1 m^3



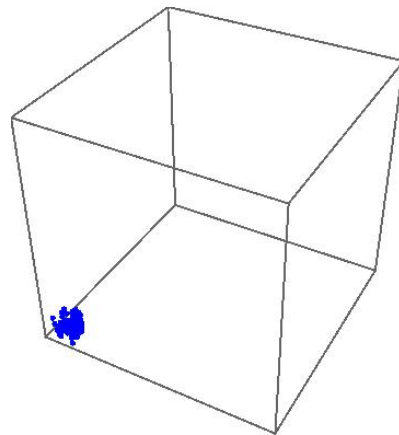
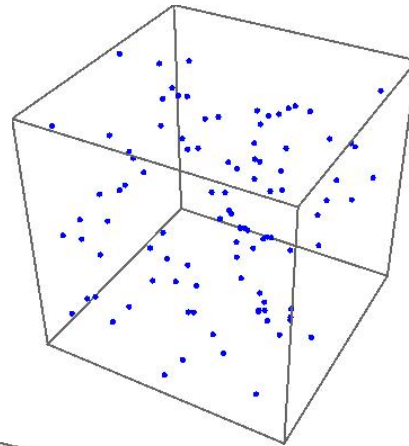
$\sim 10^{3300}$ ways to arrange in 10 m^3 !

How many more ways?

Since the big bang, there have been $\sim 10^{18}$ seconds.

Yocto = 10^{-24} is the smallest currently recognized SI unit prefix.

If we pick one configuration of molecules from $\sim 10^{3300}$ ways to arrange them in a 10 m^3 every yoctosecond,



then it will take 10^{2028} times
the lifetime of the universe
before we have even half a chance of
hitting a configuration where they all sit
in this corner!

Preview of The Laws of Thermodynamics

A system at equilibrium has maximum entropy, that is, it is in the most common state.

Quantum mechanically:

Once we set the **macroscopic quantities** temperature, volume, charge, etc the system has equal probability of being in any **microscopic state** that matches the macroscopic requirements.

But sometimes there are many more microscopic states of one kind than another!

In our example, we knew the number of molecules, and the total volume available.

The molecules will take up all the space, rather than crowding into a corner, because there are **so many more ways for them to be arranged in the full space.**

"What we don't know" = exact arrangement ("microstate") the molecules are in.

Non-quantum mechanically..

This is a little like the teenager who can't keep their room clean. So many more ways to be messy than to put everything back where it goes!

(The quantum mechanics matters here, but the analogy is ok as an analogy).

Intuition about Entropy



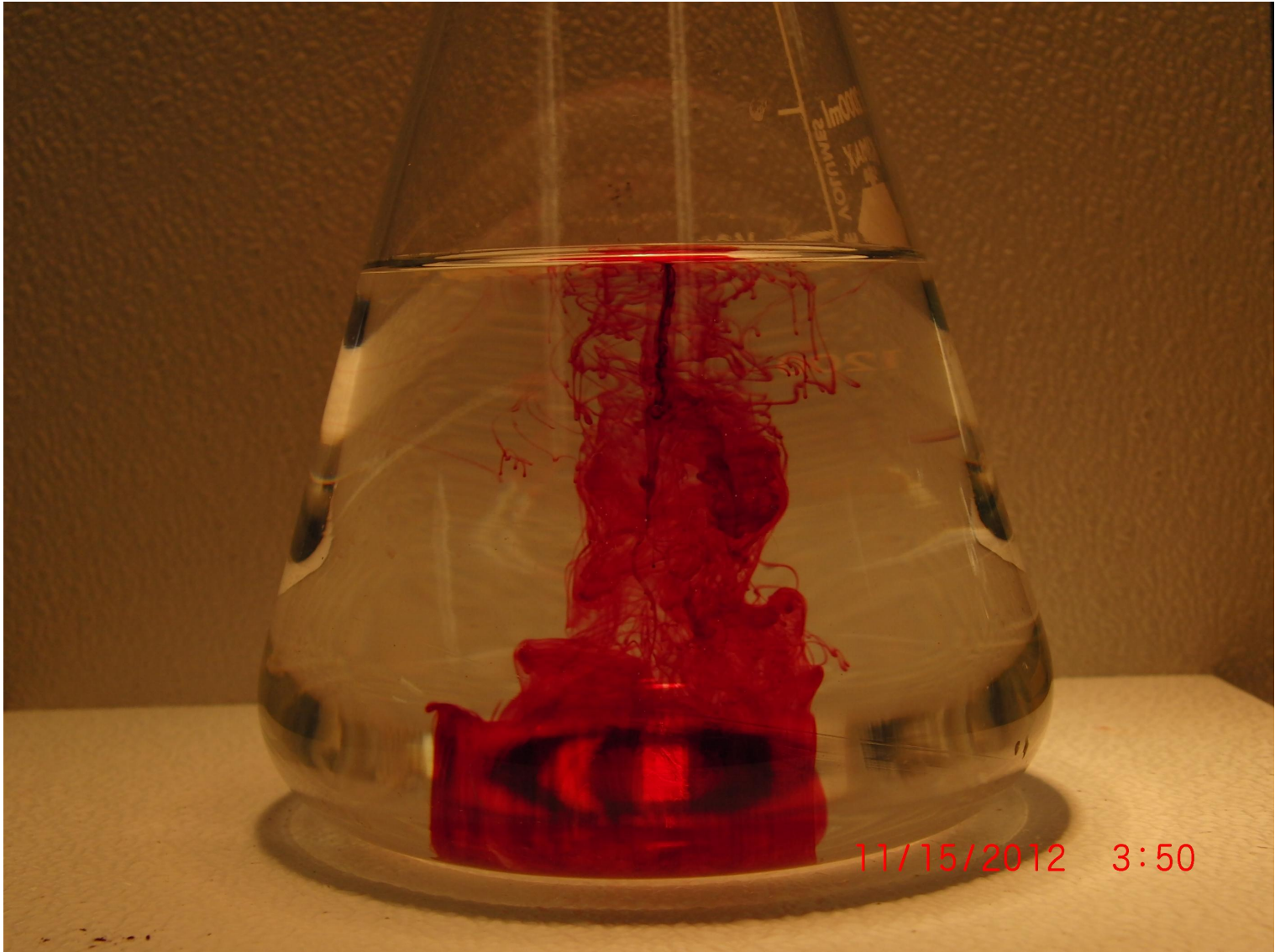
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Intuition about Entropy

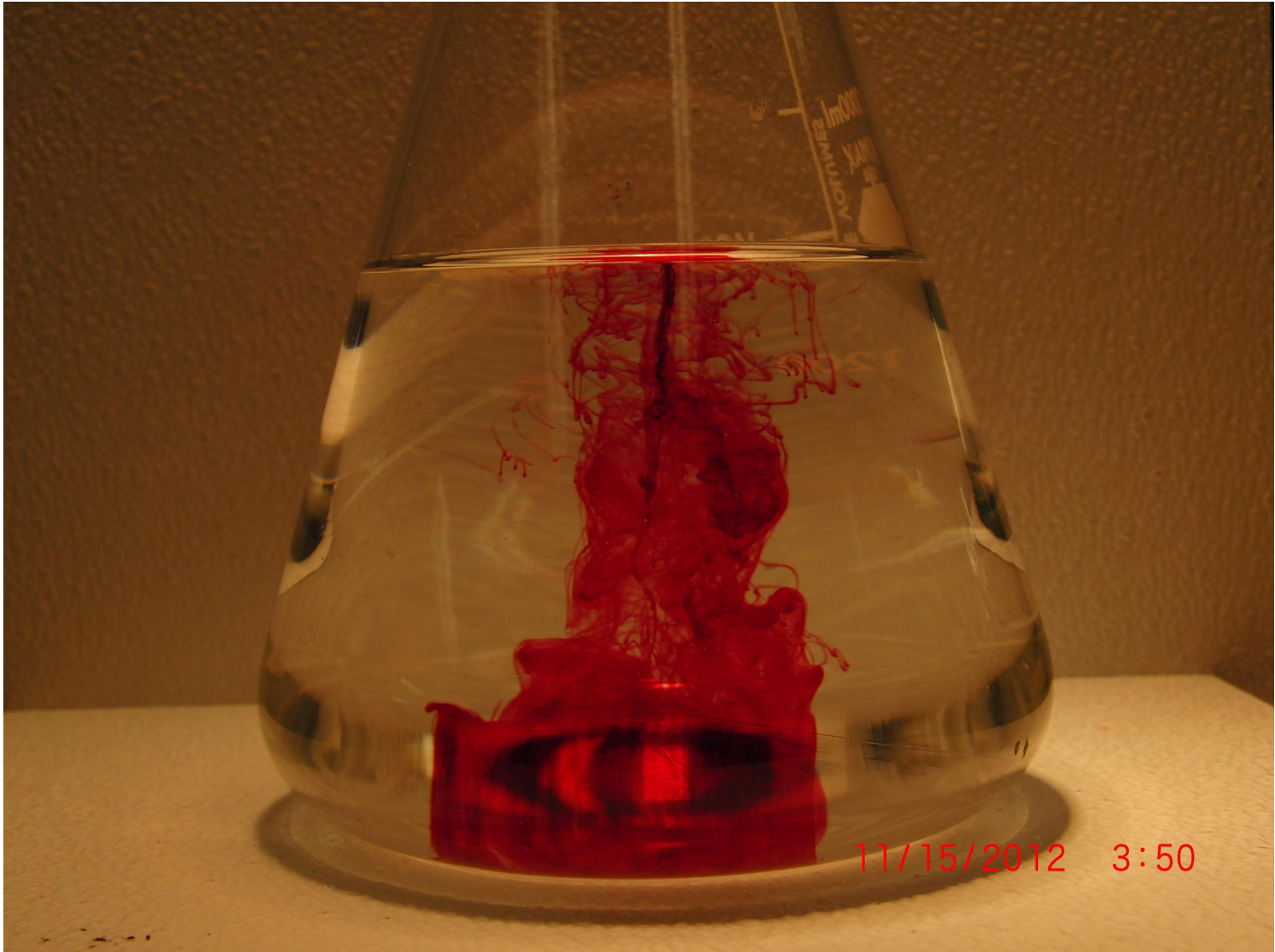


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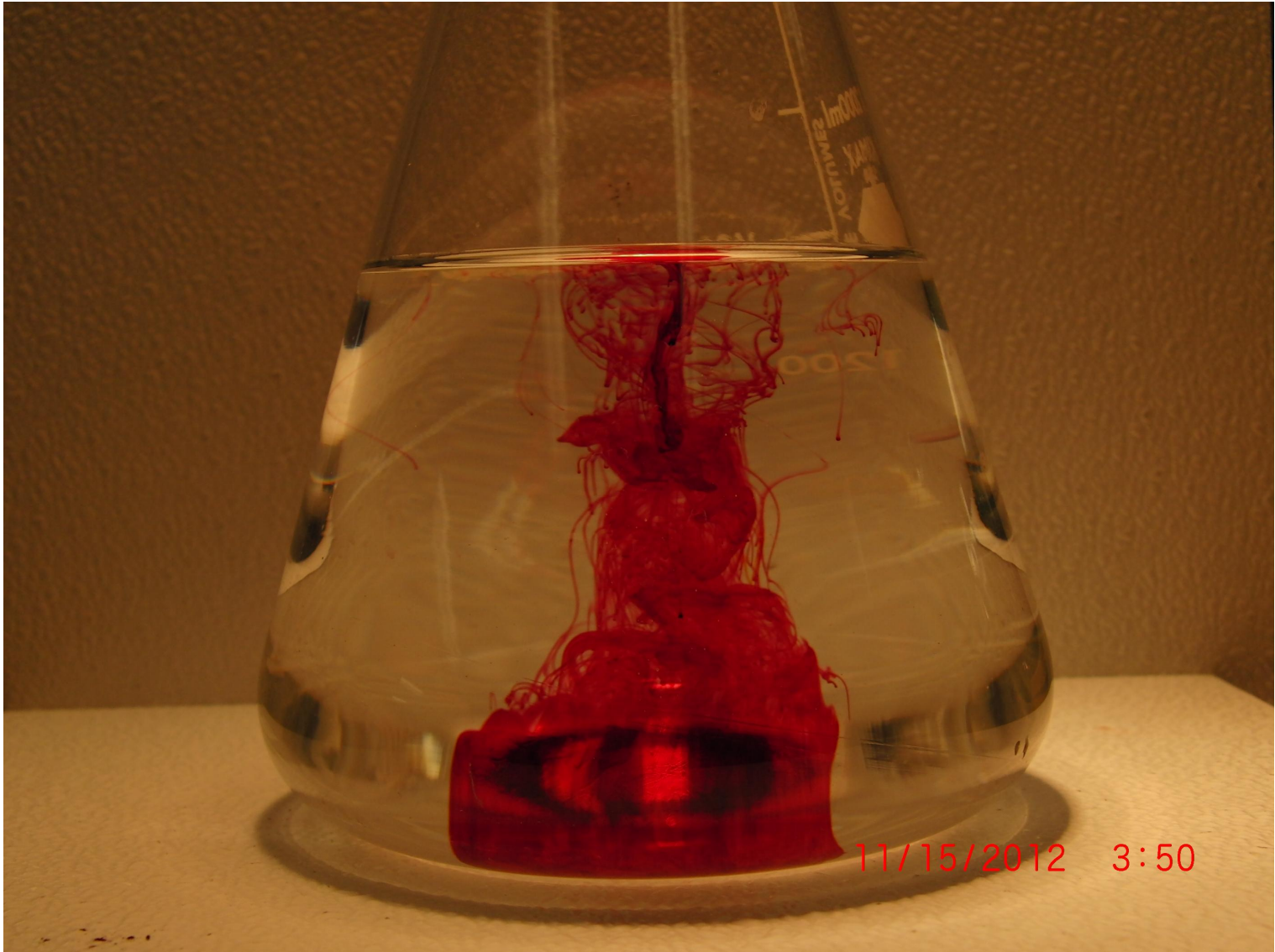
Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Wait a minute..
something's not right
here.

Wait a minute..
something's not right
here.

We want to go *forwards*
in time.

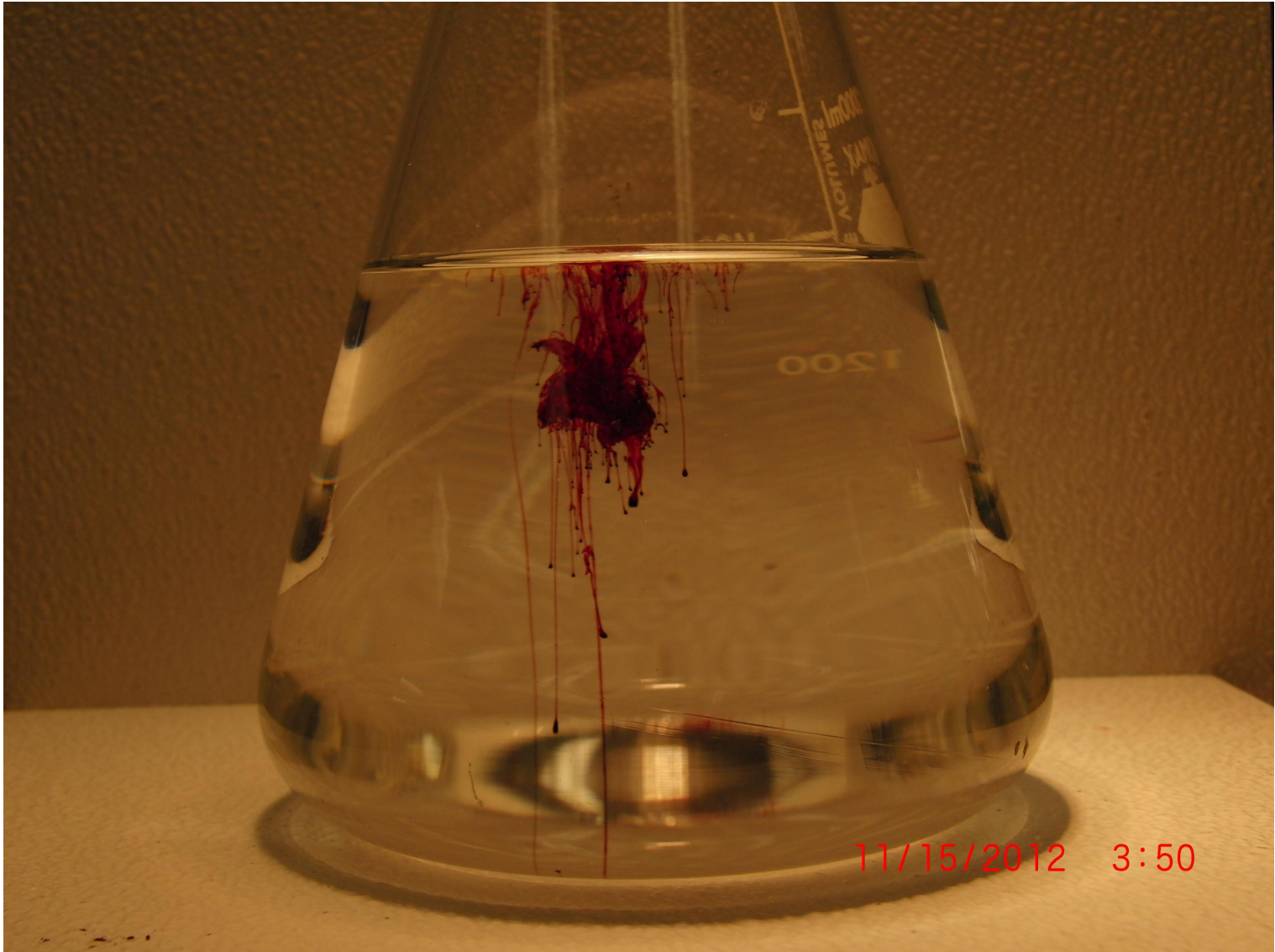
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Intuition about Entropy



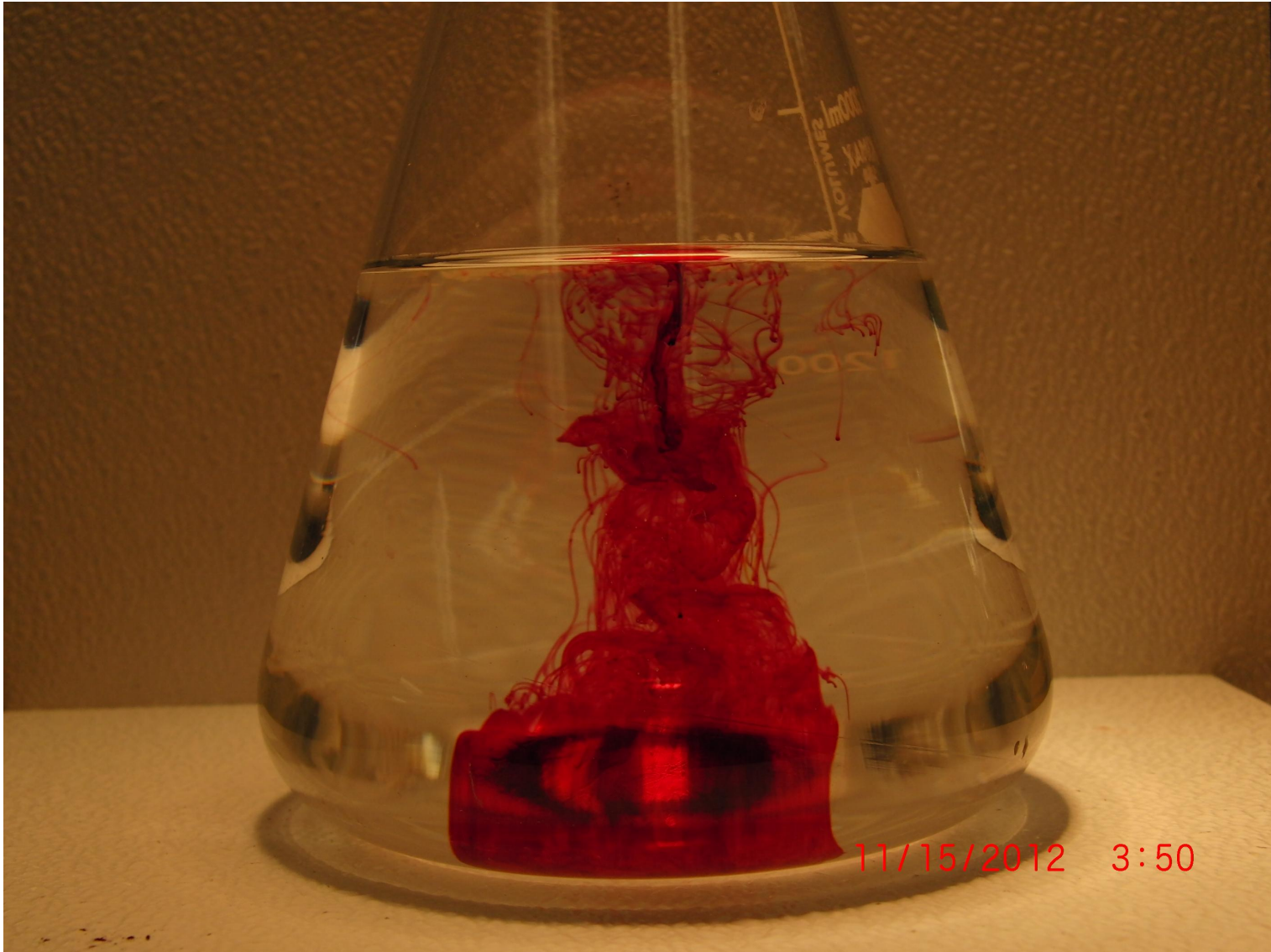
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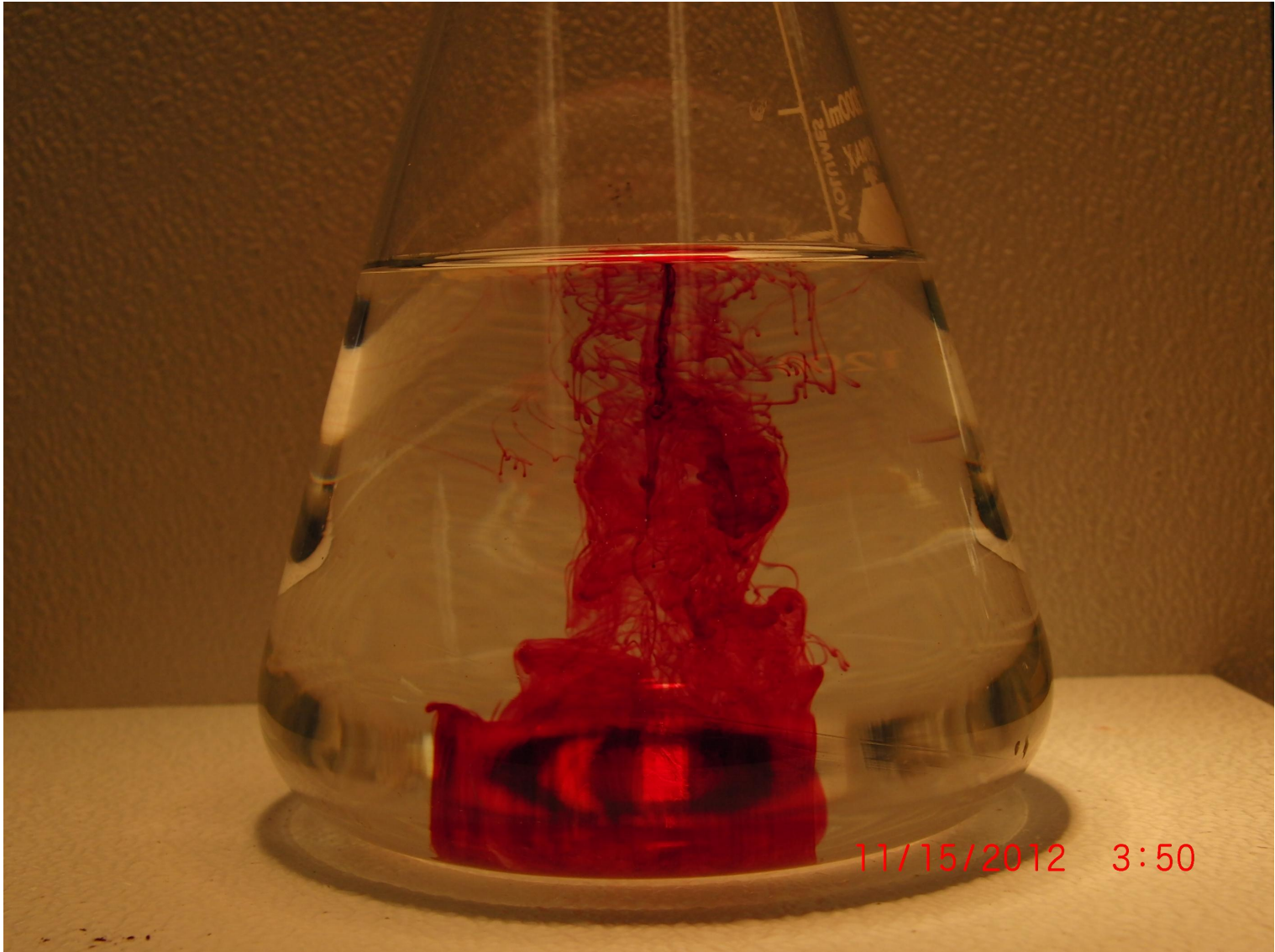
Intuition about Entropy



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Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Intuition about Entropy



Things Mix, Not Unmix

This is the Second Law of Thermodynamics.

Entropy increases.

Let us try now with dye!

(Video available at https://www.youtube.com/watch?v=p08_KITKP50)

Things Mix, Not Unmix

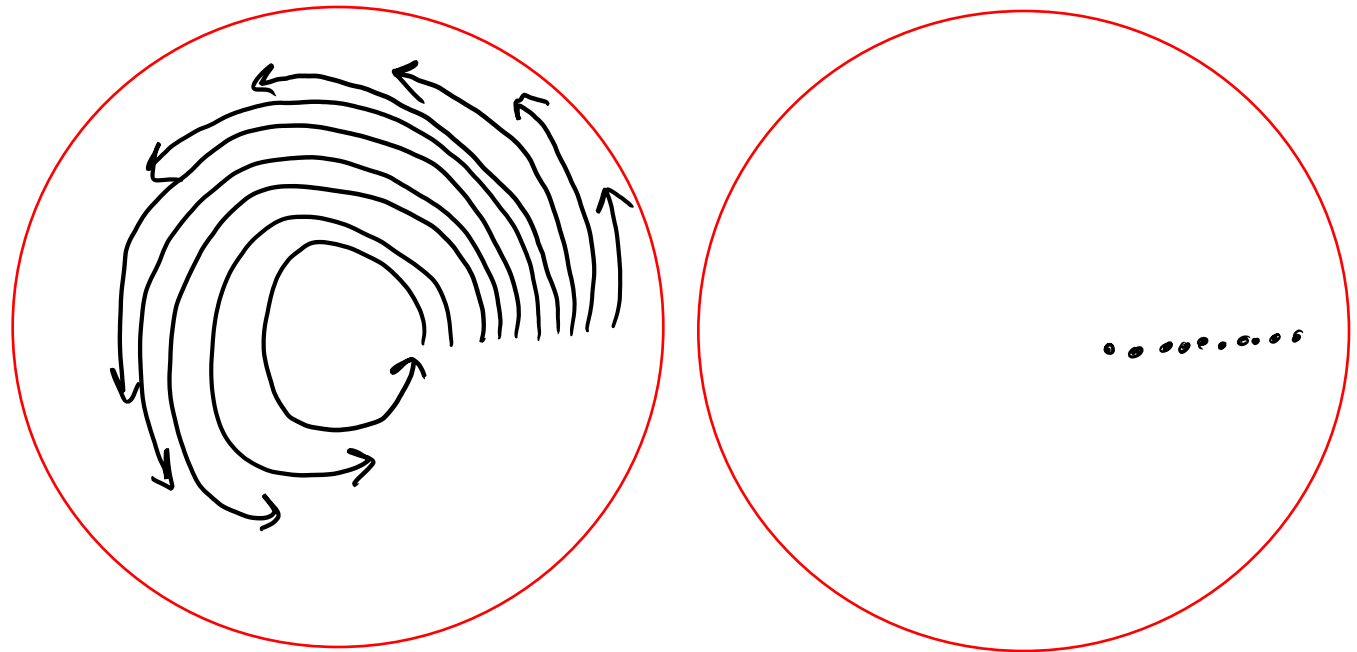
This is the Second Law of Thermodynamics.

Entropy increases (for closed systems).

Equilibrium= largest entropy

So what happened?

Laminar flow!



The Laws of Thermodynamics

- **Second: You can't break even.**

(Entropy increases over time for any closed system, so you're always closer to equilibrium overall than you were when you started).

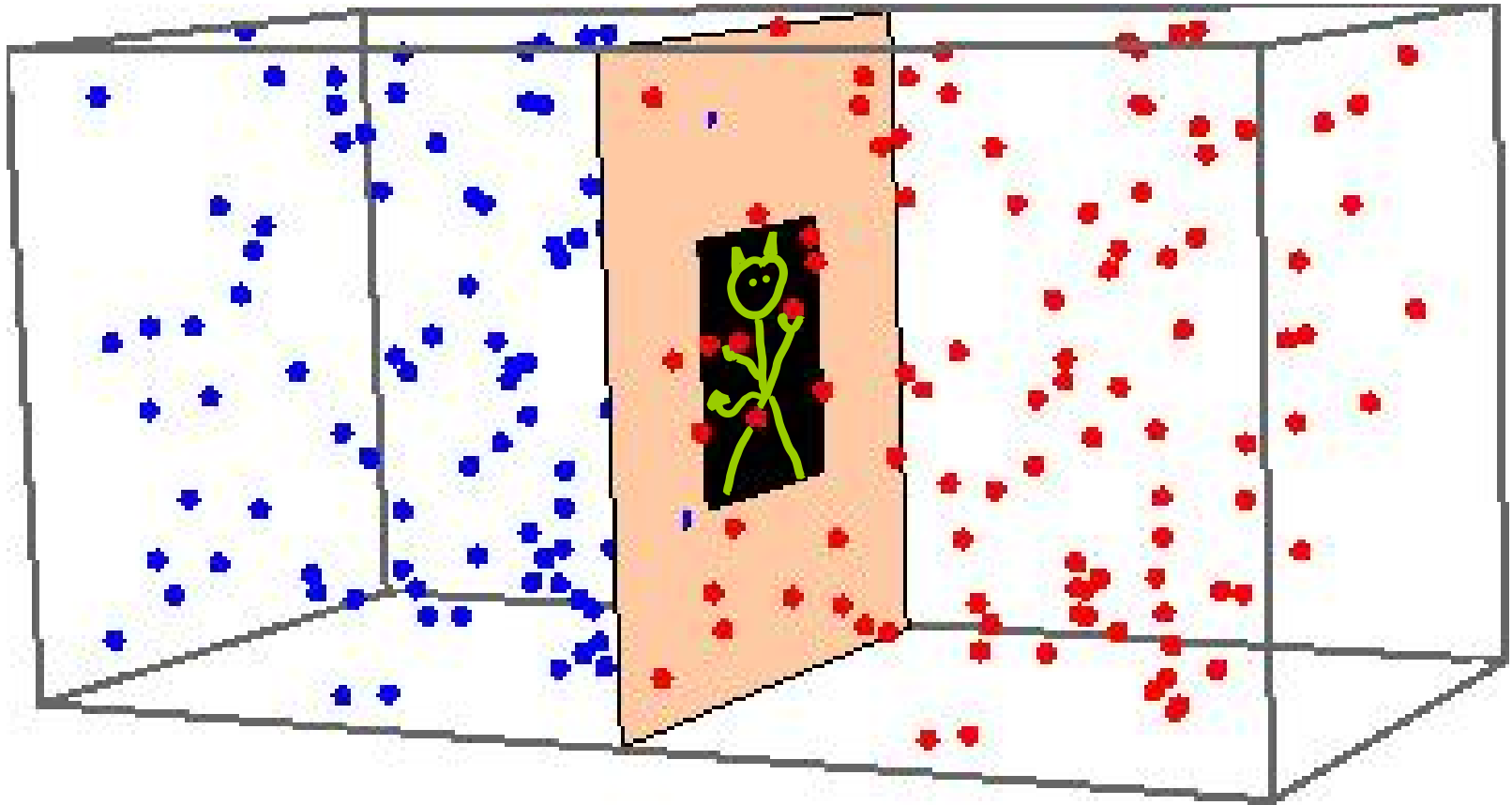
This matches with our intuition before. As time goes forward in a closed system:

gas molecules spread to fill the room
dye distributes throughout the liquid
temperature becomes even

How can we break this law?

Maxwell's Demon

What if we found Maxwell's Demon?

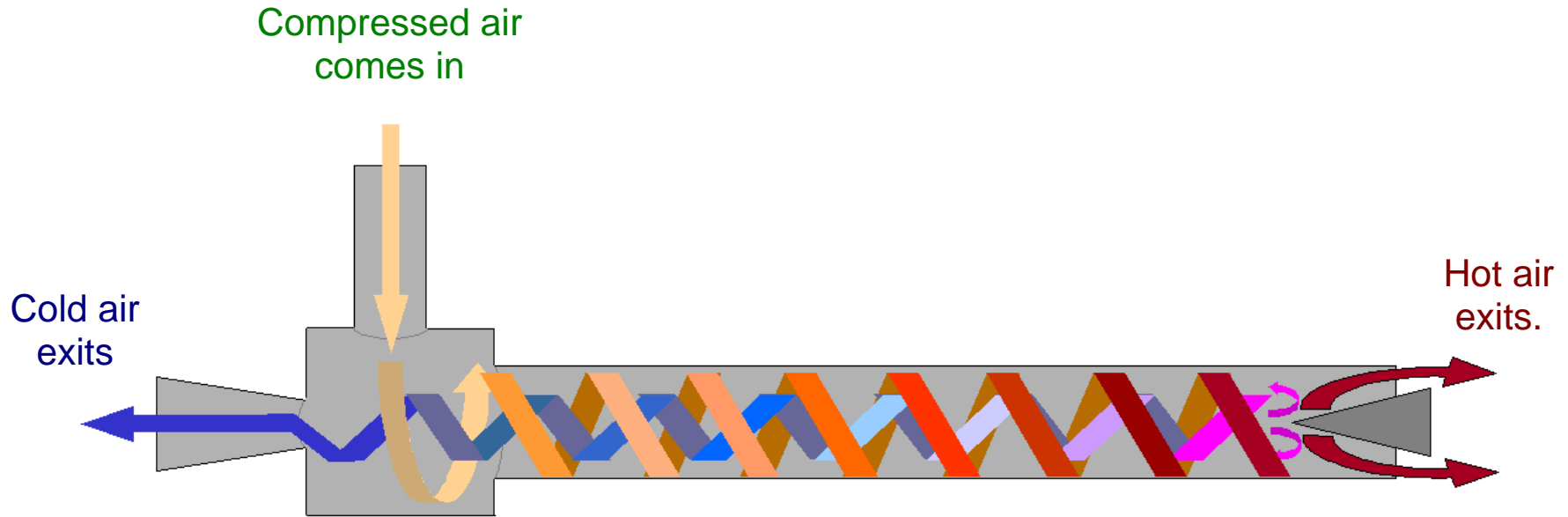


Hilsch tube



Put compressed air in and "Maxwell's Demon" will sort the molecules, pushing hot air one way and cold air the other!

Hilsch Tube



Sorts air into hot and cold streams.

But how? We haven't trapped any demons here.

The Laws of Thermodynamics

- First: You can't win.
(To do work, you have to put in energy.)
- Second: You can't break even.
(Entropy increases over time for any closed system, so you're always closer to equilibrium overall than you were when you started).

A system at equilibrium has maximum entropy, that is, it is in the most common state.

To move it out of equilibrium, we have to do work!

The upside: if a system is out of equilibrium, we can use it to do work.

Sterling Engines

Sterling engines convert a temperature difference into mechanical energy.



These engines can be used in industrial applications (e.g. old sewing machines) or they can be small enough to work on just the heat of your hand!

The Laws of Thermodynamics

- First: You can't win.
(To do work, you have to put in energy)
- Second: You can't break even.
(Entropy increases in any process)
- Third: You can't quit the game.
(except at absolute zero, but you can't get there.)
- Zeroth: You have to play the game.
(Thermal equilibrium happens!)

The Laws of Thermodynamics

Even physicists have to play by the rules..



The Laws of Thermodynamics



Well, most of them.

Broader Uses of Entropy

Information Theory



Stock Analysis



and more!

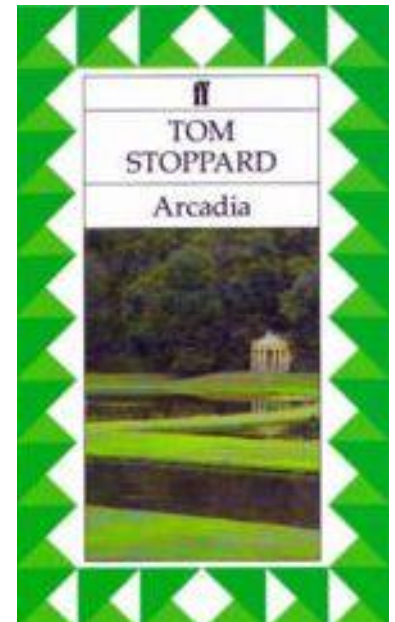
Linguistics



Ecology



Literature

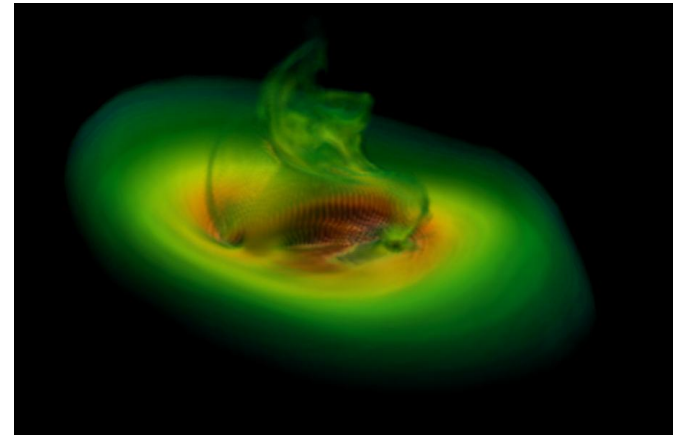
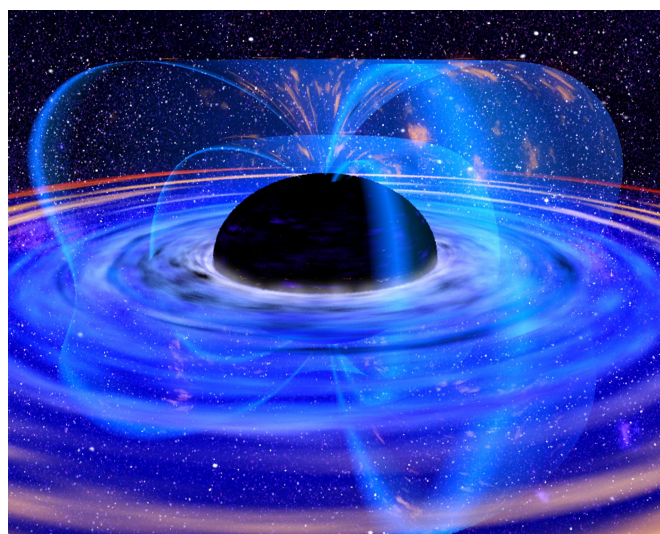


A glass flask is the central focus of the image. It is partially filled with a clear liquid. A small amount of red liquid has spilled from the top of the flask, dripping down its side. The background is a textured, brownish surface. The text is overlaid on the flask.

End of Part 1:
Entropy: Quantifying
What We Don't Know

PAUSE

Part 2: Black Hole Thermodynamics



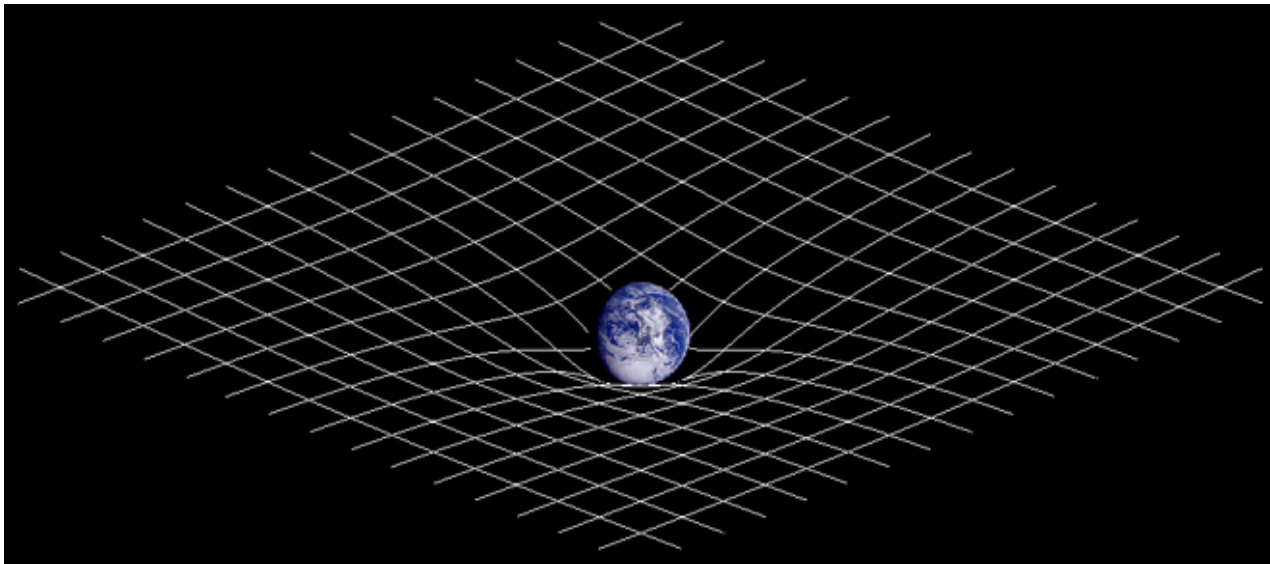
General Relativity

Recall from last week:

Einstein Equation connects **Curvature** to **Energy Density**.

$$G_{mn} = (8 \cdot \pi \cdot G / c^4) T_{mn}$$

so as we put in more matter=energy, space gets more curved:

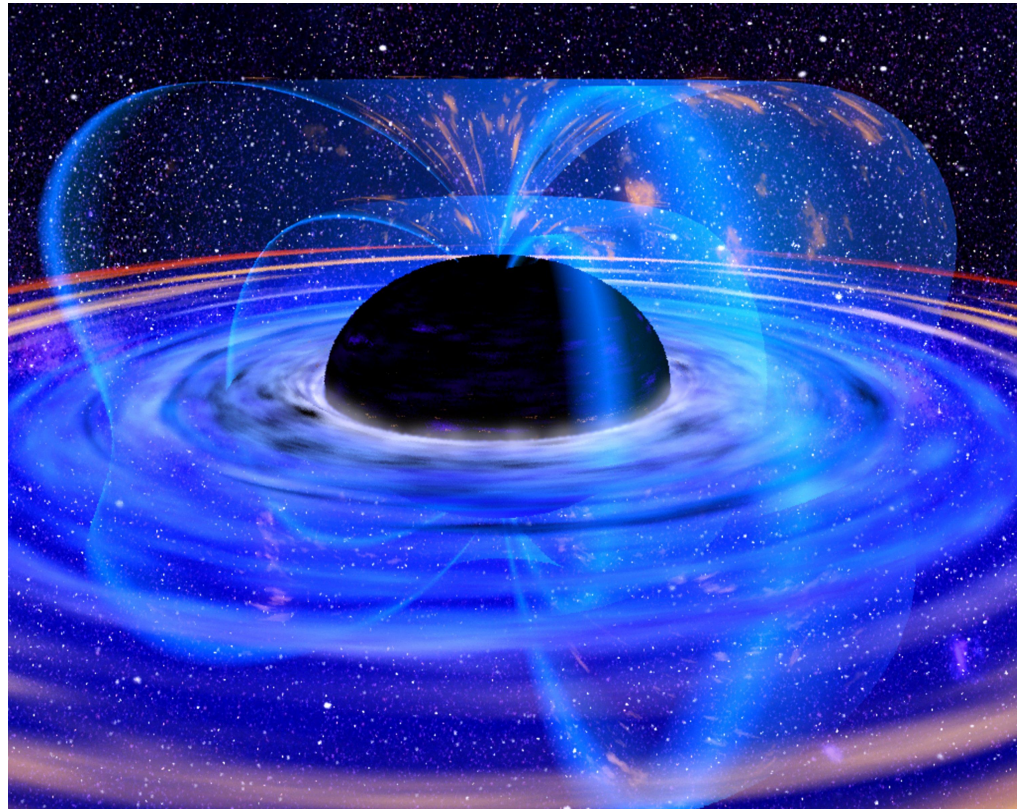


Light follows this curvature.

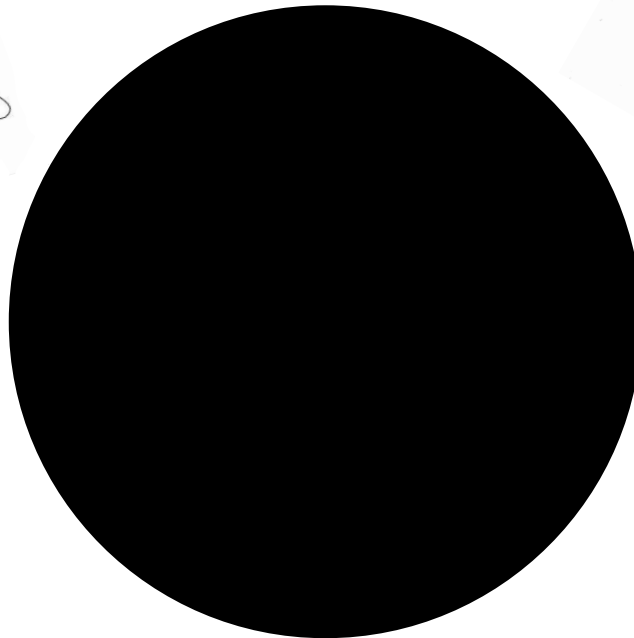
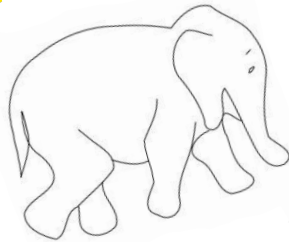
But what if spacetime gets so curved, light can't get out?

What is a Black Hole?

- They have an event horizon: Gravity is so strong not even light can escape
- Black Holes have no hair: only charge, mass, and spin
- a great laboratory for theoretical physicists!
- They're also real..
e.g. at the center of the Milky Way



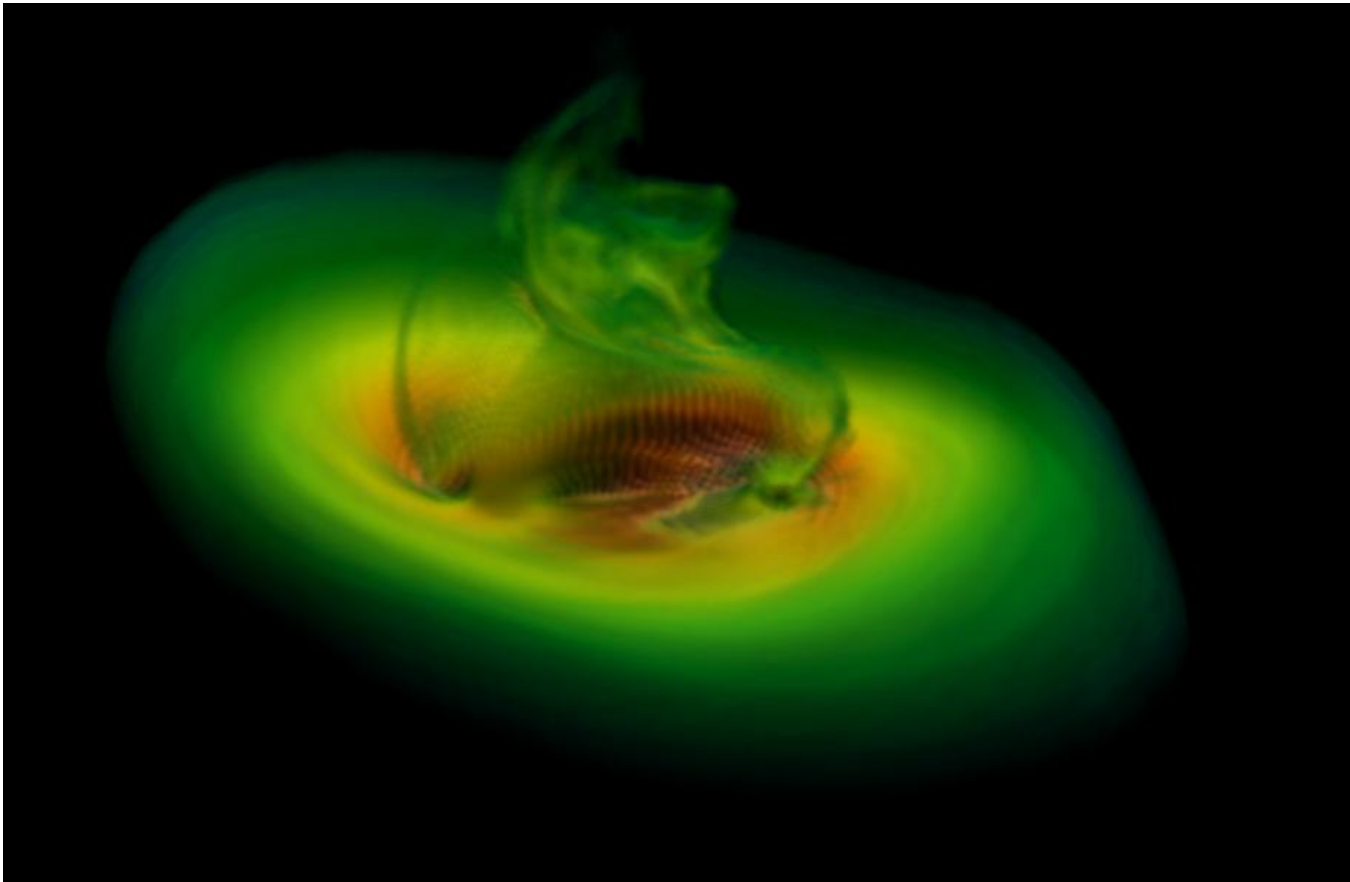
We can throw things in. . .



VAOFCBØINÅ

And the surface area of the event horizon gets bigger.
(and still we only know the mass, charge, and spin)

Black Hole Merger



Credit: NASA's Goddard Space Flight Center; P. Cowperthwaite, University of Maryland

After the merger, the surface area of the event horizon is bigger! (and still we only know the mass, charge, and spin)

Generalized Second Law of Thermodynamics

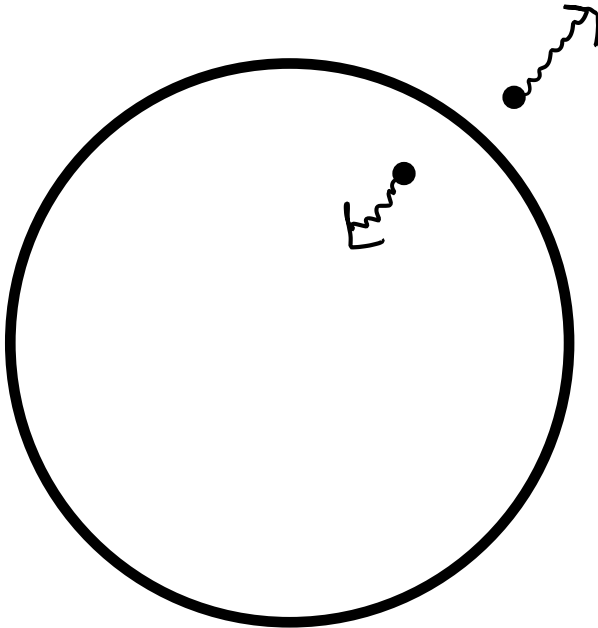
All processes increase the sum of
(entropy of all matter in universe + surface area of all black holes).

Black hole temperature

Surface gravity = temperature

(Force required to keep an object at the event horizon from falling in)

And they Hawking radiate:



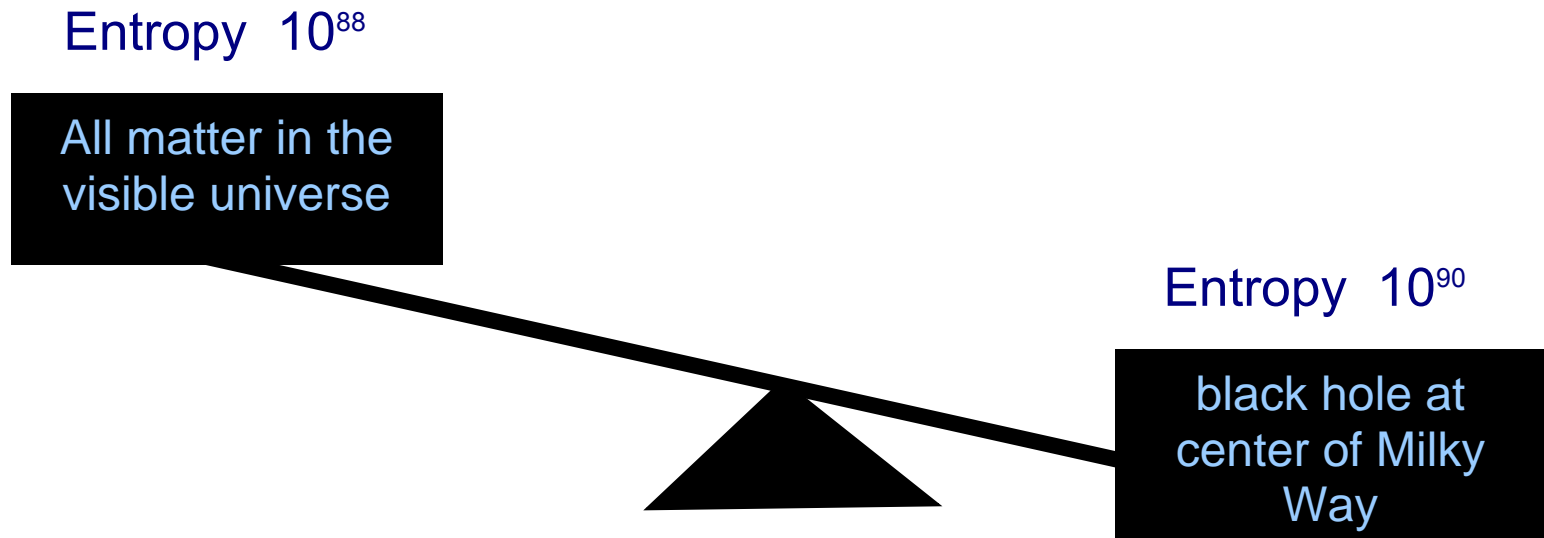
Temperature is tiny:
for million-solar-mass black hole,
 $T \sim 10^{-13}$ K

Laws of Black Hole Thermodynamics

- First: You can't win.
(To do work, you have to put in energy)
change in energy =
temp times change in entropy
- Second: You can't break even.
(Entropy increases in any process)
- Third: You can't quit the game.
(except at absolute zero, but you
can't get there.)
- Zeroth: You have to play the game.
(Thermal equilibrium happens!)

- First: You can't win.
change in mass = surface
gravity times change in area of
event horizon
- Second: You can't break even.
(Area of event horizon increases
in any process)
- Third: You can't quit the game.
(except at zero surface gravity,
but you can't get there.)
- Zeroth: You have to play the
game.
(Hawking radiation happens!)

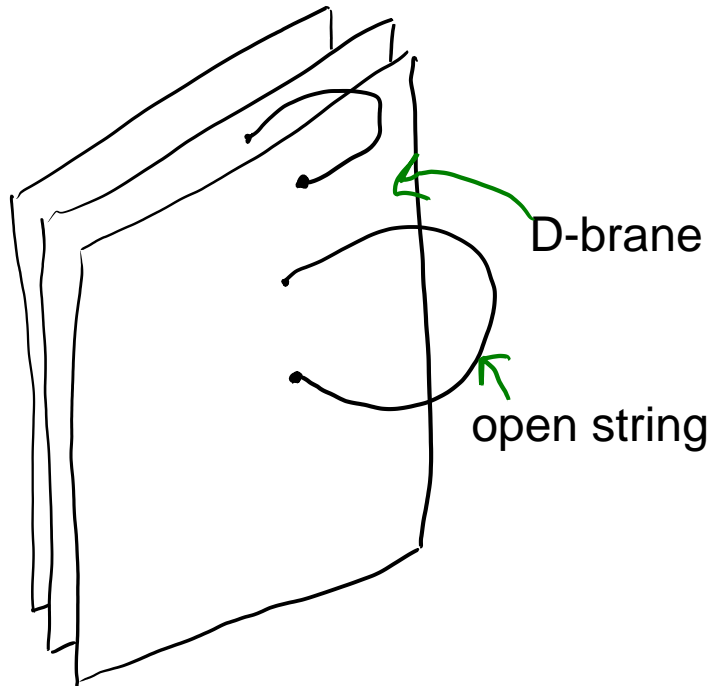
Black Holes have loads of entropy



A black hole is the MOST entropy you can put in a given region

Where are the Microstates?

String theory provides a way to count them.



Black Hole Entropy vs. Escape

Hard to get out because there is so much entropy!

Might as well try to unmix the dye from the water...

Is information lost?

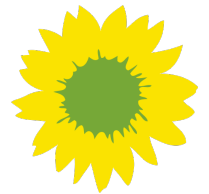
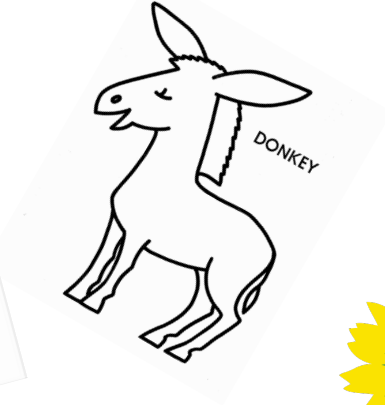
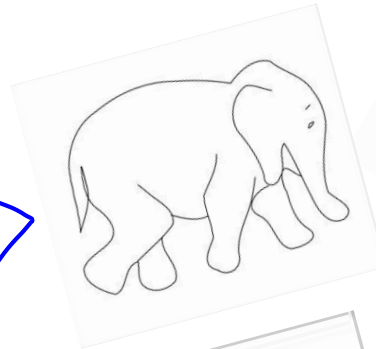
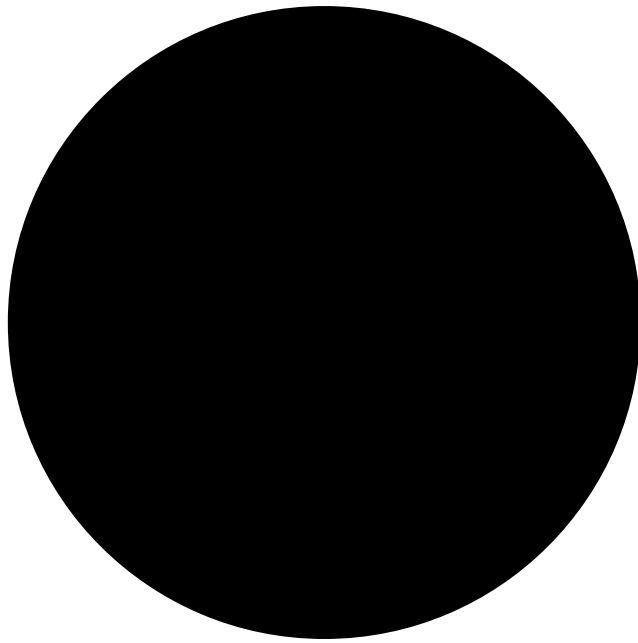
We think not, but that's the central question behind the Hawking Paradox.

The Hawking Paradox

If black holes have a temperature, then they radiate.

If they radiate, they must eventually come to thermal equilibrium with the rest of the universe.

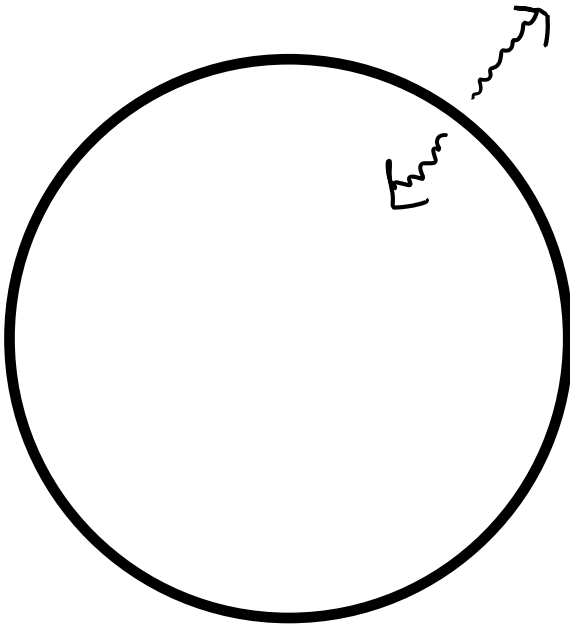
But how does the information that we threw in... come back out?



VAOFCBØINÅ

Hawking Radiation

"semiclassical" approximation
from pair production?
is exactly thermal = only information is the temperature



So if a black hole radiates away to nothing,
then where did the information we threw in
go?

All we wanted was:

- 1) weak-curvature approximation = No Drama
- 2) effective field theory = semiclassical is ok
- 3) unitarity = preservation of information.

But we can't have it all...

Proposed solutions to the Hawking Paradox

- remnants

Perhaps information really is stuck inside the black hole, and black holes do not evaporate completely.

- complementarity

Perhaps there is an extra copy of the information: one copy goes into the black hole, the other is accessible from the outside
(formerly favored by string theorists and holographists)

- fuzzballs

Perhaps black holes only appear to be black-- instead perhaps they are actually more like the water mixed into the dye. Big clumsy observers can't tell the difference.

- firewalls

Drama! Terrible things happen to anyone who tries to enter a black hole-- they burn up when they hit the horizon.

Proposed solutions to the Hawking Paradox

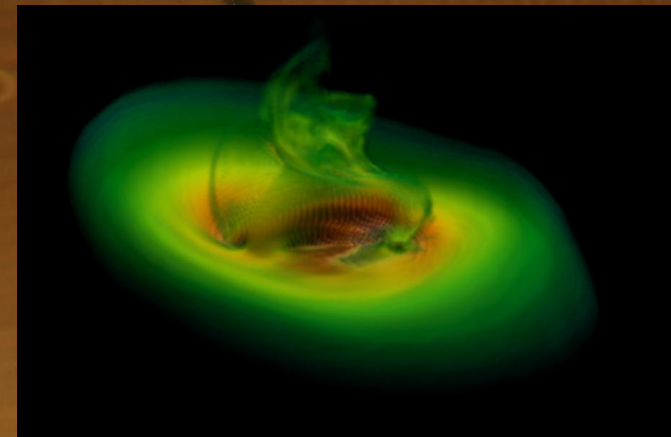
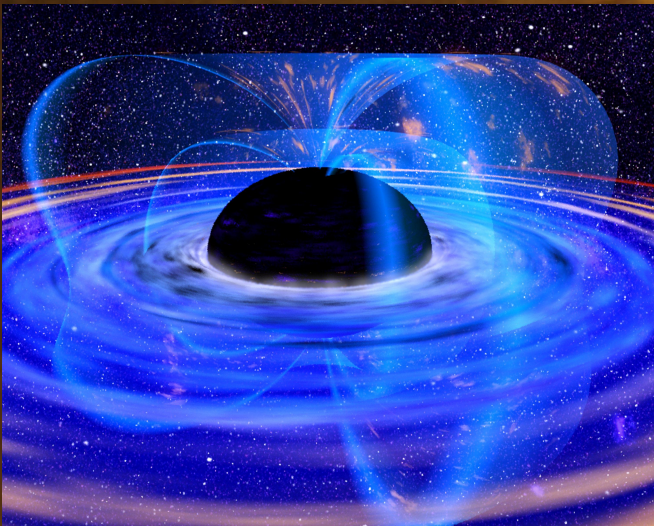
- remnants
- complementarity
- fuzzballs
- firewalls
- quantum information

The AMPS refinement of the Hawking Paradox requires us to do a difficult computational problem. Maybe there isn't time to do the computation before something happens to the computer itself.

- recently: Hawking/Perry/Strominger

So far, we don't know the answer.

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